

***SEARCHING FOR SIMPLICITY AND  
UNITY IN THE COMPLEXITY OF LIFE***

***CELLS TO CITIES, COMPANIES TO  
ECOSYSTEMS, MILLISECONDS TO  
MILLENNIA***

***GEOFFREY WEST***

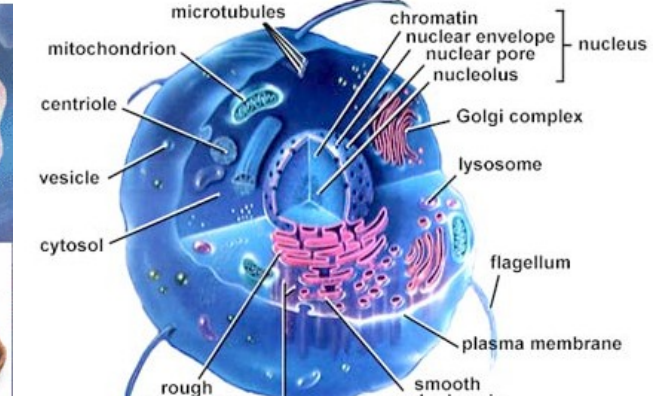
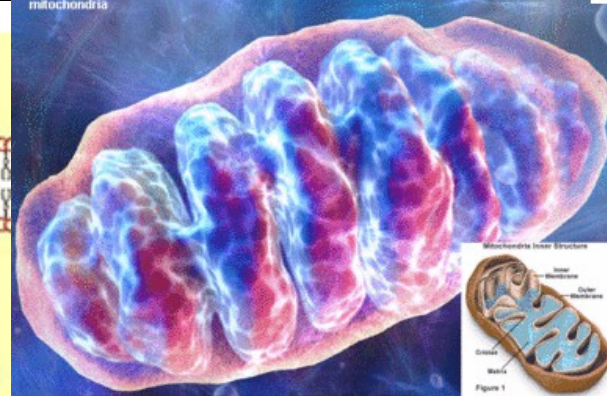
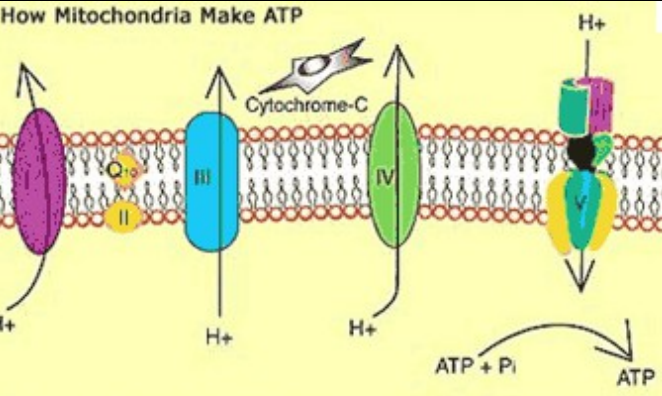
***SANTA FE INSTITUTE***



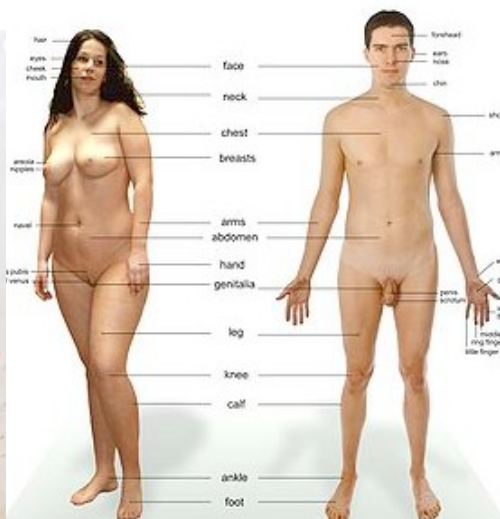
# THE FUTURE OF THE PLANET:

LIFE, GROWTH AND DEATH  
IN ORGANISMS, CITIES  
& COMPANIES

Geoffrey West | Santa Fe Institute



The  $H^+$  "pumps" and the "Turbine" of Complex V. Redra







***EQUIVALENT TO URBANISING  
OVER ONE MILLION PEOPLE  
EVERY WEEK FROM NOW TILL  
2050***

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***OR.....TO ADDING A NEW YORK  
METROPOLITAN AREA EVERY TWO MONTHS  
FROM NOW TO 2050***

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FROM NOW TO 2050***

***OR..... A SANTA FE EVERY 12 HOURS!***

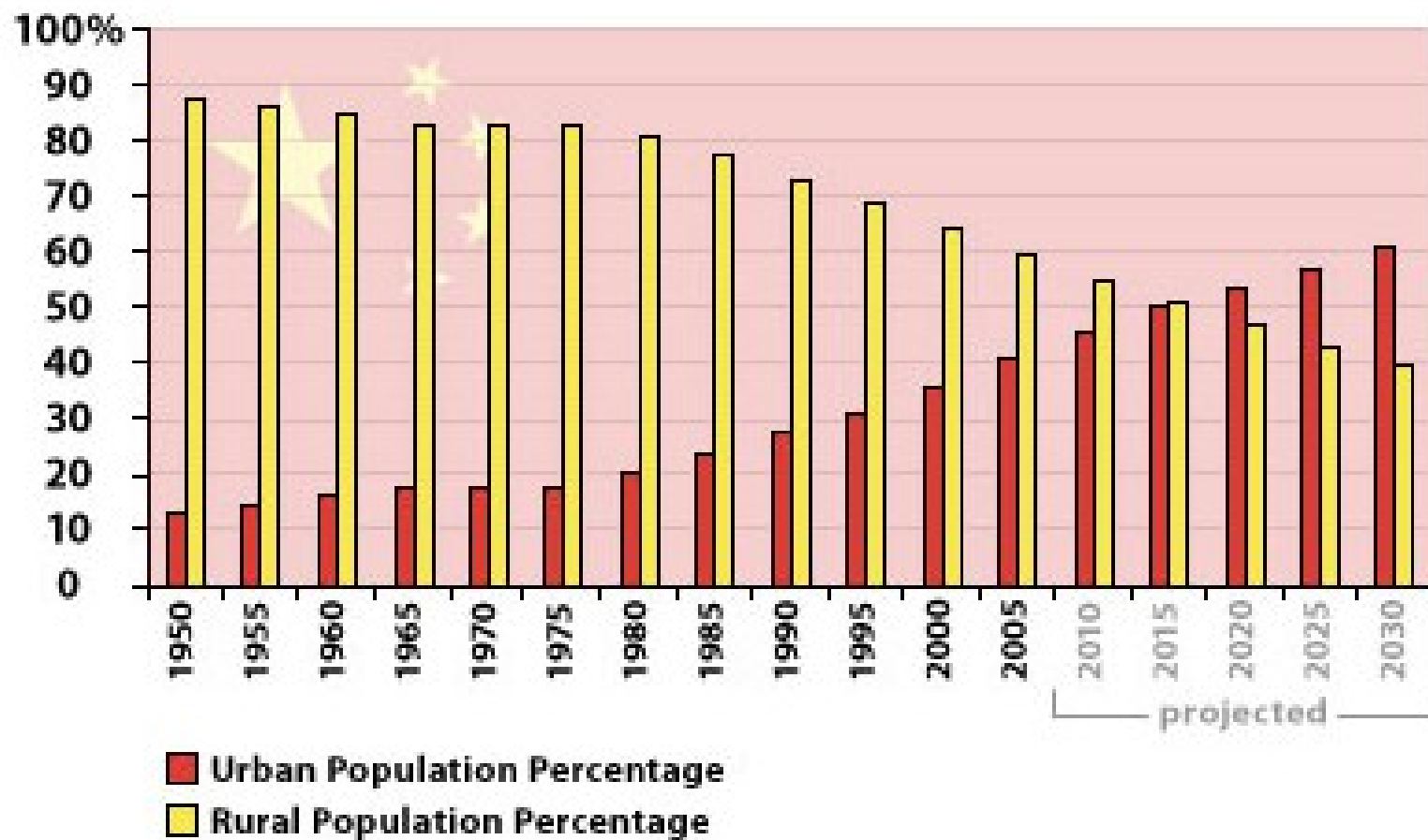
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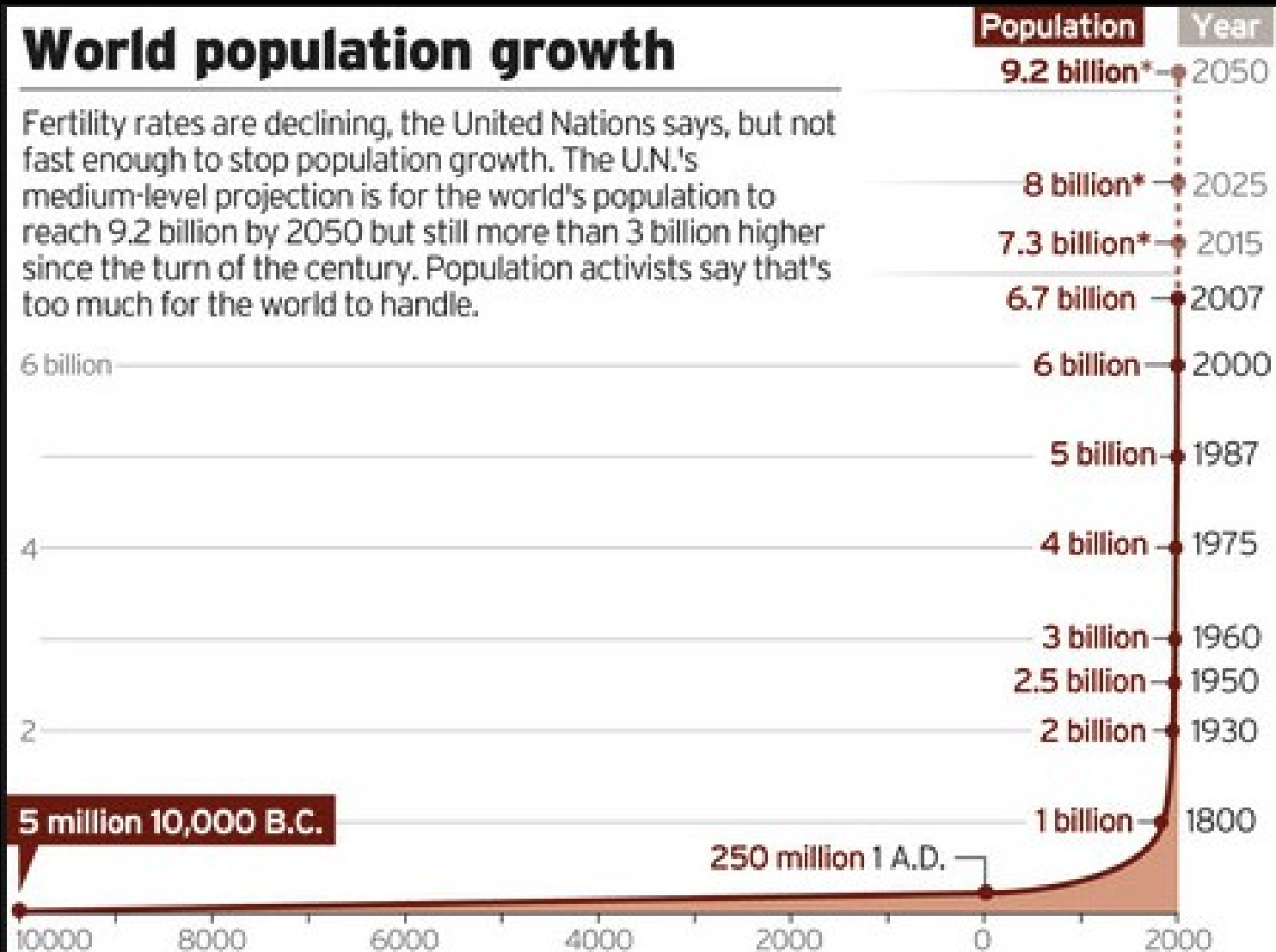
***OR.....A FINLAND EVERY MONTH!***

## CHINA URBAN/RURAL POPULATION GROWTH 1950-2030



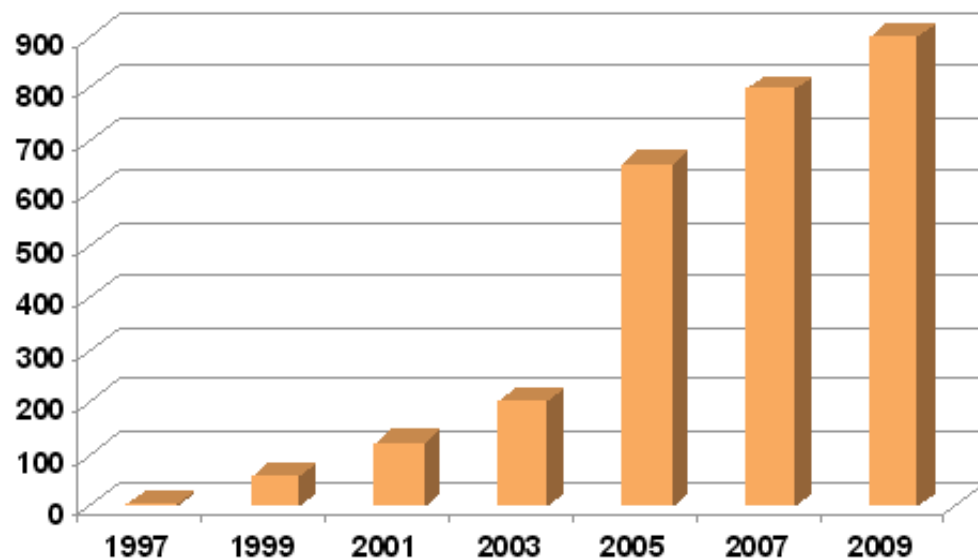
# World population growth

Fertility rates are declining, the United Nations says, but not fast enough to stop population growth. The U.N.'s medium-level projection is for the world's population to reach 9.2 billion by 2050 but still more than 3 billion higher since the turn of the century. Population activists say that's too much for the world to handle.



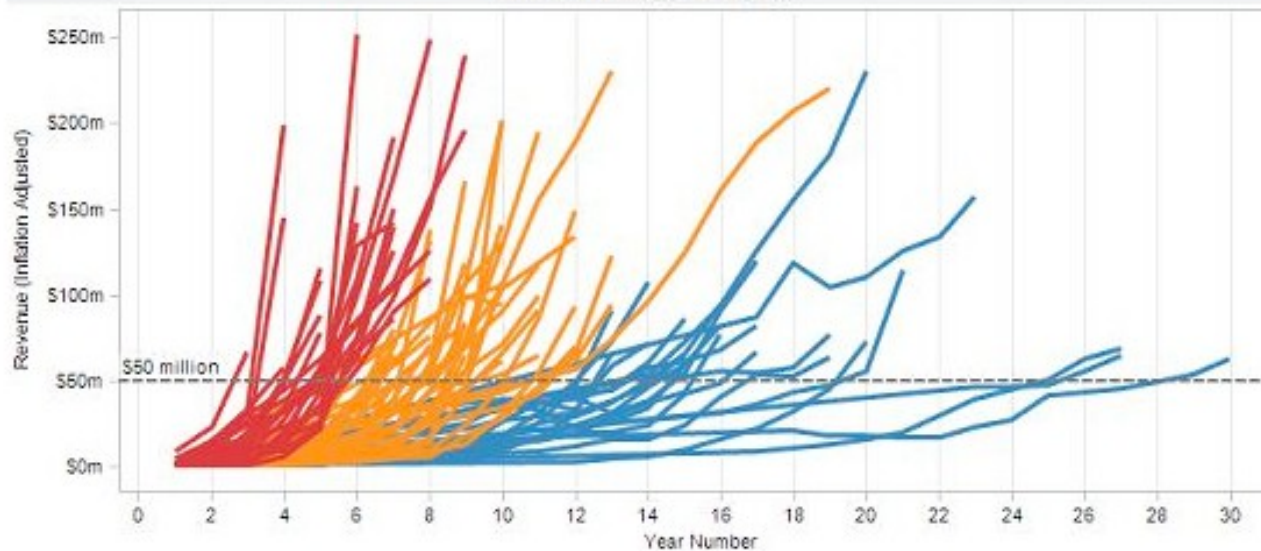
Sources: United Nations; Sustainable Scale Project; World Resources Institute; NationMaster.com

\* Projection



Click to interact

Growth History by Company



Growth rates of 100 software companies from IPO Dashboard

## BRIDGE CAPITAL

Bridge funding, as its name implies, bridges the gap between your current financing and the next level of financing.



## MEZZANINE CAPITAL

Mezzanine capital is also known as expansion capital, and is funding to help your company grow to the next level, purchase bigger and better equipment, or move to a larger facility.

## STARTUP CAPITAL

Start-up, or working capital is the funding that will help you pay for equipment, rent, supplies, etc. for the first year or so of operation.

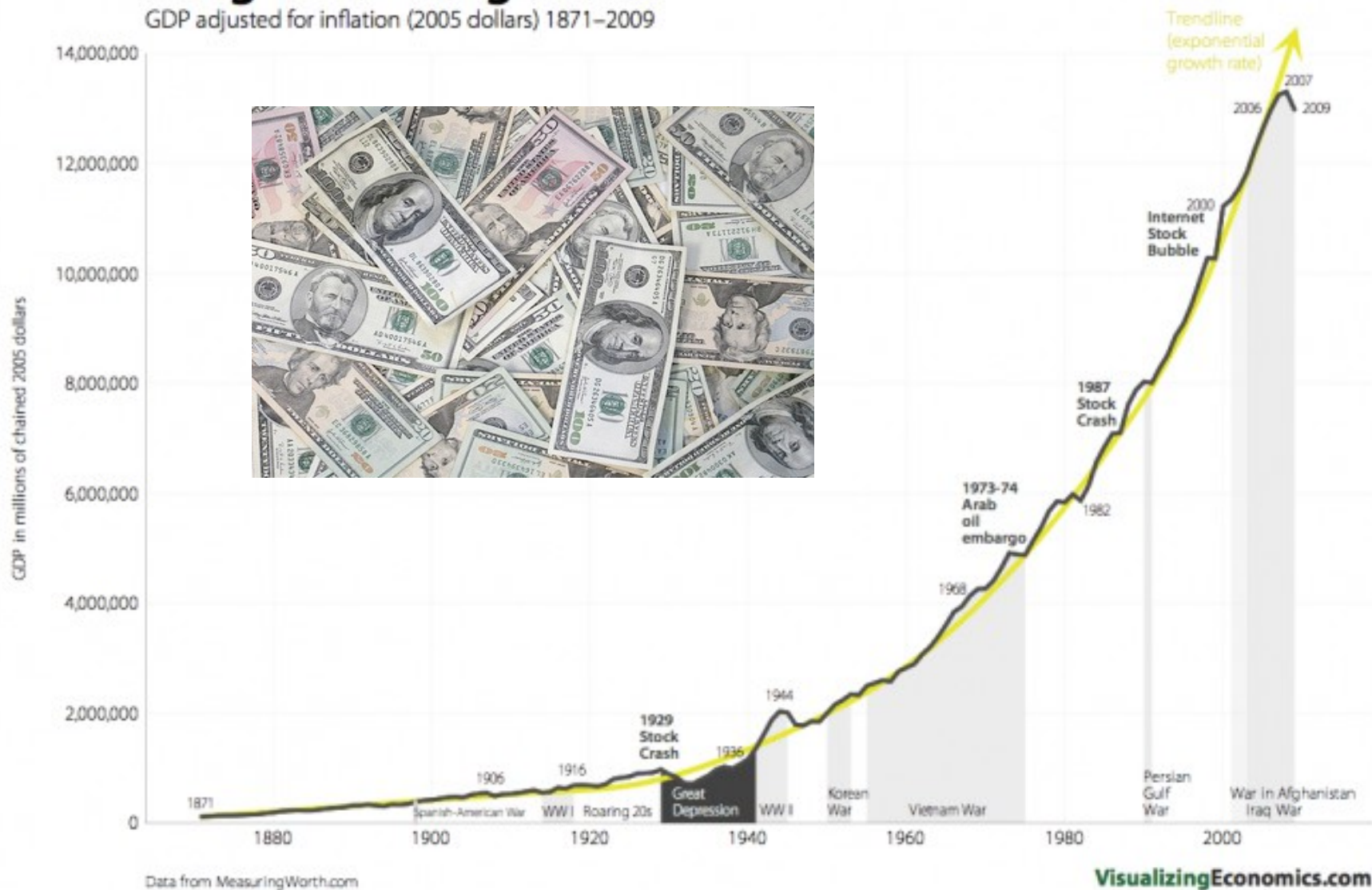
## SEED CAPITAL

Seed capital is the money you need to do your initial research and planning for your business.



# Long-term real growth in US GDP

GDP adjusted for inflation (2005 dollars) 1871–2009



**FATE OF OUR PLANET IS**  
*the fate of our cities*

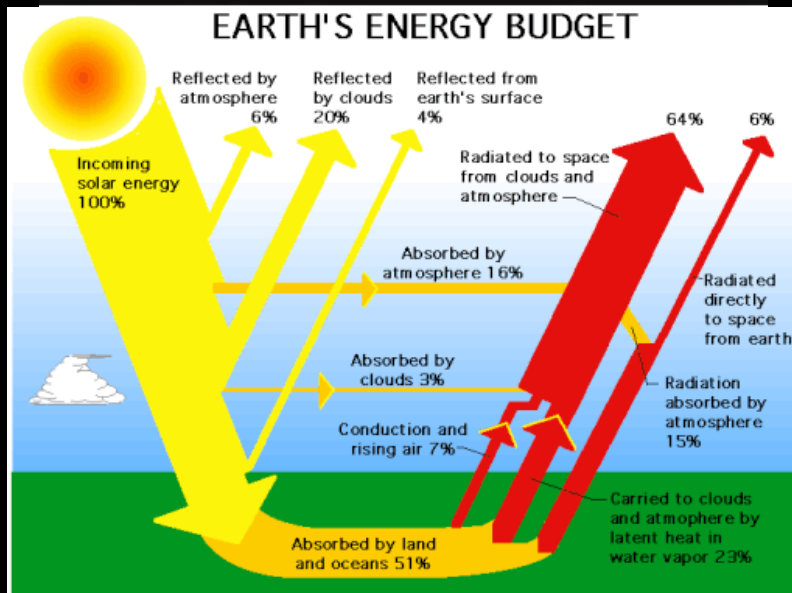














***SOCIO-ECONOMIC  
ENTROPY!!***







 Evening  
Standard 

**Classified** 79383

FRIDAY  
**MASSIVE  
INCREASE  
IN KNIFE  
CRIME  
SURVEYS**

 Evening  
Standard  
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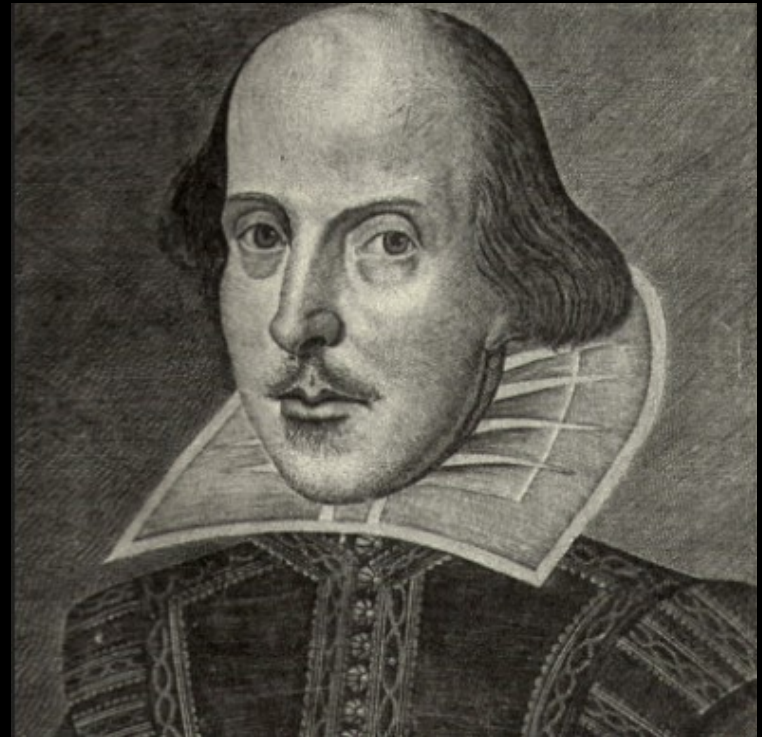


# London After Climate Change?



**“What is the city but the  
people?”**

*William Shakespeare*









**ENERGY & RESOURCES  
(METABOLISM, INFRASTRUCTURE)**

**VS.**

**INFORMATION  
(GENOMICS, INNOVATION)**

***CITIES AND URBANISATION  
ARE THE PROBLEM***

***CITIES AND URBANISATION  
ARE THE PROBLEM***

***BUT THEY ARE ALSO THE  
SOLUTION!!***

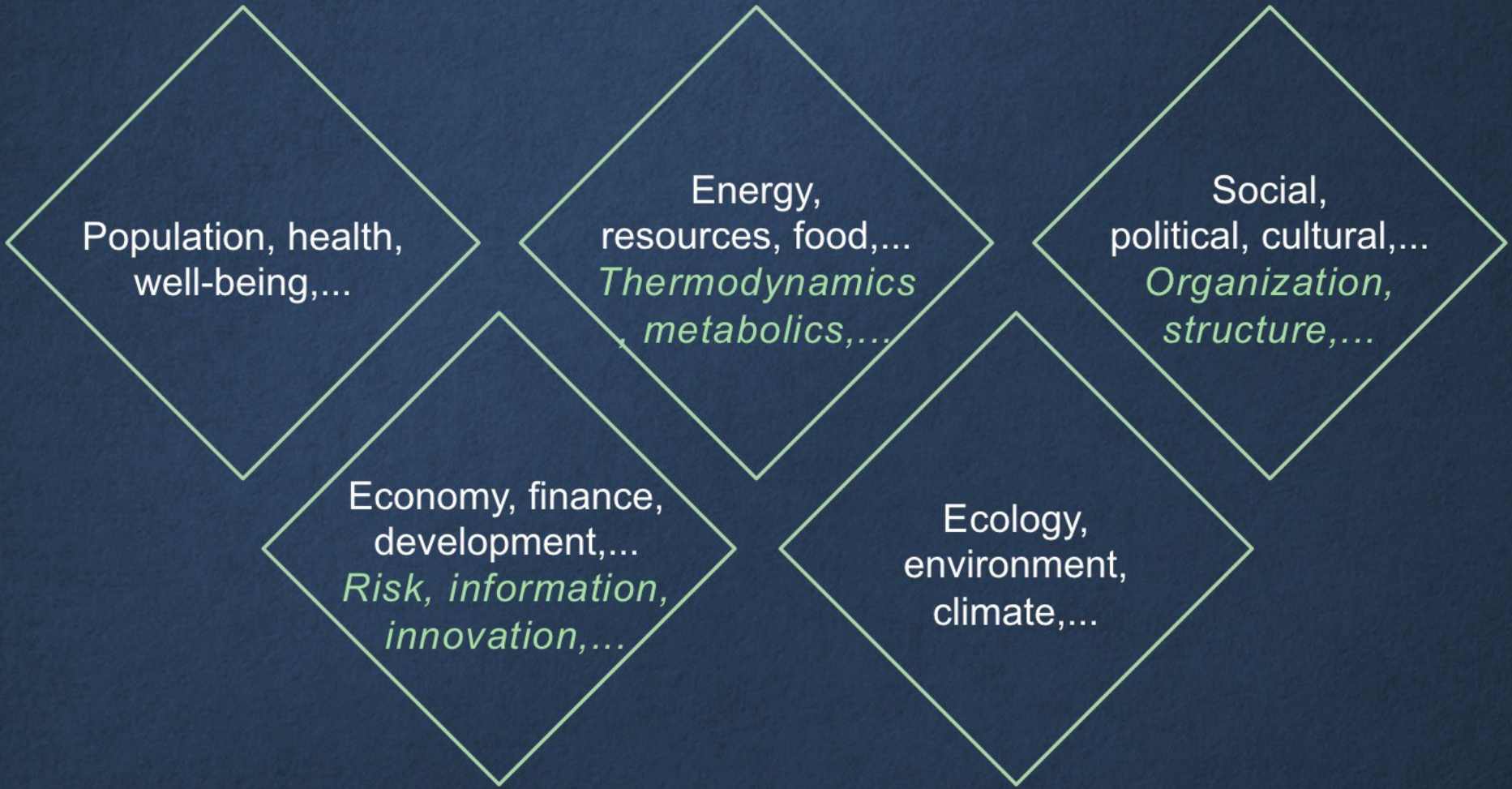
# ***URGENTLY NEED A QUANTITATIVE, PREDICTIVE SCIENCE OF CITIES***

***RESILIENCE***

***EVOLVABILITY***

***GROWTH***

***SCALABILITY***



Population, health,  
well-being,...

Energy,  
resources, food,...  
*Thermodynamics*  
*, metabolics,...*

Social,  
political, cultural,...  
*Organization,*  
*structure,...*

Economy, finance,  
development,...  
*Risk, information,*  
*innovation,...*

Ecology,  
environment,  
climate,...

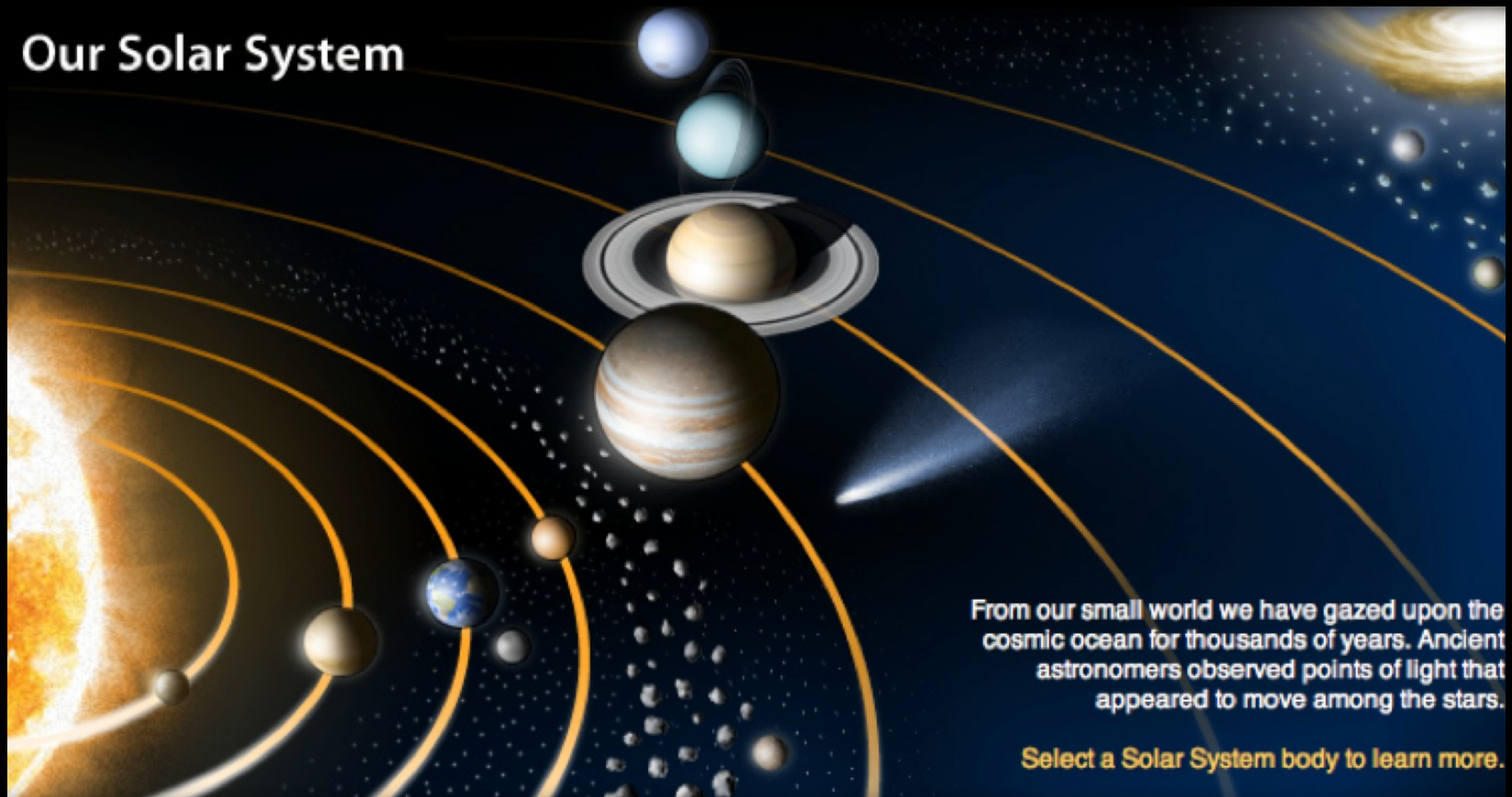


THESE ARE NOT INDEPENDENT

They are all highly coupled, inter-related,  
multi-scale *complex adaptive systems*.

***CAN THERE BE “NEWTON’S  
LAWS OF COMPLEX ADAPTIVE  
SYSTEMS”?***

## Our Solar System



From our small world we have gazed upon the cosmic ocean for thousands of years. Ancient astronomers observed points of light that appeared to move among the stars.

Select a Solar System body to learn more.

# NEWTON'S LAWS OF MOTION:

$$F = ma = m \frac{d^2 \vec{r}}{dt^2}$$

# NEWTON'S LAW OF GRAVITATION:

$$F = G \frac{m_1 m_2}{r^2}$$

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# NEWTON'S LAW OF GRAVITATION:

$$F = G \frac{m_1 m_2}{r^2}$$

**ENERGY!!**

**ENCODES AN “INFINITE” AMOUNT  
OF DATA**

# MAXWELL'S EQUATIONS

## UNIFICATION OF ELECTRICITY AND MAGNETISM

Name	"Microscopic" equations	"Macroscopic" equations
<b>Gauss's law</b>	$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$	$\nabla \cdot \mathbf{D} = \rho_f$
<b>Gauss's law for magnetism</b>	$\nabla \cdot \mathbf{B} = 0$	
<b>Maxwell–Faraday equation (Faraday's law of induction)</b>	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	
<b>Ampère's circuital law (with Maxwell's correction)</b>	$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$	$\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}$

# MAXWELL'S EQUATIONS

## UNIFICATION OF ELECTRICITY AND MAGNETISM

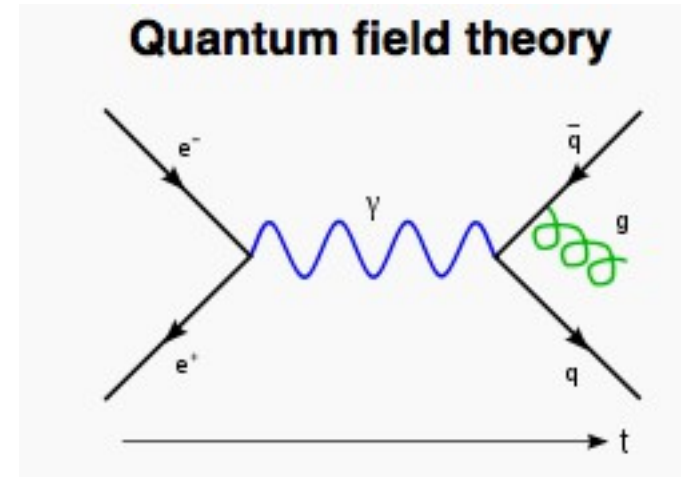
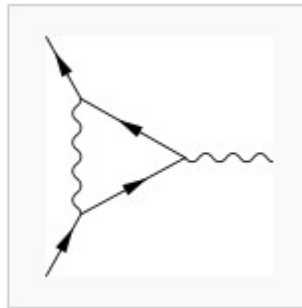
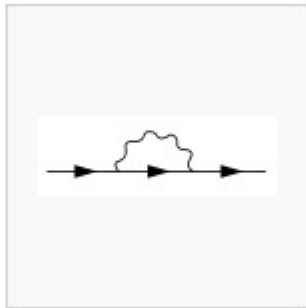
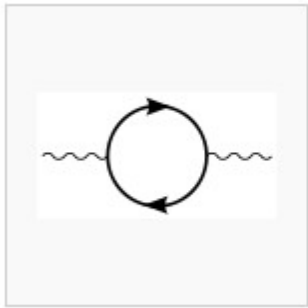
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# ELECTROMAGNETIC WAVES!!

