The Network Graph in Power Grid Phenomena: Unifying Power Flow Structure from Locational Marginal Prices to ~~Electromechanical Coherency~~

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Synopsis

- Review power flow Jacobian, demonstrating Laplacian structure extends beyond just DC power flow approx.

- Show role of an augmented Jacobian in the KKT conditions dictating market equilibrium → its Null Space determines admissible market prices (patterns of LMPs).

- ~~Show role of Jacobian in linearized differential/algebraic equations that determine "swing" dynamics; (time permitting…) show symmetry conditions and phenomenon of electromechanical coherency~~

Acknowledgements

- Role of Jacobian in network interpretation of power market KKT conditions draws on PhD thesis of Daniel Chéverez-González, now University of Puerto Rico.

- Support for this work provided through National Science Foundation and U.S. Department of Energy.

- D. Cheverez-Gonzalez; C.L. DeMarco; “Admissible Locational Marginal Prices via Laplacian Structure in Network Constraints,” *IEEE Transactions on Power Systems*, vol. 24, no. 1, pp. 125 - 133, Feb. 2009.

- D. Cheverez-Gonzalez, C.L. DeMarco, "Mutually orthogonal LMP decompositions: Congestion decomposes, losses do not," *Proc. of the 6th International Conference on the European Energy Market*, May 2009.

Motivation: LMP Patterns

What are Locational Marginal Prices, and how are they computed in typical U.S. markets? (illustrate via MISO)

Simplified version of MISO optimization problem:

- MISO collects offers to sell ("bids") from each generator: can be interpreted as per unit time cost ($/hr) to be charged, as function of generator's MW output level. Objective function to be minimized is sum of these.
- MISO imposes (using DC approximation)
  - equality constraints on active power for every bus;
  - inequality constraints on max flow for each line.

Motivation: LMP Patterns

- LMPs are simply the Lagrange multipliers (“shadow prices”) associated with active power balance equality constraints at each bus.

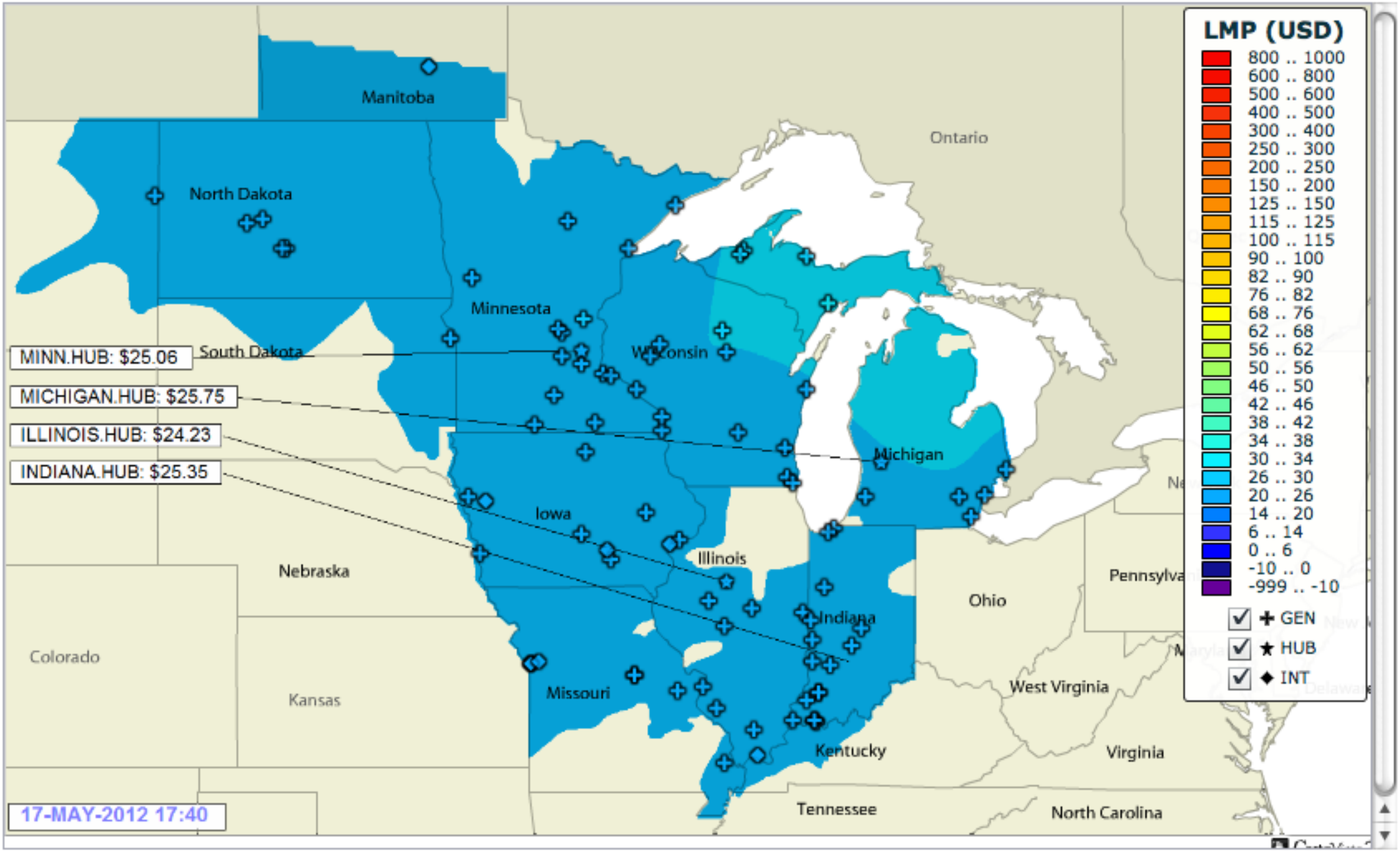
- The objective function has units of $/hr, the equality constraints have units of MW, so the resulting LMPs have units of $/hr/MW.

