The Collective Behaviors of Multi-Agent Systems

Jing HAN
Complex Systems Research Center, http://complex.amss.ac.cn/
Institute of System Sciences, Academy of Mathematics and Systems Science, CAS

What I want to talk

- Collective Behavior of multi-agent systems
  - Agent: individual, particle, variable, bird, ant …
- Interdisciplinary approach
  - Computer Science
  - Game theory
  - Math and Control
  - Statistical physics
  - Biology…

What is Collective Behavior?
Collective behavior

- many agents (individual/part), local and simple interactions. New properties emerge:
  - Examples in different systems:
    - Spins (avg. direction of neighbors) / "Magnetization"
    - Ant - ant colony / swarm intelligence;
    - Node - network / "small-world", "scale-free";
    - Variables - combinatorial problems / solution state, phase transition;
    - Bird/flocking
    - Internet scale-free, small-world
    - Crowd panic
    - Liquid & gas pattern
    - Ant colony swarm intelligence

Examples in different systems:

Three Categories of Research on Collective Behavior

1. **Analysis:** Given the local rules of agents, what is the collective behavior of the system? spin glasses, constraint networks, panic model, network dynamics,

2. **Design:** Given the desired collective behavior, what are the local rules for agents? swarm intelligence, decentralized control

3. **Control:** Given the local rule of the agents, how we control the collective behavior? soft control

Two Lectures

**Lecture 1:**
- example: Boid Model (flocking of birds)
- focus: synchronization (move to the same direction)
- problems:
  - Analysis: in what condition, the group will converge to synchronization?
  - Control: how we intervene the group to get to synchronization?

**Lecture 2:**
- example: Coloring Problems (constraint network)
- focus: solution
- problems:
  - Analysis: easy-hard phase transition, predict
  - Design: individual, compartment and the whole (evolution, Nash equilibrium, hierarchy)

Notes: John Pepper's lecture on ABM.
A Group of Birds


Flocking of birds is a kind of collective behavior

Question 1: Why they fly to the same direction?

- In what condition, the group will converge to synchronization?

Note: I will only give a very briefly review about this question. For those who like to know more, please read the references.

Question 2: How we drive them away?

- Can we guide their flight if we know how birds fly?
- How we 'control' the collective behavior of a group of autonomous agents?

Outline

- Models for a Group of Birds
- Part 1: Analysis
  Current results about analysis of synchronization
- Part 2: Soft control
  - The Key Points of ‘Soft Control’
  - Computer Simulations
  - A Case Study
  - Soft Control of Complex Systems
**Boid (1987): A Model for Birds**

A bird's Neighborhood

Alignment: steer towards the average heading of neighbors
Cohesion: steer to move toward the average position of neighbors

Separation: steer to avoid crowding neighbors

Note: John Pepper's lecture on ABM has mentioned this model.

**BOID Simulations**

http://www.red3d.com/cwr/boids/applet

http://www.abs2net.com/robert/3d_boids/

---

**A Simple Model* (1995, Vicsek et al.)**

A bird's Neighborhood

Alignment: steer towards the average heading of neighbors

- n Agents: \( (X, \theta) \)
- \( x(t), i=1,...,n \) (\( x \) is 2D vector)
- \( \theta(t) \): heading (moving direction) of the \( i \)th agent.
- \( v \): the constant speed of birds
- \( r \): radius of neighborhood, so \( N(t) = \{ |x-x(t)| \leq r \} \)
- \( \theta(t+1) = \langle \theta(t+1) \rangle \), where
  - \( \langle \theta(t+1) \rangle = \arctan(\frac{\sin(\theta(t))}{\cos(\theta(t))}) \)
  - \( x(t+1) = x(t) + v(t+1) \theta(t+1) \)

Every bird only knows the local information, and follows a local rule to make decisions.
Part 1: In what condition, the group will converge to synchronization?

Not synchronized

synchronized

Toward a Theoretical Analysis

- Some Mathematical Tools
  - Spectral graph theory,
  - Random geometric graphs,
  - Product of stochastic matrices,
  - Large deviation estimation,
  - Double-array martingales

- Some Existing Results
  - It is not solved totally yet!

Result 1:

Joint connectivity of the neighbor graphs on each time interval $[th, (t+1)h]$ with $h > 0$.

Synchronization of the linearized Vicsek model, i.e., there exists $\theta_0$, such that

$$\lim_{t \to \infty} \theta_i(t) = \theta_0, \quad \forall i$$

A. Jadbabaie et al., IEEE TAC, 2003
J. N. Tsitsiklis, et al., IEEE TAC, 1984

Result 2:

The Vicsek’s model will synchronize, if

1) The initial headings belong to $(-\pi/2, \pi/2)$;
2) The initial neighbor graph $(V, E_0)$ is connected;
3) The speed $v$ and radius $r$ satisfy:

$$v \leq \frac{d}{\Delta} \left( \cos \frac{\theta}{\pi} \right)$$

where

$$d = r - \max_{i,j} \left| x_i(t) - x_j(t) \right|$$

$$\theta = \max_i \left| \theta_i(0) \right|, \quad \Delta = \max_{i,j} (\tan \theta_i(0) - \tan \theta_j(0)).$$

Zhixin Liu & Lei Guo, CCC2006
More and it is going on…

- More results
  - in the Stochastic Framework
  - with noise in the model
  - …
- Researchers are still working hard
  - a necessary and sufficient initial condition for synchronization?
  - …