# QUANTUM ELECTRODYNAMICS

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

## FEYNMAN DIAGRAMS



## MAGNETIC MOMENT OF THE ELECTRON

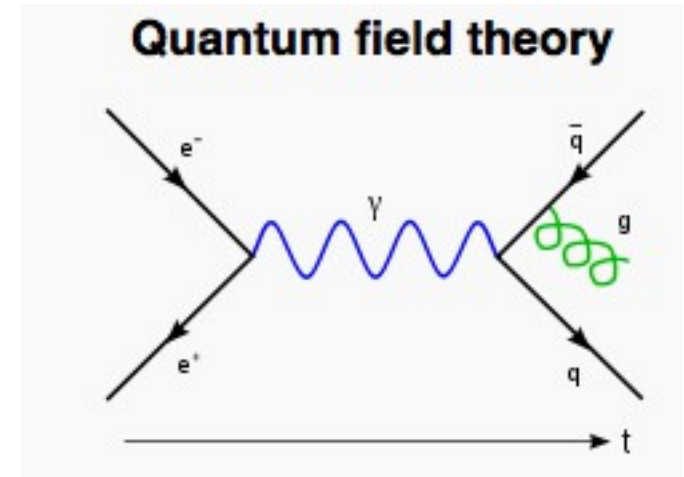
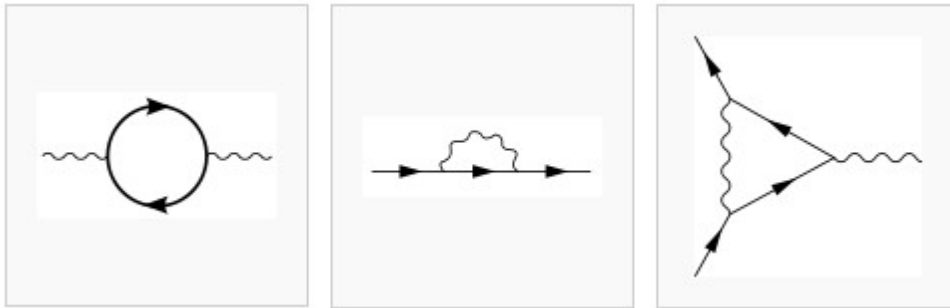
### ***THEORY***

$$g/2 = 1.001\,159\,652\,177\,60\,(520)\,[4.4\,\text{ppt}]$$

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## FEYNMAN DIAGRAMS



## MAGNETIC MOMENT OF THE ELECTRON

### THEORY

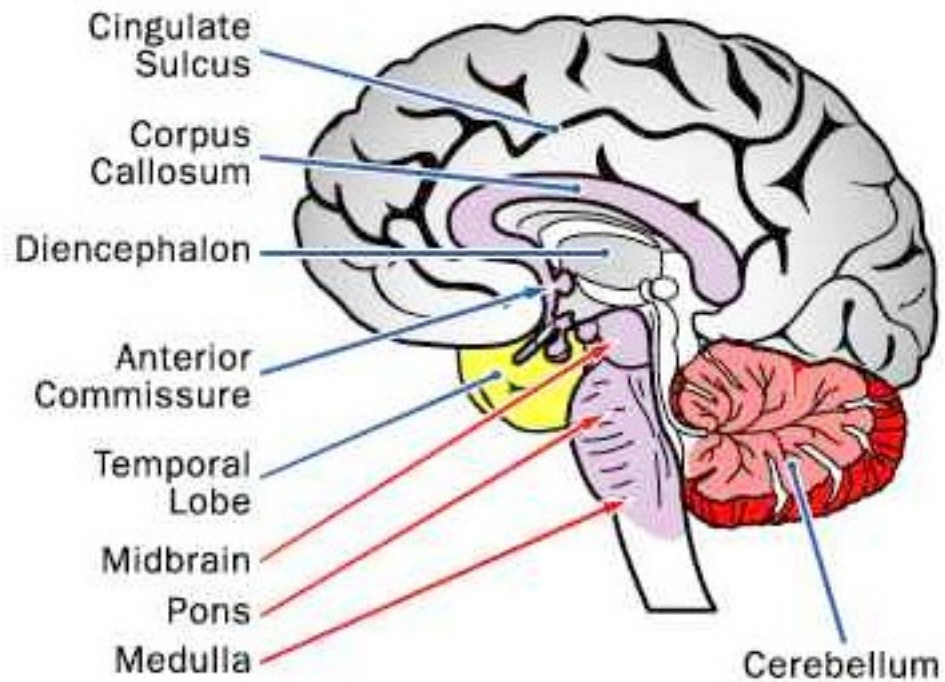
$$g/2 = 1.001\,159\,652\,177\,60\,(520) [4.4 \text{ ppt}]$$

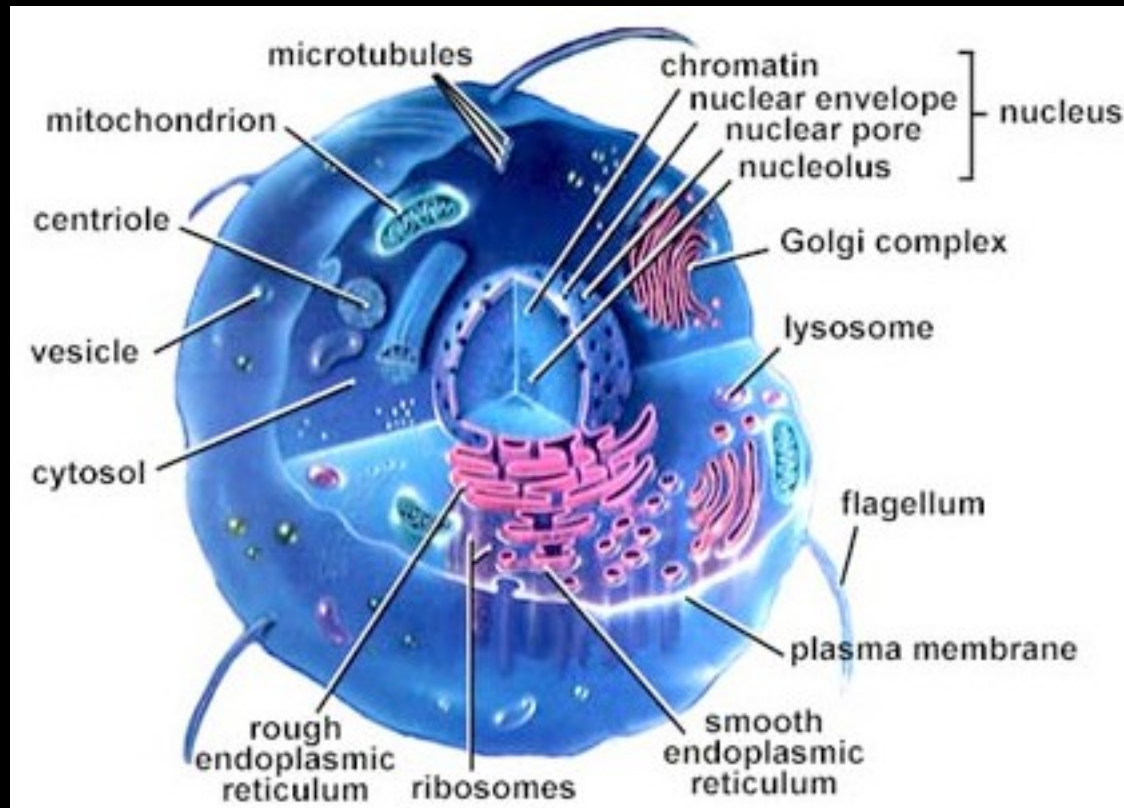
### EXPERIMENT

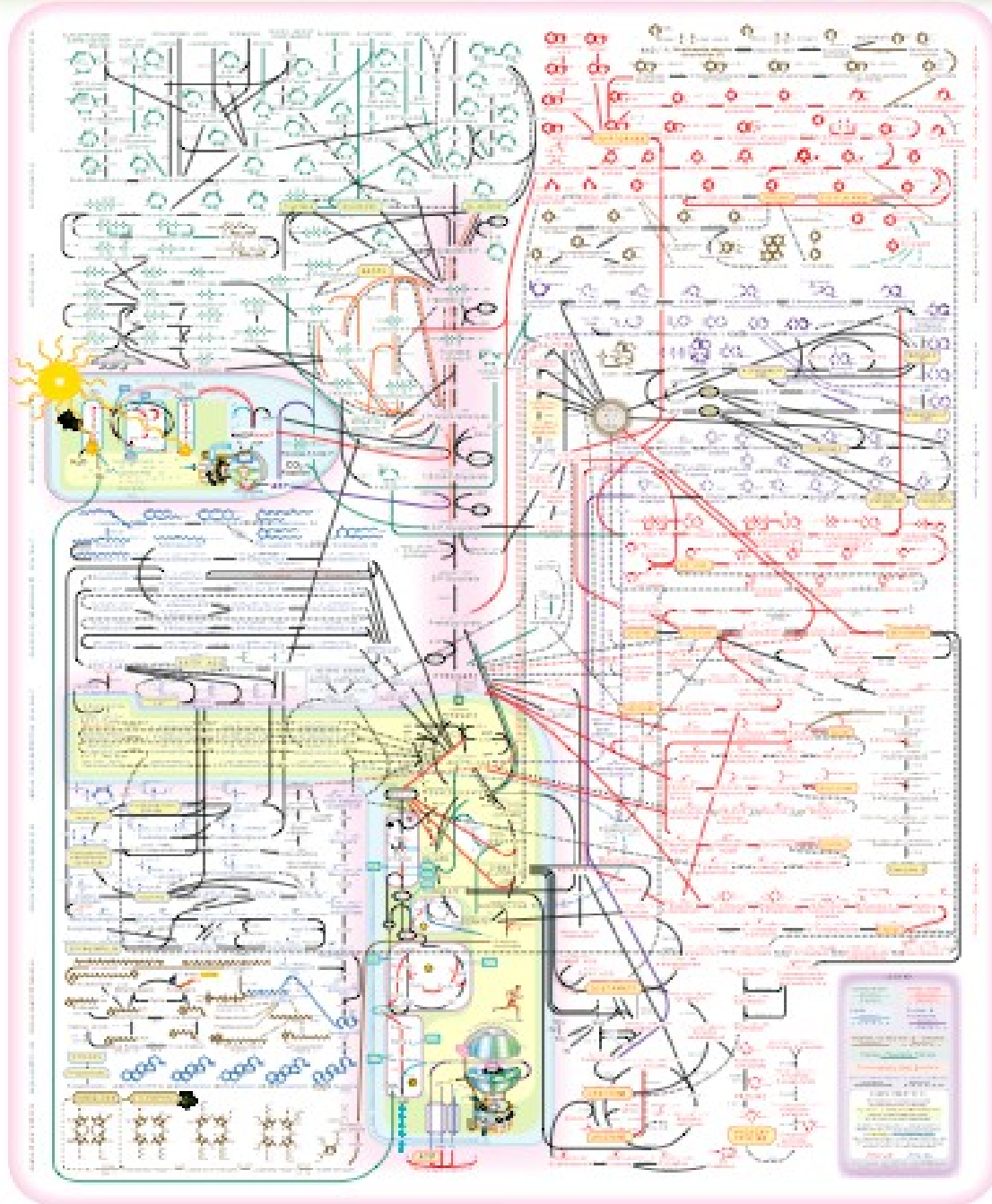
$$g/2 = 1.001\,159\,652\,180\,73\,(28) [0.28 \text{ ppt}]$$



## Major Internal Parts of the Human Brain













# ***SOME CHARACTERISTICS OF COMPLEX SYSTEMS***

- ***MANY COMPONENTS***
- ***MANY INDIVIDUAL ACTORS / AGENTS***
- ***MULTI SPATIAL AND TEMPORAL SCALES***
- ***STRONGLY COUPLED / INTERACTING***
- ***NON-LINEAR***
- ***SENSITIVITY TO BOUNDARY CONDITIONS (CHAOS)***
- ***MORE IS DIFFERENT***
- ***EMERGENT PHENOMENA / MULTIPLE PHASES***
- ***UNINTENDED CONSEQUENCES***
- ***ADAPTIVE / EVOLVING***
- ***ENERGY/INFORMATION***
- ***HISTORICALLY CONTINGENT / PATH DEPENDENT***
- ***ROBUST / RESILIENT***
- ***NON-EQUILIBRIUM***
- ***UNDERLYING SIMPLICITY***
- ***COMPLICATED vs COMPLEX***

- ***SEARCH FOR UNDERLYING LAWS AND PRINCIPLES LEADING TO A QUANTITATIVE (MATHEMATISABLE) PREDICTIVE CONCEPTUAL FRAMEWORK***

- ***CAN THERE BE “NEWTON’S LAWS OF COMPLEX ADAPTIVE SYSTEMS”?***

***COARSE - GRAINED DESCRIPTION***

***WITH INCREASING RESOLUTION  
AND GRANULARITY***

***STATISTICAL/PROBABILISTIC***

***QUANTITATIVE, PREDICTIVE***

***WHY DO WE STOP GROWING?***

***WHY DO WE LIVE ~100 YEARS AND NOT 1000, OR 2-3 YEARS LIKE A MOUSE?***

***WHERE DOES A TIME-SCALE OF 100 YEARS COME FROM?***

***HOW IS IT GENERATED FROM  
FUNDAMENTAL MICROSCOPIC  
MOLECULAR TIME-SCALES OF GENES  
AND RESPIRATORY ENZYMES?***





***WHY DO WE SLEEP ~8 HOURS A DAY AND NOT 15 LIKE MICE AND BABIES OR JUST 3 LIKE ELEPHANTS?***

***WHY DO CITIES KEEP GROWING WHEREAS ALL COMPANIES EFFECTIVELY STOP?***

***WHY DO (ALMOST) ALL COMPANIES EVENTUALLY DISAPPEAR - LIKE WE DO - WHEREAS (ALMOST) ALL CITIES SURVIVE?***



***WHY DOES THE PACE OF LIFE CONTINUE  
TO GET FASTER?***

***IS ANY OF THIS SUSTAINABLE?***

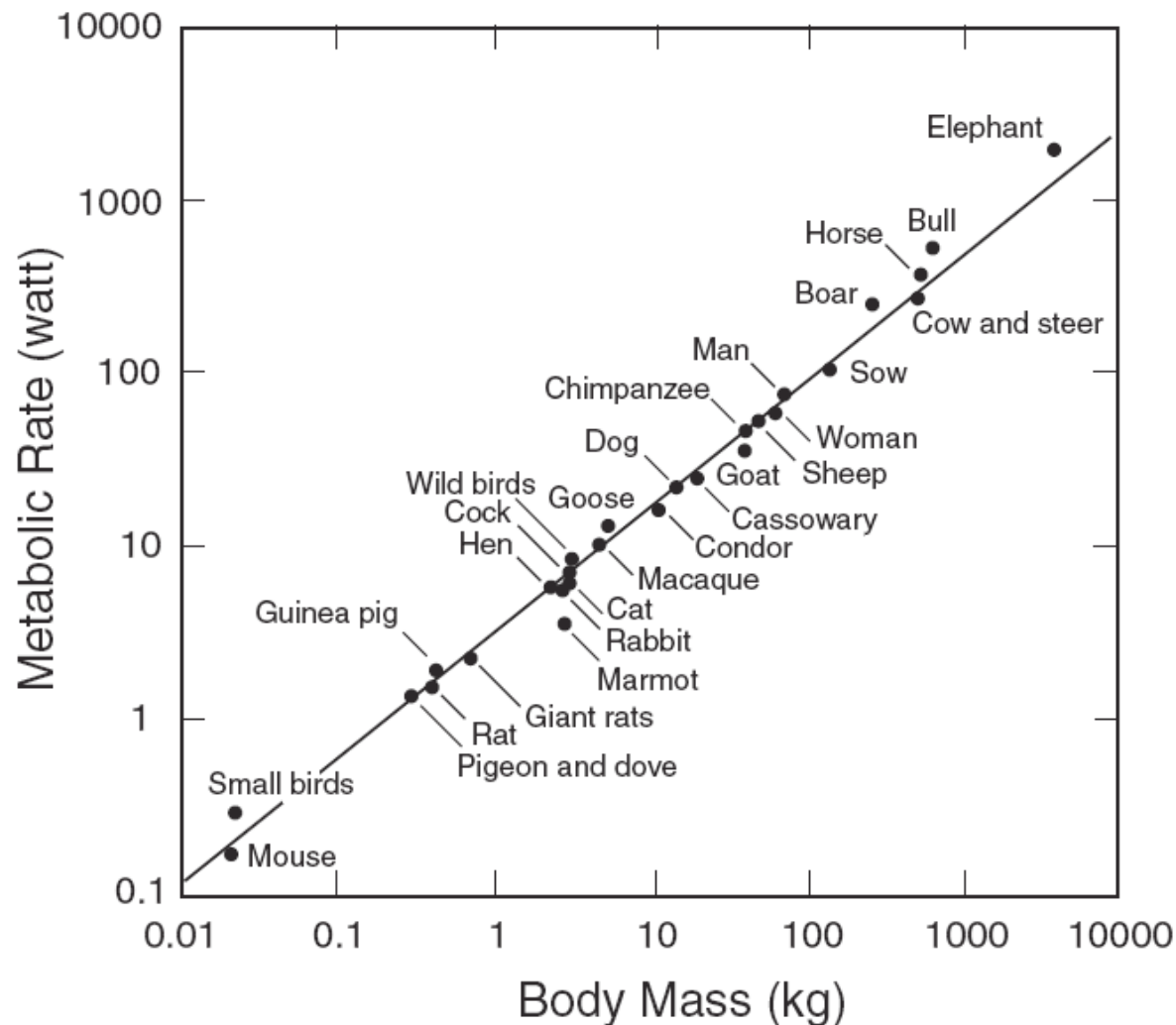
***IS THERE AN END TO (SOCIO-ECONOMIC)  
TIME?***

# Mammals vary in size by 8 orders of magnitude



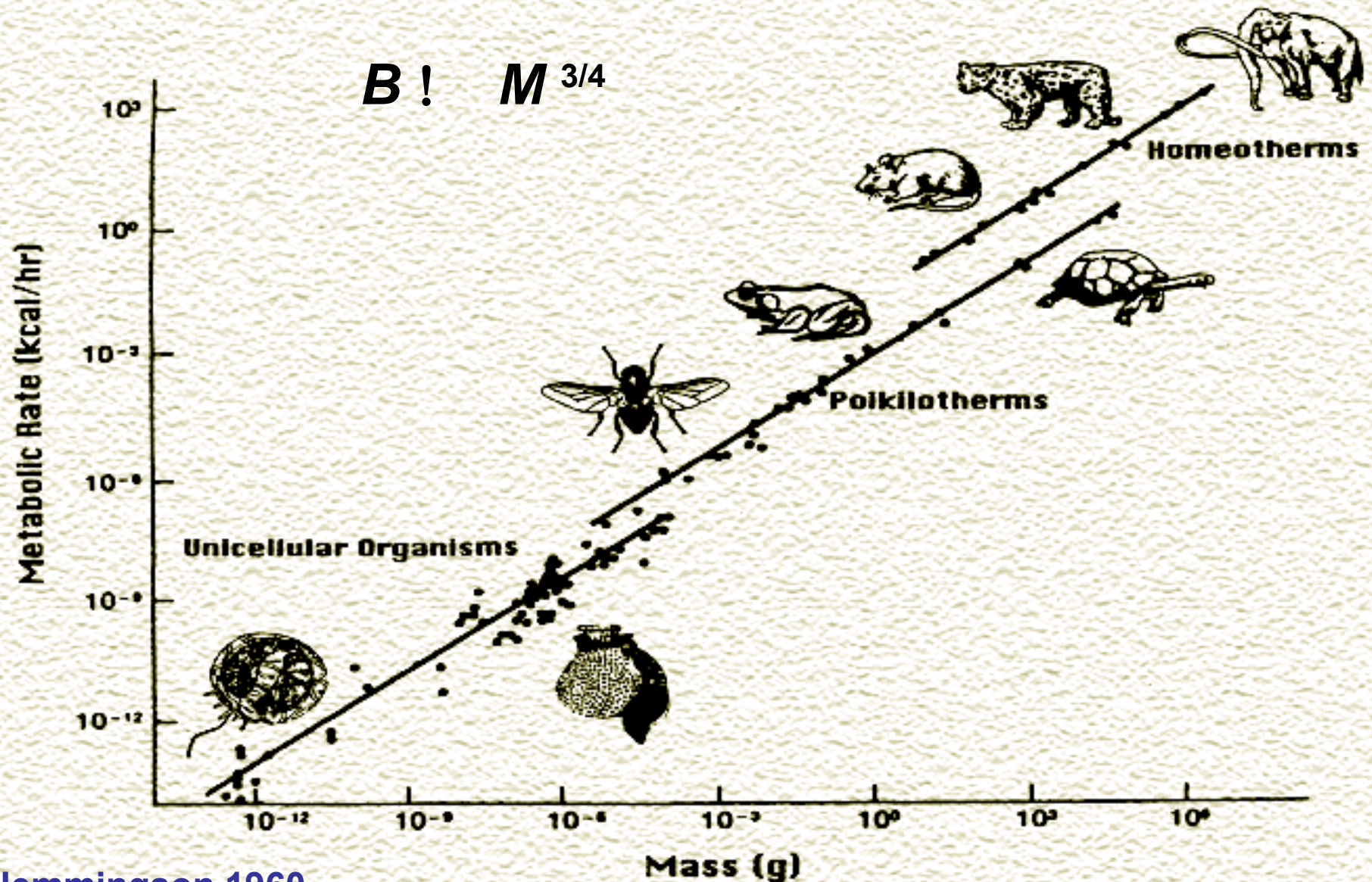
**Blue Whale**  
**200,000,000g**

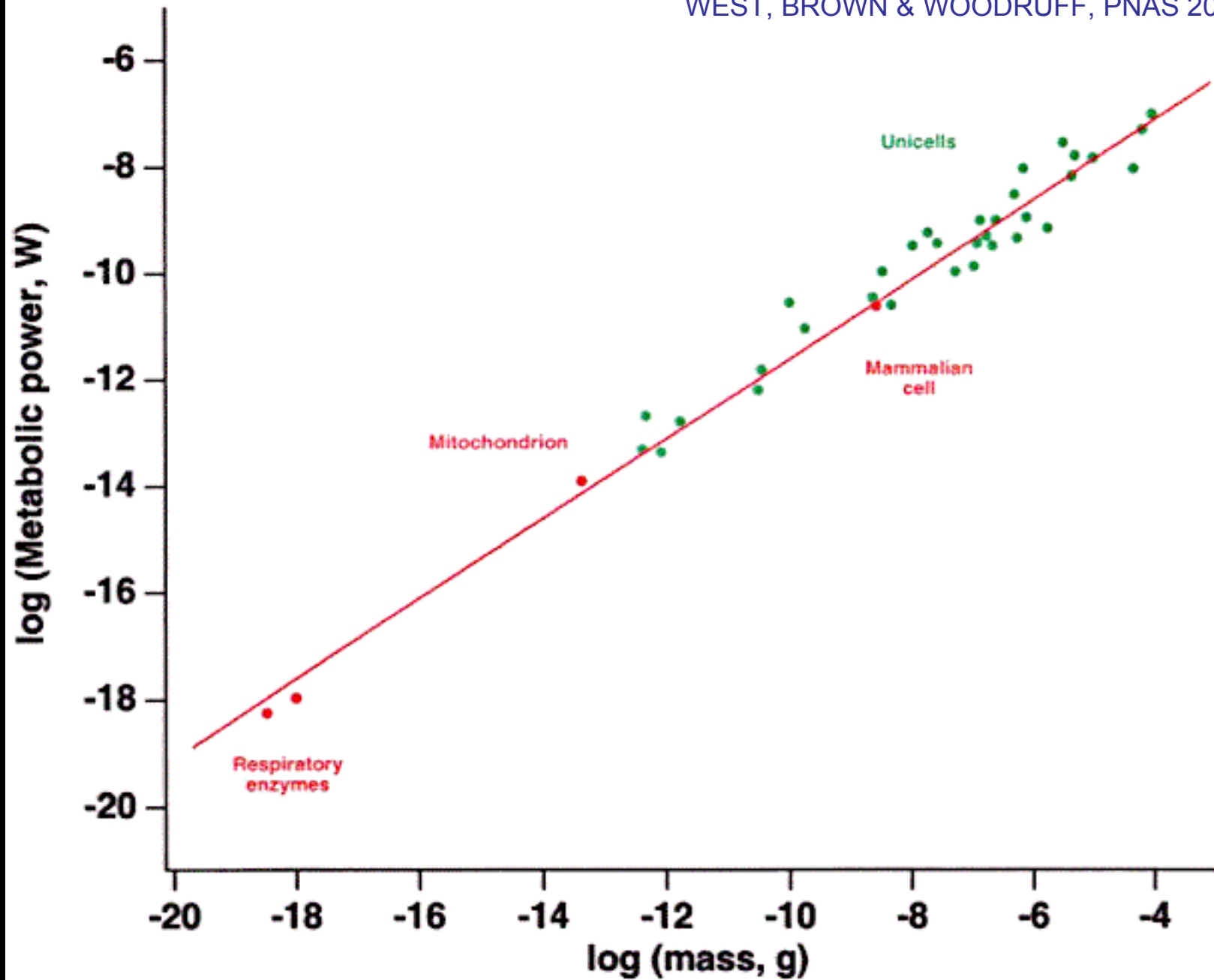




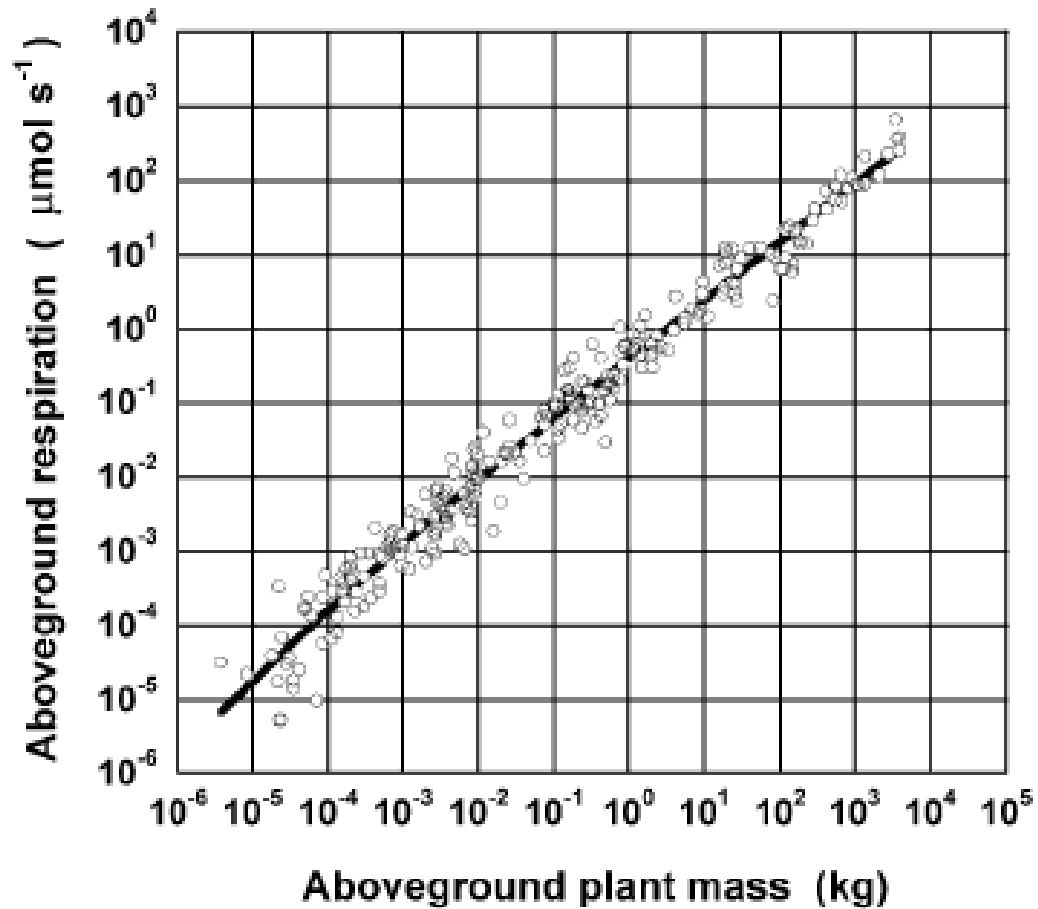
**SLOPE =  $\frac{3}{4} < 1$  SUB-LINEAR ECONOMY OF SCALE**

# Whole-organism metabolic rate ( $B$ ) scales as the $3/4$ power of body mass ( $M$ )





# PLANTS/TREES



$$B \propto M^{0.780 \pm 0.037}$$

**SINCE  $N_{\text{cells}} \sim M$  NAIVELY MIGHT EXPECT  $B \sim M$**

**HOWEVER,  $B \sim M^{3/4}$**

**OVER 27 ORDERS OF MAGNITUDE**

**SPECIFIC METABOLIC RATE (PER UNIT MASS)**

$$\frac{B}{M} \propto M^{-1/4}$$

**SO METABOLIC RATE OF AVERAGE CELL**

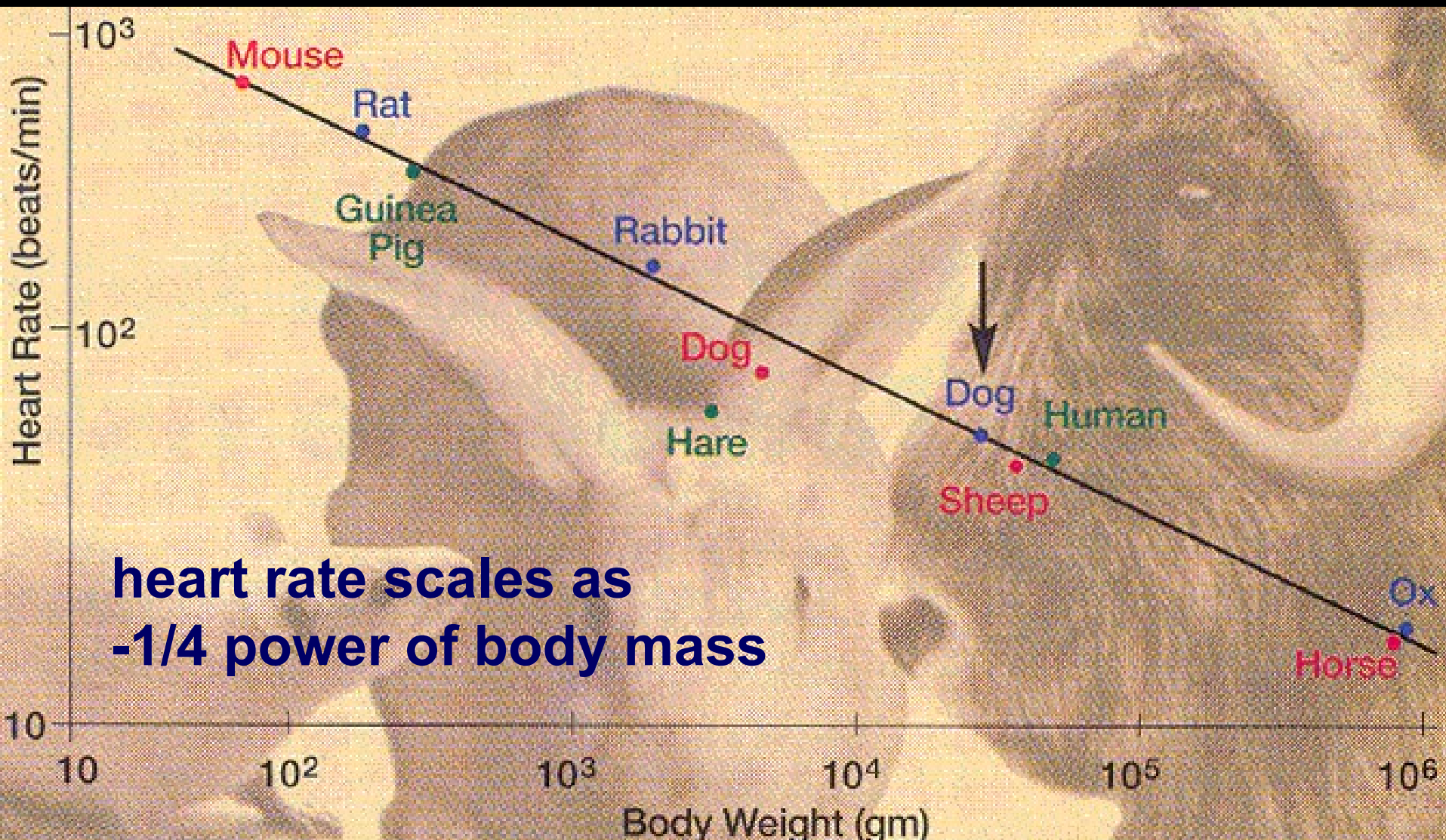
$$B_{\text{cell}} \propto M^{-1/4}$$

***EXTRAORDINARY SYSTEMATIC  
ECONOMY OF SCALE (THE BIGGER  
YOU ARE, THE LESS NEEDED PER  
“CAPITA”)***

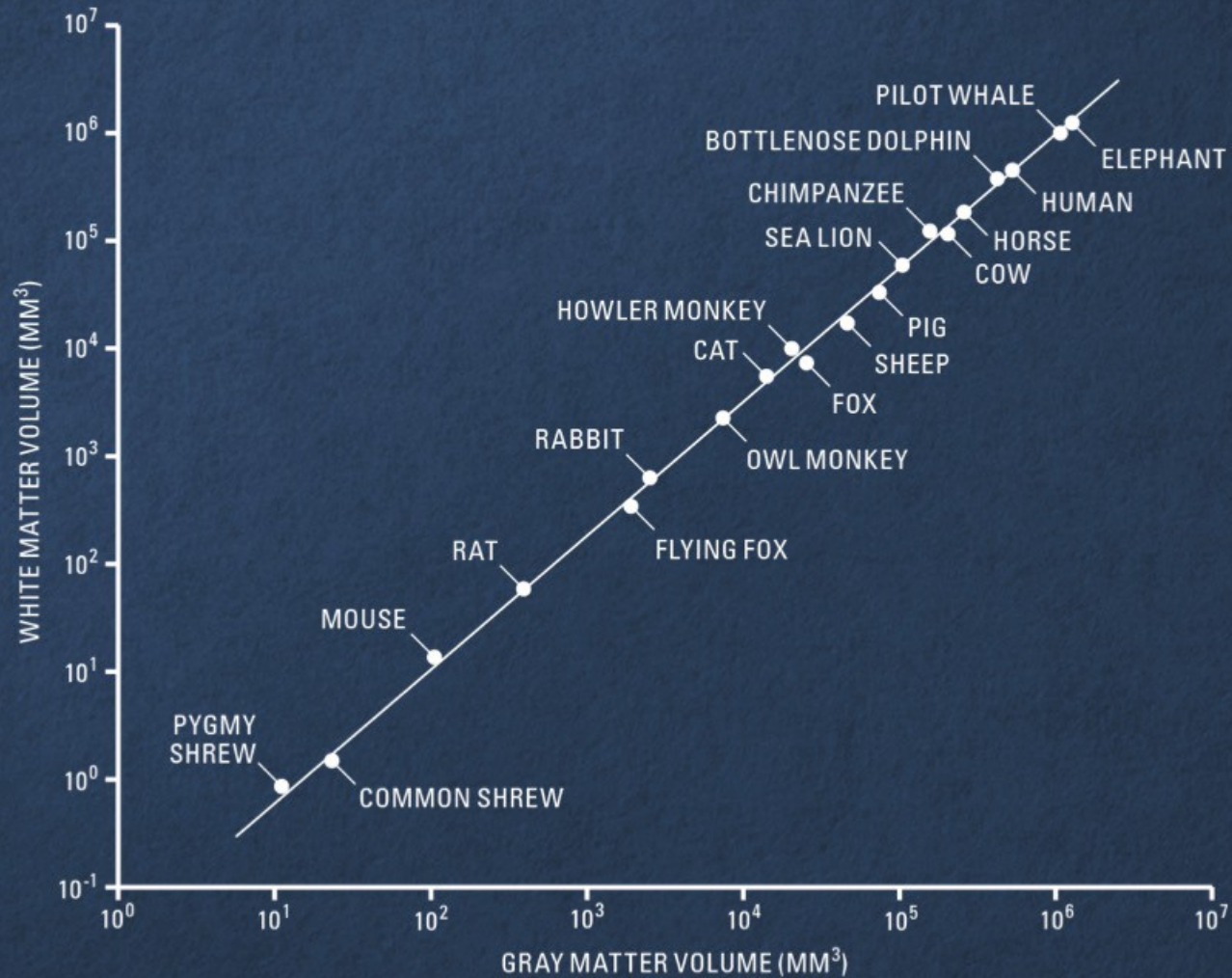
***SIMILAR SCALING HOLDS TRUE  
FOR ALL PHYSIOLOGICAL  
PROCESSES AND LIFE HISTORY  
EVENTS OVER THE ENTIRE  
SPECTRUM OF LIFE***