- **Editorial Comment**: LMPs are almost universally written in units of $/MWhr. Invites misinterpretation of LMPs as price for energy. They are per unit time operating cost, incremental w.r.t. MWs.

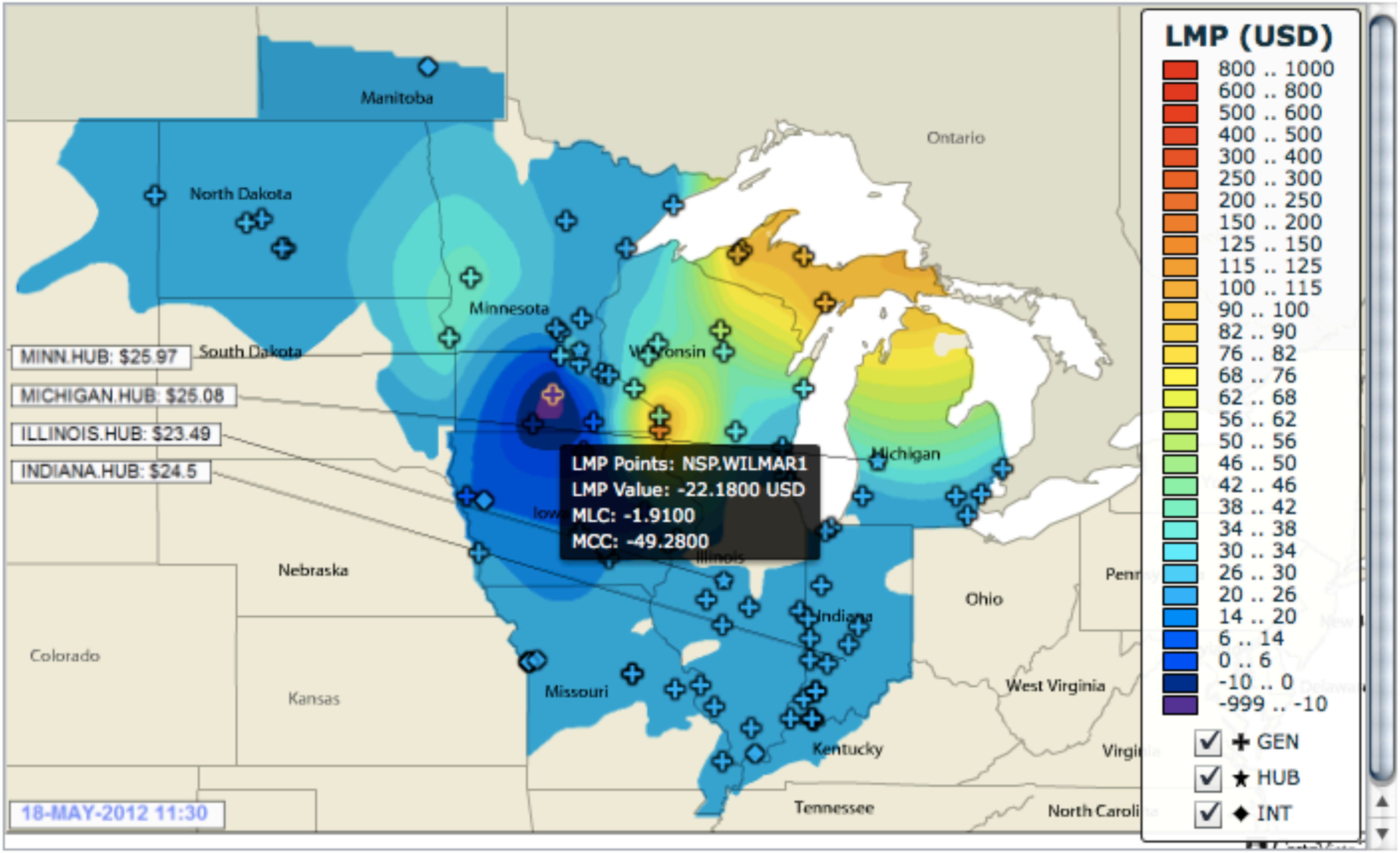
Motivation: LMP Patterns

- If no line flow inequality constraint active (i.e., all power flows on lines within limits), have ***only*** shadow prices associated with power balance
- As line constraints becomes binding, clearly additional shadow prices associated.
- Question here: with no knowledge of offer information (objective function), only knowledge of network, can one predict admissible patterns of LMPs?