Part 2: Soft Control

* How we ‘control’ the collective behavior of a group of autonomous agents?
  - The Key Points of ‘Soft Control’
  - Computer Simulations
  - A Case Study
  - Soft Control of Complex Systems

Based on collaboration with Prof. Lei GUO and Dr. Ming LI

Key Points of ‘Soft Control’

- The system:
  - Many agents
    - more is different
  - Each agent follows a local rule
    - Autonomous, distributed
  - Agents locally interact with each other, not isolated (most of the time)
    - The local effect will spread and may affect the whole

- The “Control”:
  - Not to change the local rule of existing agents
    - Can not implement by changing adjustable global parameters
  - Add one (or a few) agent
    - might not affect the collective behavior
    - will not increase the complexity of the system
Key Points of 'Soft Control'

- The system:
  - Many agents
  - More is different
  - Each agent follows a local rule
  - Autonomous, distributed
  - Agents locally interact with each other, not isolated (most of the time)
  - The local effect will spread and may affect the whole

- The "Control":
  - Not to change the local rule of existing agents
  - Can not implement by changing adjustable global parameters
  - Add one (or a few) special agent – shill, control interface
    - Shill is not a leader, not leader-follower type.
    - Shill is controlled by us, not following the local rule, not following the local rule.
    - Is treated as an ordinary agent by the existing agents.

Computer Simulation
A Case Study

Problem:

- System:
  A group of $n$ agents with initial headings $\theta_i(0) \in [0, \pi)$
- Goal: all agents move to the direction of $\pi$ eventually.
- Control: What is the control law (strategy) of the shill?

Assumptions:

- The local rule about the ordinary agents is known
- The position $x_i(t)$ and heading $\theta_i(t)$ of the shill can be controlled at any time step $t$
- The state information (headings and positions) of all ordinary agents are observable at any time step

The Control Law $u_\beta$

$u_\beta = \begin{cases} \theta_{s}(t) + \beta & \text{if } \theta_{s}(t) \leq \pi - \beta \\ \pi & \text{if } \theta_{s}(t) > \pi - \beta \end{cases}$

When $\beta \in (0, \pi)$, and $x(t)$ satisfies $\theta_{s}(t) = \min_{i \neq s}(\theta_i(t))$

Theorem 1: For any initial headings and positions $\theta_i(0) \in [0, \pi)$, $x_i(0) \in \mathbb{R}^2$, $1 \leq i \leq n$, the update rule and the control law $u_\beta$ will lead to the asymptotic synchronization of the group, i.e., $\lim_{t \to \infty} \Delta(t) = 0$.

Simulation

How we make use of the property of the self-organized collective behavior?

1. Given the local rules of the agent, what is the collective behavior of the overall system? Spin Glasses, panic model, network dynamics,
2. Given the desired collective behavior, what are the local rules for agents? Swarm Intelligence, BA model for internet
3. Given the local rule of the agent, how we control the collective behavior?

- The shill is jumping all the time (too much energy!) Is there a better control law?
Simulations

Switching between $u_\beta$ and $u_r$

Control Law $u_\beta$

More Questions …

- Better control laws?
  - Dig out more advantages of soft control
  - Also depend on better understanding of the self-organized collective behaviors.

- What if the shill can only see locally?
- How much information we need to know about the system?
- How about if we don’t know the local rule of birds, how does the shill learn and lead the group? (Learning and Adaptation)
- …

Soft-control is not just for …

the Vicsek’s model

- Would be applied to systems:
  - many autonomous agents with local interactions

  - Claytronics (CMU)
  - Evolution of Language
  - Multi-player Game
  - Panic in Crowd
  - …

Claytronics

- a new form of programmable matter

  - An ensemble of Catoms
    - (millimeter scale)
  - Simple local rule
  - Can form interesting dynamic shapes and configurations.

How do you design, program, maintain, and use a billion component system?

How to design and program a collection of micro/nanorobots to create a useful macroscale robot?

www.cs.cmu.edu/~claytronics
Language Evolution*

- Language is a kind of collective behavior relates to social behavior, culture. Evolves.
- Computational study of language evolution
- Lexical (grammar) diffusion model, multi-agent model
- Soft Control
  - Language diffusion model: guide diffusion, save dying languages
  - Language acquisition model: how to use shill to guide

*Working group of Language Evolution and Acquisition, SFI, Feb. 28-Mar 11
William S-Y Wang, Jyh-Lang Tzeng, Helena Hong Gao, John H. Holland, Tom Schoenemann

Social and Economic System

- Multi-Agent Model of Class Norms *
  - Repeated Nash Demand Game
- Transition time between Nash equilibria
  - Usually, exponential growing
  - Small proportions of special agents (no feedback): radical, revolutionary, reactionary
dramatically alter the expected transition times
- Soft Control:
  - How a shill changes (class/equity) norm?

* Robert L. Axtell, Shubha Chakravorty. <Radicals, Revolutionaries and Reactionaries in a Multi-Agent Model of Class Norms>
  Discussion with Prof. Samuel Bowles (SFI)

Future work:
Soft Control of Complex Systems

- Try on other systems
- “Add a shill” is just one way.
  - should be other ways for different systems:
    - Remove agents
    - Put obstacle
    - ...

  Intervention to complex systems.
  We need further study for Soft Control!

Thank You !

It is just the beginning!