# Metabolic rate sets the pace of life

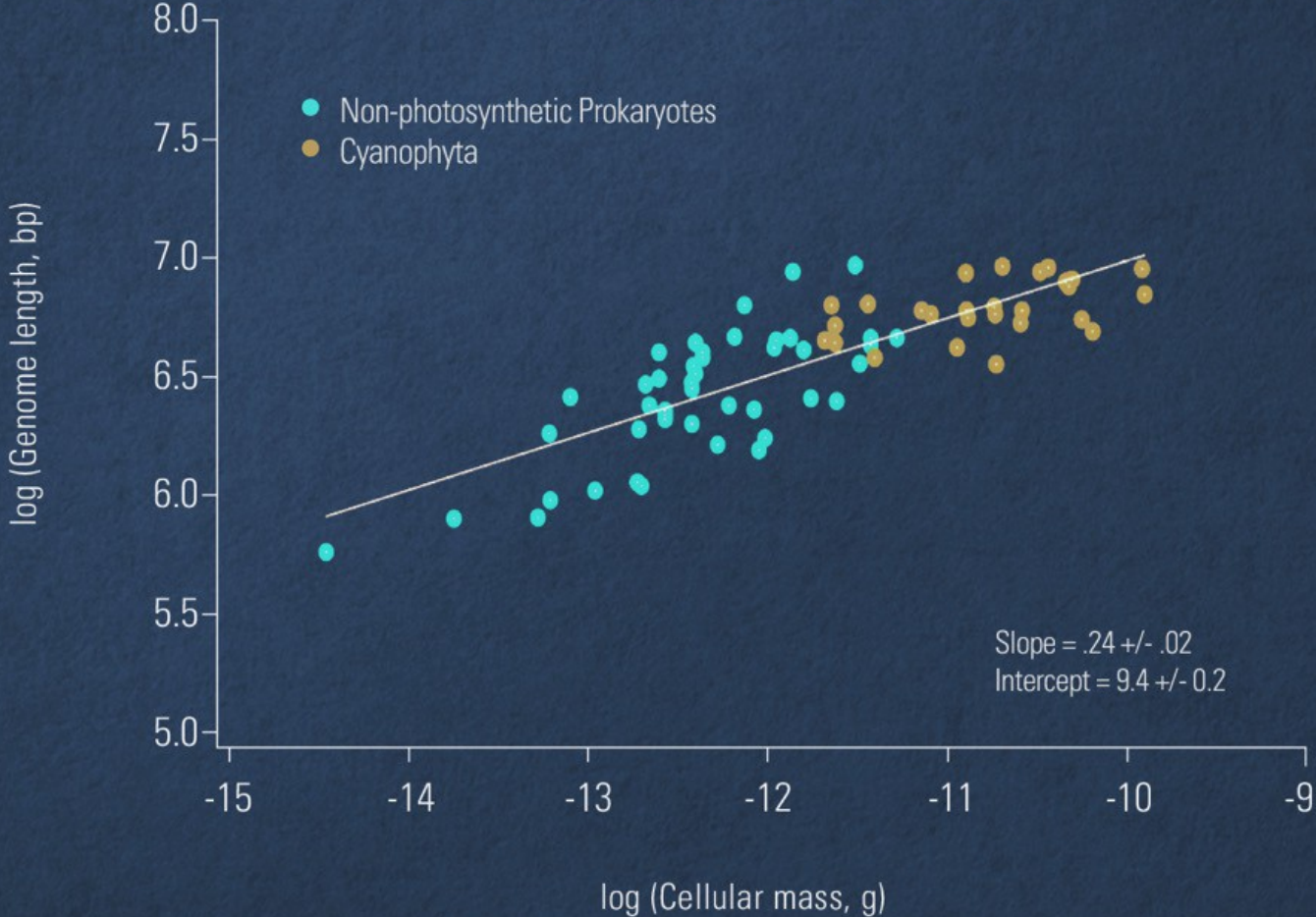
## Small animals live fast and die young



# WHITE AND GRAY MATTER OF BRAINS



# DEPENDENCE OF GENOME LENGTH ON CELLULAR MASS



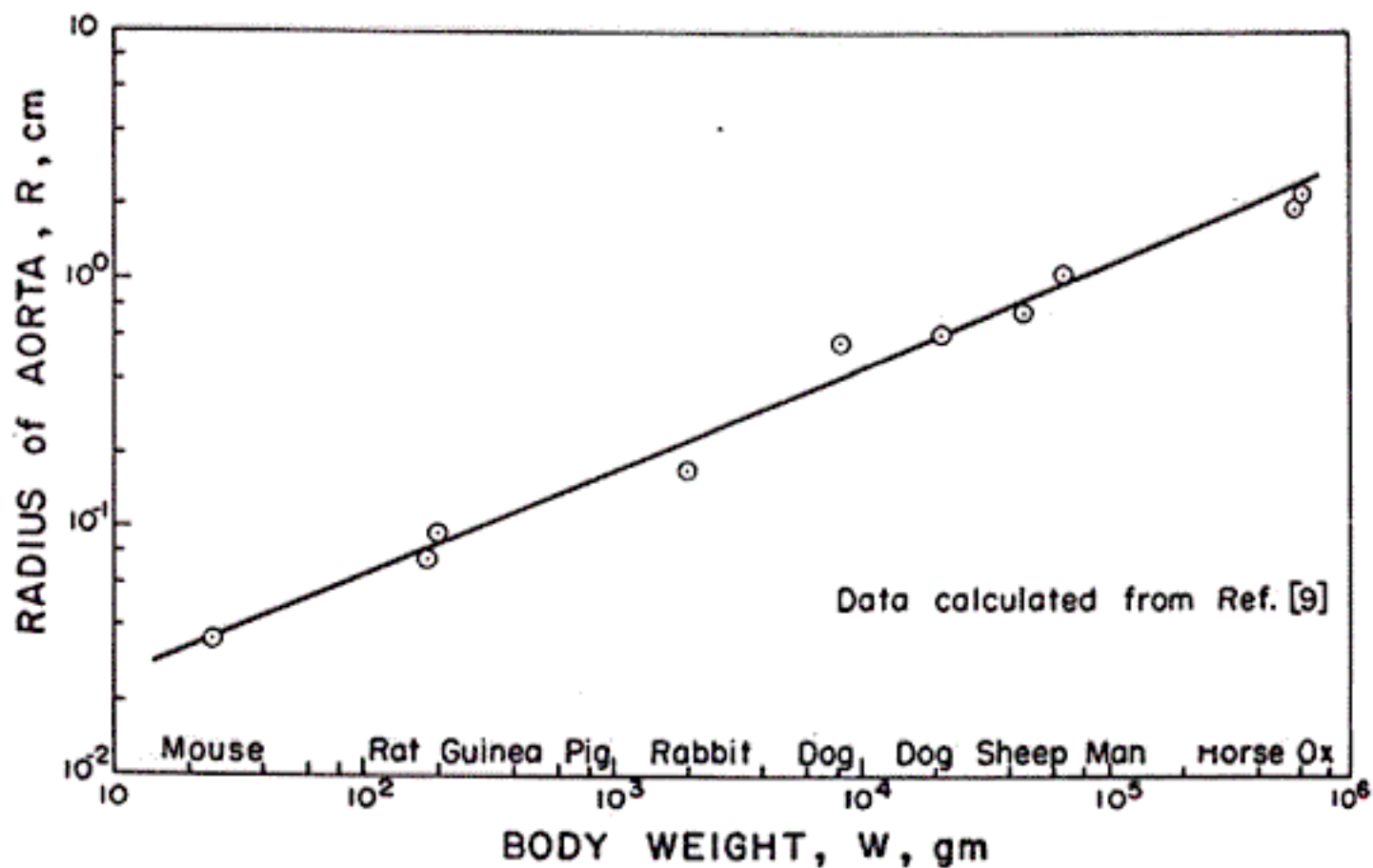


FIG. 4 - VARIATION IN RADIUS OF AORTA WITH BODY WEIGHT

$$r \sim M^{3/8}$$

SAME SCALING FOR TREE TRUNKS

Slopes (exponents) are typically  
sub-linear and simple multiples of  $\frac{1}{4}$

*“quarter-power scaling”*

## LIFESPAN

$$T \sim M^{1/4}$$

IF HEART-RATE (NUMBER OF BEATS PER SEC.)

$$\sim M^{-1/4}$$

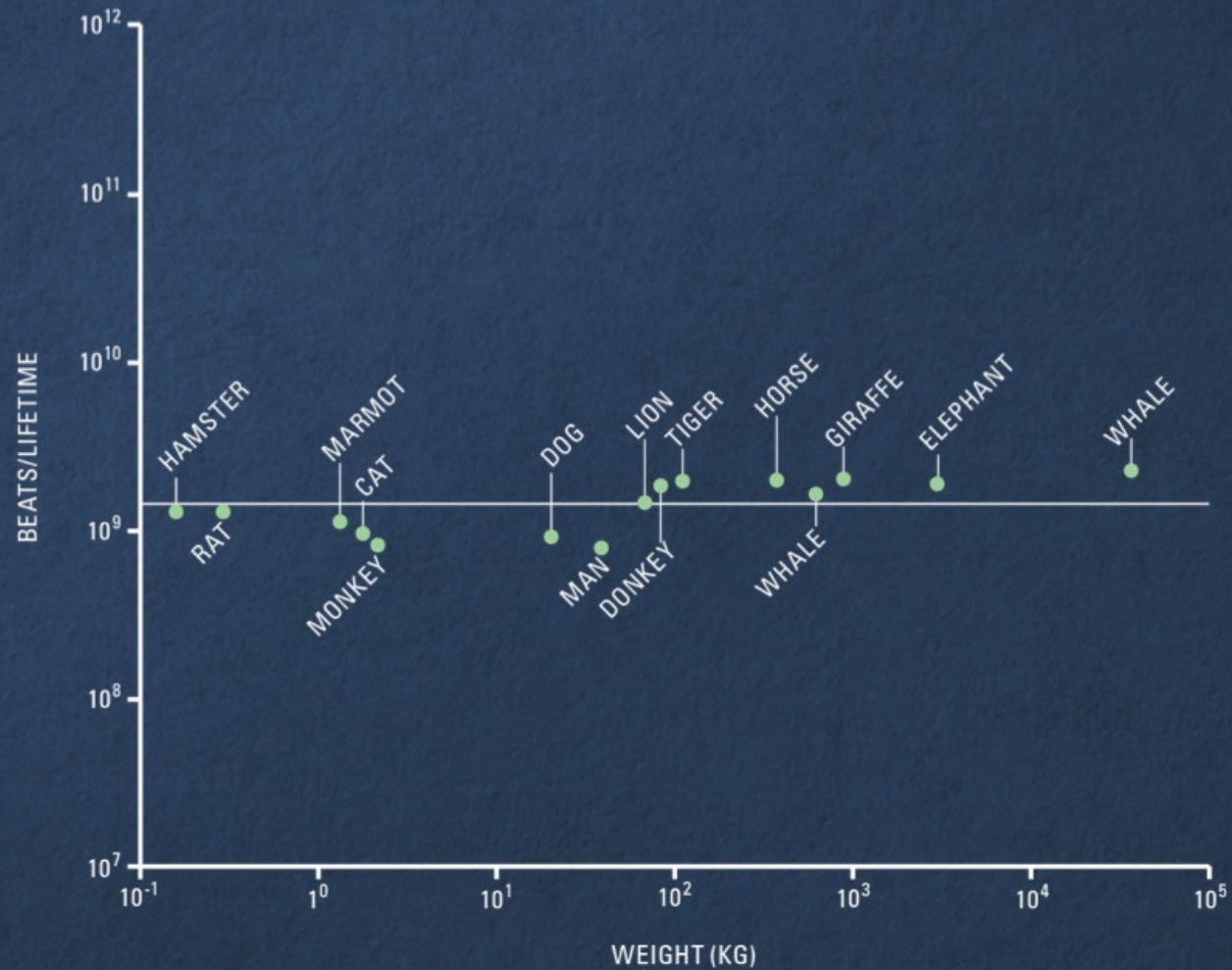
⇒ TOTAL NUMBER OF HEART-BEATS IN A

TYPICAL LIFE-TIME IS INDEPENDENT OF SIZE!

$$\approx 1.5 \times 10^9$$

EACH ANIMAL SPECIES REGARDLESS OF SIZE  
HAS APPROXIMATELY THE SAME NUMBER OF HEART-  
BEATS IN ITS LIFE-TIME (ROUGHLY 1 BILLION)

# NUMBER OF HEARTBEATS PER LIFETIME OF ANIMALS

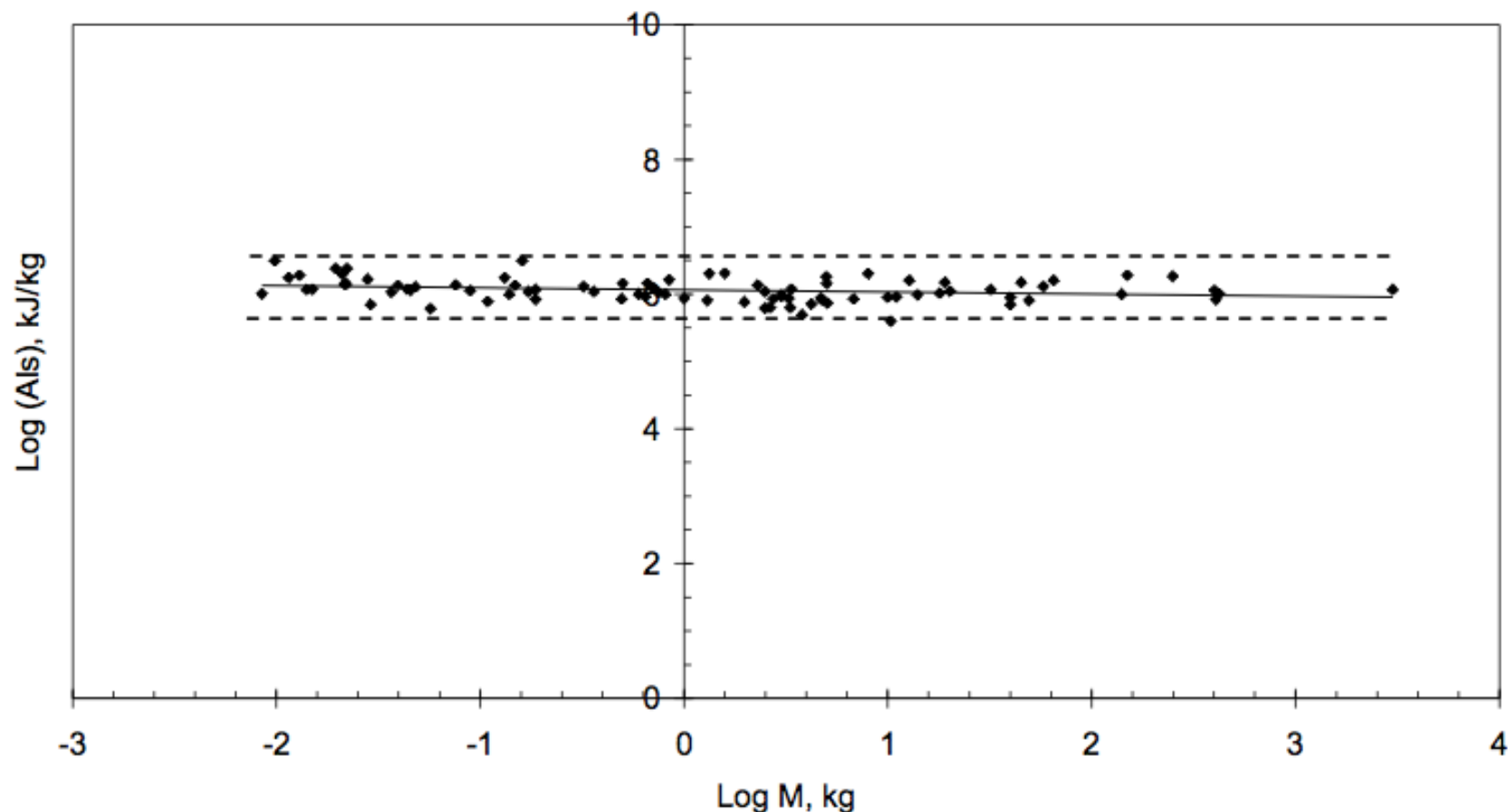


RECALL SPECIFIC METABOLIC RATE

$$\bar{B} = \frac{B}{M} \propto M^{-1/4}$$

⇒ TOTAL ENERGY NEEDED TO SUPPORT UNIT  
MASS OF AN ANIMAL DURING A LIFETIME  
IS THE SAME FOR ALL ANIMALS REGARDLESS  
OF SIZE :

$$\begin{aligned} E_{TOT} &\approx 1.2 \times 10^6 \text{ JOULES / gm} \\ &\approx 300 \text{ kcals / gm} \end{aligned}$$



**Fig. 2.** Relationship between the total metabolic energy per life span per unit body mass ( $A_{ls}=PT_{ls}/M$  kJ/kg) and the body mass ( $M$ , kg) for 86 terrestrial mammals in captivity (Prototheria, Metatheria and Eutheria). The 95% confidence limits are shown by dashed lines.

LIFE IS THE MOST COMPLEX SYSTEM

SCALING LAWS ARE REMARKABLE BECAUSE

i) THEY EXIST

ii) THEY ARE VERY SIMPLE

iii) THEY ARE UNIVERSAL

} DOMINANCE OF  
 $\frac{1}{4}$  POWER

iv)  $\Rightarrow$  BIGGER IS MORE EFFICIENT

v) FEW QUANTITATIVE "LAWS" IN BIOLOGY

***EXPLAINED BY THE GENERIC  
MATHEMATICAL AND  
PHYSICAL PROPERTIES OF.....***



# NETWORKS



**Large vessels  
branch into  
smaller ones**

**Beating heart**

**Pulse wave  
propagates  
through elastic  
vessels**

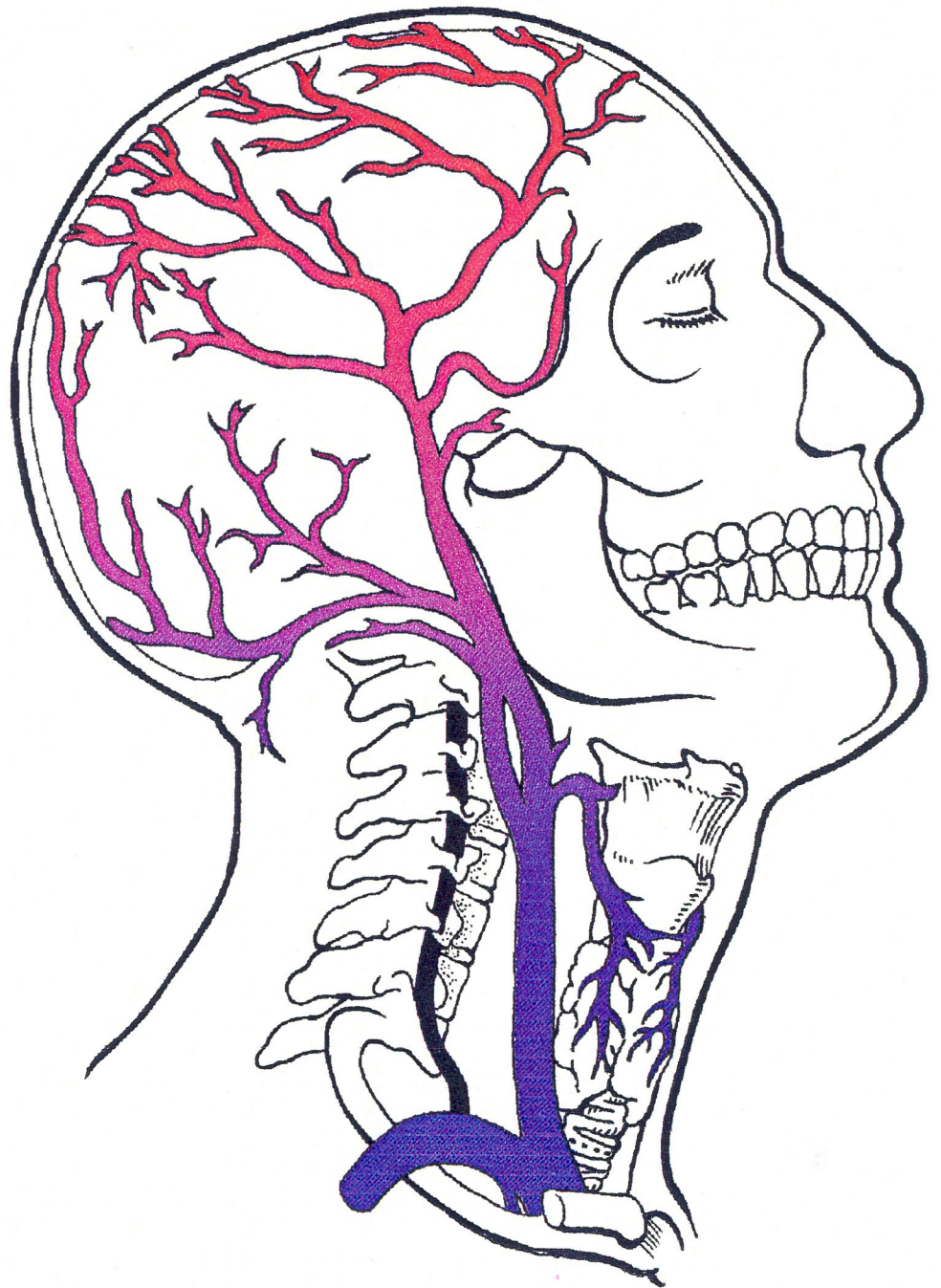
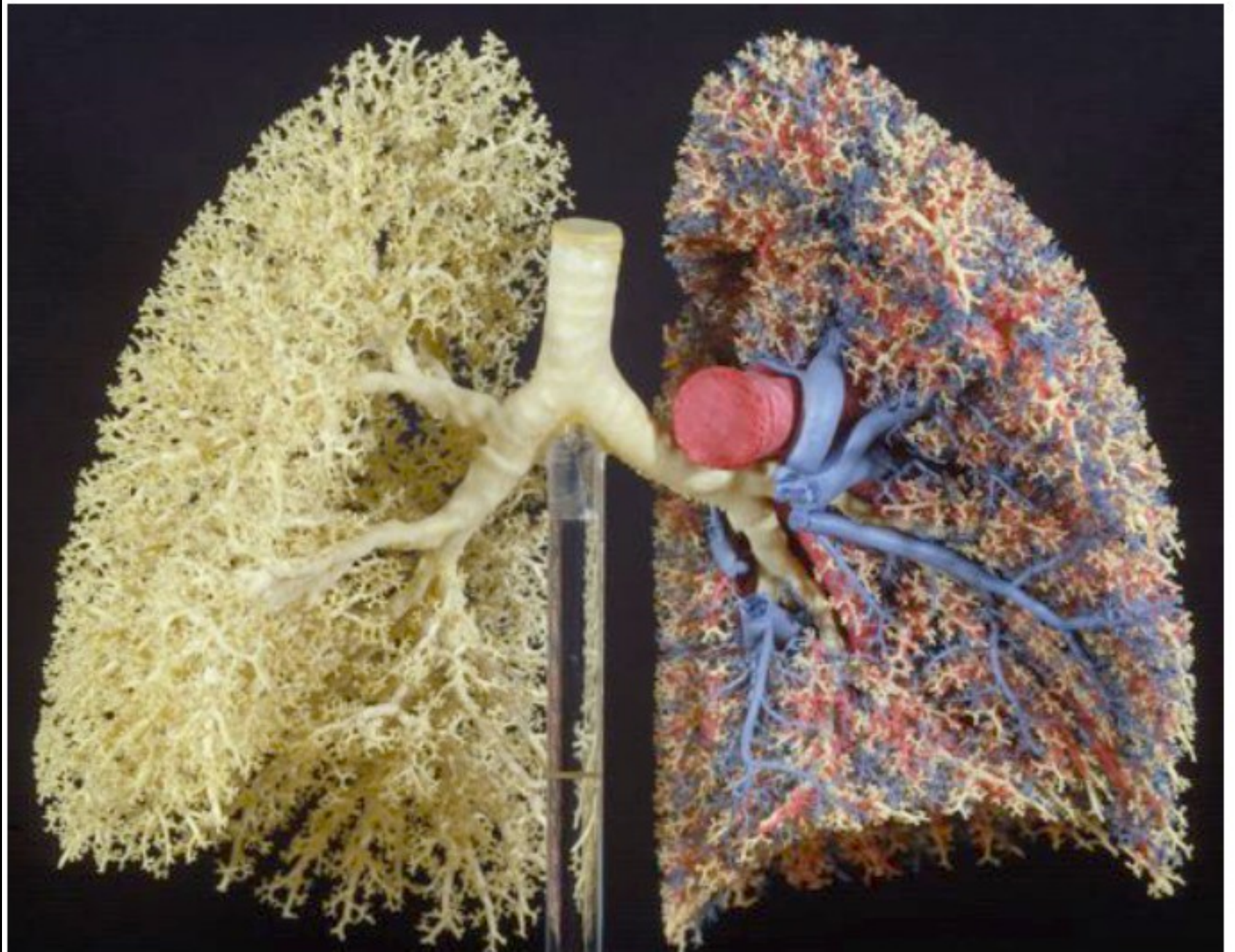


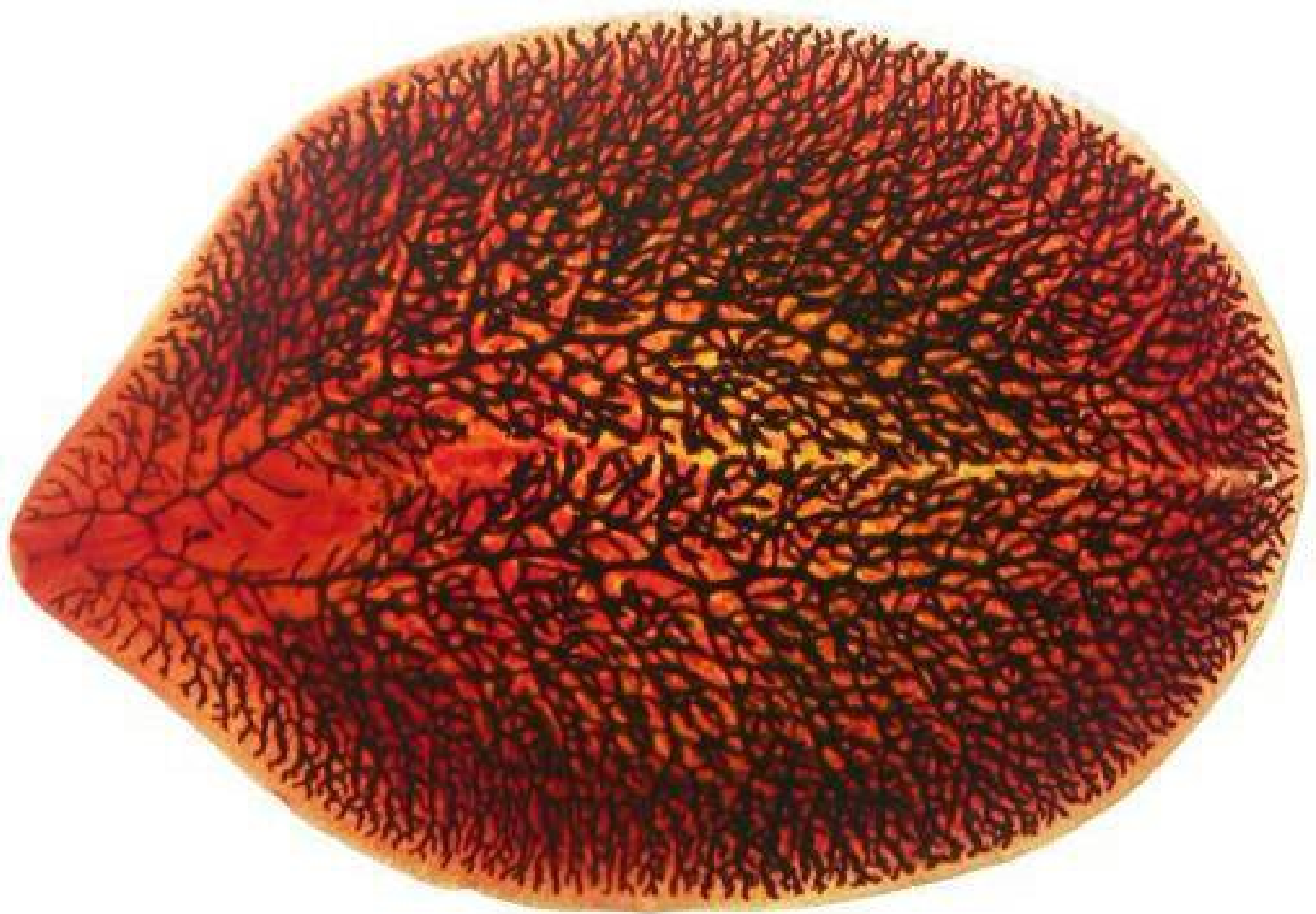


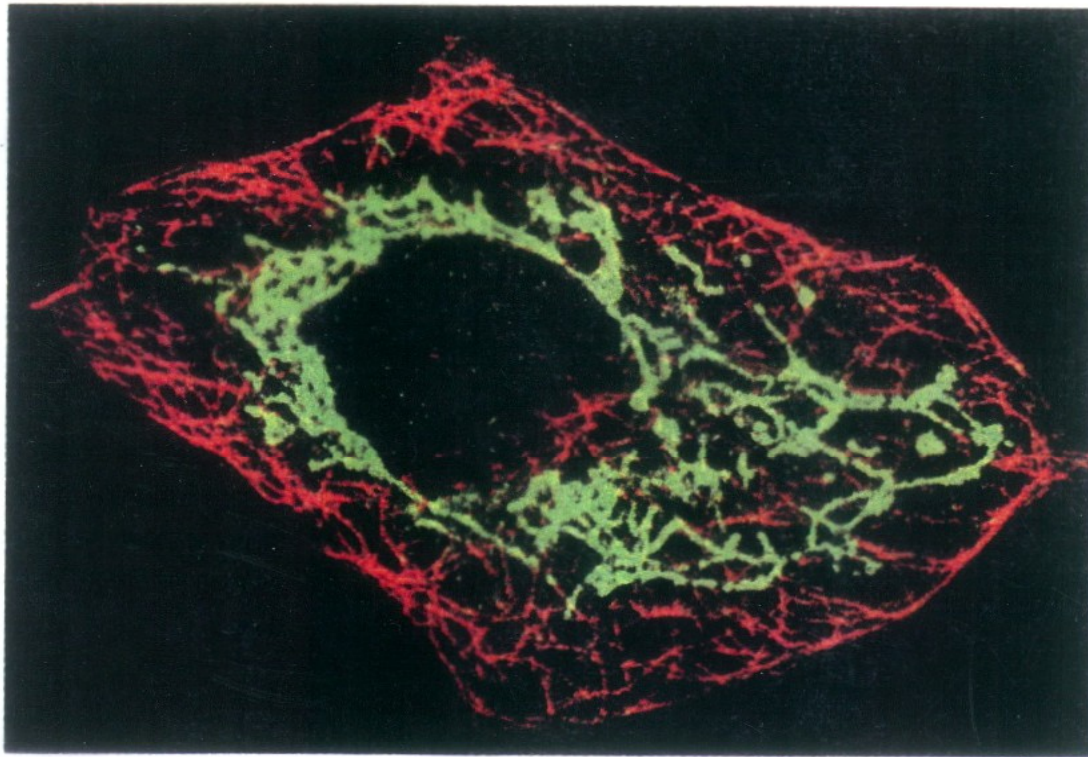
Fig. 5 A slice through the cerebellum showing the progressive branching structure. The white matter is distributed throughout the cerebellar volume. The geometric complexity of these structures provides for rapid dissemination of information (or energy) via a large surface area in a compact space. This feature is a hallmark of structures which maximize the surface area within a finite volume.



# Relation between number and size of branches within a tree

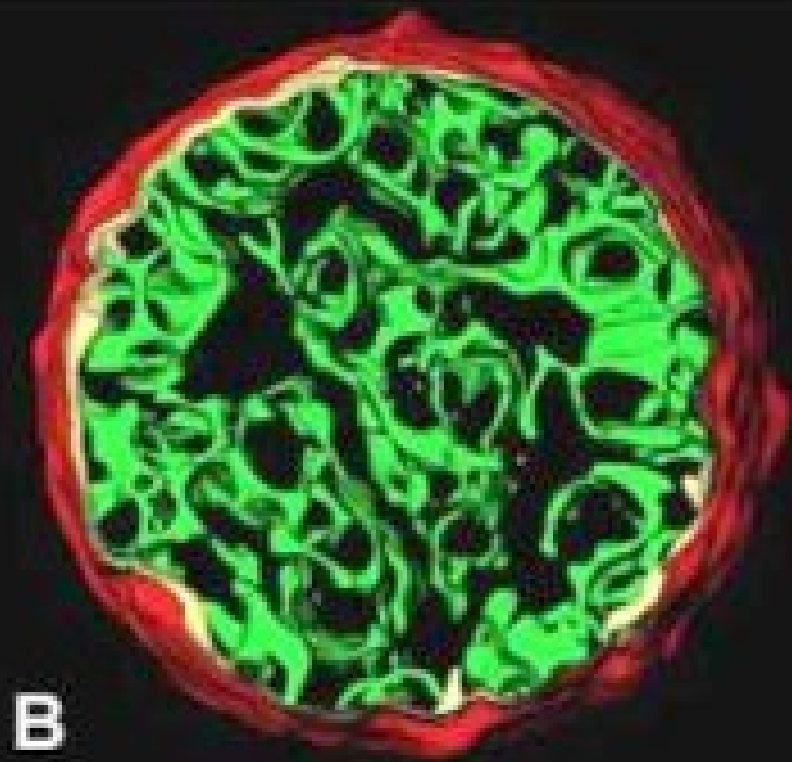
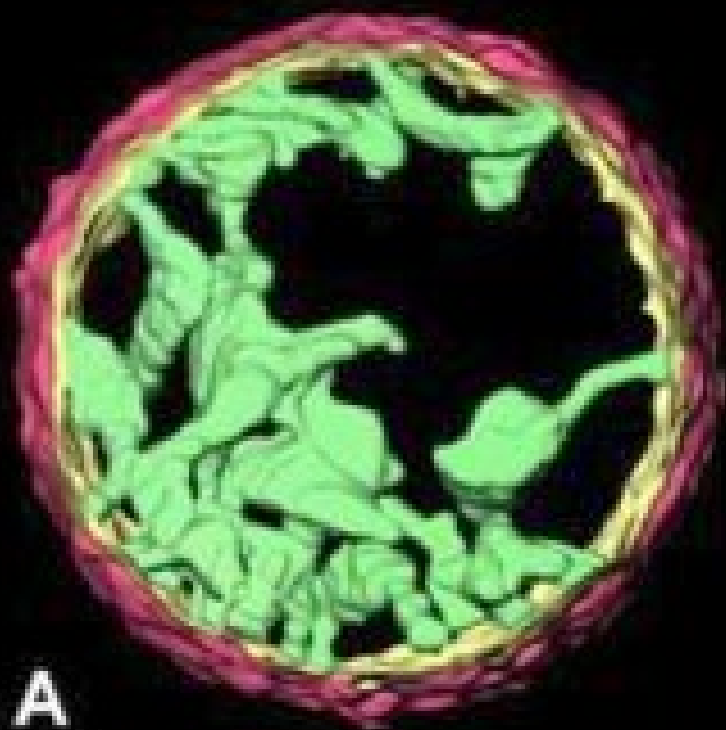






**Fig. 1.** Mitochondrial network in a mammalian fibroblast. A COS-7 cell labeled to visualize mitochondria (green) and microtubules (red) was analyzed by indirect immunofluorescence confocal microscopy. Mitochondria were labeled with antibodies to the  $\beta$  subunit of the  $F_1$ -ATPase and a rhodamine-conjugated secondary antibody. Microtubules were labeled with antibody to tubulin and a fluorescein-conjugated secondary antibody. Pseudocolor was added to the digitized image. Scale: 1 cm = 10  $\mu$ m.