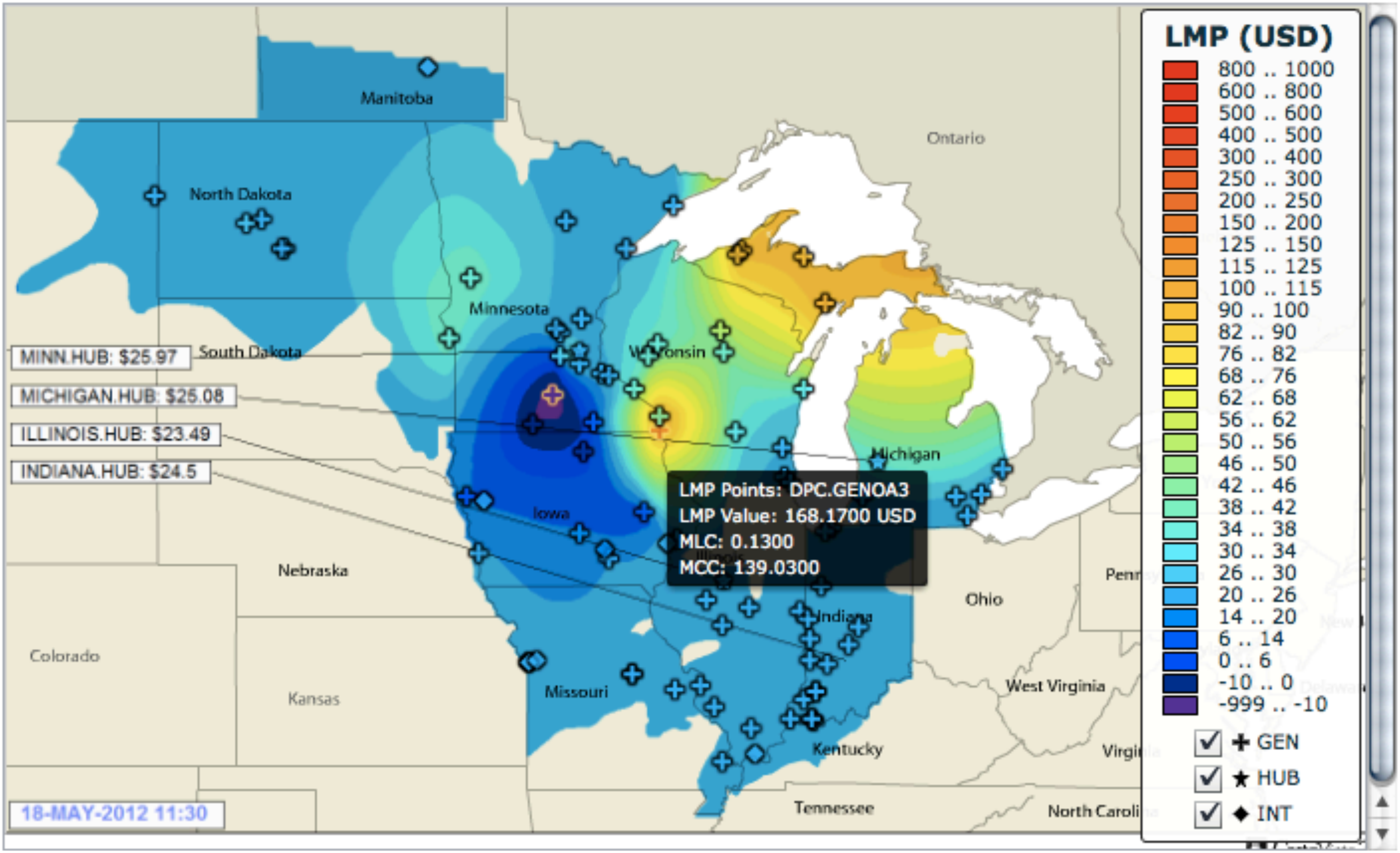
Motivation: LMP Patterns



Motivation: LMP Patterns



Motivation: LMP Patterns



Genoa Generating Station

From Wikipedia, the free encyclopedia

Genoa Generating Station is a base load, coal fired, electrical power station located south of Genoa, Wisconsin in Vernon County.

Genoa 3 uses low sulfur western coal from Utah and the Powder River Basin shipped via rail to the Mississippi River, then to Genoa via barge.

Units [edit]

Unit	Capacity	Commissioning	Notes
1	14 MW (originally coal, later fuel oil)	1941	Retired 1985
2	50 MW (nuclear)	1969	Retired 1987, La Crosse Boiling Water Reactor (nuclear)
3	379 MW (coal)	1969	Tangentially-fired, pulverized coal-fired dry bottom boiler, rated at 376 net MW

See also [edit]

Genoa Station (left)

- List of power stations in Wisconsin
- La Crosse Boiling Water Reactor (Genoa 2)

External links [edit]

- http://www.dairynet.com/energy_resources/genoa.php
- http://www.dairynet.com/energy_resources/genoa.pdf

This article about a United States power station is a stub. You can help Wikipedia by expanding it.

http://en.wikipedia.org/wiki/Genoa_Generating_Station

Motivation: LMP Patterns

- As illustrated, sharply differentiated prices can exist across short geographic and network distances
- Such differentiated prices obviously arise from binding line constraints. Previously literature explored these via methodologies that mixed offer and network information.
