The Network Graph in Power Grid Phenomena: Unifying Power Flow Structure from Locational Marginal Prices to Electromechanical Coherency

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Synopsis

• Review power flow Jacobian, demonstrating Laplacian structure extends beyond just DC power flow approx.

• Show role of an augmented Jacobian in the KKT conditions dictating market equilibrium → its Null Space determines admissible market prices (patterns of LMPs).

• Show role of Jacobian in linearized differential/algebraic equations that determine “swing” dynamics; (time permitting...) show symmetry conditions and phenomenon of electromechanical coherency.
Acknowledgements

• Role of Jacobian in network interpretation of power market KKT conditions draws on PhD thesis of Daniel Chéverez-González, now University of Puerto Rico.

• Support for this work provided through National Science Foundation and U.S. Department of Energy.


Motivation: LMP Patterns

What are Locational Marginal Prices, and how are they computed in typical U.S. markets? (illustrate via MISO)

Simplified version of MISO optimization problem:

• MISO collects offers to sell (“bids”) from each generator: can be interpreted as per unit time cost ($/hr) to be charged, as function of generator’s MW output level. Objective function to be minimized is sum of these.

• MISO imposes (using DC approximation)
  - equality constraints on active power for every bus;
  - inequality constraints on max flow for each line.
Motivation: LMP Patterns

• LMPs are simply the Lagrange multipliers ("shadow prices") associated with active power balance equality constraints at each bus.

• The objective function has units of $/hr, the equality constraints have units of MW, so the resulting LMPs have units of $/hr/MW.

• Editorial Comment: LMPs are almost universally written in units of $/MWhr. Invites misinterpretation of LMPs as price for energy. They are per unit time operating cost, incremental w.r.t. MWs.
Motivation: LMP Patterns

- If no line flow inequality constraint active (i.e., all power flows on lines within limits), have only shadow prices associated with power balance.

- As line constraints become binding, clearly additional shadow prices associated.

- Question here: with no knowledge of offer information (objective function), only knowledge of network, can one predict admissible patterns of LMPs?
Motivation: LMP Patterns
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**Genoa Generating Station**

From Wikipedia, the free encyclopedia

**Genoa Generating Station** is a base load, coal fired, electrical power station located south of Genoa, Wisconsin in Vernon County.

Genoa 3 uses low sulfur western coal from Utah and the Powder River Basin shipped via rail to the Mississippi River, then to Genoa via barge.

### Units

<table>
<thead>
<tr>
<th>Unit</th>
<th>Capacity</th>
<th>Commissioning</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14 MW (originally coal, later fuel oil)</td>
<td>1941</td>
<td>Retired 1985</td>
</tr>
<tr>
<td>2</td>
<td>50 MW (nuclear)</td>
<td>1969</td>
<td>Retired 1987, La Crosse Boiling Water Reactor (nuclear)</td>
</tr>
<tr>
<td>3</td>
<td>379 MW (coal)</td>
<td>1969</td>
<td>Tangentially-fired, pulverized coal-fired dry bottom boiler, rated at 376 net MW</td>
</tr>
</tbody>
</table>

### See also

- List of power stations in Wisconsin
- La Crosse Boiling Water Reactor (Genoa 2)

### External links


This article about a United States power station is a stub. You can help Wikipedia by expanding it.

http://en.wikipedia.org/wiki/Genoa_Generating_Station
Motivation: LMP Patterns

• As illustrated, sharply differentiated prices can exist across short geographic and network distances.

• Such differentiated prices obviously arise from binding line constraints. Previously literature explored these via methodologies that mixed offer and network information.

• Seeking extract maximum amount of information obtainable from network exclusively; necessitates return to examining network structure in power flow.
The Power Flow & Its Jacobian

• Several presenters displayed active power flow, in its commonly used summation form. For active power balance at a bus(node) \( i \) :

\[
\sum_{k=1}^{n} V_i V_k G_{ik} \cos(\delta_i - \delta_k) + \sum_{k=1}^{n} V_i V_k B_{ik} \sin(\delta_i - \delta_k) - P^I_i = 0
\]

• \( P^I_i \) denotes exogenous power injection (– for load) \emph{independent} of network. Remaining terms identify power absorbed by network from bus \( i \); denote that \emph{network} portion \( P^N_i(\delta, V) \).
The Power Flow & Its Jacobian

- Alternate functional form better illustrates dependence on network topology:

\[
\begin{align*}
\mathbf{P}^N (\delta, \mathbf{V}) &= \text{diag} \{\mathbf{V}\} \text{diag} \{|\mathbf{A}|g_l\} \mathbf{V} - \mathbf{A} \cdot \text{diag} \{b_l\} \text{diag} \{e^{|\mathbf{A}|^T \ln(\mathbf{V})}\} \sin(\mathbf{A}^T \delta) \\
&- |\mathbf{A}| \text{diag} \{g_l\} \text{diag} \{e^{|\mathbf{A}|^T \ln(\mathbf{V})}\} \cos(\mathbf{A}^T \delta)
\end{align*}
\]