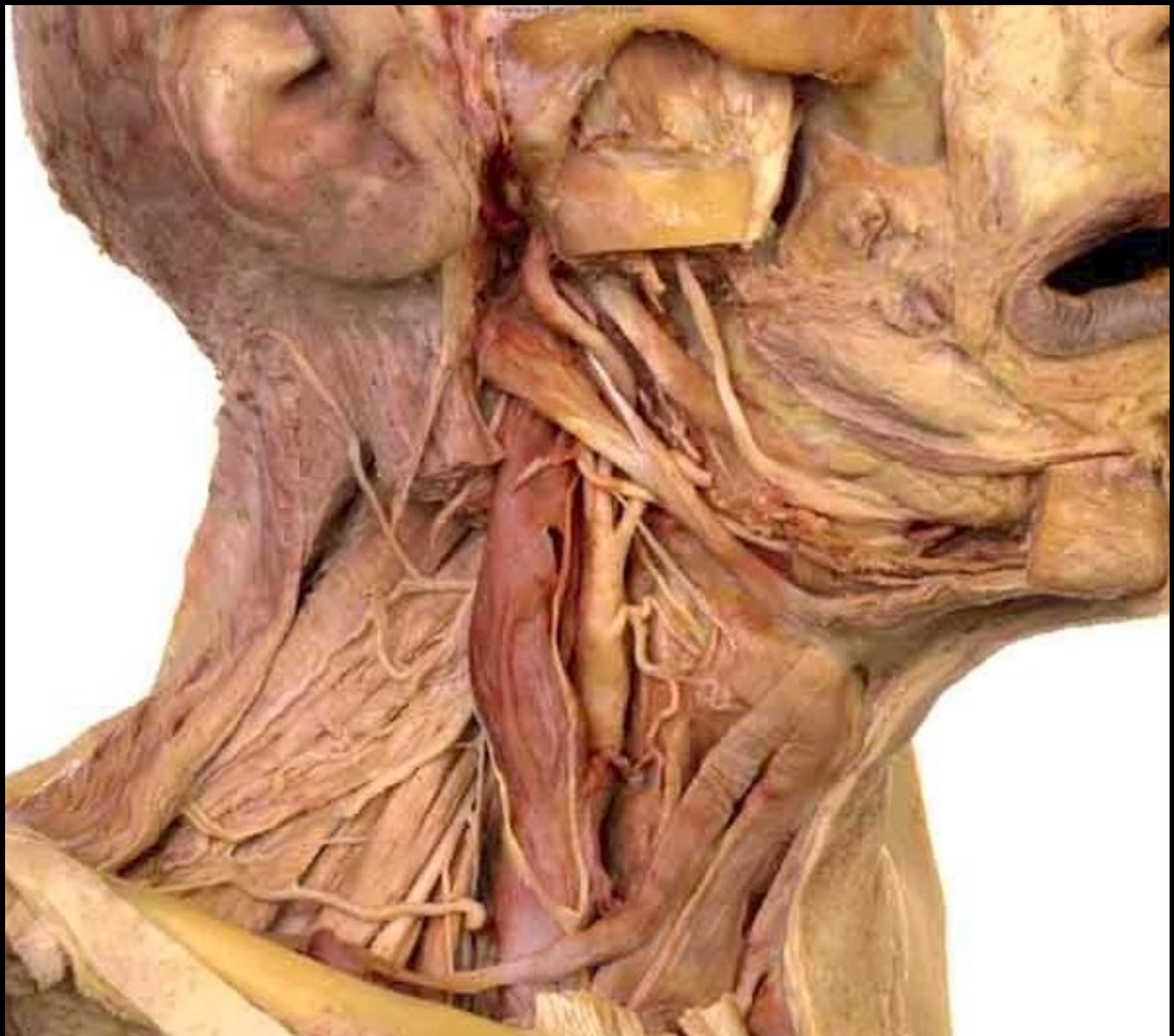
**From M. P. Yaffe, *Science*, 283, 1493 (1999).**

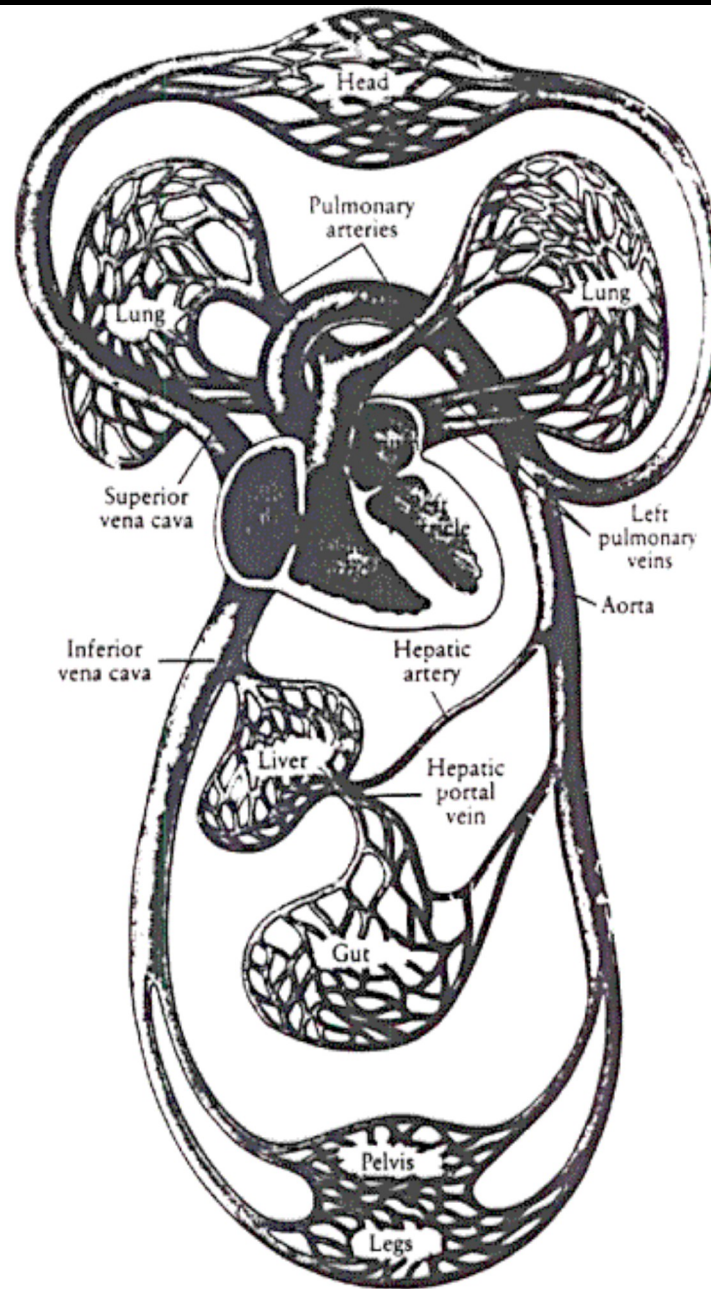


## FUNDAMENTAL PRINCIPLES

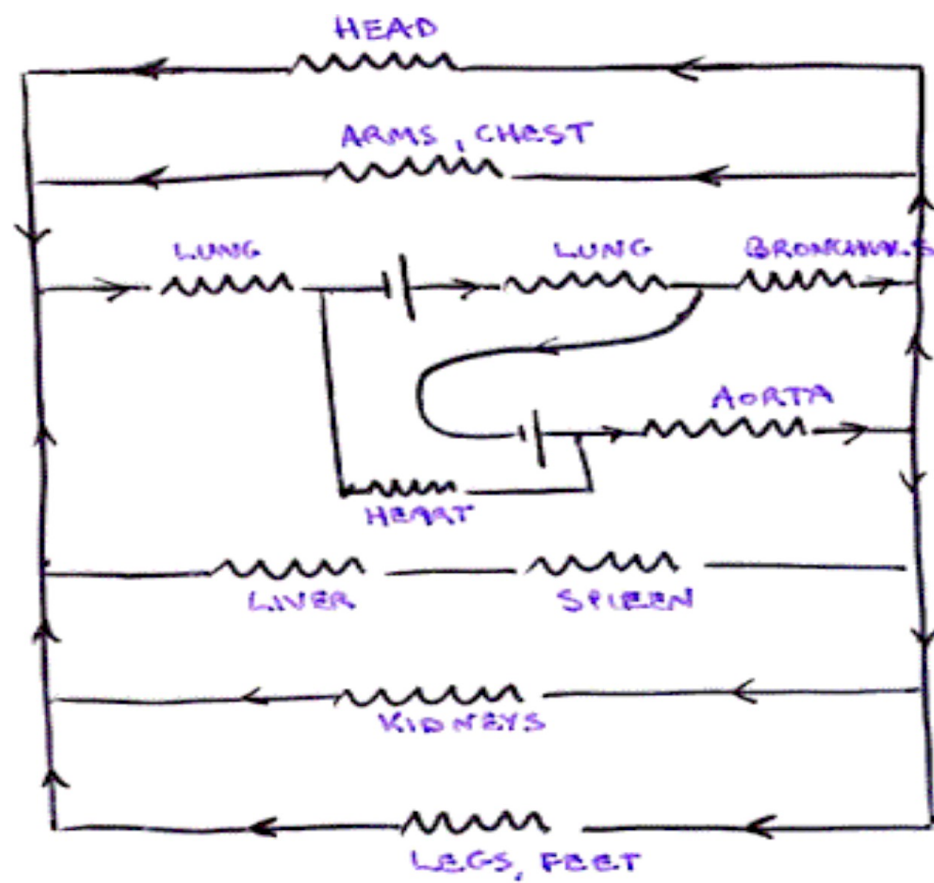
(NATURAL SELECTION)

- I. AT ALL SCALES ORGANISMS ARE SUSTAINED BY THE TRANSPORT OF ENERGY AND ESSENTIAL MATERIALS THROUGH HIERARCHICAL BRANCHING NETWORK SYSTEMS IN ORDER TO SUPPLY ALL LOCAL PARTS OF THE ORGANISM
- II. THESE NETWORKS ARE SPACE-FILLING
- III. THE TERMINAL BRANCHES OF THE NETWORK ARE INVARIANT UNITS
- IV. ORGANISMS HAVE EVOLVED BY NATURAL SELECTION SO AS TO
  - i) MINIMISE ENERGY DISSIPATED IN THE NETWORKS
  - AND/OR ii) MAXIMISE THE SCALING OF THEIR AREA OF INTERFACE WITH THEIR RESOURCE ENVIRONMENT





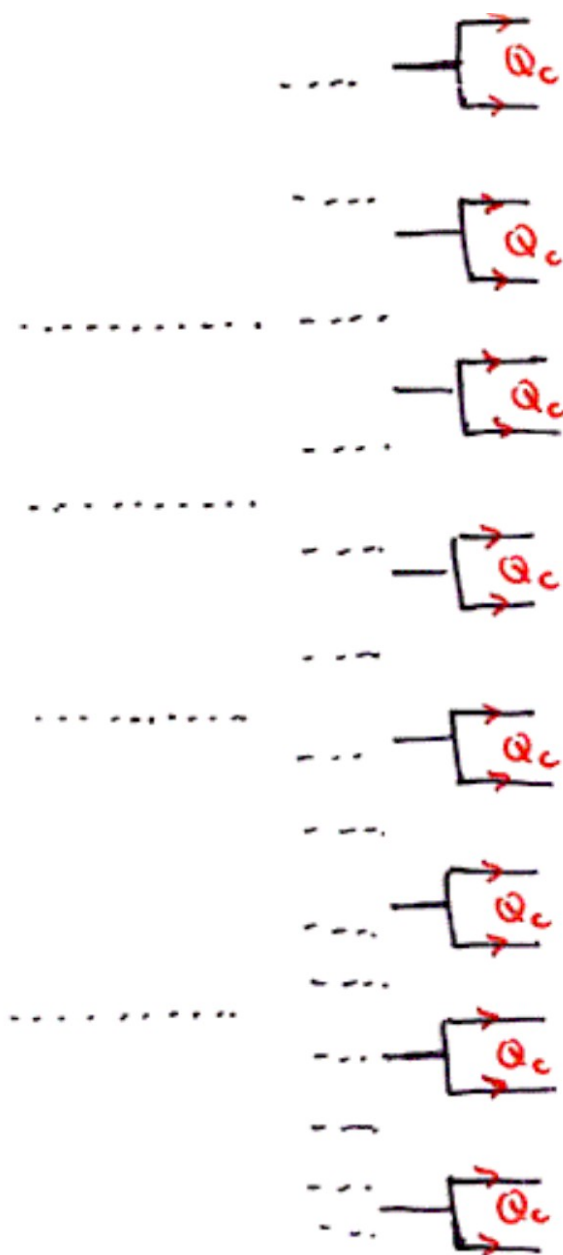
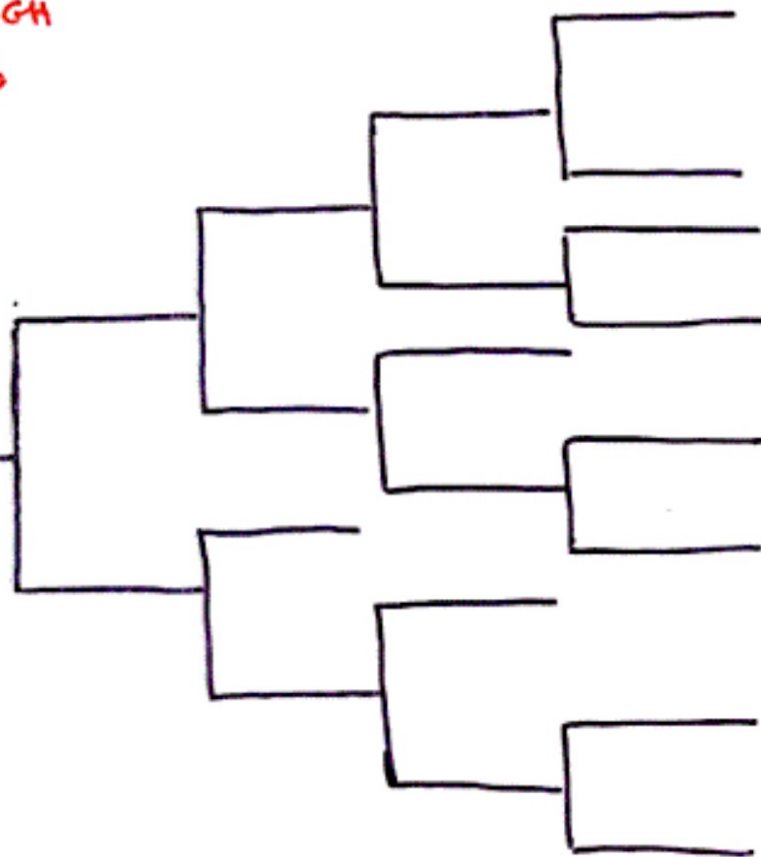
THE VASCULAR CIRCUITRY



FLUID FLOW THROUGH  
AORTA  $Q_0 \propto B$



AORTA  
TRUNK  
CEO



CAPILLARIES  
PETIOLES  
MITOCHONDRIA

SINCE THE FLUID (BLOOD) TRANSPORTS OXYGEN,  
NUTRIENTS, ETC FROM THE AORTA TO THE  
CAPILLARIES

METABOLIC RATE  $\propto$  VOLUME FLOW RATE

$$B \propto Q_0$$

BUT THE CONSERVATION OF FLUID (BLOOD)

$$\Rightarrow Q_0 = N_c Q_c$$

TOTAL NUMBER OF CAPILLARIES      VOLUME FLOW RATE IN AVERAGE CAPILLARY

CAPILLARY IS AN INVARIANT UNIT

( $Q_c$  IS SAME FOR ALL MAMMALS)

$\Rightarrow$  NUMBER OF CAPILLARIES ( $N_c$ ) MUST SCALE IN SAME  
WAY AS THE METABOLIC RATE ( $B \propto Q_0$ )

SO, IF  $B \sim M^{3/4}$  THEN

$$N_c \sim M^{3/4} \quad (\text{NOT } N_c \sim M)$$

TOTAL NUMBER OF CELLS

$$N_{\text{cell}} \sim M \quad (\text{LINEAR})$$

TOTAL NUMBER OF CAPILLARIES

$$N_c \sim M^{3/4}$$

MISMATCH !

⇒ NUMBER OF CELLS FED BY A SINGLE  
CAPILLARY INCREASES AS  $M^{1/4}$

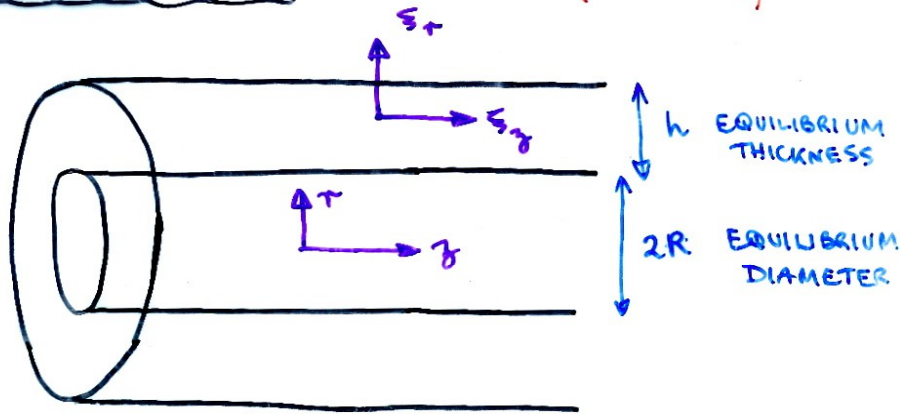
(ANOTHER MANIFESTATION THAT EFFICIENCY  
INCREASES WITH SIZE)

IMPORTANT IMPLICATIONS FOR GROWTH AND DEATH !

# 1) PULSATILE TREATMENT

(WOMERSLEY)

(19)



## a) FLUID :

STRESS TENSOR:  $\theta_{ij} = \lambda e_{kk} \delta_{ij} + 2\mu e_{ij} - p \delta_{ij}$

(NEWTONIAN)

COEFFICIENTS OF VISCOSITY

PRESSURE

STRAIN TENSOR:  $e_{ij} = \frac{1}{2} (\partial_i v_j + \partial_j v_i)$

VELOCITY

EQ<sup>n</sup>. OF MOTION:  $\rho \frac{Dv_i}{Dt} = \partial_j \theta_{ij}$

NAVIER-STOKES EQ<sup>n</sup>.

COVARIANT DERIVATIVE  $\frac{D}{Dt} = \frac{\partial}{\partial t} + v_i \partial_i$

EQ<sup>n</sup>. OF CONTINUITY:  $\frac{\partial \rho}{\partial t} + \partial_i (\rho v_i) = 0$

b) WALLS :

$$\theta_{ij}^w = \lambda e_{kk}^w \delta_{ij} + 2B e_{ij}^w - p \delta_{ij} \quad \text{HOOKE'S LAW}$$

ELASTIC MODULI

$$e_{ij}^w = \frac{1}{2} (\partial_i \xi_j + \partial_j \xi_i)$$

$$\rho_w \frac{Du_i}{Dt} = \partial_j \theta_{ij}^w \quad ; \quad u_i \equiv \frac{\partial \xi_i}{\partial t} \quad \text{NAVIER, EQ. 4.5.}$$

$$\partial_i \xi_i = 0$$

NEGLECT NON-LINEARITIES :

$$\rho_w \frac{\partial^2 \xi_i}{\partial t^2} = B \partial^2 \xi_i - \partial_i p$$

SOLVE USING FOURIER AS WITH FLUID, WALLS AND

FLUID COUPLED VIA BOUNDARY CONDITIONS : CONTINUITY

OF VELOCITY AND FORCE :  $u_r = \frac{\partial \xi_r}{\partial t}$

$$\text{AND } \int_{\text{SURFACE}} \theta_{ij} dS_j \quad \text{CONTINUOUS}$$

CAN BE SOLVED : BIG MESS!

SIMPLIFY USING THIN WALL APPROXIMATION

$$\text{i.e. } \frac{h}{R} \ll 1$$

IMPEDANCE :  $\bar{Z}(\omega) = \frac{\rho c_0^2}{\pi R^2 c} \frac{1}{J_0(ikR)}$

WHERE  $c = \frac{\omega}{k}$  PHASE VELOCITY

AND  $c_0 = \left( \frac{Bh}{2\rho R} \right)^{1/2}$

$B = \frac{E}{1 - \sigma^2}$

YOUNG'S MODULUS  
POISSON RATIO

[ KORTEWEG - MOENS VELOCITY,  
FIRST DERIVED BY YOUNG ]

TYPICALLY  $kR = \frac{2\pi R}{\lambda} \ll 1$  so

$\bar{Z}(\omega) \approx \frac{\rho c_0^2}{\pi R^2 c}$

WITH  $\left( \frac{c}{c_0} \right)^2 = - \frac{J_2(i^{3/2}\alpha)}{J_0(i^{3/2}\alpha)}$  DISPERSION RELATION

WHERE  $\alpha \equiv \left( \frac{\omega \rho}{\mu} \right)^{1/2} R$  DIMENSIONLESS WOMERSLEY NUMBER

$\Rightarrow$  ATTENUATION AND DISPERSION

ii) OPTIMISATION (ENERGY MINIMISATION)

(17)

EX: FOR GIVEN VOLUME OF BLOOD MINIMISE ENERGY

OUTPUT (THEREFORE MINIMISE RESISTANCE) SUBJECT

TO SPACE FILLING GEOMETRY, USE LAGRANGE

MULTIPLIERS AND CONSIDER:

$$F(r_k, l_k, n_k) = Z(r_k, l_k, n_k) + \lambda V_b(r_k, l_k, n_k) + \sum_{k=0}^N \lambda_k n_k^k l_k^3$$

DEMAND  $\frac{\partial F}{\partial r_k} = \frac{\partial F}{\partial l_k} = \frac{\partial F}{\partial n_k} = 0$

LAGRANGE EQ<sup>ns</sup>

USE:  $Z(r_k, l_k, n) = \sum_{k=0}^N \frac{8\mu l_k}{\pi r_k^4 n_k}$

$$V_b(r_k, l_k, n) = \sum_{k=0}^N \pi r_k^2 l_k n^k$$

e.g.  $\frac{\partial F}{\partial r_k} = 0 \Rightarrow r_k^6 n^{2k} = \text{CONSTANT}$

i.e.  $\beta_k = \frac{r_{k+1}}{r_k} = \frac{1}{n^{1/3}}$  INDP. OF  $k$  NOT  $\frac{1}{n^{1/2}}$  !

NOW VARY MASS : MINIMISE ENERGY LOSS ( $\dot{Q}_0$  CHANGES)

$$F(r_k, l_k, n, M) = \underset{\downarrow M^a}{\dot{E}(M)} + \lambda \left[ \sum_{k=0}^N \pi r_k^2 l_k n^k - \underset{\downarrow M^b}{V_b(M)} \right] \\ + \sum_{k=0}^N \lambda_k [n^k l_k^3 - \underset{\downarrow N_0 l_0^3 \sim M^a}{V_n(M)}] + \lambda_m M$$

FOR FIXED  $M$ , AS ABOVE ;

$$\frac{\partial F}{\partial M} \sim a M^{a-1} + \lambda b M^{b-1} + (\sum \lambda_k) a M^{a-1} + \lambda_m$$

$$= 0 \quad \Rightarrow \quad \underline{b=1}$$

$$\text{i.e.} \quad \underline{V_b \sim M}$$

IN  $d$  DIMENSIONS

$$B \propto M^{\frac{d}{d+1}}$$

WE LIVE IN 3 SPATIAL DIMENSIONS SO  $B \propto M^{3/4}$

⇒ "3" REPRESENTS DIMENSIONALITY OF SPACE

"4" INCREASE IN DIMENSIONALITY DUE TO  
FRACTAL-LIKE SPACE FILLING

LIFE HAS TAKEN ADVANTAGE OF THE POSSIBILITY OF  
USING SPACE-FILLING FRACTAL-LIKE SURFACES  
(WHERE ENERGY AND RESOURCES ARE EXCHANGED)

TO MAXIMISE ENERGY TRANSFER FROM THE  
ENVIRONMENT

NON-FRACTAL :

AREA

$M^{2/3}$

DIMENSIONALITY OF SPACE (VOLUME)

BIOLOGICAL (FRACTAL)

$$M^{3/4}$$

BY ANALOGY : LIFE EFFECTIVELY OPERATES IN  
FOUR SPATIAL DIMENSIONS

[FIVE IF TIME IS INCLUDED]

# Cardiovascular

Variable	Exponent	
	Predicted	Observed
Aorta radius $r_o$	$3/8 = 0.375$	0.36
Aorta pressure $\Delta p_o$	$0 = 0.00$	0.032
Aorta blood velocity $u_o$	$0 = 0.00$	0.07
Blood volume $V_b$	$1 = 1.00$	1.00
Circulation time	$1/4 = 0.25$	0.25
Circulation distance $l$	$1/4 = 0.25$	ND
Cardiac stroke volume	$1 = 1.00$	1.03
Cardiac frequency $\omega$	$-1/4 = -0.25$	-0.25
Cardiac output $\dot{E}$	$3/4 = 0.75$	0.74
Number of capillaries $N_c$	$3/4 = 0.75$	ND
Service volume radius	$1/12 = 0.083$	ND
Womersley number $\alpha$	$1/4 = 0.25$	0.25
Density of capillaries	$-1/12 = -0.083$	-0.095
O <sub>2</sub> affinity of blood $P_{50}$	$-1/12 = -0.083$	-0.089
Total resistance $Z$	$-3/4 = -0.75$	-0.76
Metabolic rate $B$	$3/4 = 0.75$	0.75

## Respiratory

Variable	Exponent	
	Predicted	Observed
Tracheal radius	$3/8 = 0.375$	0.39
Interpleural pressure	$0 = 0.00$	0.004
Air velocity in trachea	$0 = 0.00$	0.02
Lung volume	$1 = 1.00$	1.05
Volume flow to lung	$3/4 = 0.75$	0.80
Volume of alveolus $V_A$	$1/4 = 0.25$	ND
Tidal volume	$1 = 1.00$	1.041
Respiratory frequency	$-1/4 = -0.25$	-0.26
Power dissipated	$3/4 = 0.75$	0.78
Number of alveoli $N_A$	$3/4 = 0.75$	ND
Radius of alveolus $r_A$	$1/12 = 0.083$	0.13
Area of alveolus $A_A$	$1/6 = 0.083$	ND
Area of lung $A_L$	$11/12 = 0.92$	0.95
O <sub>2</sub> diffusing capacity	$1 = 1.00$	0.99
Total resistance	$-3/4 = -0.75$	-0.70
O <sub>2</sub> consumption rate	$3/4 = 0.75$	0.76

**Table 1 Predicted values of scaling exponents for physiological and anatomical variables of plant vascular systems.**


Variable	Plant mass		Branch radius		
	Exponent predicted	Symbol	Symbol	Exponent	
				Predicted	Observed
Number of leaves	$\frac{3}{4}$ (0.75)	$n_0^L$	$n_k^L$	2 (2.00)	2.007 (ref. 12)
Number of branches	$\frac{3}{4}$ (0.75)	$N_0$	$N_k$	-2 (-2.00)	-2.00 (ref. 6)
Number of tubes	$\frac{3}{4}$ (0.75)	$n_0$	$n_k$	2 (2.00)	n.d.
Branch length	$\frac{1}{4}$ (0.25)	$l_0$	$l_k$	$\frac{2}{3}$ (0.67)	0.652 (ref. 6)
Branch radius	$\frac{3}{8}$ (0.375)	$r_0$			
Area of conductive tissue	$\frac{7}{8}$ (0.875)	$A_0^{CT}$	$A_k^{CT}$	$\frac{7}{3}$ (2.33)	2.13 (ref. 8)
Tube radius	$\frac{1}{16}$ (0.0625)	$a_0$	$a_k$	$\frac{1}{8}$ (0.167)	n.d.
Conductivity	1 (1.00)	$K_0$	$K_k$	$\frac{8}{3}$ (2.67)	2.63 (ref. 12)
Leaf-specific conductivity	$\frac{1}{4}$ (0.25)	$L_0$	$L_k$	$\frac{2}{3}$ (0.67)	0.727 (ref. 17)
Fluid flow rate			$\dot{Q}_k$	2 (2.00)	n.d.
Metabolic rate	$\frac{3}{4}$ (0.75)	$\dot{Q}_0$			
Pressure gradient	$-\frac{1}{4}$ (-0.25)	$\Delta P_0/l_0$	$\Delta P_k/l_k$	$-\frac{2}{3}$ (-0.67)	n.d.
Fluid velocity	$-\frac{1}{8}$ (-0.125)	$u_0$	$u_k$	$-\frac{1}{3}$ (-0.33)	n.d.
Branch resistance	$-\frac{3}{4}$ (-0.75)	$Z_0$	$Z_k$	$-\frac{1}{3}$ (-0.33)	n.d.
Tree height	$\frac{1}{4}$ (0.25)	$h$			
Reproductive biomass	$\frac{3}{4}$ (0.75)				
Total fluid volume	$\frac{26}{24}$ (1.0415)				

## PLANTS

**VERY DIFFERENT  
EVOLVED  
ENGINEERING  
DESIGN (NON-  
PULSATILE FIBRE  
BUNDLES) BUT  
SAME NETWORK  
PRINCIPLES**

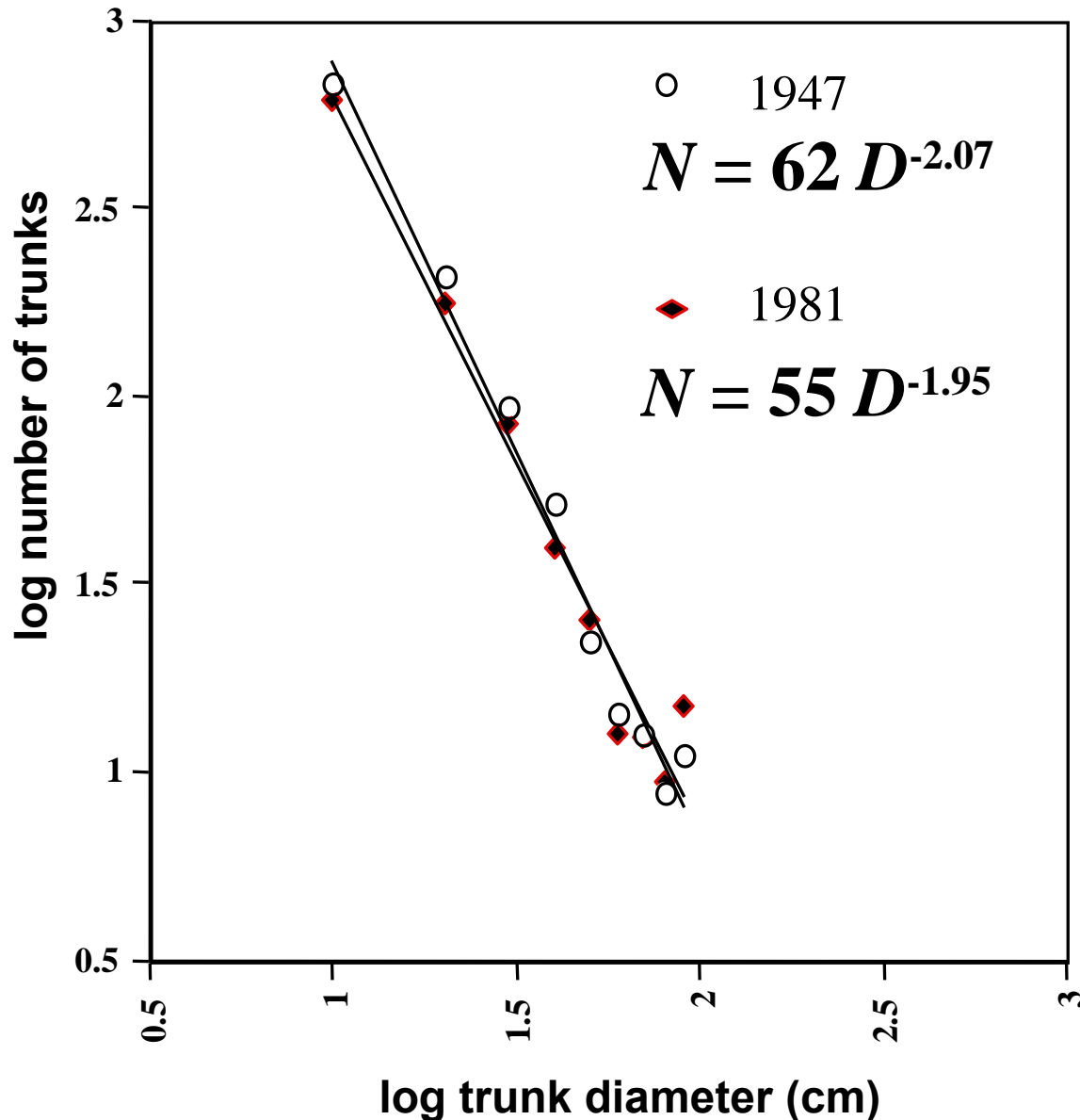
Table 1. Similarity of predicted scaling relations for branches within a tree [quantities denoted by uppercase symbols and subscripts  $i$  (20)], and for trees within a forest (denoted by lowercase symbols and subscripts  $k$ )\*

Scaling quantity	Individual tree	Entire forest
Area preserving	$\frac{R_{i+1}}{R_i} = \frac{1}{n^{1/2}}$	$\frac{r_{k+1}}{r_k} = \frac{1}{\lambda^{1/2}}$
Space filling	$\frac{L_{i+1}}{L_i} = \frac{1}{n^{1/3}}$	$\frac{l_{k+1}}{l_k} = \frac{1}{\lambda^{1/3}}$
Biomechanics	$R_i^2 = L_i^3$	$r_k^2 = l_k^3$
Size distribution*	$\Delta N_i \propto R_i^{-2} \propto M_i^{-3/4}$	$\Delta n_k \propto r_k^{-2} \propto m_k^{-3/4}$
Energy and material flux*	$B_i \propto R_i^2 \propto N_i^L \propto M_i^{3/4}$	$B_k \propto r_k^2 \propto n_k^L \propto m_k^{3/4}$

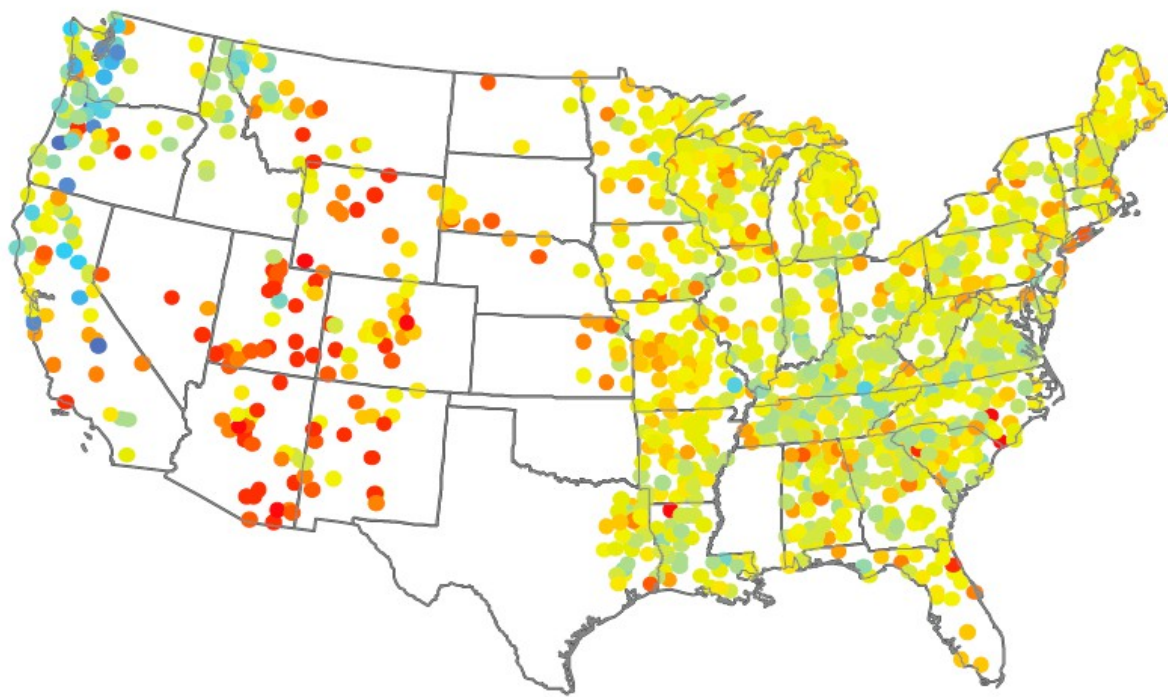
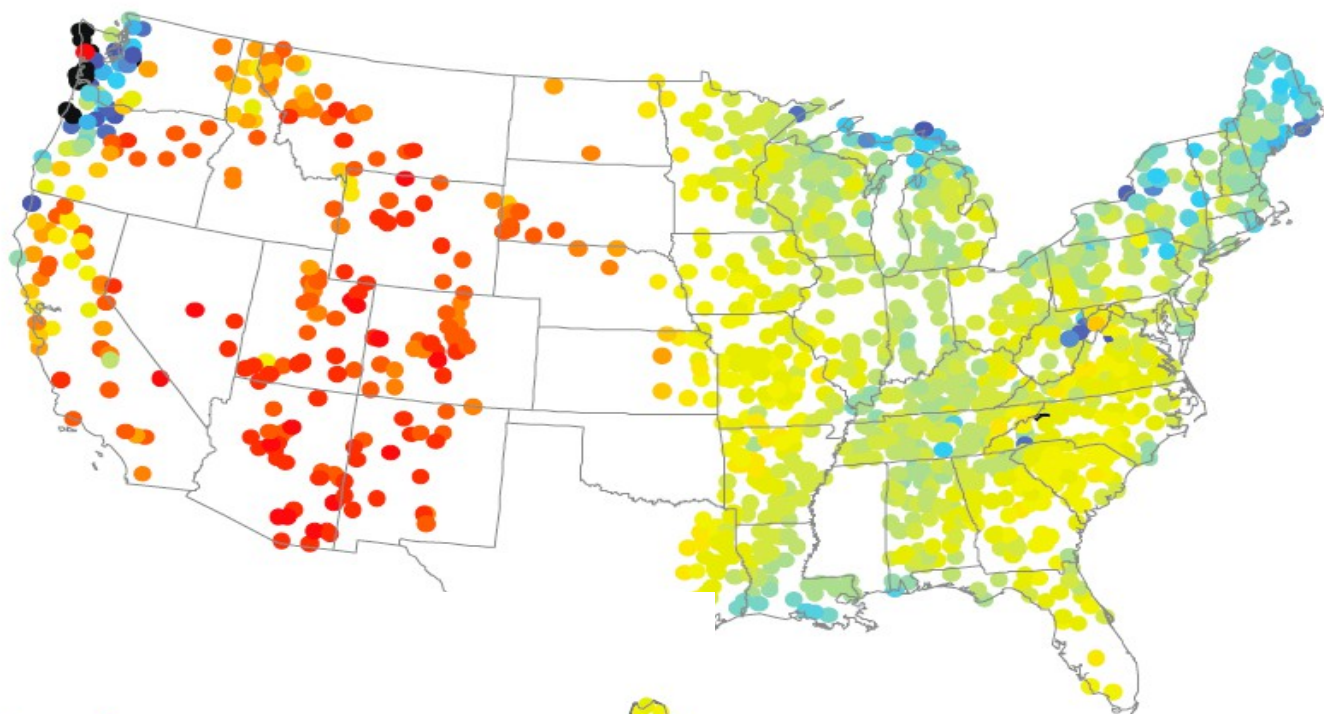
Stand property	Predicted stem radius,  based scaling function
Size class neighbor separation	$d_k \propto r_k$
Canopy scaling	$r_k^{\text{can}} \propto r_k^{2/3}$
Canopy spacing	$d_k^{\text{can}} = c_1 r_k \left[ 1 - \left( \frac{r_k}{r_{\bar{k}}} \right)^{1/3} \right]$
Energy Equivalence	$\Delta n_k B_k \propto r_k^0$
Total forest resource use	$B_{\text{Tot}} \propto \sum \Delta n_k r_k^2 \leq \dot{R}$
Mortality rate	$\mu_k \approx A r_k^{-2/3}$
Size distribution	$N_k \approx \frac{\dot{R}}{(K+1)b_0} r_k^{-2}$

# INTERSPECIFIC SIZE DISTRIBUTION

## All species in a Malaysian Rainforest



Manokaran and  
Kochummen (1987)



## HYDRODYNAMIC RESISTANCE OF THE NETWORK

$$\sim \frac{1}{M^{3/4}}$$

TOTAL RESISTANCE DECREASES WITH SIZE !!