- Seeking extract maximum amount of information obtainable from network exclusively; necessitates return to examining network structure in power flow.

The Power Flow & Its Jacobian

- Several presenters displayed active power flow, in its commonly used summation form. For active power balance at a bus(node) *i* :

$$\sum_{k=1}^{n} V_i V_k G_{ik} \cos(\delta_i - \delta_k) + \sum_{k=1}^{n} V_i V_k B_{ik} \sin(\delta_i - \delta_k) \ - P_i^I = 0$$

- P^I_i denotes exogenous power injection (– for load) ***independent*** of network. Remaining terms identify power absorbed by network from bus *i* ; denote that ***network*** portion $P^N_i(\boldsymbol{\delta}, \boldsymbol{V})$.

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The Power Flow & Its Jacobian

- Alternate functional form better illustrates dependence on network topology:

$$\underline{P}^N(\underline{\delta},\underline{V}) = diag\{\underline{V}\}diag\{|A|\underline{g}_l\}\underline{V} - A \cdot diag\ \{\underline{b}_l\}\ diag\ \{e^{|A|^T \ln(\underline{V})}\}\sin(A^T\underline{\delta})$$

$$-|A|\ diag\{\underline{g}_l\}diag\ \{e^{|A|^T \ln(\underline{V})}\}\cos(A^T\underline{\delta})$$

- Most terms above familiar from this week:
  A denotes network incidence matrix;
  b_l and g_l are line susceptances & conductances.
  One nonstandard term:
  $|A|$ as component-wise absolute value of A.