- Most terms above familiar from this week:
  - \(\mathbf{A}\) denotes network incidence matrix;
  - \(b_l\) and \(g_l\) are line susceptances & conductances.
  - One nonstandard term:
    - \(|\mathbf{A}|\) as component-wise absolute value of \(\mathbf{A}\).
The Power Flow & Its Jacobian

• Analogous expression for reactive power absorbed by network:

\[
Q^N(\delta, V) = |A| \text{diag}\{b_l\} \text{diag}\{e^{A^T \ln(V)}\} \cos(A^T \delta) \\
- \text{diag}\{V\} \text{diag}\{|A|b_l\}V - A \text{diag}\{g_l\} \text{diag}\{e^{A^T \ln(V)}\} \sin(A^T \delta)
\]
The Power Flow & Its Jacobian

Several facts follow simply by direct calculation:

- If $g_i=0$, Jacobian matrix of partial derivatives of $P^N$ and $Q^N$ w.r.t. $\delta$ and $V$ is symmetric.

- Block associated with $\partial P^N/\partial \delta$ has weighted Laplacian structure often cited this week.

- Linearization at $\delta=0$, $y$, and taking all (normalized) $V=1$ yields “DC” approximation:

$$\partial P^N/\partial \delta = \text{Adiag}\{b_y\} A^T$$
Structure of Lagrangian for LMPs

Current markets ignore reactive balance, so Lagrangian with no line limits binding is simple:

\[
\text{Original objective} + \lambda^T[P^N(\delta) - P^I]
\]

Key observation: Original objective is function of \(P^I\) only, independent of \(\delta\).
Structure of Lagrangian for LMPs

Consequence: Necessary condition for optimality associated with partial of Lagrangian w.r.t. $\delta$ is

$$\lambda^T[\partial P^N/\partial \delta] = 0^T$$

And in DC approximation

$$Adiag\{b_i\} A^T \lambda = 0$$
First provides alternate view on most obvious, well-known feature of LMPs:

- In lossless model, with no line constraints binding, all LMPs must be equal.

- Alternate view: any weighted Laplacian is rank deficient by one, with vector of all equal entries forming its null space.
More interesting: what happens as line constraints become binding?

• Suppose single line, index $k$, has binding constraint. Denoting line flow function as $p_k(\delta)$, one has additional term in Lagrangian:

$$\mu \left[ p_k(\delta) - p_k^{lim} \right] = 0$$
Network Topology
Features in LMPs

Key observation: line flow function intimately related to overall powerflow. In particular:

\[ \frac{\partial p_k}{\partial \delta} = \text{diag}\{b_L\} \times \text{[column } k \text{ of } \{A^T\}] \]

• Resulting subset of KKT conditions associated with partial of Lagrangian w.r.t. \( \delta \):

\[
\begin{align*}
\text{Adiag}\{b_L\} A^T & \quad \lambda = 0 \\
\text{diag}\{b_L\} \text{ col}_k\{A^T\} & \quad \mu = 0
\end{align*}
\]
Network Topology Features in LMPs

• Consequence: vector of Lagrange multipliers lies in null space of matrix rank deficient by 2; iLMPs now have 2 degrees of freedom.

• Not surprising: each added (independent) constraint reduces degrees of freedom in primal variables by one, increases degrees of freedom in dual variables by one.
Network Topology
Features in LMPs

• Very attractive computational feature: for number of binding line constraints (up to n), establish orthonormal basis for null space, one basis vector per line (+ uniform vector).

• Basis vector associated with given line constraint can be computed from case of that single constraint active.

• Any admissible LMP vector must be formed as (partition of) linear combinations of these.
Conclusions

• Set of LMPs that can possibly be achieved ("admissible" LMPs) associated with a given set of binding line constraints is entirely determined by network, completely independent of generators’ offers.

• Whether a differentiated price pattern is realized of course does depend on generator offers.

• Impact: extremely easy to identify if a given set of binding line constraints creates opportunity for market power.
Real-Time Binding Constraints

Transmission Binding Constraints

The Real-Time Binding Transmission Constraints table* shows a transmission element that is experiencing congestion and the related market cost. Information on this webpage is updated every five minutes.

- Download CSV data
- Download XML data

<table>
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<th>Name</th>
<th>Shadow Price</th>
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</thead>
<tbody>
<tr>
<td>WECWPS04_NAPPLETO_NAPP6LOST1_1</td>
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</tr>
<tr>
<td>NSP34046_SWAN_LK_SWAN_WILMA11_1_1</td>
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<tr>
<td>MEC16036_WELSBGCB_TR1_TR1</td>
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<tr>
<td>Lansing_Genoa161_FLO_Harmony_Genoa161</td>
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<tr>
<td>WEC13310_NLAKEGE1_65411_65</td>
<td>-0.2</td>
</tr>
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