SMALL MAY BE BEAUTIFUL BUT LARGE IS  
MORE EFFICIENT !!

BLOOD PRESSURE  $\sim M^0$

AORTA BLOOD VELOCITY  $\sim M^0$

} INVARIANT !

RADIUS OF A WHALE'S AORTA  $\sim 30$  cm

RADIUS OF A SHREW'S AORTA  $\sim \frac{1}{10}$  mm

THIS DECREASE OF  $B_c$  WITH SIZE IS DRIVEN

BY THE HEGEMONY OF THE NETWORK

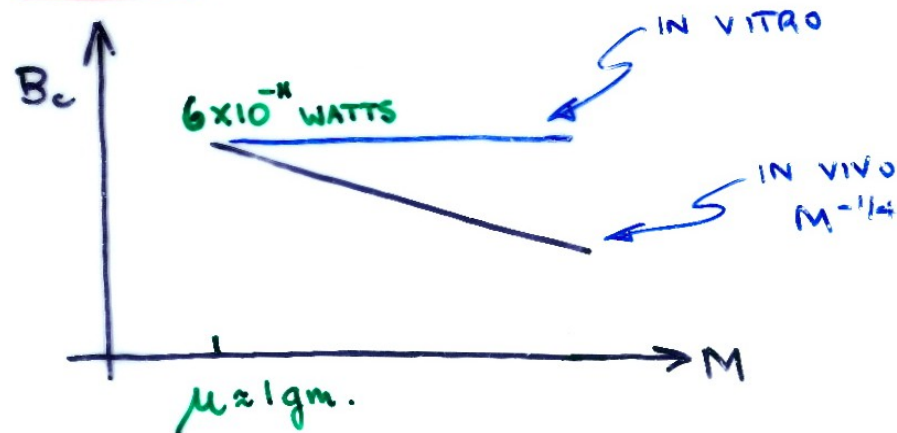
(CONTROLS FUNDAMENTAL BIOCHEMICAL RATES)

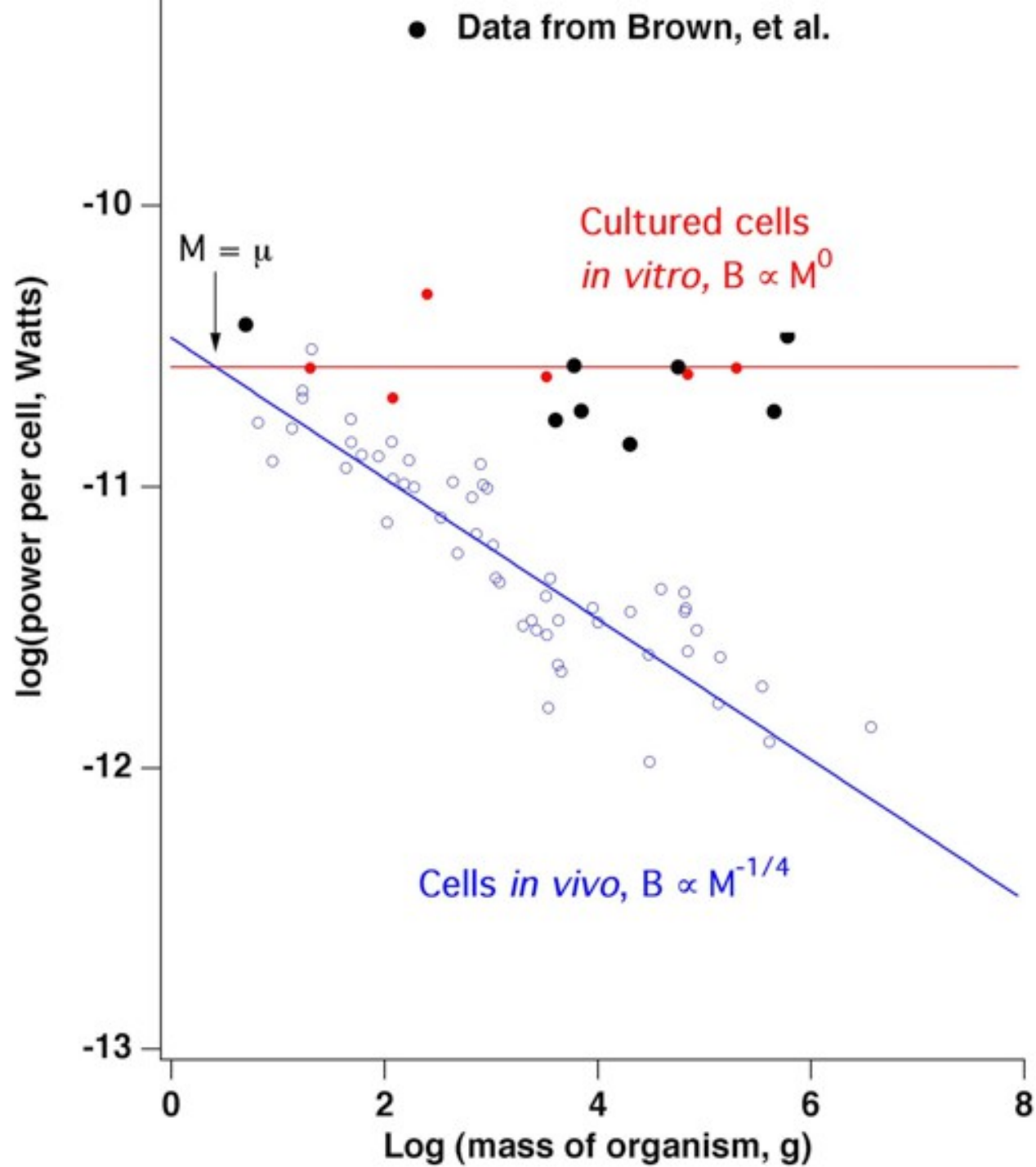
⇒ IF THE NETWORK WERE REMOVED SO CELLS

BECOME FREE (IN VITRO)  $B_c$  SHOULD BECOME

INDEPENDENT OF WHAT MAMMAL THEY ORIGINATED

IN: PREDICT





***NETWORK GEOMETRY AND DYNAMICS  
CONTROLS THE PACE OF LIFE AT ALL  
SCALES LEADING TO AN EMERGENT  
“UNIVERSAL” TIME SCALE***

$$B_{cell} \propto \frac{B}{M} = B_0 M^{21/4}$$

***THE PACE OF LIFE SYSTEMATICALLY  
SLOWS WITH INCREASING SIZE***

***ALL RATES ~  $M^{-1/4}$***

***METABOLISM***

***GROWTH***

***EVOLUTION***

***LONGEVITY***

***DIFFUSION***

***FLUXES***

***.....***

***ALL TIMES ~  $M^{1/4}$***

***LIFESPANS***

***TURNOVER TIMES***

***TIMES TO MATURITY***

***CIRCULATION TIMES***

***.....***

# TEMPERATURE DEPENDENCE

**METABOLIC RATE IS THE SUM OF ALL CONTRIBUTING REACTION SUB-PROCESSES (IN PARALLEL):**

$$B = \sum_i P_i$$

**$P_i \sim (\text{CONCENTRATIONS}) \times (\text{FLUXES}) \times (\text{KINETICS})$**

**$(\text{CONCENTRATIONS}) \times (\text{FLUXES}) \sim \text{NETWORK} \sim M^{3/4}$**

**$(\text{KINETICS}) \sim \text{BOLTZMANN - ARRENIUS} \sim e^{-E/kT}$**

**E = AVERAGE ACTIVATION ENERGY FOR RATE-LIMITING  
PROCESS IN RESPIRATORY COMPLEX  
(PRODUCTION OF ATP)  $\sim 0.7 \text{ eV} \sim 2 \times 10^{-20} \text{ cal}$**

***ALL RATES ~  $M^{-1/4}$***

***METABOLISM***

***GROWTH***

***EVOLUTION***

***LONGEVITY***

***DIFFUSION***

***FLUXES***

***.....***

***ALL TIMES ~  $M^{1/4}$***

***LIFESPANS***

***TURNOVER TIMES***

***TIMES TO MATURITY***

***CIRCULATION TIMES***

***.....***

# ***TEMPERATURE***

## ***REACTION RATES GOVERNED BY STATISICAL PHYSICS (BOLTZMANN-ARRENIHIUS)***

***ALL RATES  $\sim M^{-1/4}e^{-E/kT}$***     ***ALL TIMES  $\sim M^{1/4}e^{E/kT}$***

***METABOLISM***

***GROWTH***

***EVOLUTION***

***LONGEVITY***

***DIFFUSION***

***FLUXES***

***.....***

***LIFESPANS***

***TURNOVER TIMES***

***TIMES TO MATURITY***

***CIRCULATION TIMES***

***.....***

**MASS AND TEMPERATURE ARE THE MAJOR  
DETERMINANTS OF THE MEASURABLE TRAITS OF  
ORGANISMS**

**IF THE MASS AND TEMPERATURE DEPENDENCIES ARE  
ACCOUNTED FOR THEN:**

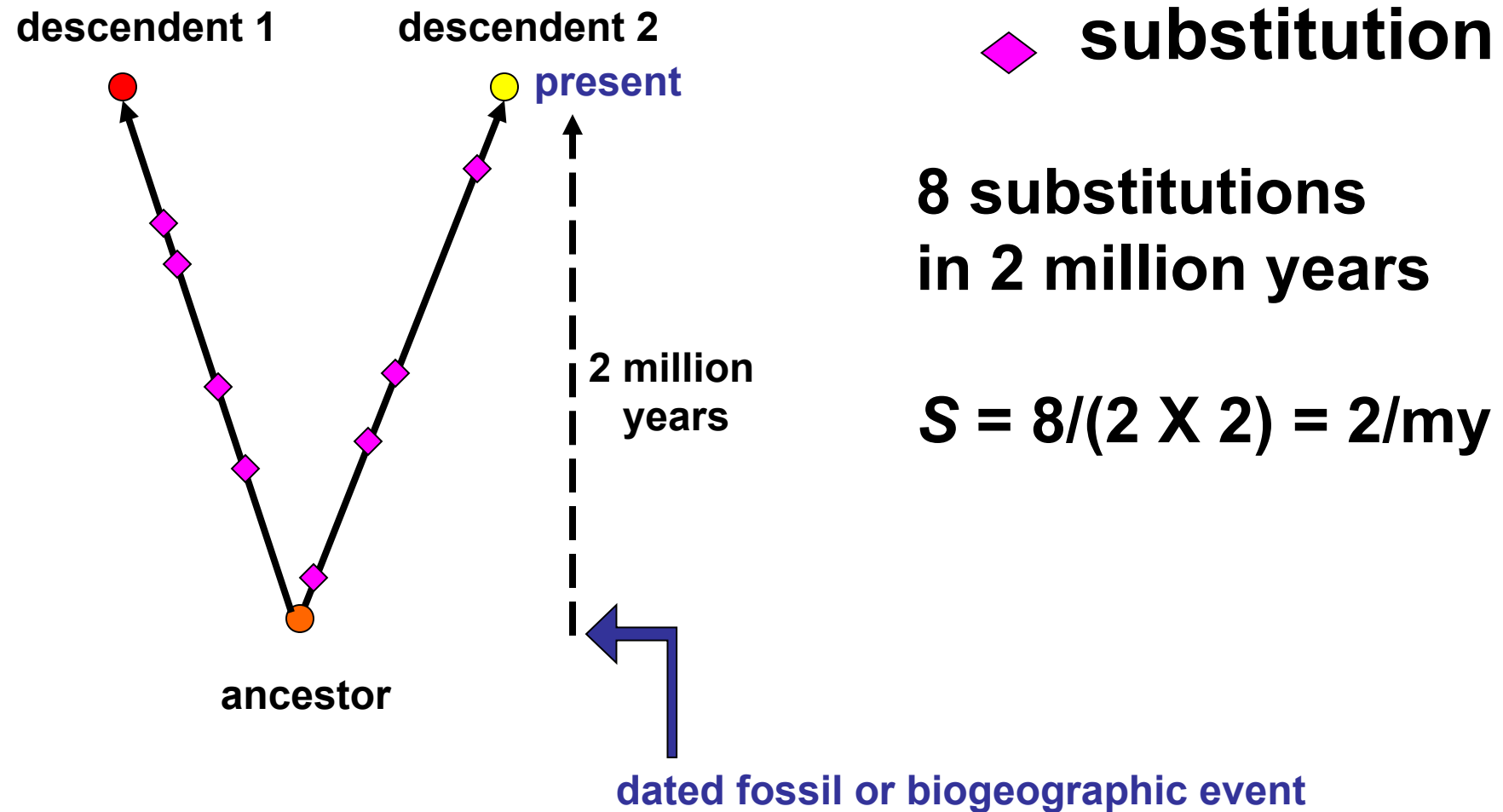
$$(TIMES) \times M^{-1/4} e^{-E/kT}$$

$$(RATES) \times M^{1/4} e^{E/kT}$$

**ARE INVARIANT, IMPLYING A “UNIVERSAL” RATE OF  
LIVING, DYING, GROWING, REPRODUCING, EVOLVING,  
.....GOVERNED BY JUST TWO PARAMETERS:**

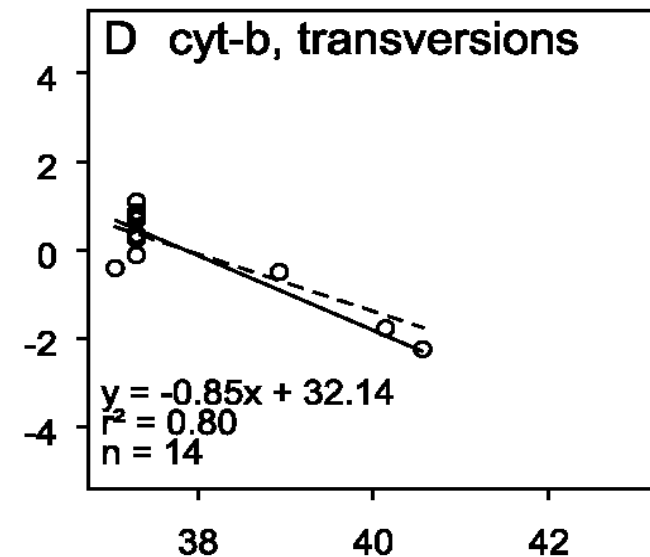
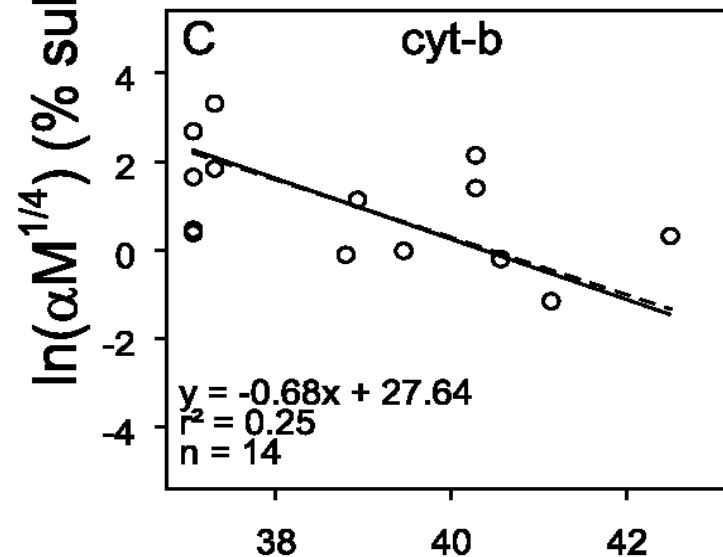
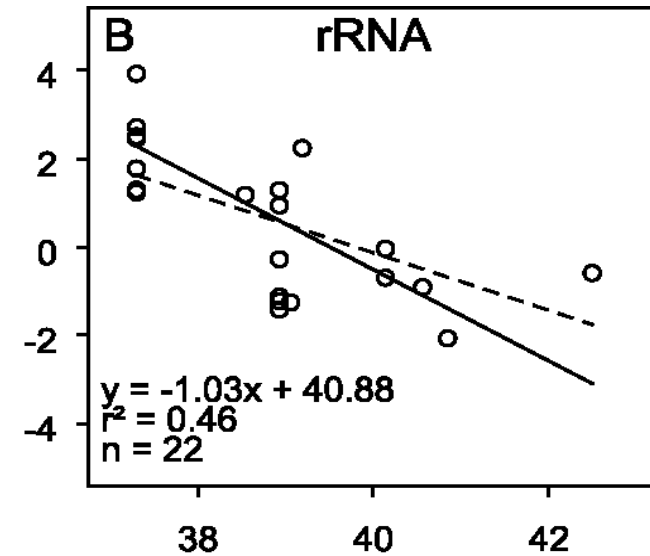
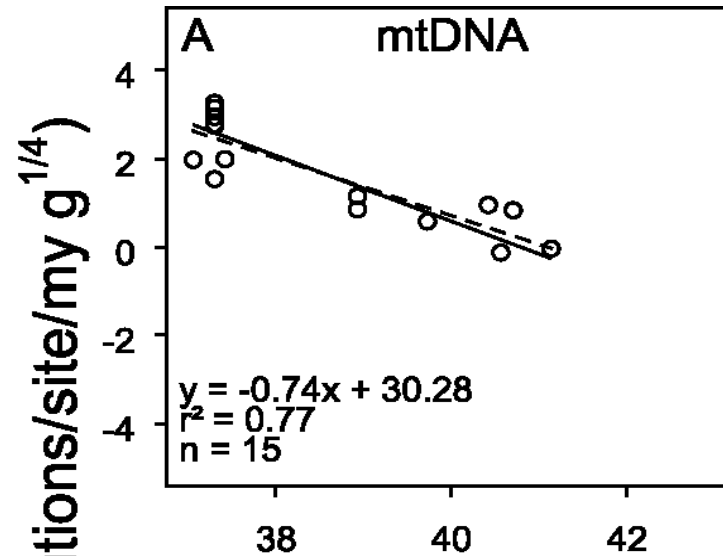
$$1/4 \text{ AND } E \sim 0.7 \text{ ev}$$

# Evolution: measuring nucleotide substitution rate (S)



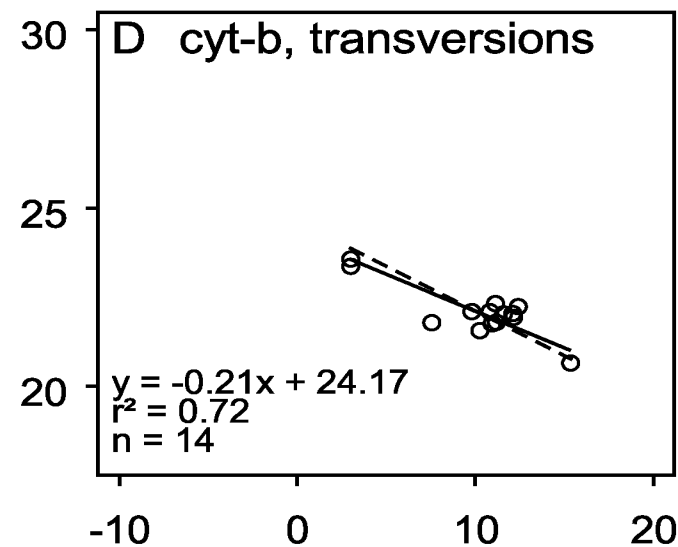
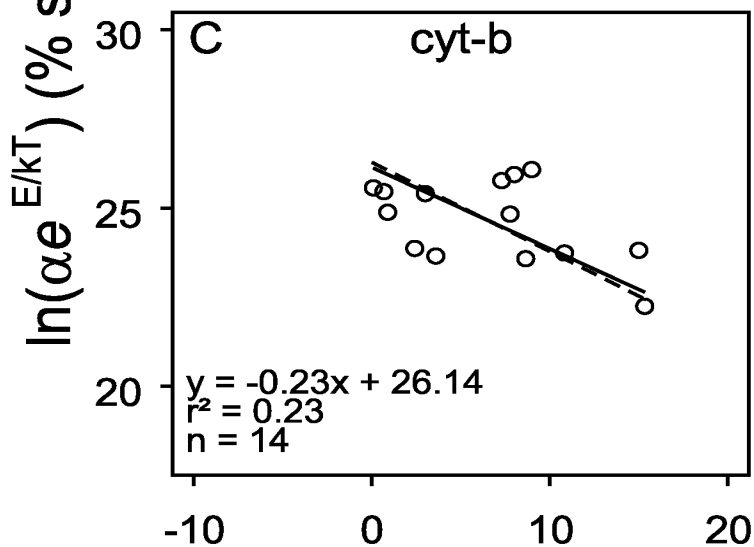
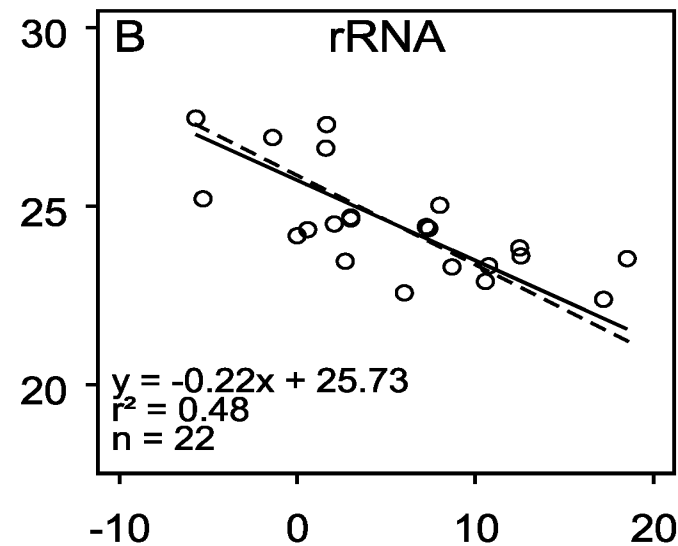
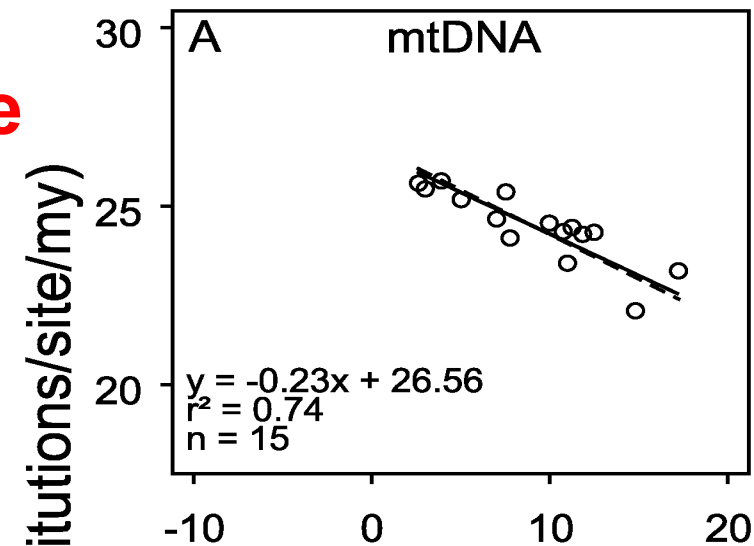
# Rates of molecular evolution

temperature  
dependence

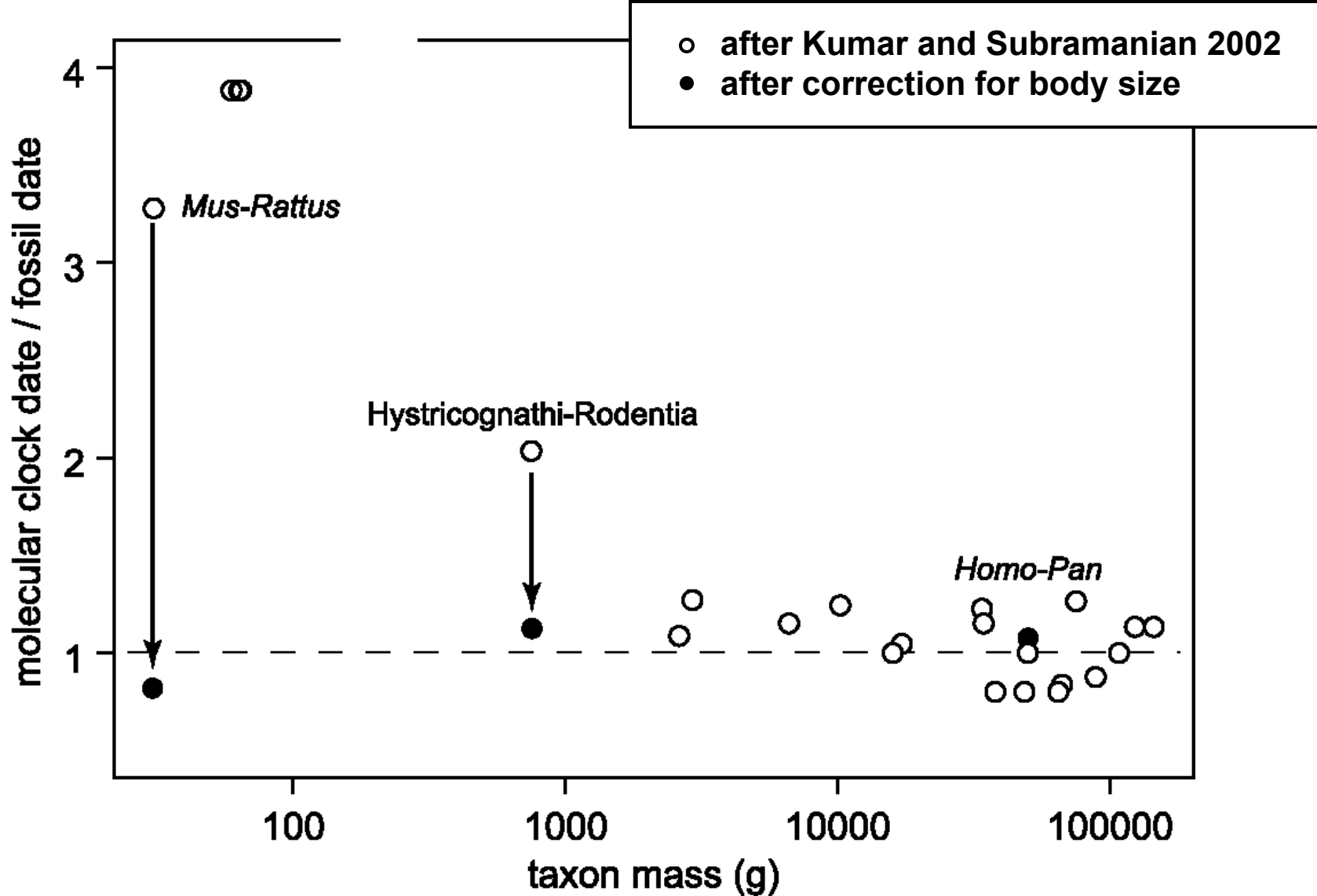


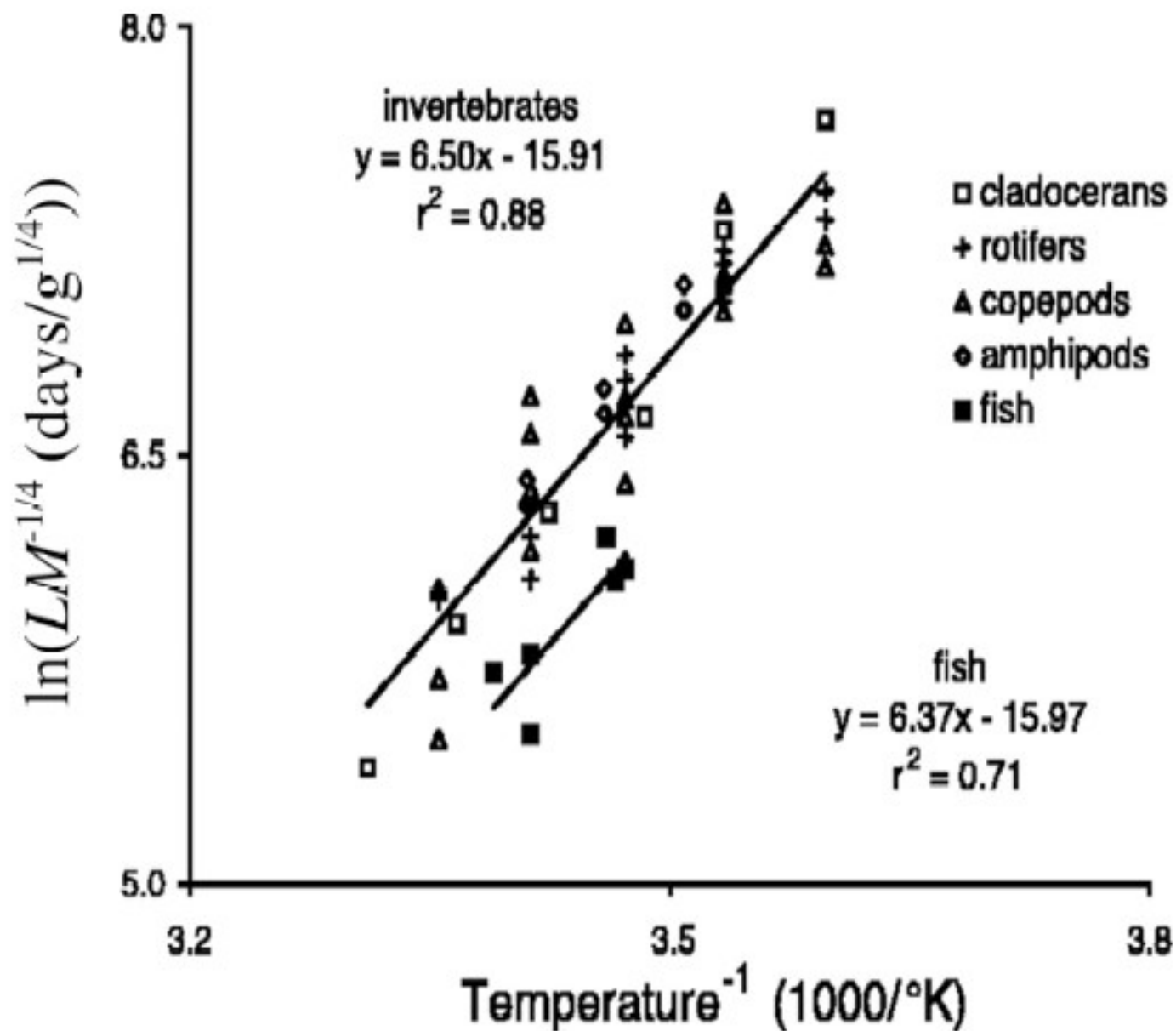
# Rates of molecular evolution

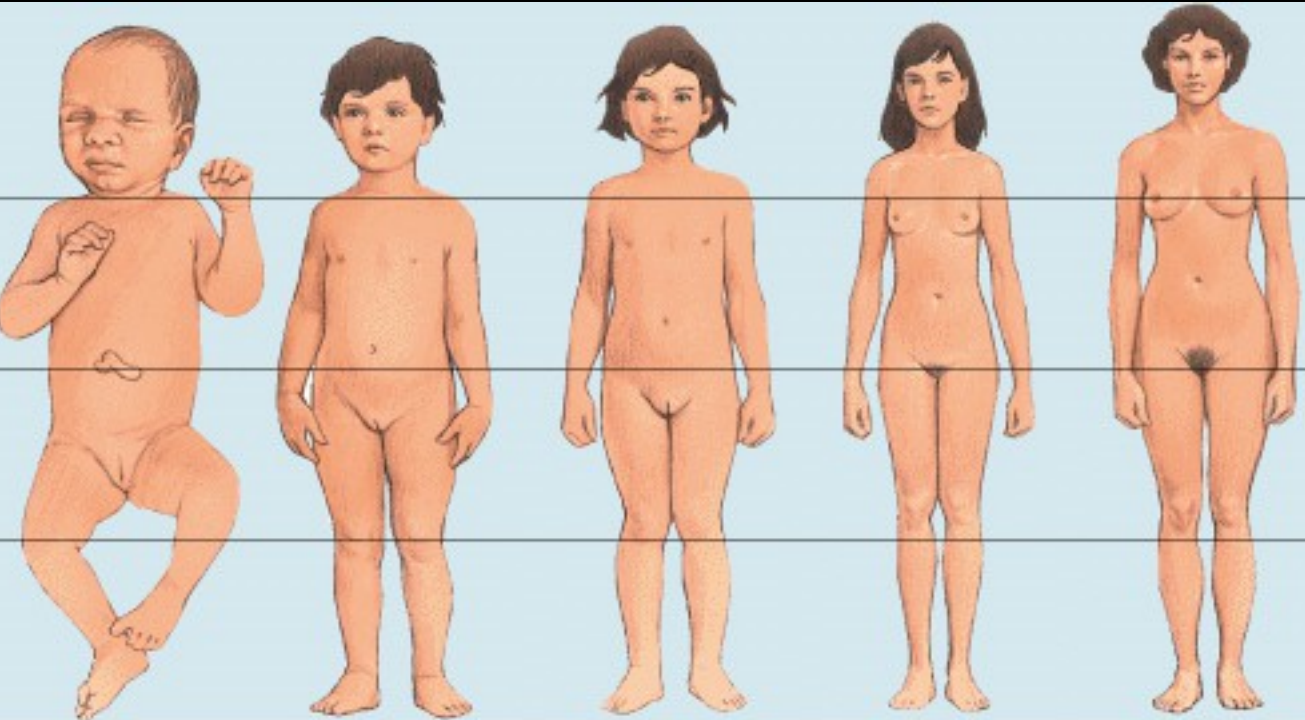
body size  
dependence



# Rates of molecular evolution: size correction reconciles molecular clock with fossil dates







Newborn

2 years

5 years

15 years

Adult

# GROWTH

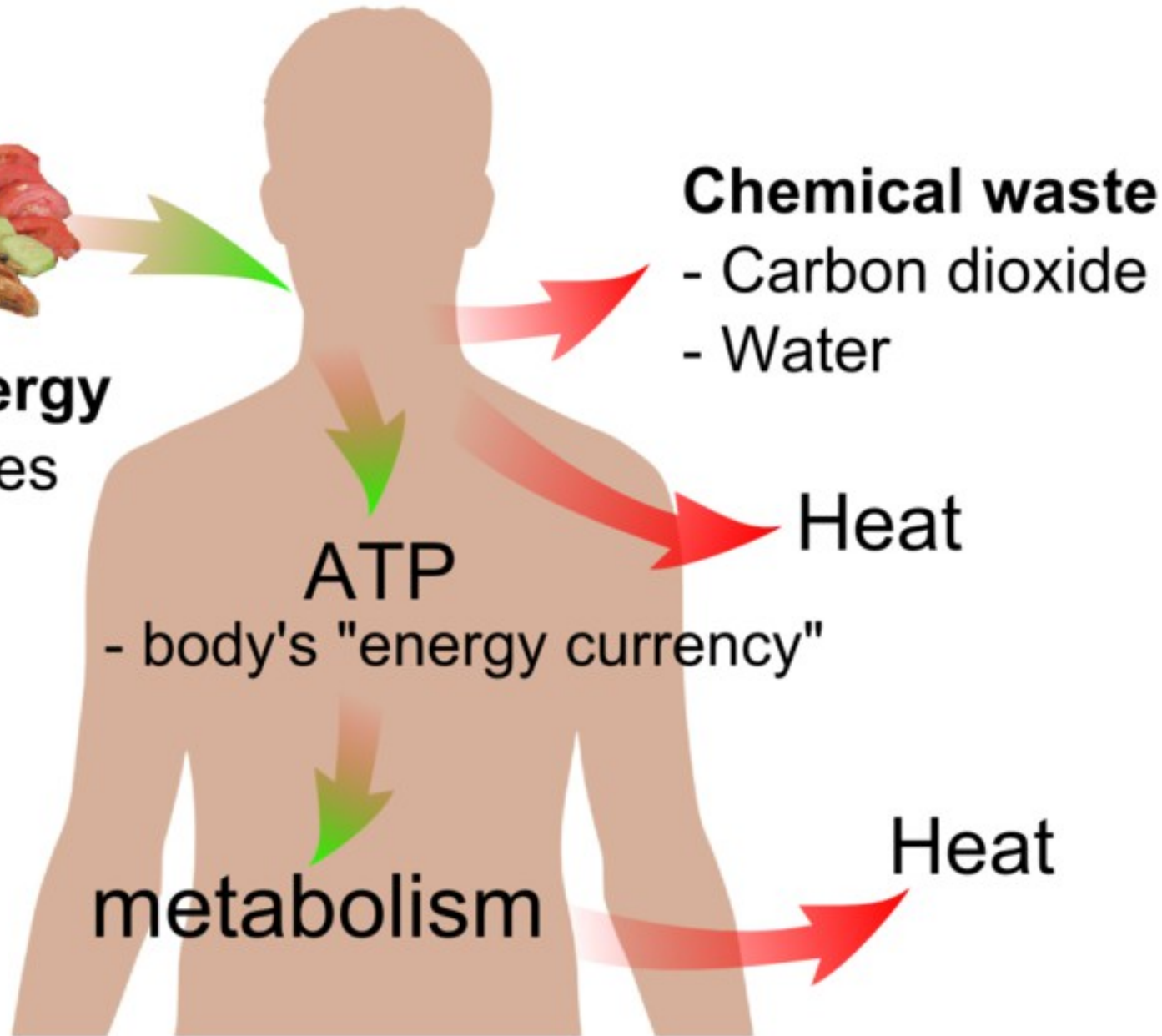


# Energy and human life



## **Chemical energy**

- Carbohydrates
- Fats
- Others



## **Chemical waste**

- Carbon dioxide
- Water

**ATP**

- body's "energy currency"