The Power Flow & Its Jacobian

- Analogous expression for reactive power absorbed by network:

$$\underline{Q}^N(\underline{\delta},\underline{V}) = |A|\, diag\{\underline{b_l}\} diag\{e^{|A|^T \ln(\underline{V})}\}\cos(A^T\underline{\delta})$$

$$-diag\{\underline{V}\} diag\{|A|\underline{b_l}\}\underline{V} - A\, diag\{\underline{g_l}\} diag\{e^{|A|^T \ln(\underline{V})}\}\sin(A^T\underline{\delta})$$

The Power Flow & Its Jacobian

Several facts follow simply by direct calculation:

- If g_l=0, Jacobian matrix of partial derivatives of P^N and Q^N w.r.t. δ and V is symmetric.

- Block associated with $\partial P^N/\partial \boldsymbol{\delta}$ has weighted Laplacian structure often cited this week

- Linearization at $\delta = 0$ y, and taking all (normalized) V =1 yields "DC" approximation

$$\partial P^N/\partial \boldsymbol{\delta} = A diag\{b_l\} A^T$$

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Structure of Lagrangian for LMPs

Current markets ignore reactive balance, so Lagrangian with no line limits binding is simple:

$$\{\text{Original objective}\} + \lambda^{\mathsf{T}}[P^N(\delta) - P^I]$$

Key observation: Original objective is function of P^I only, independent of $\boldsymbol{\delta}$.

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Structure of Lagrangian for LMPs

Consequence: Necessary condition for optimality associated with partial of Lagrangian w.r.t. $\boldsymbol{\delta}$ is

$$\lambda^{T}[\partial P^{N}/\partial \boldsymbol{\delta}] = \boldsymbol{0}^{T}$$

And in DC approximation

$$A\,diag\{b_l\}\,A^{T}\lambda = \boldsymbol{0}$$

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Network Topology Features in LMPs

First provides alternate view on most obvious, well-known feature of LMPs:

- In lossless model, with no line constraints binding, all LMPs must be equal.

- Alternate view: any weighted Laplacian is rank deficient by one, with vector of all equal entries forming its null space.

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Network Topology Features in LMPs

More interesting: what happens as line constraints become binding?

- Suppose single line, index k, has binding constraint. Denoting line flow function as $p_k(\boldsymbol{\delta})$, one has additional term in Lagrangian:

$$\boldsymbol{\mu} \,[\, p_k(\boldsymbol{\delta}) - p_k^{lim} \,] = 0$$

Network Topology Features in LMPs

Key observation: line flow function intimately related to overall powerflow. In particular:

$$\partial p_k/\partial \boldsymbol{\delta} = diag\{b_l\} \times [column\ k\ of\ \{A^T\}]$$

- Resulting subset of KKT conditions associated with partial of Lagrangian w.r.t. $\boldsymbol{\delta}$:

$$A diag\{b_l\} A^T \quad \lambda = \mathbf{0}$$

$$diag\{b_l\}\ col_k\{A^T\} \quad \boldsymbol{\mu} = \mathbf{0}$$

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Network Topology Features in LMPs

- Consequence: vector of Lagrange multipliers lies in null space of matrix rank deficient by 2; iLMPs now have 2 degrees of freedom.

- Not surprising: each added (independent) constraint reduces degrees of freedom in primal variables by one, increases degrees of freedom in dual variables by one.

Network Topology Features in LMPs

- Very attractive computational feature: for number of binding line constraints (up to n), establish orthonormal basis for null space, one basis vector per line (+ uniform vector) .

- Basis vector associated with given line constraint can be computed from case of that single constraint active.

- Any ***admissible*** LMP vector must be formed as (partition of) linear combinations of these.

