Santa Fe Institute
2011 Complex Systems Summer School

Introduction to Nonlinear Dynamics
Lab 1

1 Logistic Map

(a) Use the applet found at `http://math.la.asu.edu/~chaos/logistic.html` to generate a ten-point trajectory of the logistic map from the initial condition 0.2 with parameter value $a = 2$. Many people call this R instead of a; this applet uses *both* of these symbols at different places on the website, which is confusing.

What kind of dynamics is this? What happens if you vary the initial condition, keeping everything else fixed?

(b) Use that applet to find a two-cycle and the onset of chaos. Note those a values.

(c) Set $a = 3.828$ and plot 500 iterates. The cobweb plot will be a mess, but the time-domain plot should show some interesting patterns. Raise a slowly to 3.829. Describe & explain what you see, relating your explanation to the bifurcation plot that was discussed in lecture.

2 Pendulum

Direct your browser to `http://www.myphysicslab.com/pendulum2.html`.

Fix m=l=g=1, beta=0.5, and drive amplitude=1.15.

(a) Set the drive frequency to 0, start a trajectory from $[\theta, \omega] = [\frac{\pi}{4}, 0]$, and plot it with ω (angular velocity) on the y-axis and θ (angular position) on the x-axis. Repeat for different initial conditions. What kind of attractor is this?

(b) Now set the drive frequency to $\frac{2}{3}$, raise the drive amplitude slowly to 1.5, and describe what you see. Be sure to try 1.35, 1.45, and 1.47 during your explorations.

3 Introduction to TISEAN

TISEAN is a useful time-series analysis package that you'll explore at more length in Lab 2. The purpose of this problem is to get you started with it.

(a) If you have your own computer with you, install TISEAN on it. Binaries for Macs can be found at:

`oldweb.ct.infn.it/auger/Members/dedomenico/main.html`

Unix and Windows binaries are available through the main TISEAN page:

`www.mpipks-dresden.mpg.de/~tisean`

Test out your installation by running `mutual -h` and call one of us over if you get an error message.

(b) The Hénon map is defined as:

$$\begin{aligned} x_{n+1} &= 1 - Ax_n^2 + y_n \\ y_{n+1} &= Bx_n \end{aligned}$$

Generate a trajectory of this map using the following TISEAN command:

`henon -l100000 -A0.8 -B0 -X.01 -Y.01 -o.`

This command sets the parameter values `A` and `B` to 0.8 and 0, respectively, iterates the map 100,000 times from the initial condition $(x_0, y_0) = (0.01, 0.01)$, and puts the resulting trajectory in a file called `henon.dat` in the current directory. Look at the last few lines of this file. Does this map have an attractor for these parameter values? What kind?

If the `henon` command does not work on your computer, you can use one of the lab computers, or just grab the data file (`HenonA0.8B0.dat`) from the CSSS wiki.

(c) Plot the data using your favorite plotting tool (Excel, Matlab, gnuplot, ...). You can find a short gnuplot tutorial at:

`www.cs.colorado.edu/~lizb/na/gnuplot-info.html`

Homework

- Let b_i be the i^{th} value of the parameter a for which a bifurcation occurs in the logistic map. The a value for which the dynamics changes from a fixed point to a periodic orbit is b_1, for example; the a value where the logistic map switches from period 2 to period 4 is b_2, and so on.

  The Feigenbaum number δ is defined as

  $$\lim_{n\to\infty} \frac{b_n - b_{n-1}}{b_{n+1} - b_n} = \delta.$$

  Use the applet from problem 1 to estimate the Feigenbaum number.

- (for experts) Code up the logistic map yourself and plot x_n vs n for some of the a interesting values that you explored in problem 1.

- (for experts) Generate a logistic map bifurcation diagram for $1 < R < 4$.

- Use the applet from problem 2 to find a value of the pendulum drive frequency that yields an attracting periodic orbit.

- Vary the Henon parameters (A and B) in the TISEAN example, plot the attractor (x_n vs y_n), and explore the dynamics:

  - Choose $-1 < B < 1$ and $A > \frac{3}{4}(1-B)^2$. What kind of attractor is this? Does it change if you change the initial conditions?
  - Now fix $B = 0.3$ and vary $0.8 \leq A \leq 1.3$. What happens at $A = 1.06$ and $A = 1.3$?
  - Now fix A but change the sign of B. How does this affect the dynamics?

  You may need to use the lab computers to do this problem if TISEAN's `henon` command doesn't work on your machine.