**Heat**

**metabolism**

**Heat**



# *Growth*

Incoming  
Metabolized Energy



Maintenance  
(of Existing Cells)



New Growth  
(of New Cells)

$$B = N_{cells} B_{cell} + E_{cell} \frac{dN_{cell}}{dt}$$

**IN TERMS OF MASS AT AGE  $t$**

$$\square \quad \frac{dm}{dt} = am^{3/4} - bm$$

where

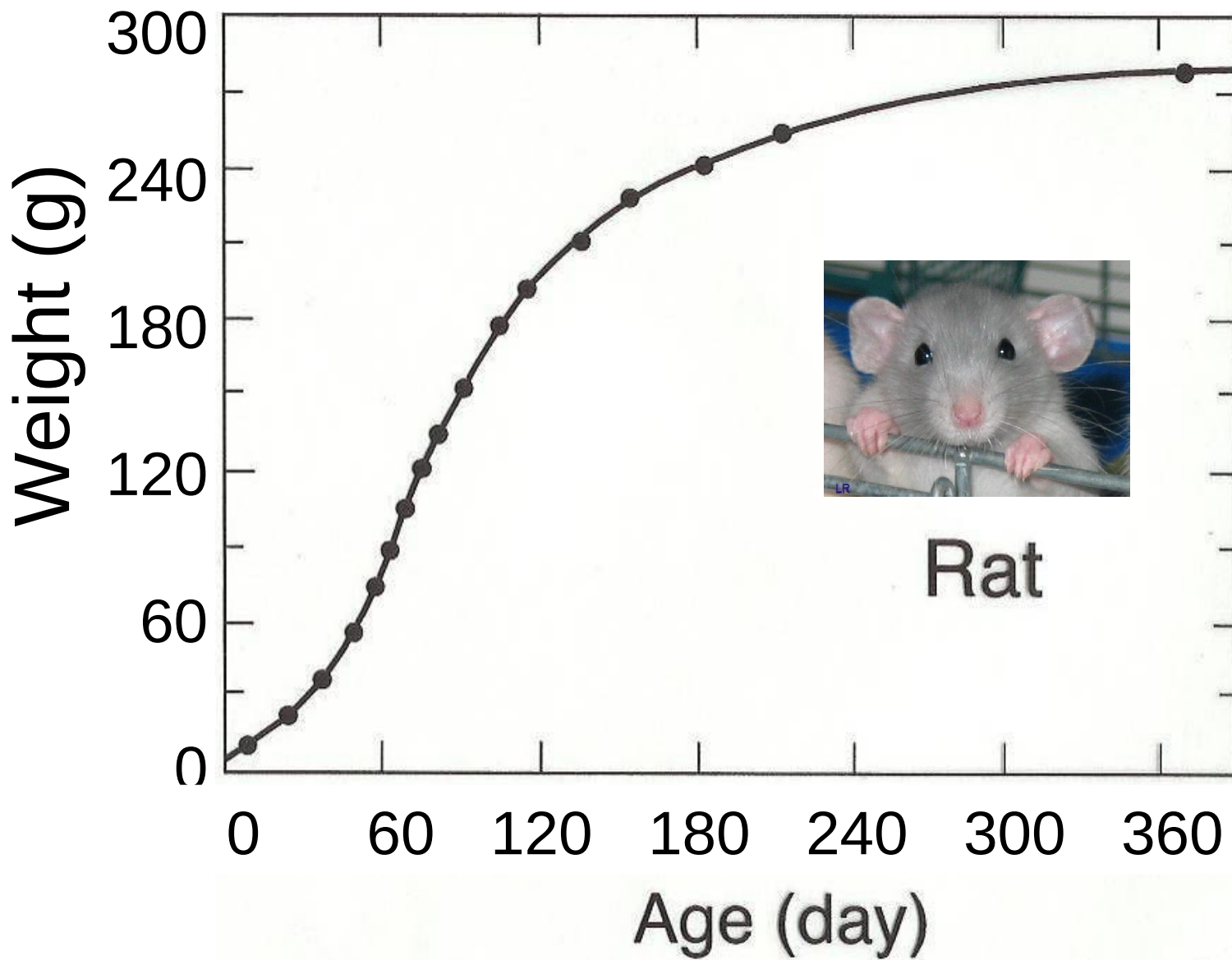
$$a \vartheta \frac{B_0 m_c}{E_c}$$

$$b \vartheta \frac{B_c}{E_c}$$

SOLUTION:

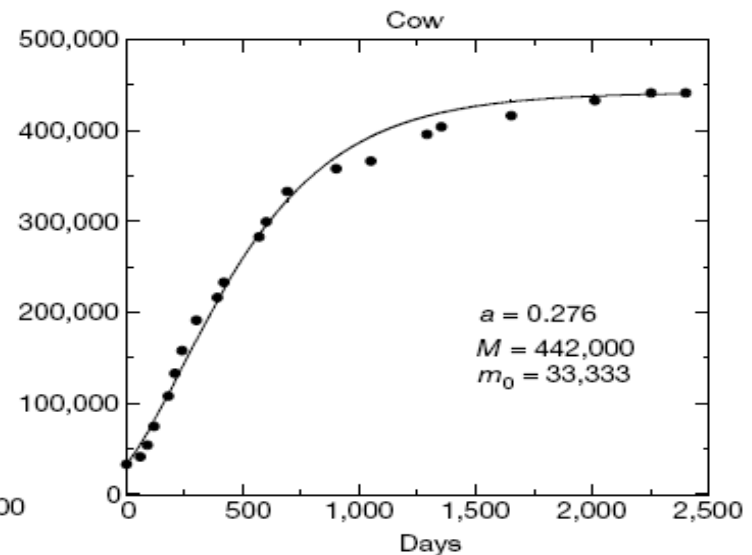
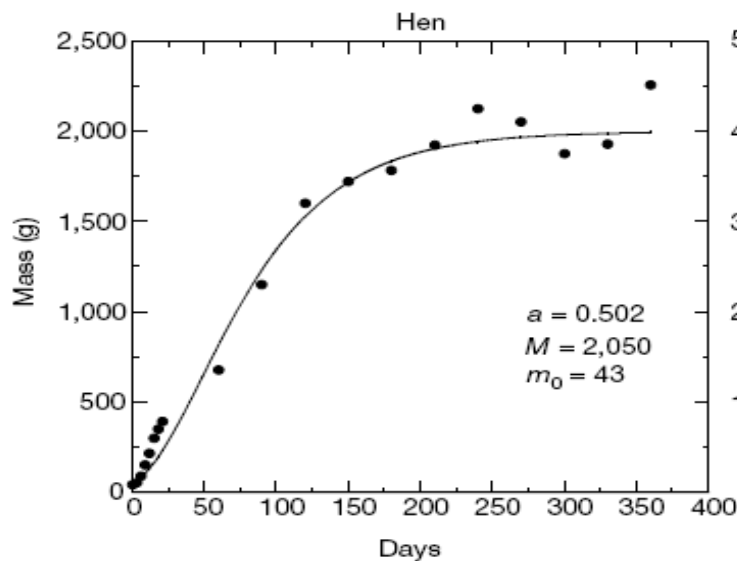
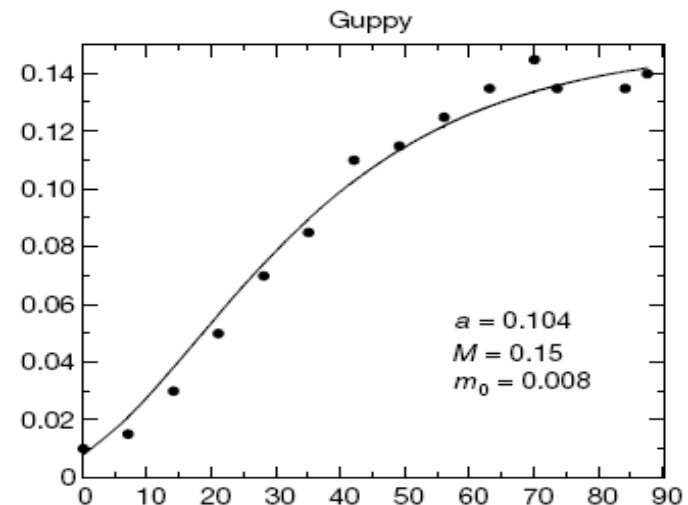
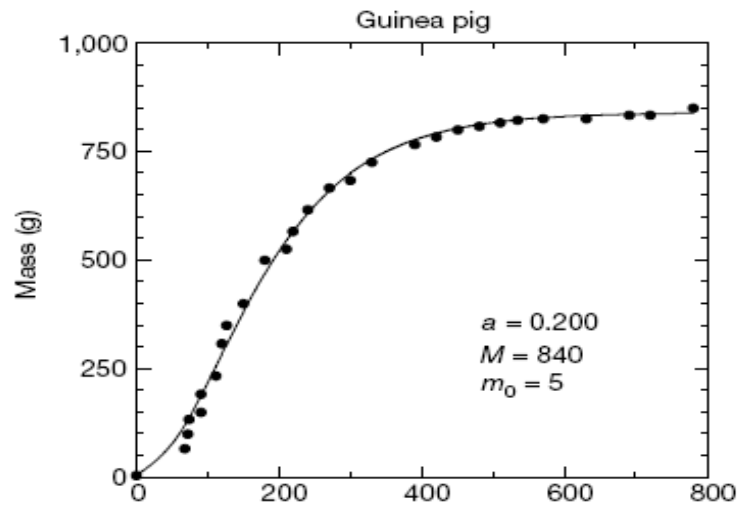
$$\left(\frac{m}{M}\right)^{1/4} = 1 - \left[1 - \left(\frac{M_0}{M}\right)^{1/4}\right] e^{-at/4M^{1/4}}$$

WHERE  $M_0$  = MASS AT BIRTH ( $m = M_0$  WHEN  $t = 0$ )



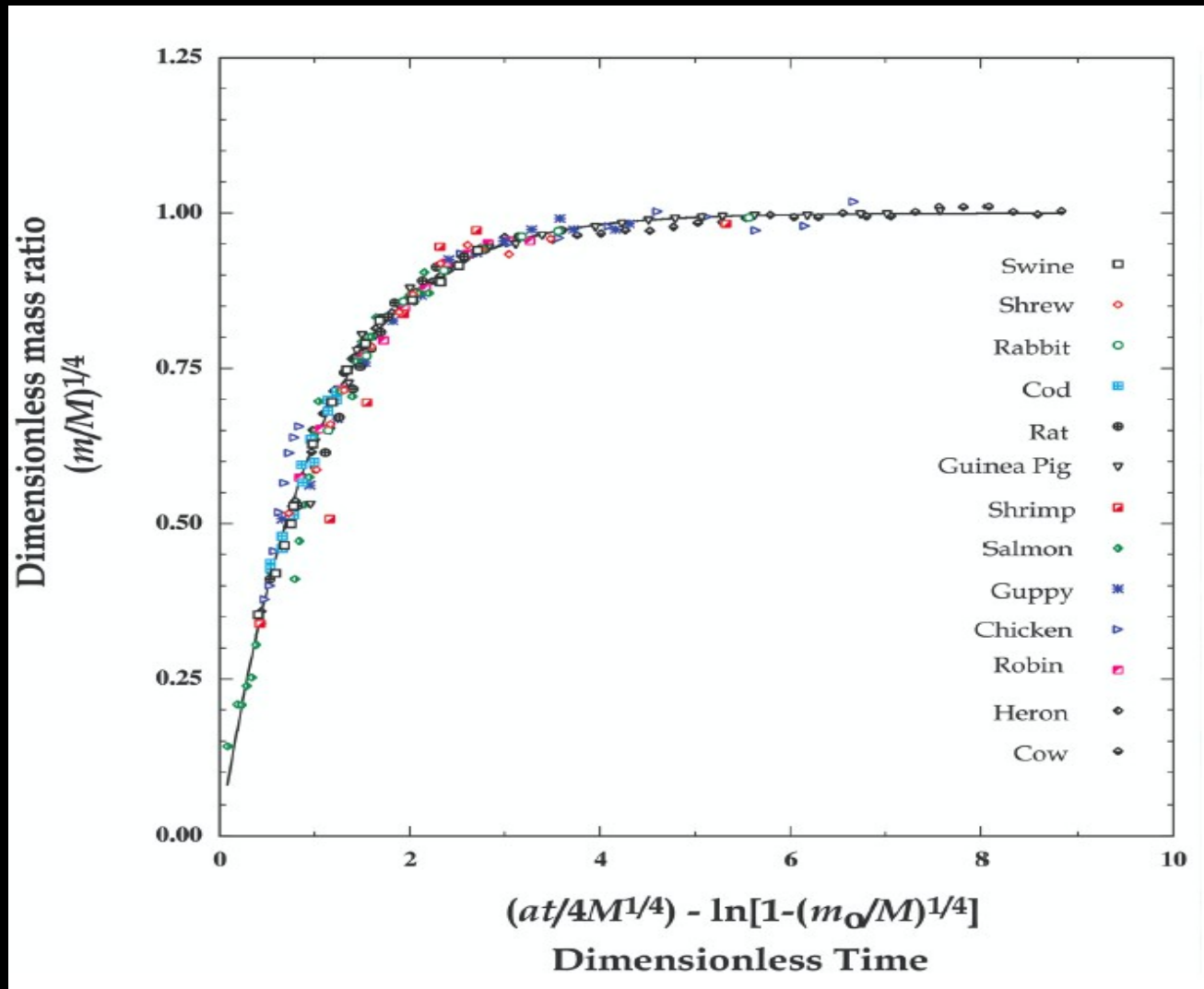
**SUB-LINEAR SCALING  
LEADS TO BOUNDED GROWTH**

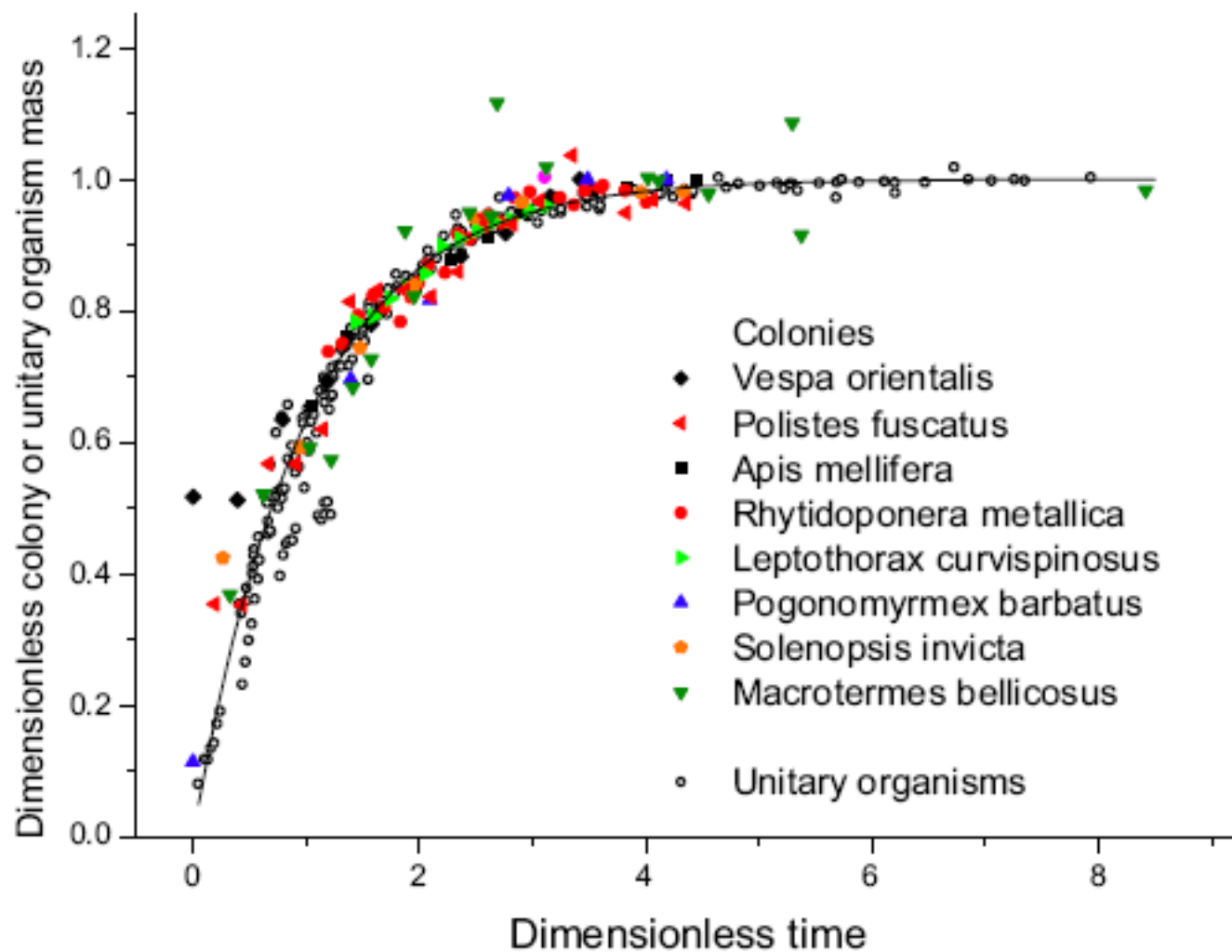
# GROWTH CURVES OF ANIMALS



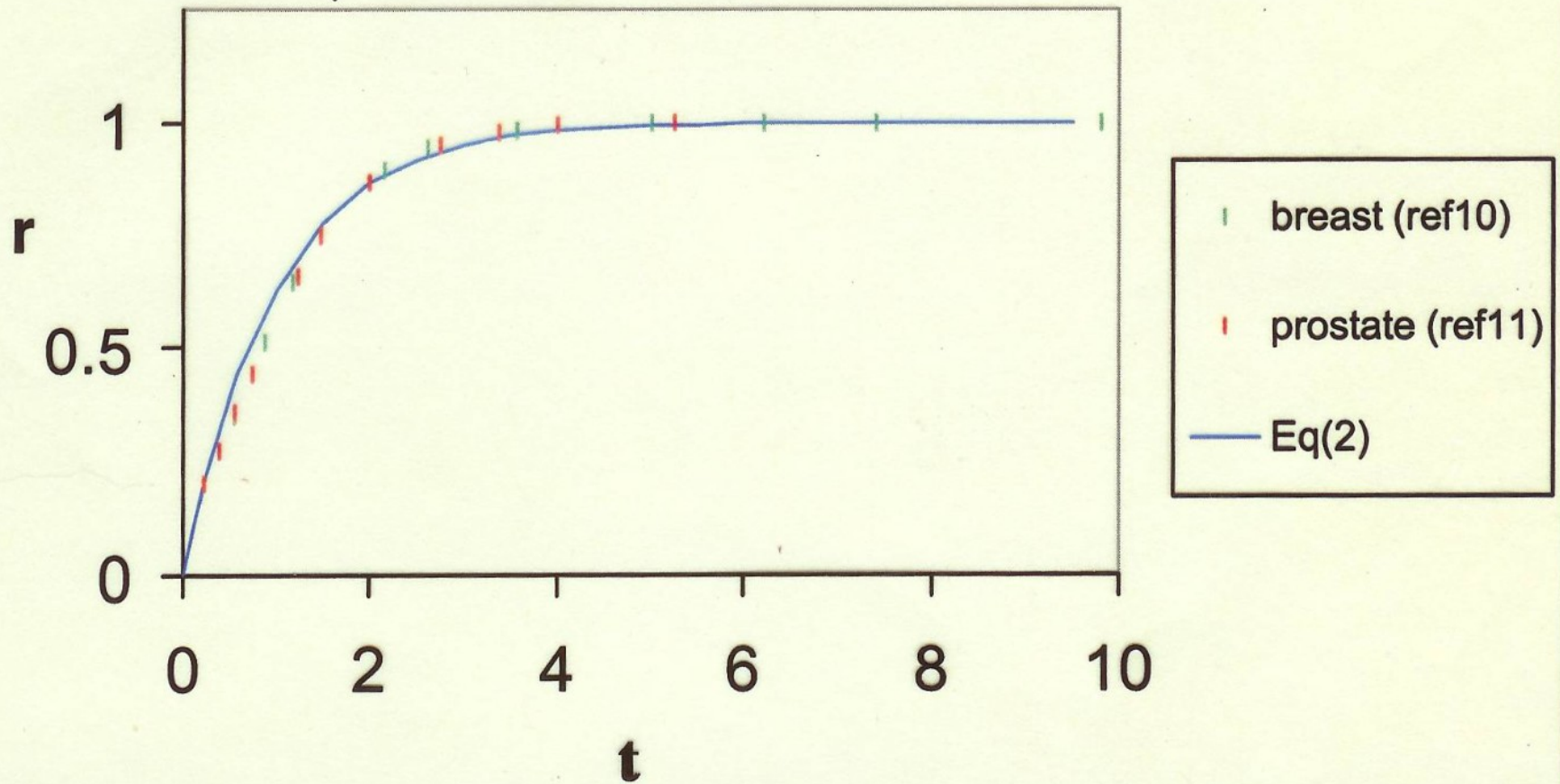
# UNIVERSAL COLLAPSED GROWTH CURVE

## RESCALED MASS VS. RESCALED AGE

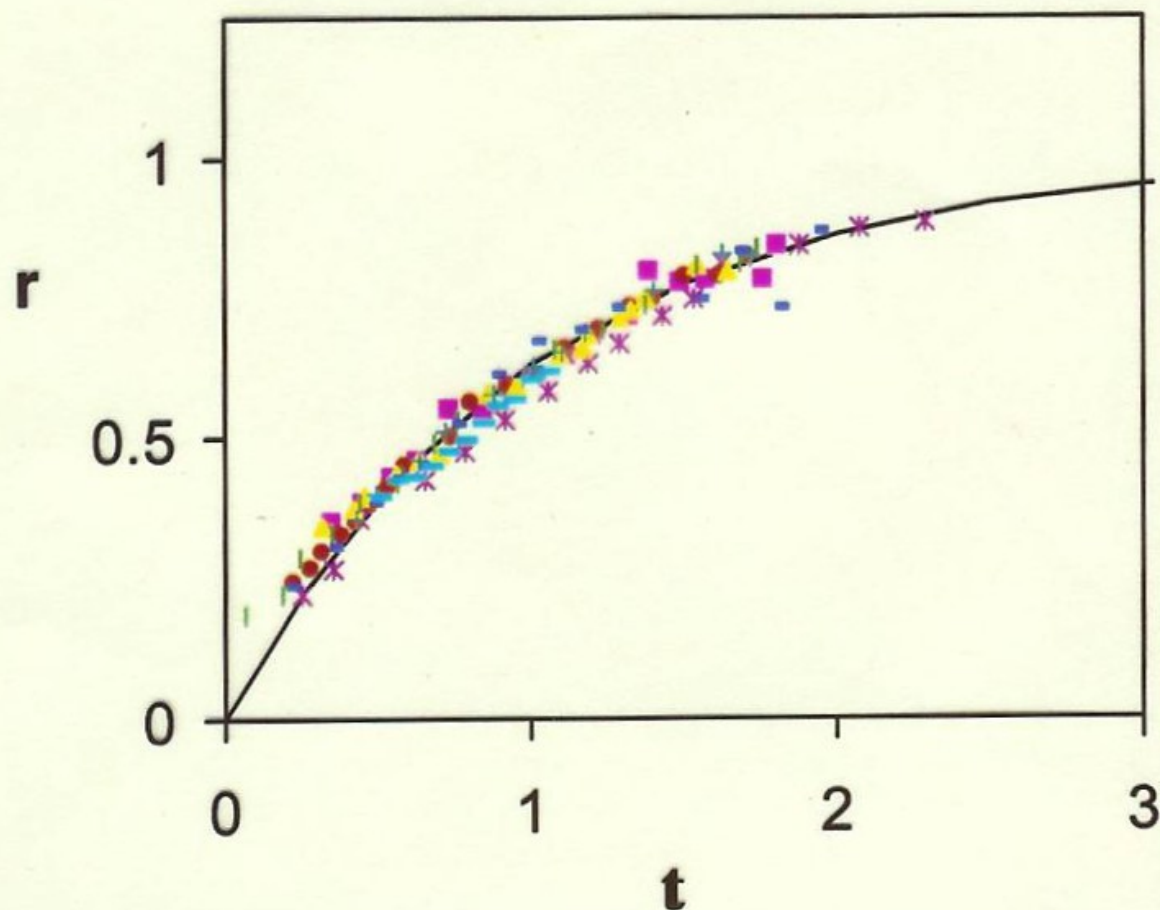




**'in vivo' data  
(patients)**



## 'in vivo' data (rodents)



■ Fibro (ref8)

× Walker (ref8)

● KHJJ (ref8)

▲ C3H (ref8)

■ EMT6 (ref8)

+ NCTC2472 (ref8)

■ Osteo (ref8)

— C33 ISS (ref9)

— Eq(2)

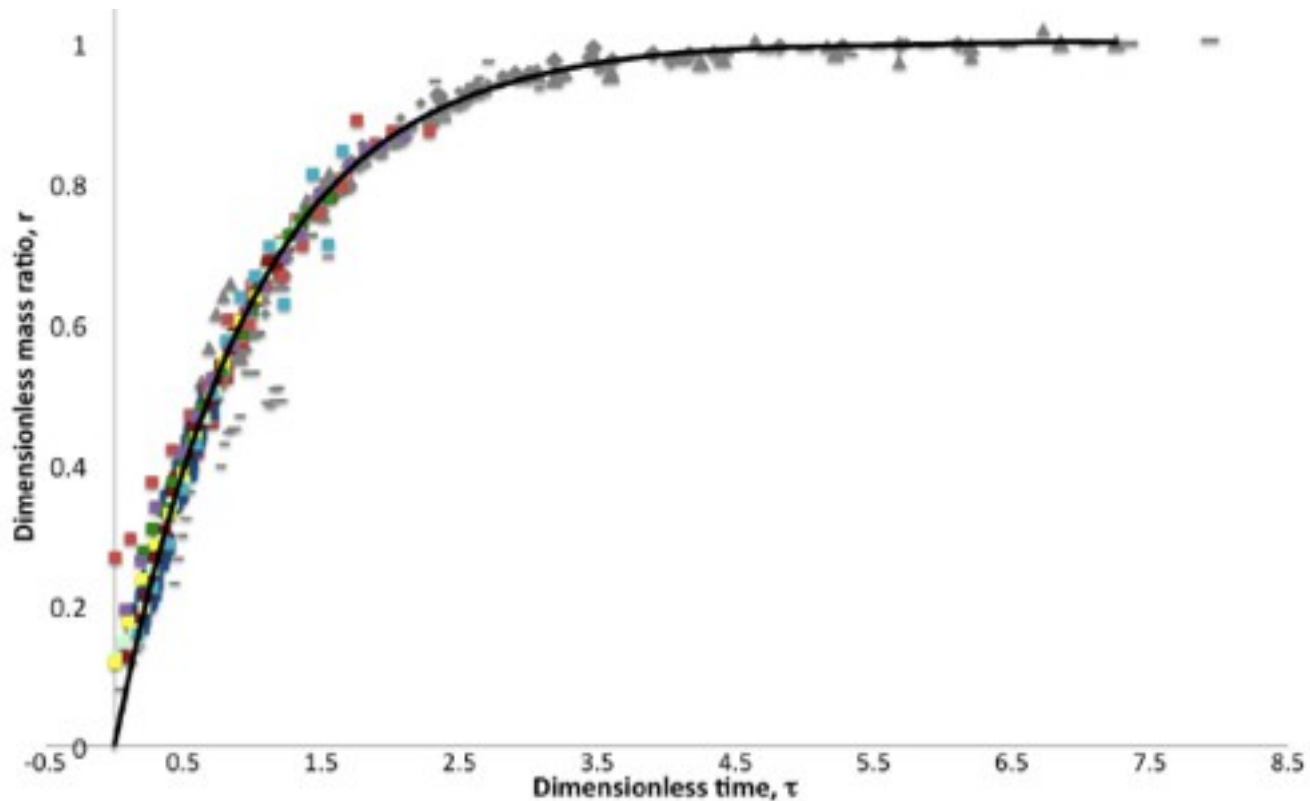


FIGURE 5. Plots of dimensionless ratio versus dimensionless time as defined by Eqs. (29)-(30). Data in grey are for ontogenetic growth from 13 species of animals (see [90] for original data sources), ranging from guppy to cod to guinea pig, and data in color with square symbols are for tumor growth trajectories for C3H mammary carcinoma (dark blue), EMT6 mammary carcinoma (dark red), KHJJ mammary carcinoma (light green), NCTC (dark green), Flank (yellow), Primary fibroadenoma (red), Primary Osteosarcoma (light blue), and Walker Carcinoma (purple) tumor

# GODZILLA

**MAY 16**

SEE IT IN REALD 3D AND IMAX 3D

**MENU**

**HOME**

**TICKETS**

**VIDEOS**

**DOWNLOADS**

**GALLERY**

**STORY**

**PARTNERS**

**ULTIMATE FAN CONTEST**

**IMAX FAN ART CONTEST**

**MOTOREsearch.net**

**WORLDWIDE RELEASE DATES**

**GODZILLA SHOP**

**LEGAL** | [f](#) [t](#) [+](#) [v](#) [t](#) [v](#)

TM & © 2014 WARNER BROS. ENT AND LICENSEE. ALL RIGHTS RESERVED.

**LENGTH**      350 ft

**WEIGHT**       $1.7 \times 10^7$  Kg =  $1.7 \times 10^4$  tons

**BASAL METABOLIC RATE**     $2 \times 10^7$  calories a day = 1 megawatt

**WEIGHT OF HEART**               $10^5$  Kg = 100 tons

**RADIUS OF HEART**              30 ft

**HEART RATE**              2.5 times a minute

**VOLUME OF BLOOD**               $2 \times 10^6$  litres

**DIAMETER OF AORTA**              10 ft

**SLEEP**    < 1 hour a day

**LIFESPAN**    2000 years

# GENERALISED SCALING

**i) SUPPOSE THE POPULATION SIZE CHANGES BY A FACTOR  $\lambda$ :**

$$N \rightarrow \lambda N$$

**ii) THIS INDUCES A CHANGE IN SOME METRIC FROM  $Y(N)$  TO  $Y(\lambda N)$ :**

$$Y(N) \rightarrow Y(\lambda N) = Z(\lambda, N)Y(N)$$

# GENERALISED SCALING

- i) SUPPOSE THE POPULATION SIZE CHANGES BY A FACTOR  $\lambda$ :

$$N \rightarrow \lambda N$$

- ii) THIS INDUCES A CHANGE IN SOME METRIC FROM  $Y(N)$  TO  $Y(\lambda N)$ :

$$Y(N) \rightarrow Y(\lambda N) = Z(\lambda, N)Y(N)$$

## **RENORMALISATION GROUP**

*M. Gell-Mann & F. E. Low (1954) Physical Review 95 (5): 1300–1312*

iii) FOR **ARBITRARY**  $Z(\lambda, N)$  THIS CAN BE SOLVED TO GIVE THE GENERAL SOLUTION:

$$Y(N) = Y_0 N^{b(N)}$$

WHERE THE GENERALISED EXPONENT,  $b(N)$ , DEPENDS ON  $N$  AND IS GIVEN BY:

$$b(N) = \frac{\int_0^{\ln N} \gamma(N) d \ln N}{\ln N}$$

WITH

$$\gamma(N) \equiv \frac{\partial Z(1, N)}{\partial \lambda}$$

iv) THE “NATURAL” VARIABLE IS  $\ln N$

v) WHEN DO WE GET SIMPLE POWER LAWS  
WITH EXPONENTS  $b(N)$  INDEPENDENT OF  $N$ ?