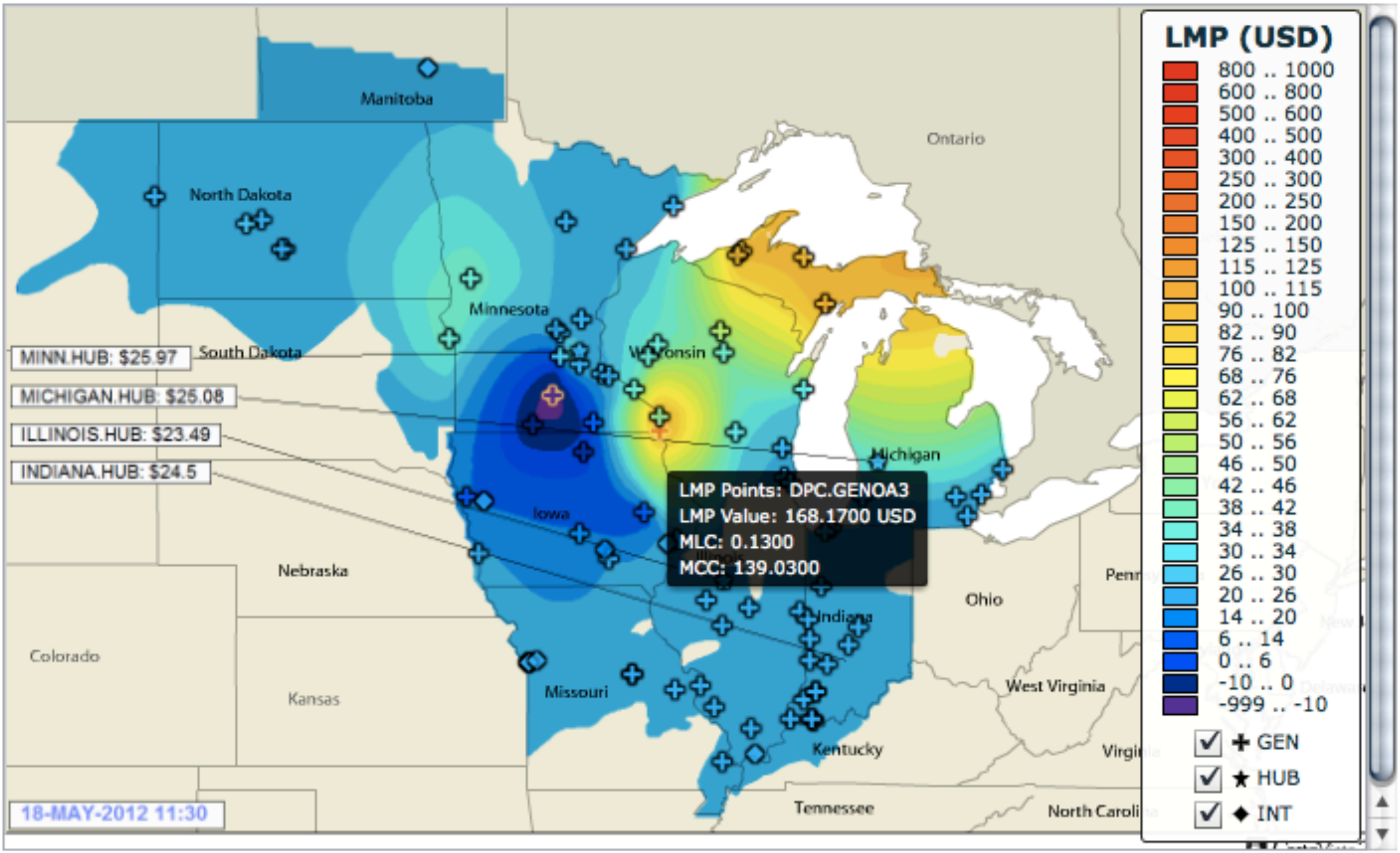
Conclusions

- Set of LMPs that can possible be achieved ("admissible" LMPs) associated with given set of binding line constraints is entirely determined by network, ***completely independent of generators' offers.***

- Whether a differentiated price pattern is ***realized*** of course does depend on generator offers.

- Impact: ***extremely*** easy to identify if a given set of binding line constraints creates opportunity for market power.

Return to MISO LMP Patterns & Constraints



Return to MISO LMP Patterns & Constraints

Real-Time Binding Constraints

Transmission Binding Constraints

The Real-Time Binding Transmission Constraints table* shows a transmission element that is experiencing congestion and the related market cost. Information on this webpage is updated every five minutes.

- Download CSV data
- Download XML data

May. 18, 2012 - Interval 11:30 EST

Name	Shadow Price
WECWPS04_NAPPLETO_NAPP6LOST1_1	-1000.0
NSP34046_SWAN_LK_SWAN_WILMA11_1_1	-544.4
MEC16036_WELSBGCB_TR1_TR1	-112.3
NSP11258_LINDE_LINDECHEMO11_1_1	-4.6
Lansing_Genoa161_FLO_Harmony_Genoa161	-487.0
WEC13310_NLAKEGE1_65411_65	-0.2