ANSWER: WHEN  $\gamma(N)$  IS INDEPENDENT OF  $N$

→ WHEN  $Z(\lambda, N)$  IS INDEPENDENT OF  $N$ :

$$Y(\lambda N) = Z(\lambda) Y(N)$$

**SELF-SIMILAR (FRACTALITY)**

# GENERALISE TO “DYNAMICAL” REPRESENTATION

$$Y(N) \rightarrow Y[N, g(N)]$$

$g(N)$  “STRENGTH OF INTERACTION”  
THEN RG SOLUTION IS

$$Y(N) \vartheta Y[N, g(N)] = Y(N_0) e^{\int_{\gamma(g)}^g \frac{B(g)}{\gamma(g)} dg} F[Ne^{\int_{\gamma(g)}^g dg}]$$

WHERE

$$\gamma(g) \vartheta \frac{\overleftarrow{g}(N)}{\overleftarrow{N}}$$

(FIXED POINTS)

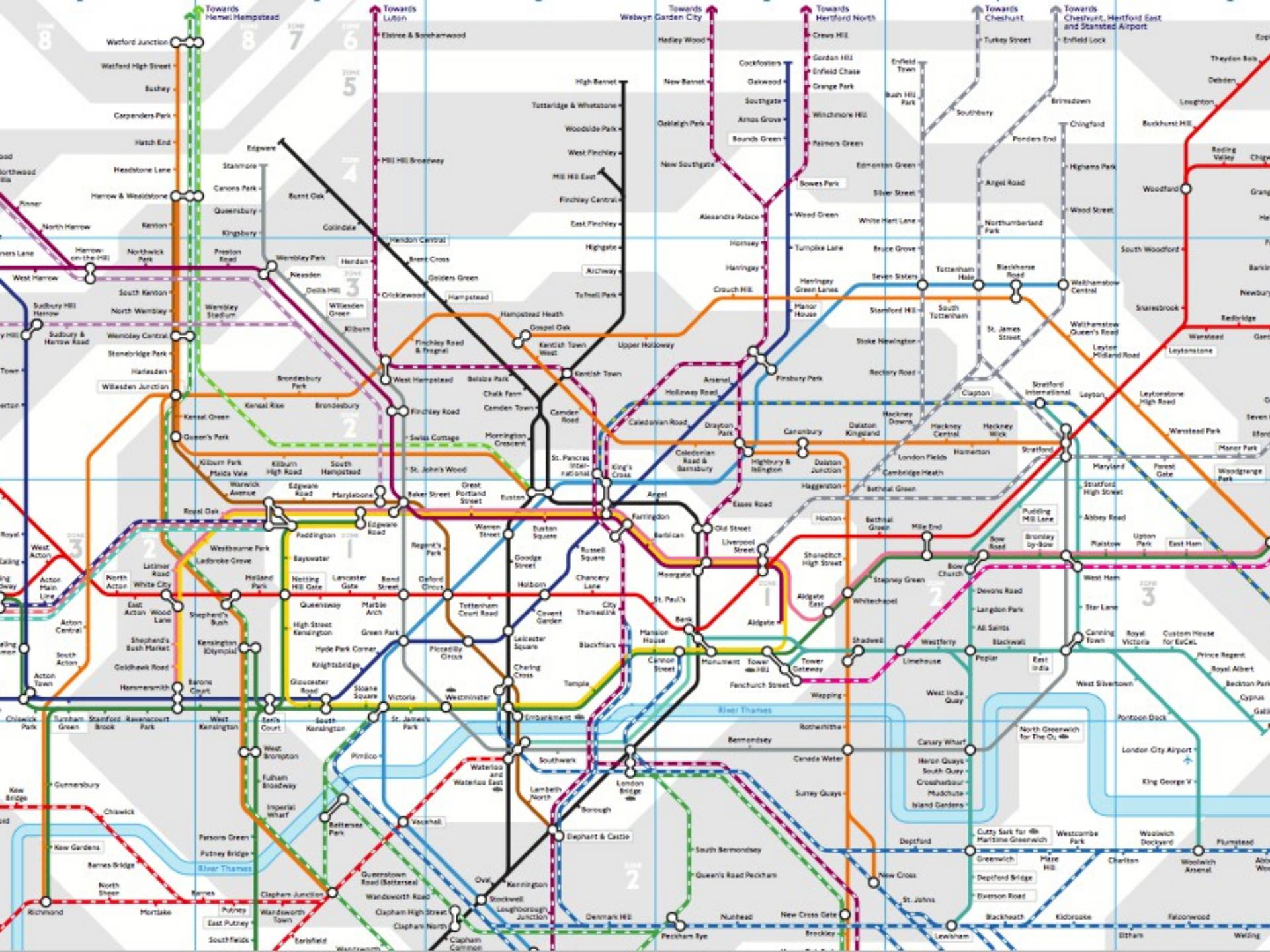
# BIOLOGY (LIFE)

- a) **DOMINATED BY SYSTEMATIC, PREDICTABLE, NON-LINEAR (UNIVERSAL) SCALING LAWS**
- b) **ECONOMIES OF SCALE** (THE BIGGER YOU ARE, THE LESS YOU NEED PER “CAPITA”) - **SUBLINEAR**
- c) **PACE OF LIFE** SYSTEMATICALLY SLOWS WITH INCREASING SIZE
- d) **GROWTH** IS SIGMOIDAL REACHING A STABLE SIZE AT MATURITY
- e) **FINITE LIFESPAN**

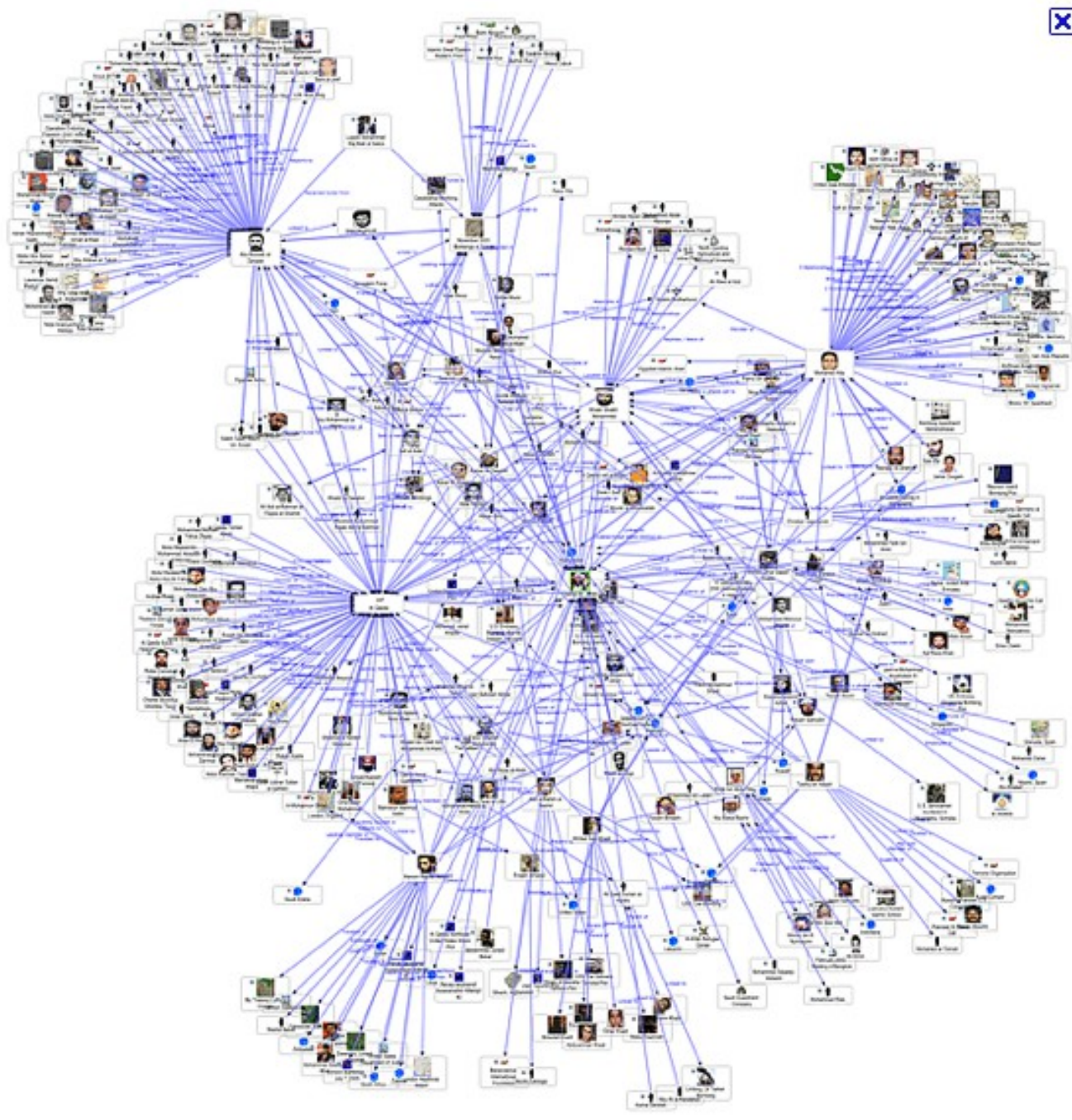
**ARE CITIES (AND COMPANIES)  
SCALED VERSIONS OF EACH  
OTHER?**

**DO THEY MANIFEST  
“UNIVERSALITY”?**



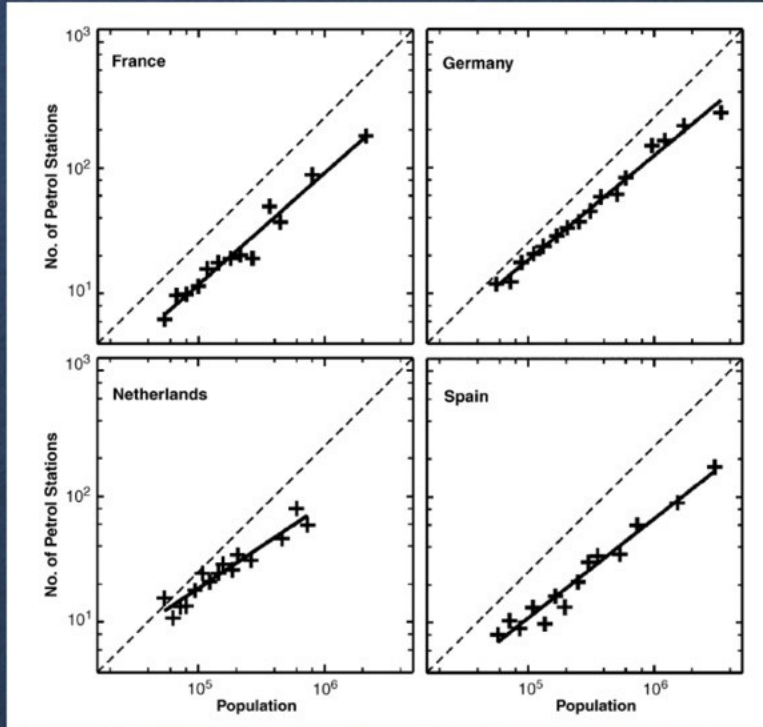








## NUMBER OF PETROL STATIONS VS. POPULATION

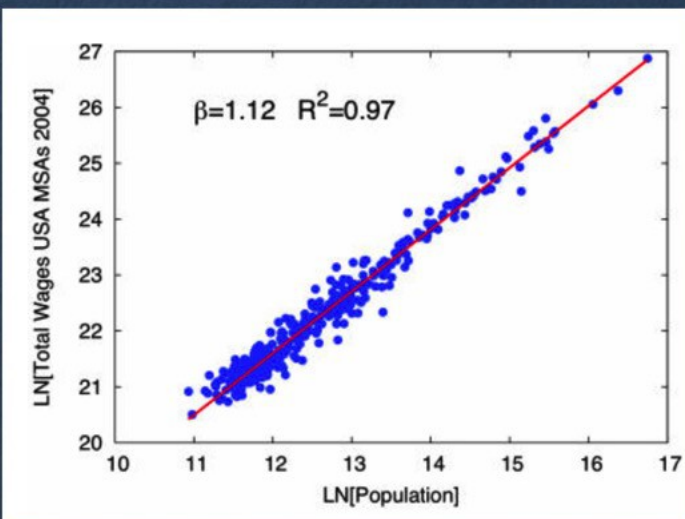


***INFRASTRUCTURE***

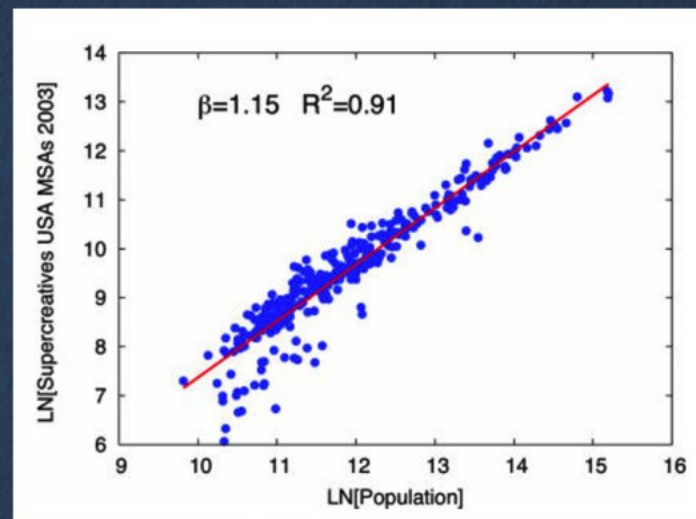
***SUB-LINEAR SCALING***

***ECONOMY OF SCALE***

# SUPER-LINEAR SCALING

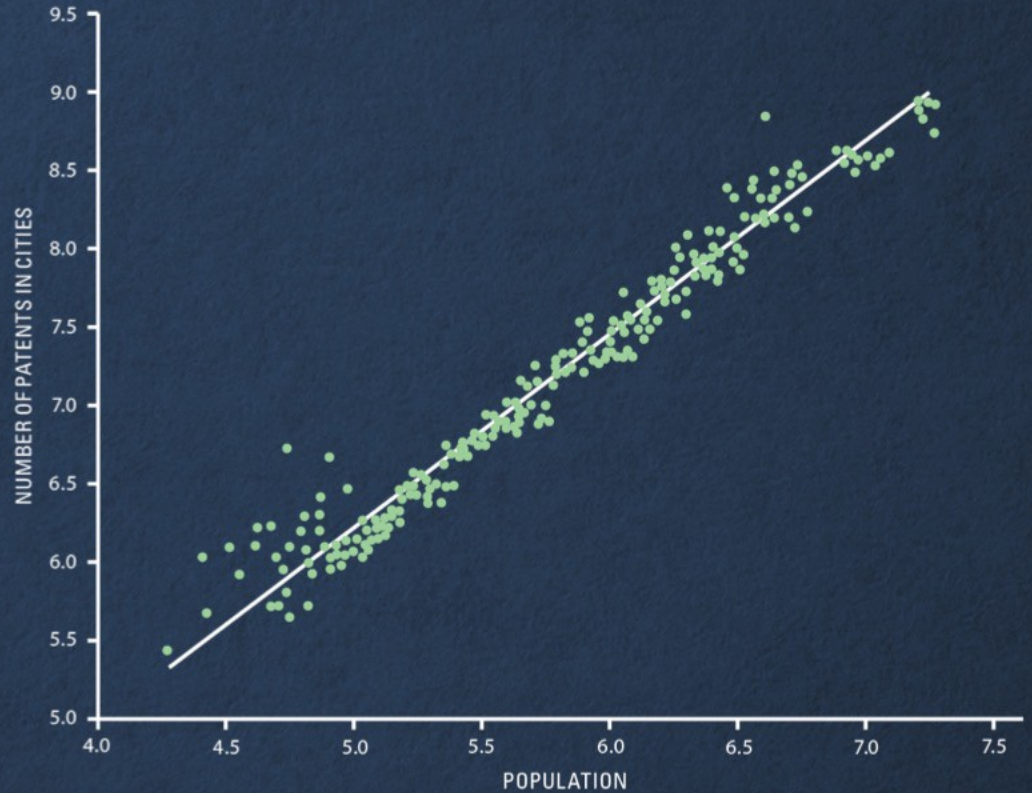


Total **wages** per MSA in 2004 for the USA vs. metropolitan population.



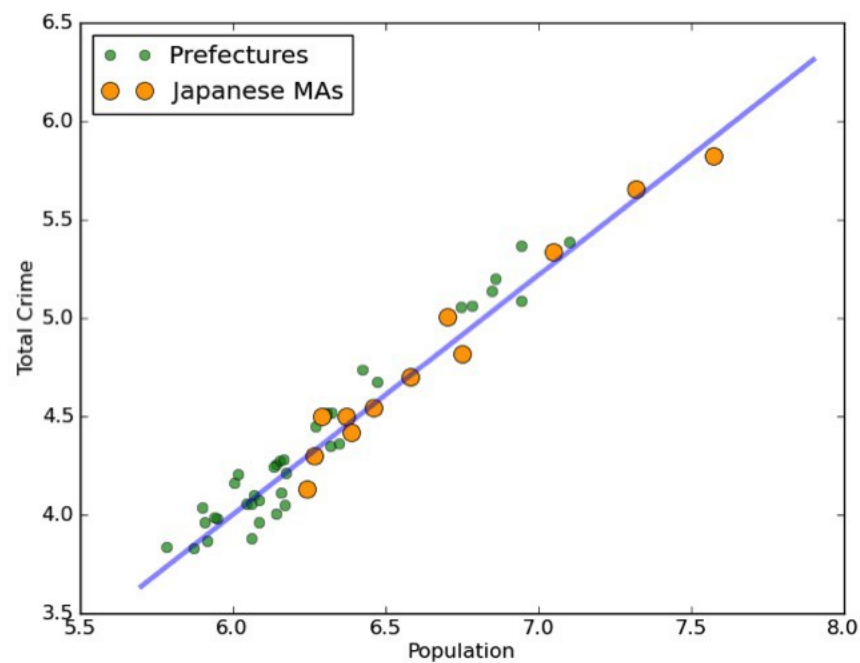
**Supercreative employment** per MSA in 2003, for the USA vs. metropolitan population.

## INNOVATION MEASURED BY PATENTS

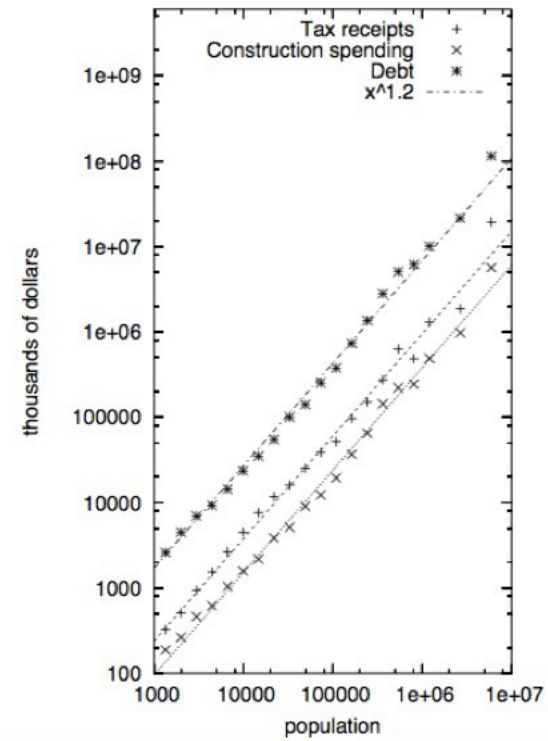
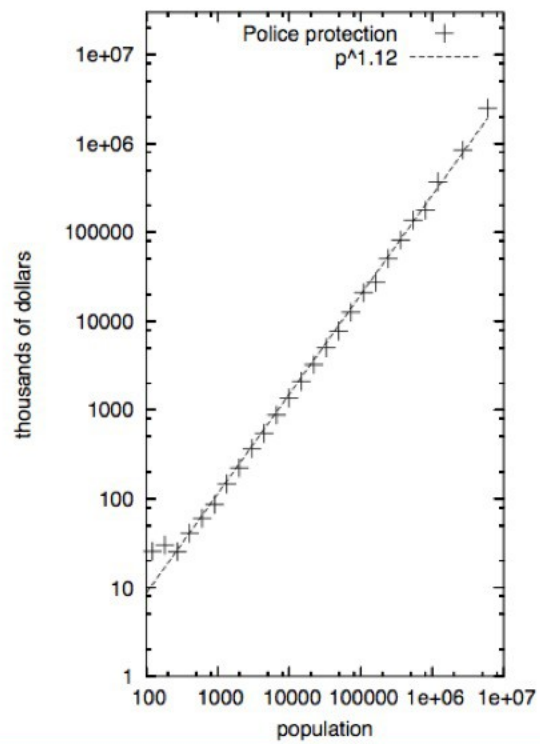




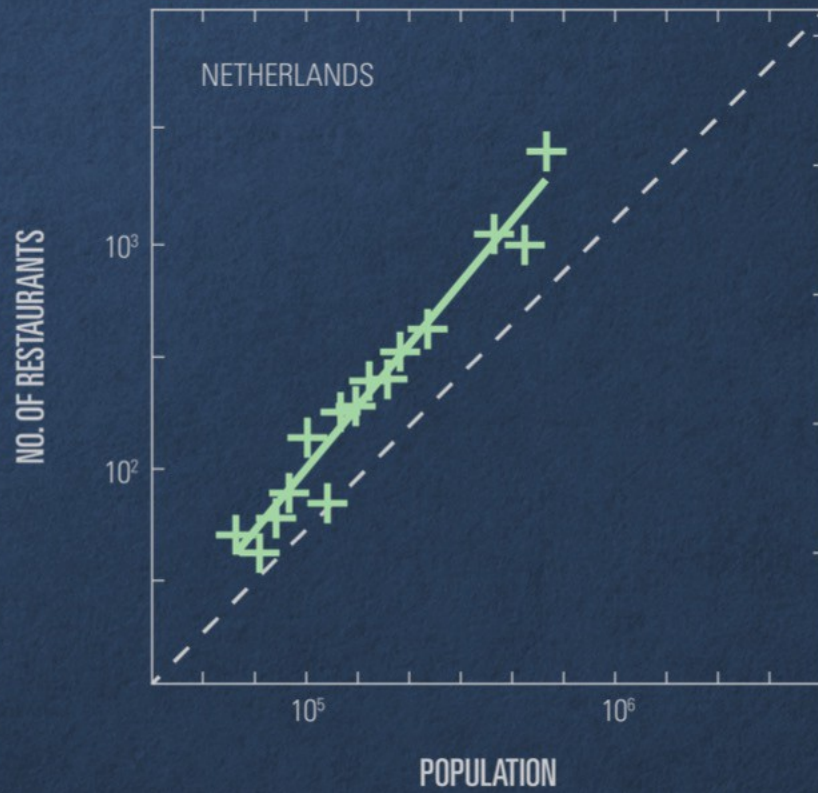
## TOTAL CRIME (JAPAN)



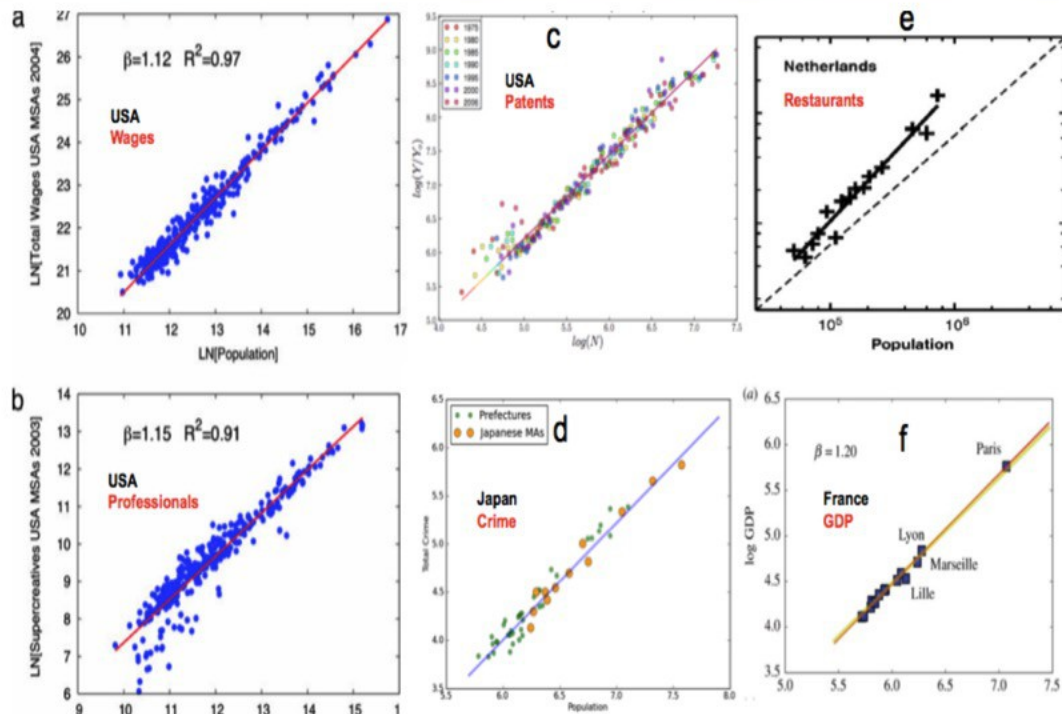
*Slope = 1.21 [1.08, 1.35]*



## RESTAURANTS IN THE NETHERLANDS



# UNIVERSALITY OF URBAN SCALING





# THE GOOD, THE BAD, THE UGLY

**ON AVERAGE DOUBLING THE SIZE OF  
A CITY  
SYSTEMATICALLY INCREASES**

**ON AVERAGE DOUBLING THE SIZE OF  
A CITY**

**SYSTEMATICALLY INCREASES  
INCOME, WEALTH, PATENTS,  
COLLEGES, CREATIVE PEOPLE,  
POLICE, AIDS & FLU, CRIME, SOCIAL  
INTERACTIONS,.....**

**ON AVERAGE DOUBLING THE SIZE OF  
A CITY**

**SYSTEMATICALLY INCREASES  
INCOME, WEALTH, PATENTS,  
COLLEGES, CREATIVE PEOPLE,  
POLICE, AIDS & FLU, CRIME, SOCIAL  
INTERACTIONS,.....**

**ALL BY APPROXIMATELY 15%  
REGARDLESS OF CITY**

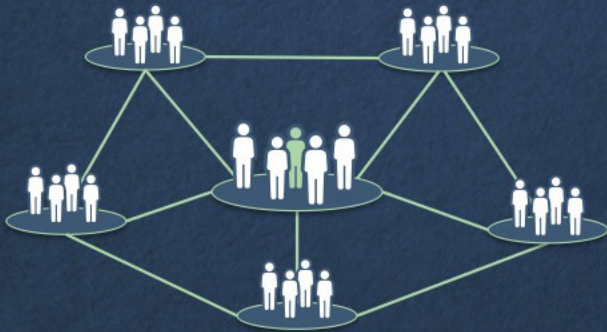
***AND.....***

***AND.....***

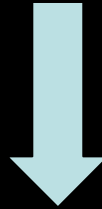
***SAVES APPROXIMATELY 15%  
ON ALL INFRASTRUCTURE  
(ROADS, ELECTRICAL LINES,  
GAS STATIONS,.....)***

# Universality of Social Networks

*(clustering hierarchies)*

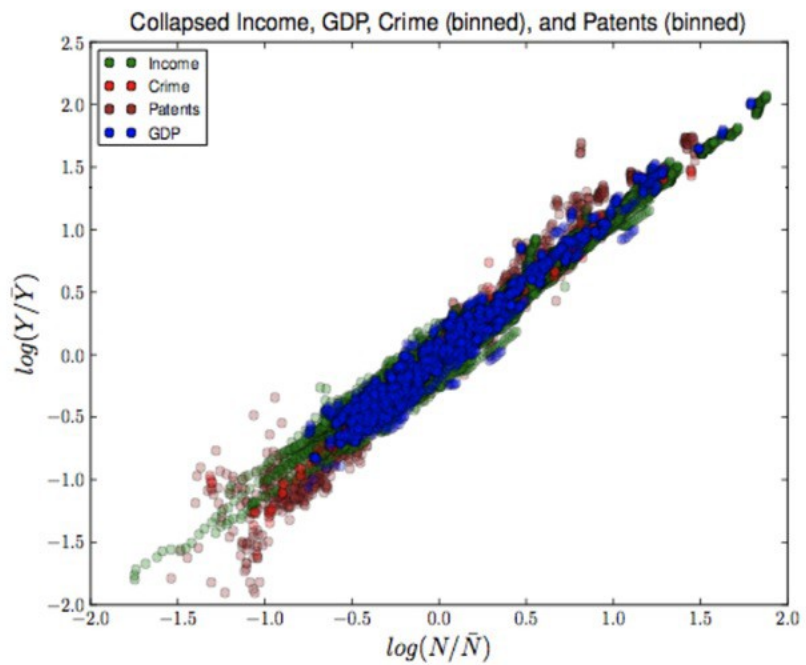


**POSITIVE FEEDBACK MECHANISM IN  
SOCIAL NETWORKS**

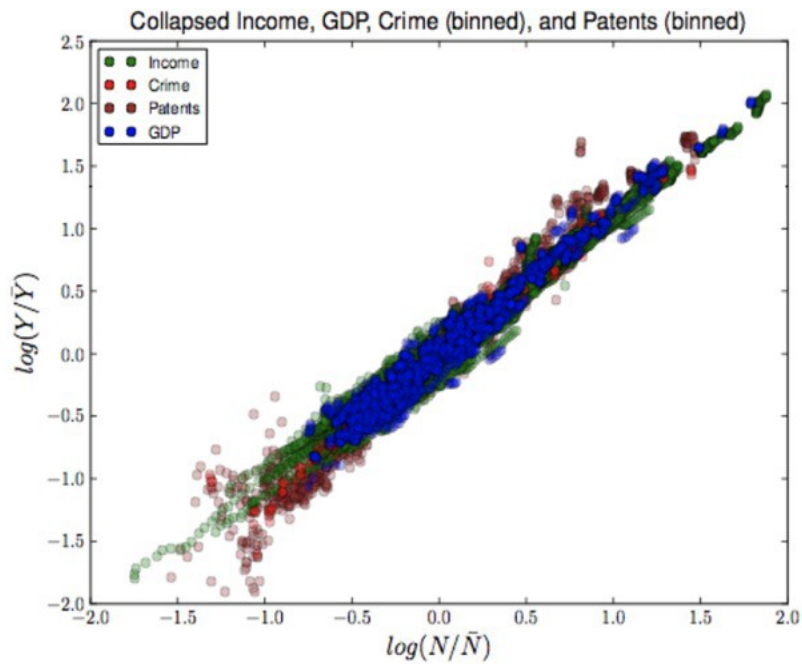


**SUPERLINEAR SCALING & INCREASING  
PACE OF LIFE**

# UNIVERSALITY

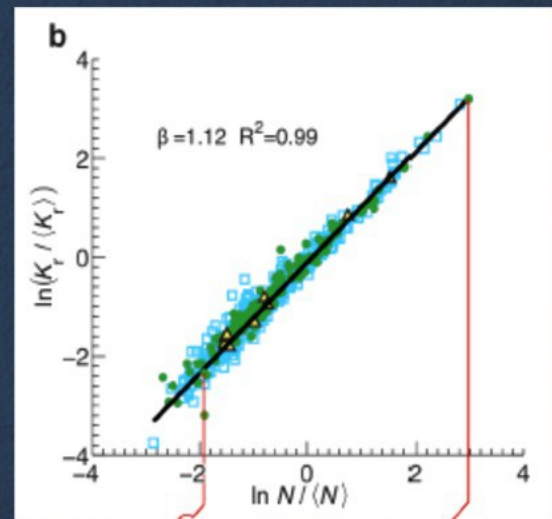


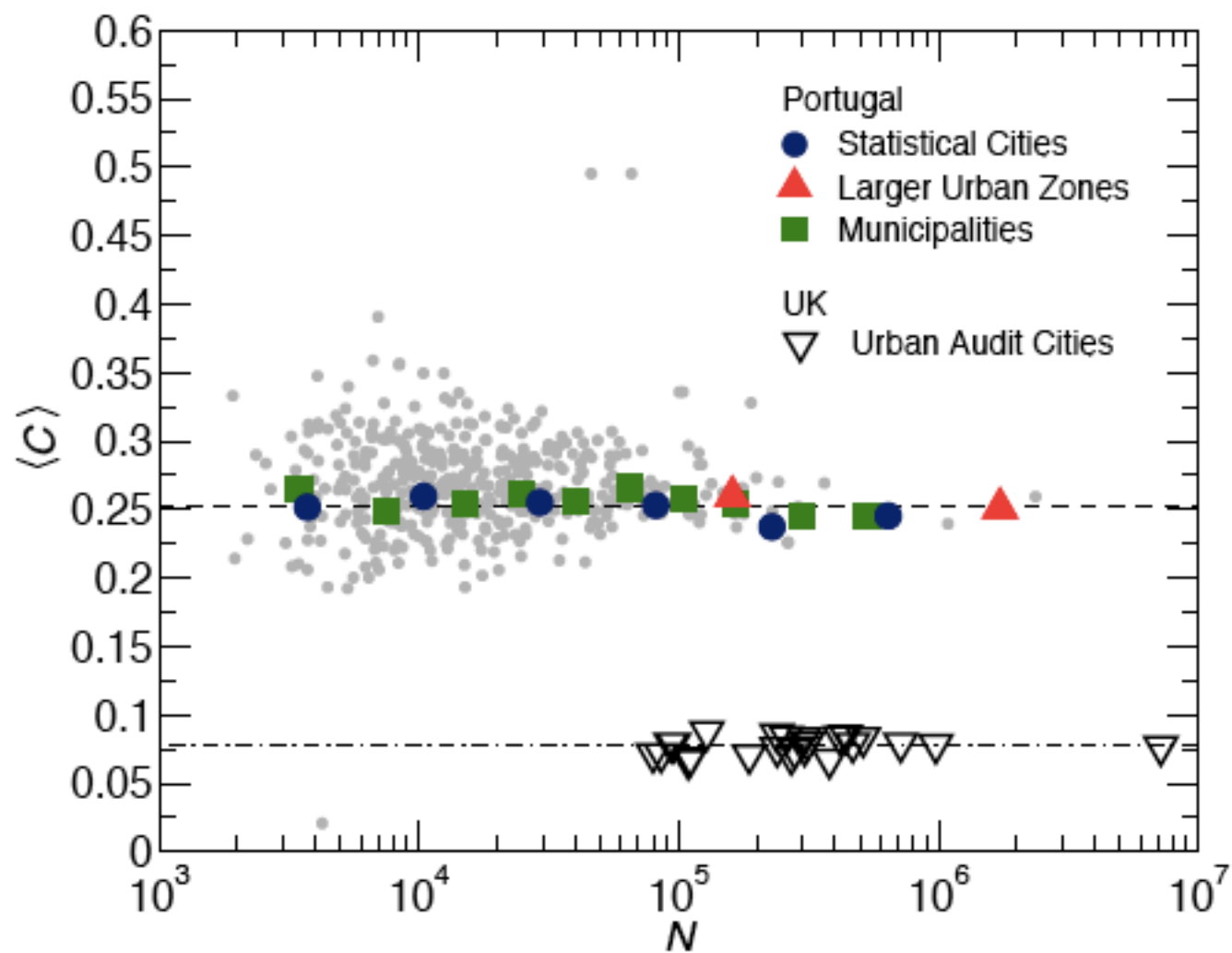
## UNIVERSALITY



## SOCIAL CONNECTIVITY

(Cell Phone Data)







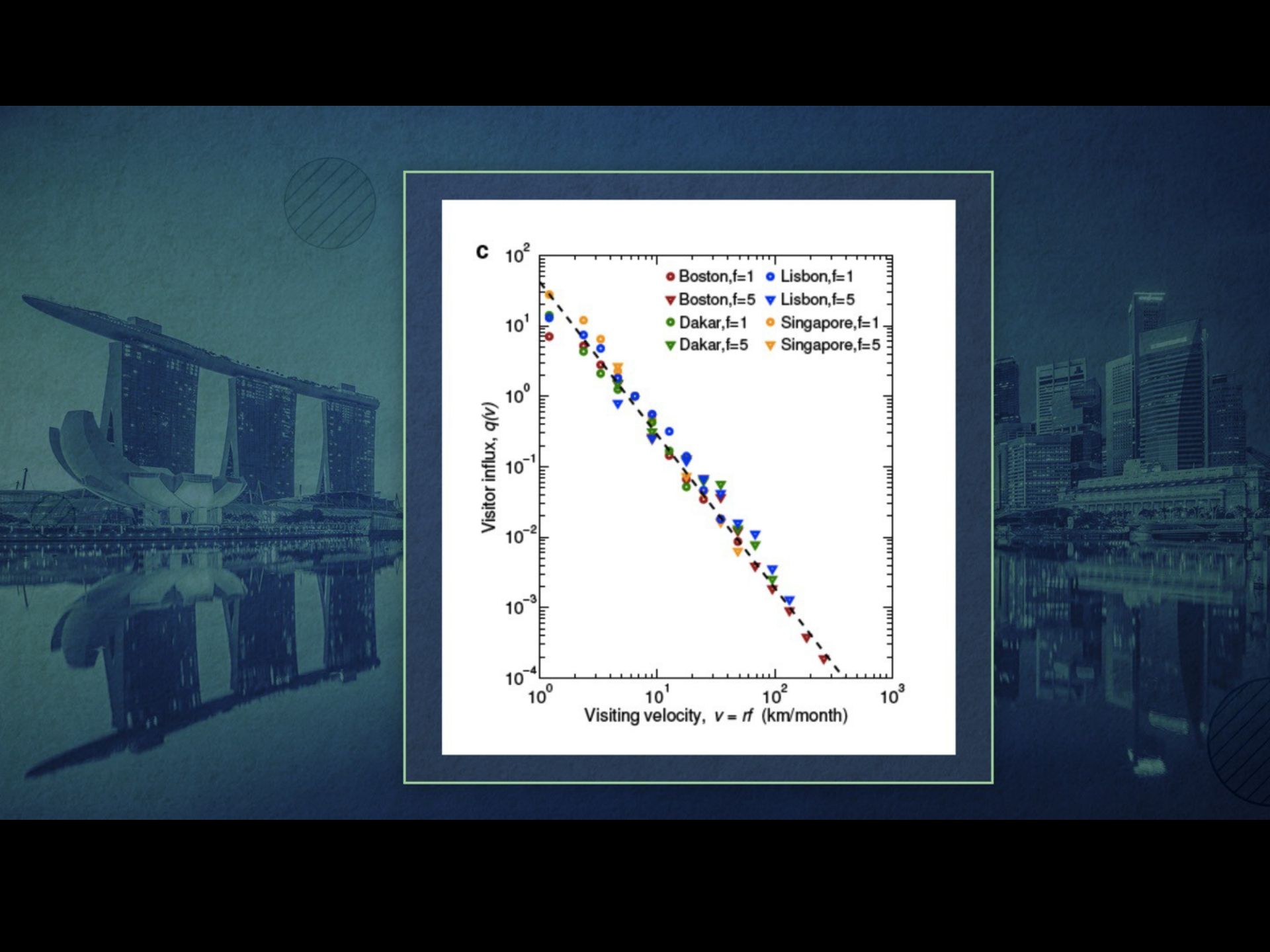
## MOVEMENT IN CITIES

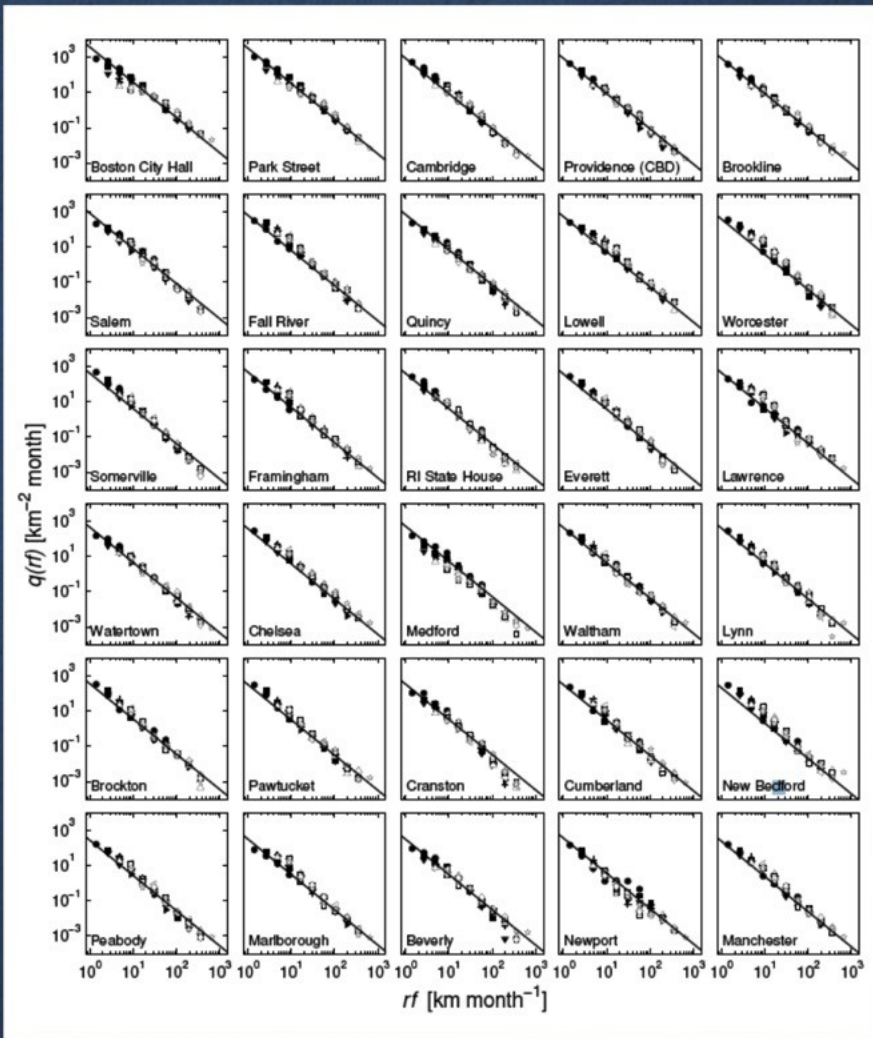
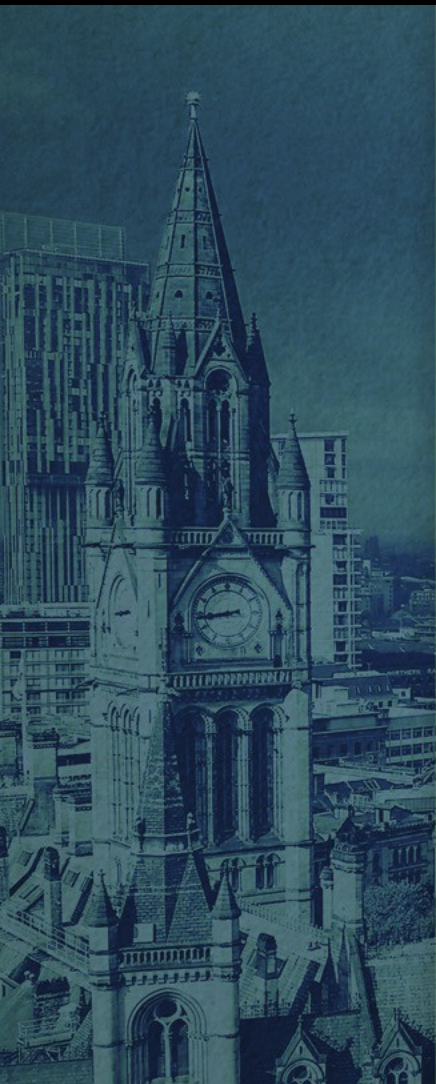


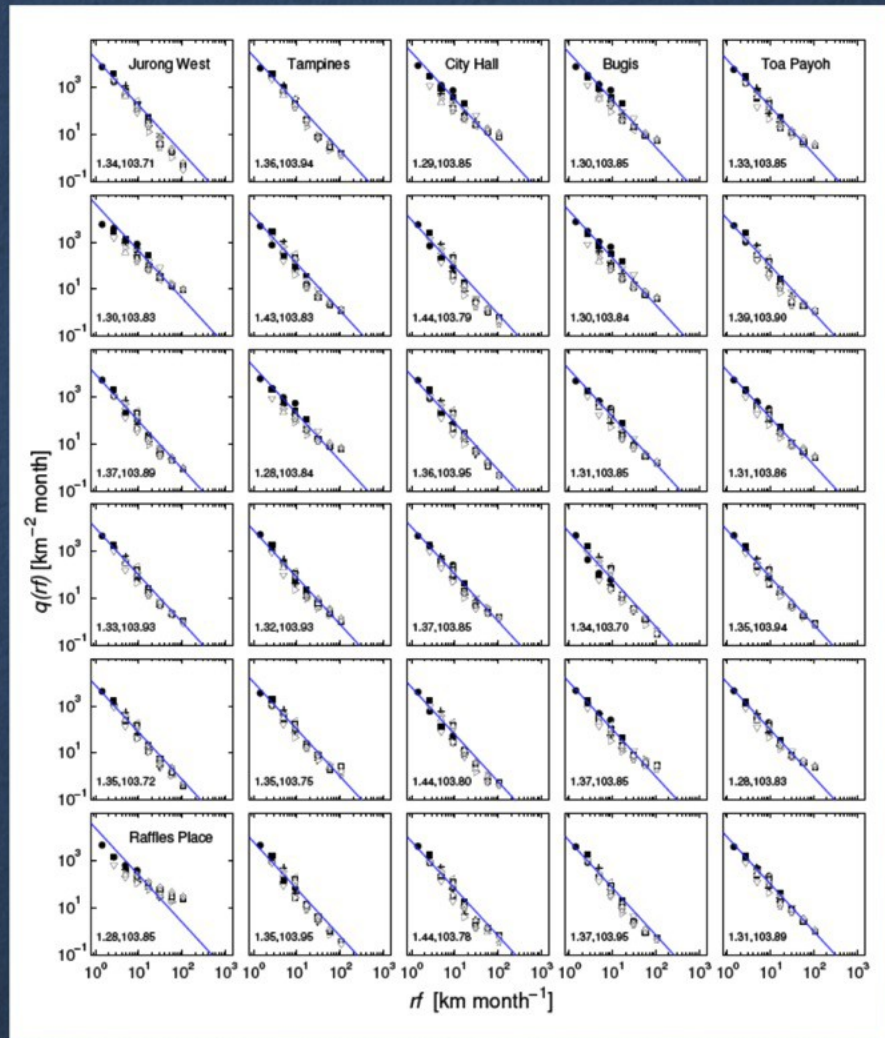
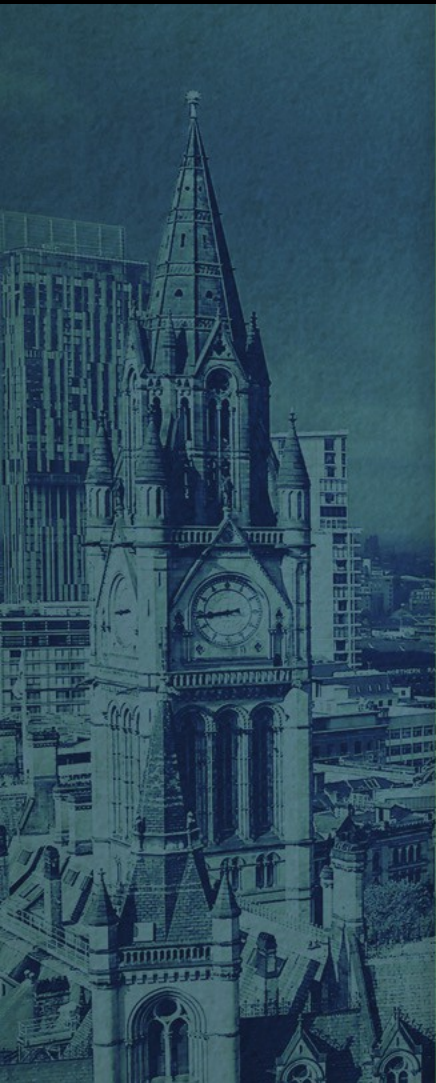
People on average minimize travel time and distance.

“Theorem”: the number traveling to any location in any city from a distance  $r$  away  $f$  times a month is:

$$q(r, f) = \frac{A}{(rf)^2}$$





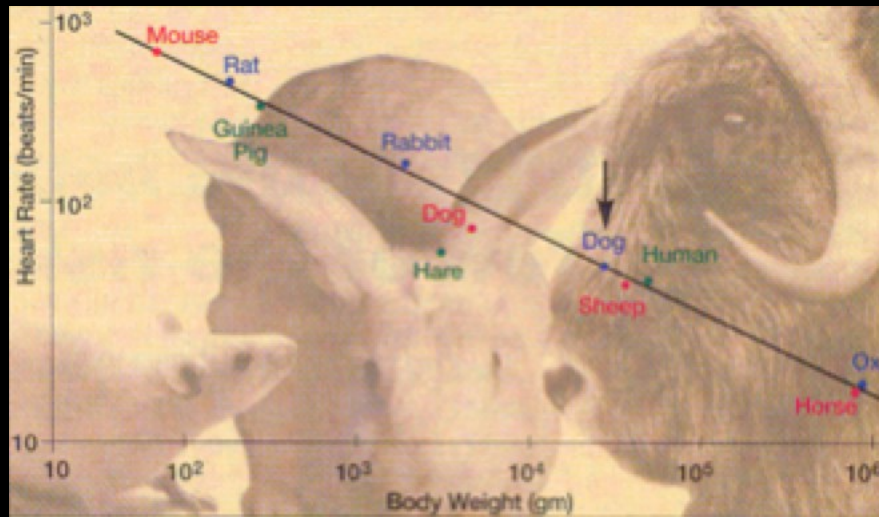


# ***NETWORK DYNAMICS DETERMINES THE PACE OF LIFE***

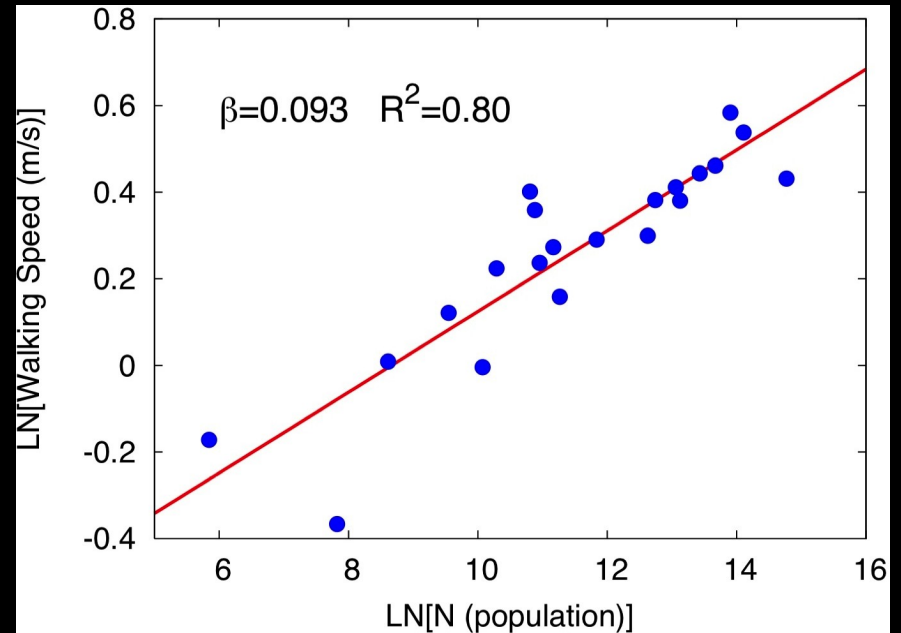
***IF THE SLOPE IS  $< 1$       PACE OF LIFE  
SLOWS DOWN***

***IF THE SLOPE IS  $> 1$       PACE OF LIFE  
SPEEDS UP***

# ***Pace of biological life vs. Pace of social life***



**Heart Rate vs Body Weight**



**Walking Speed vs. Population Size**



Research revealed almost half the nation found the slow pace of high streets to be their biggest shopping bugbear. Photo: Mercury Press

# GROWTH EQUATION

Total Incoming Rate

(resources, products, patents, . . . “energy” or “dollar” equivalent)

≈ Maintenance

(repair, replacement, sustenance, . . . )

+

Growth

$$R = \sum_{i=1}^n Y_i(N) = \sum_{j=1}^N r_j + \frac{d}{dt} \sum_{j=1}^N c_j$$

**$n$  = NUMBER OF “DRIVERS”  $Y_i$  CONTRIBUTING TO THE CITY “METABOLISM”**

**$r_j$  = RATE AT WHICH THESE RESOURCES ARE USED BY THE  $j^{\text{th}}$  INDIVIDUAL (MAINTAIN HIS/HER/ITS LIFE-STYLE, ETC)**

**$c_j$  = COST OF ADDING A NEW INDIVIDUAL TO THE CITY POPULATION**

**SCALING LAWS TELL US THAT EACH  $i$  SCALES AS**  $Y_i(N) = Y_i(1)N^{\alpha_i}$

**WITH  $\alpha_i \approx \alpha \approx 1.15$**   
**APPROXIMATELY THE SAME FOR ALL  $i$ ,**  
**SO**

$$R(N) = R(1)N^{\alpha}$$

**INTRODUCE AVERAGE COSTS:**

$$R_0 \approx \frac{1}{N} \sum_{j=1}^N r_j$$

$$E_0 \approx \frac{1}{N} \sum_{j=1}^N c_j$$

$$R \ll NR_0 + E_0 \frac{dN}{dt}$$

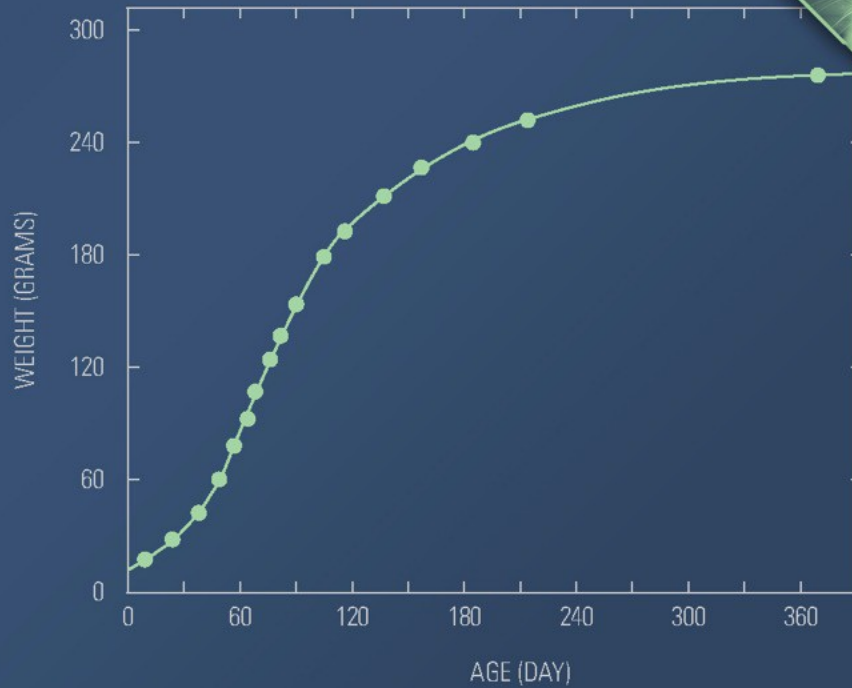
$$\frac{dN}{dt} = \frac{\varphi R_1}{\phi E_0 \varepsilon \psi} N^{\cap} \approx \frac{\varphi R_0}{\phi R_1 \varepsilon} N^{\ominus}$$

**SOLUTION:**

$$N^{12 \cap} = \frac{R_1}{R_0} + \frac{v}{\chi \psi} N^{12 \cap}(0) \approx \frac{R_1}{R_0} \oplus 2 \frac{R_0}{E_0} (12 \cap) t$$

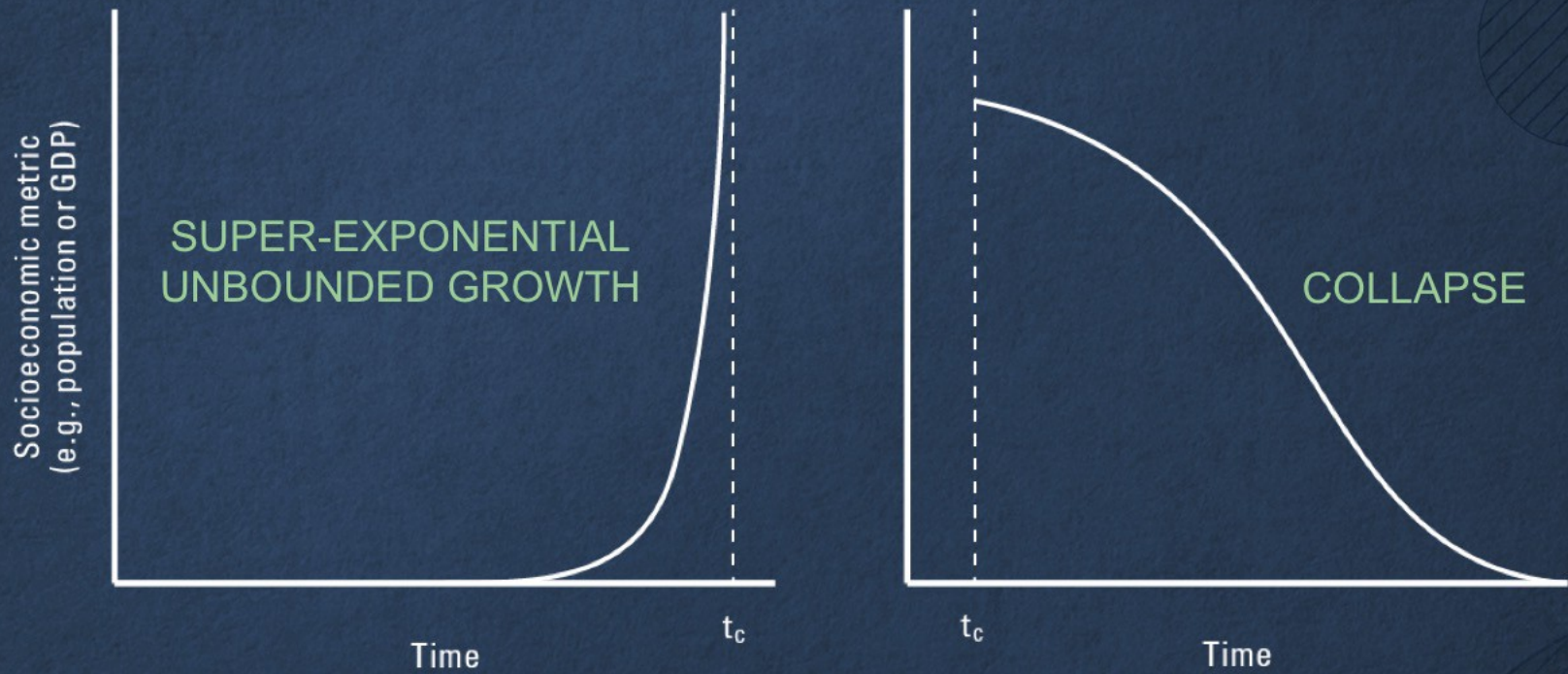
**CHARACTER OF SOLUTION SENSITIVE TO  $\cap >, =, < 1$**

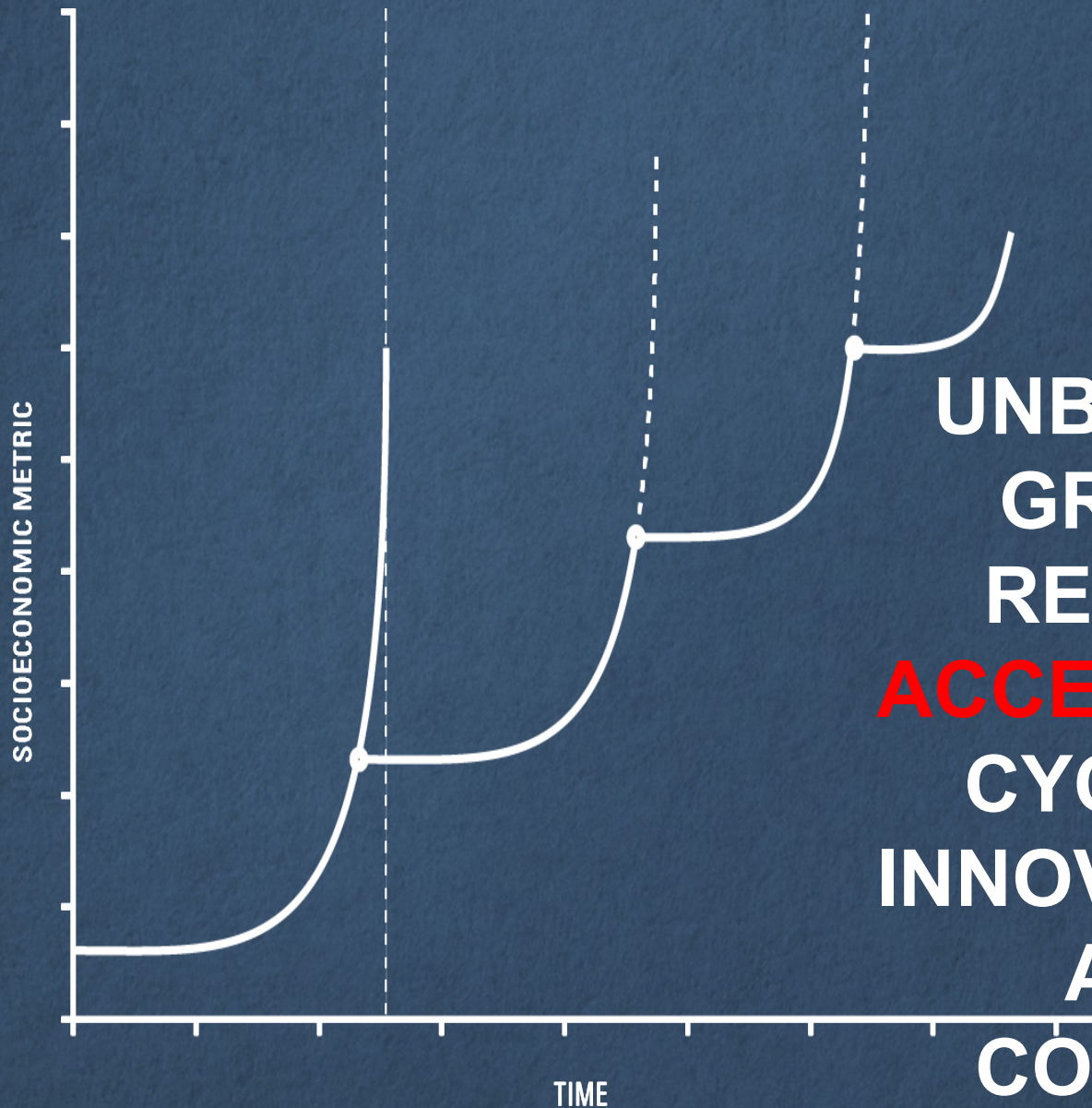
## GROWTH CURVE OF RAT



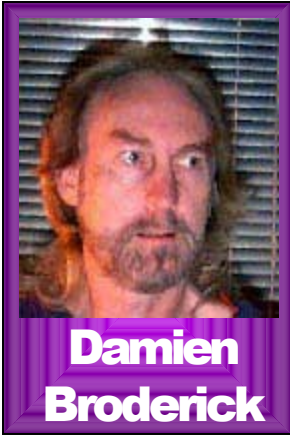
SUB-LINEAR SCALING LEADS  
TO BOUNDED GROWTH

# SUPER-LINEAR

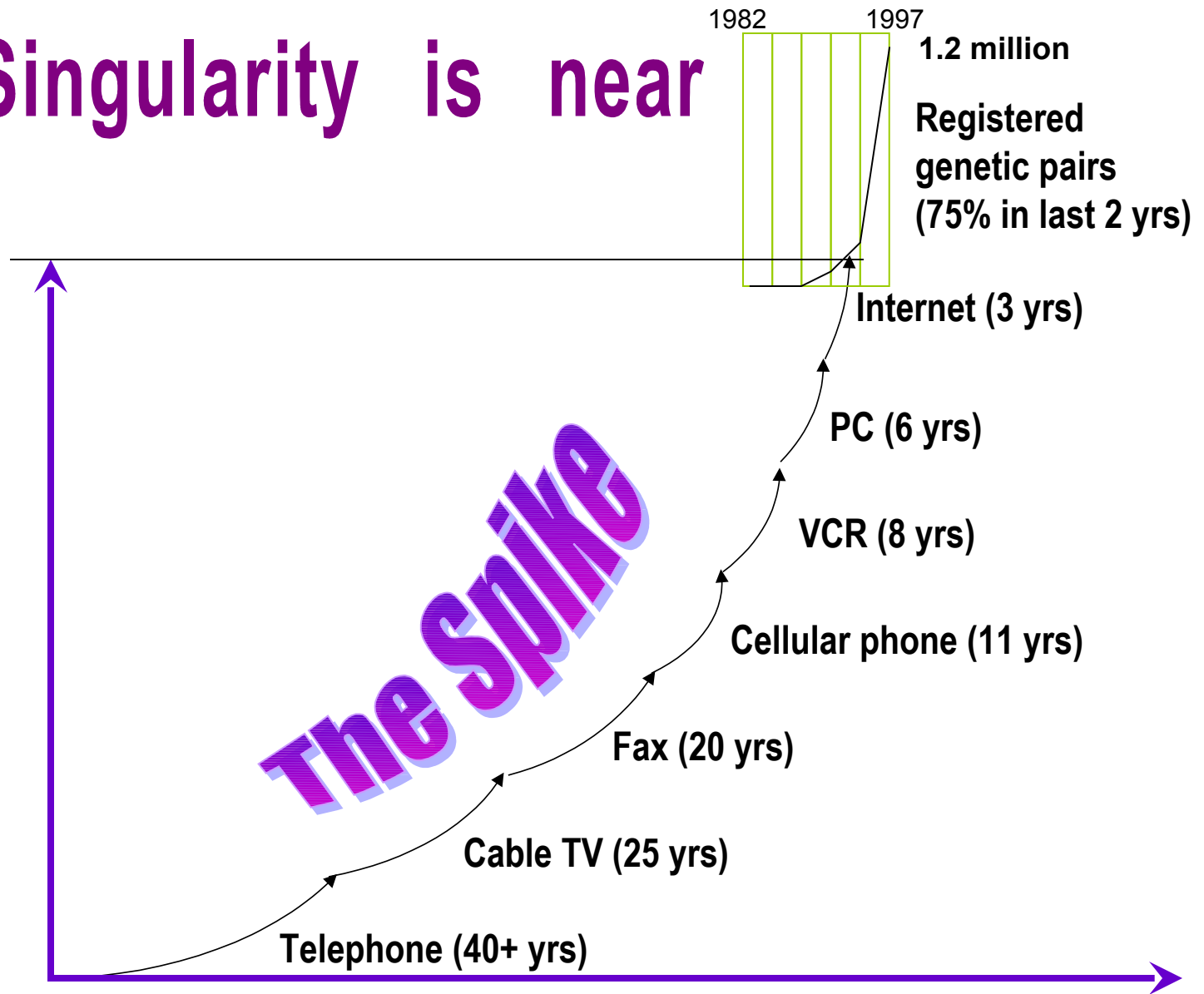




UNBOUNDED  
GROWTH  
REQUIRES  
**ACCELERATING**  
CYCLES OF  
INNOVATION TO  
AVOID  
COLLAPSE



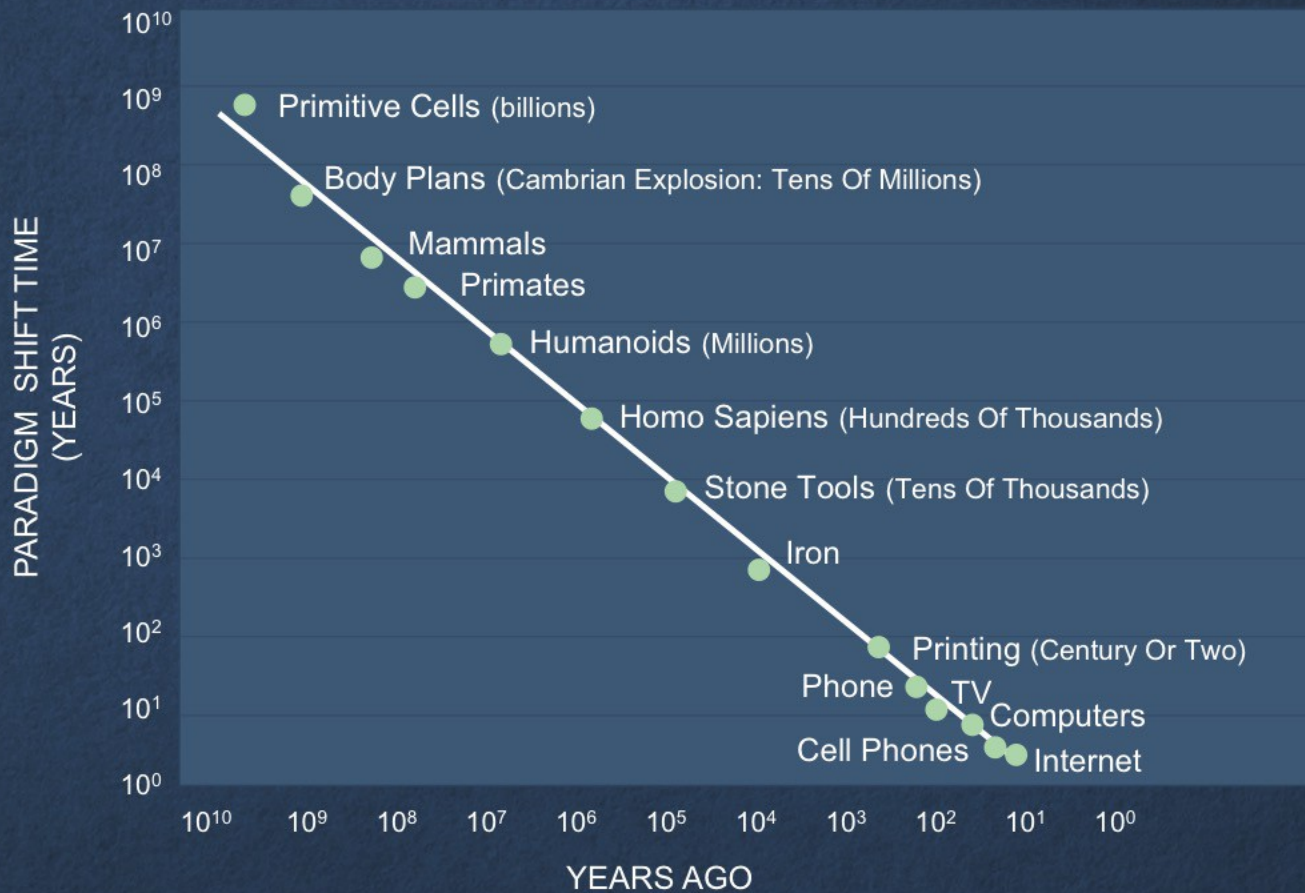
# Singularity is near



**Years to reach 10 million customers (US)**

*Time*

# SEQUENCE OF SINGULARITIES



**UNBOUNDED GROWTH LEADING TO  
“FINITE-TIME SINGULARITY” & COLLAPSE**

**UNLESS INNOVATIONS (SYSTEMATICALLY)  
OCCUR FASTER AND FASTER**

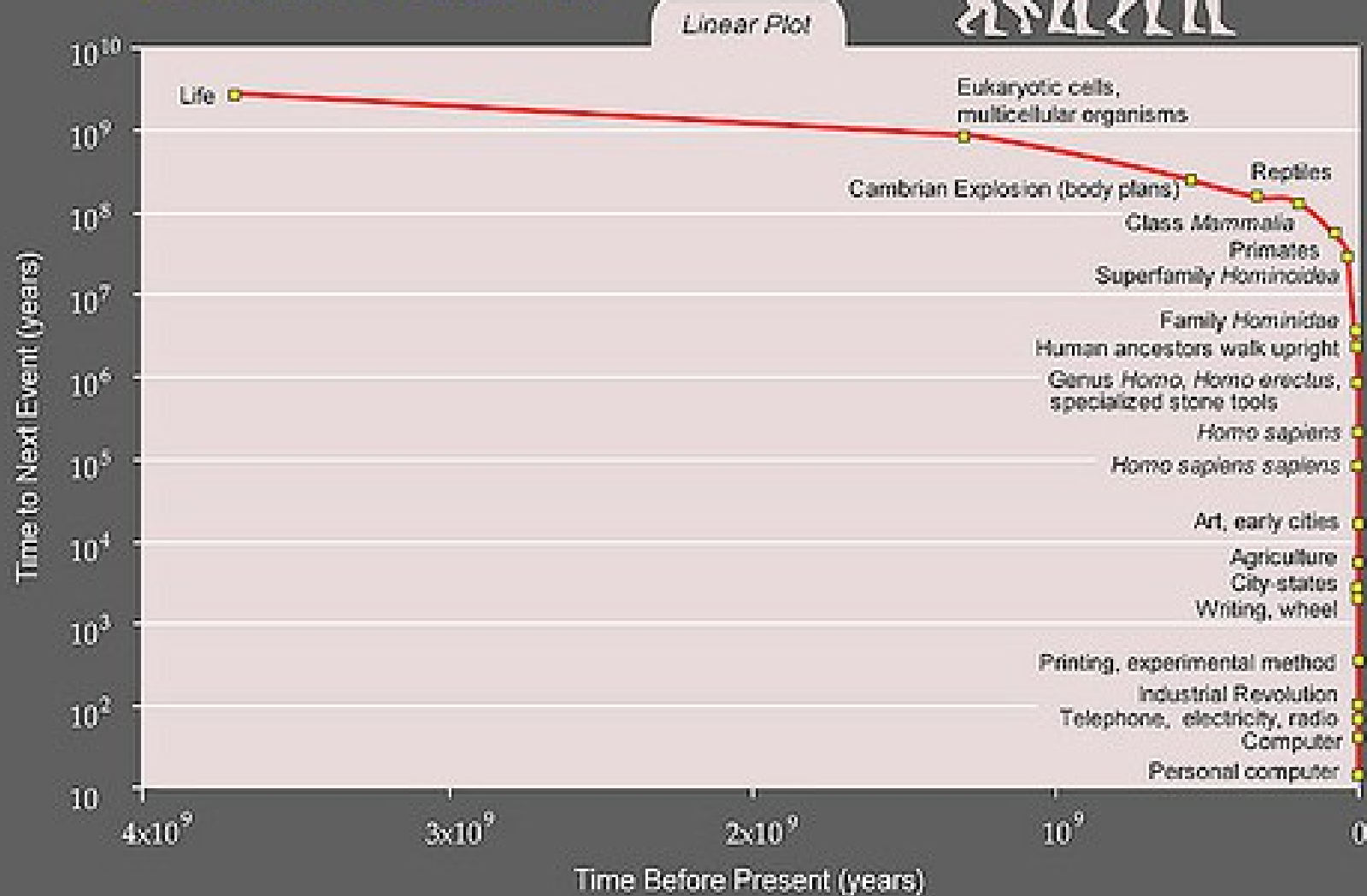
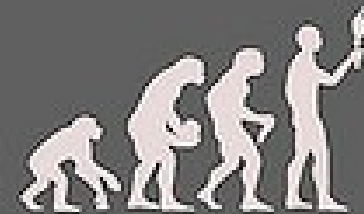
**CONTINUOUS TENSION BETWEEN:**

**INNOVATION & WEALTH CREATION vs  
ECONOMIES OF SCALE**



***SUSTAINABLE????***

## Countdown to Singularity

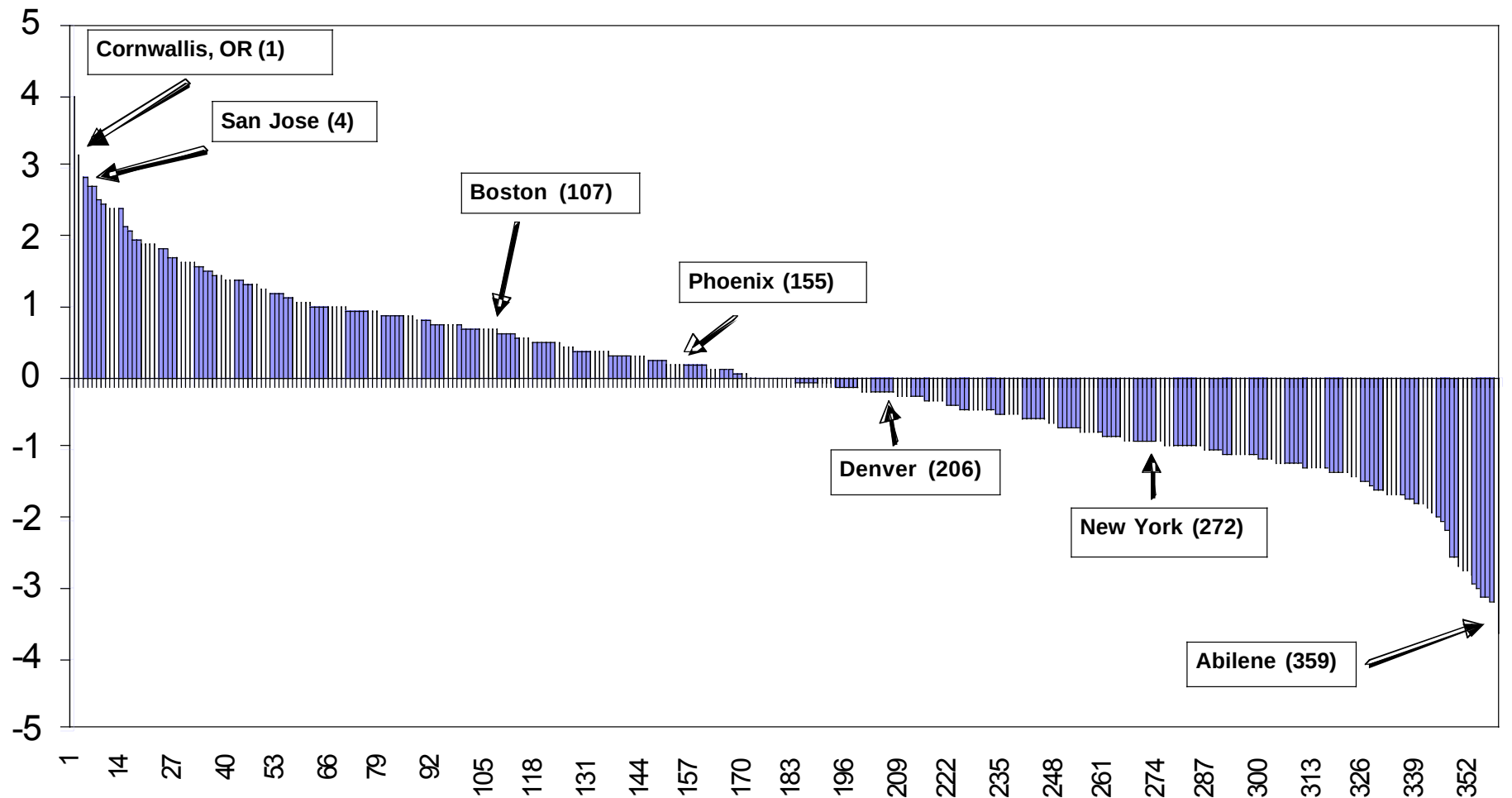


**Average “idealised, universal”  
characteristics of cities and companies of a  
given size (constrained by underlying  
principles and dynamics of network  
structures) as manifested in scaling laws**

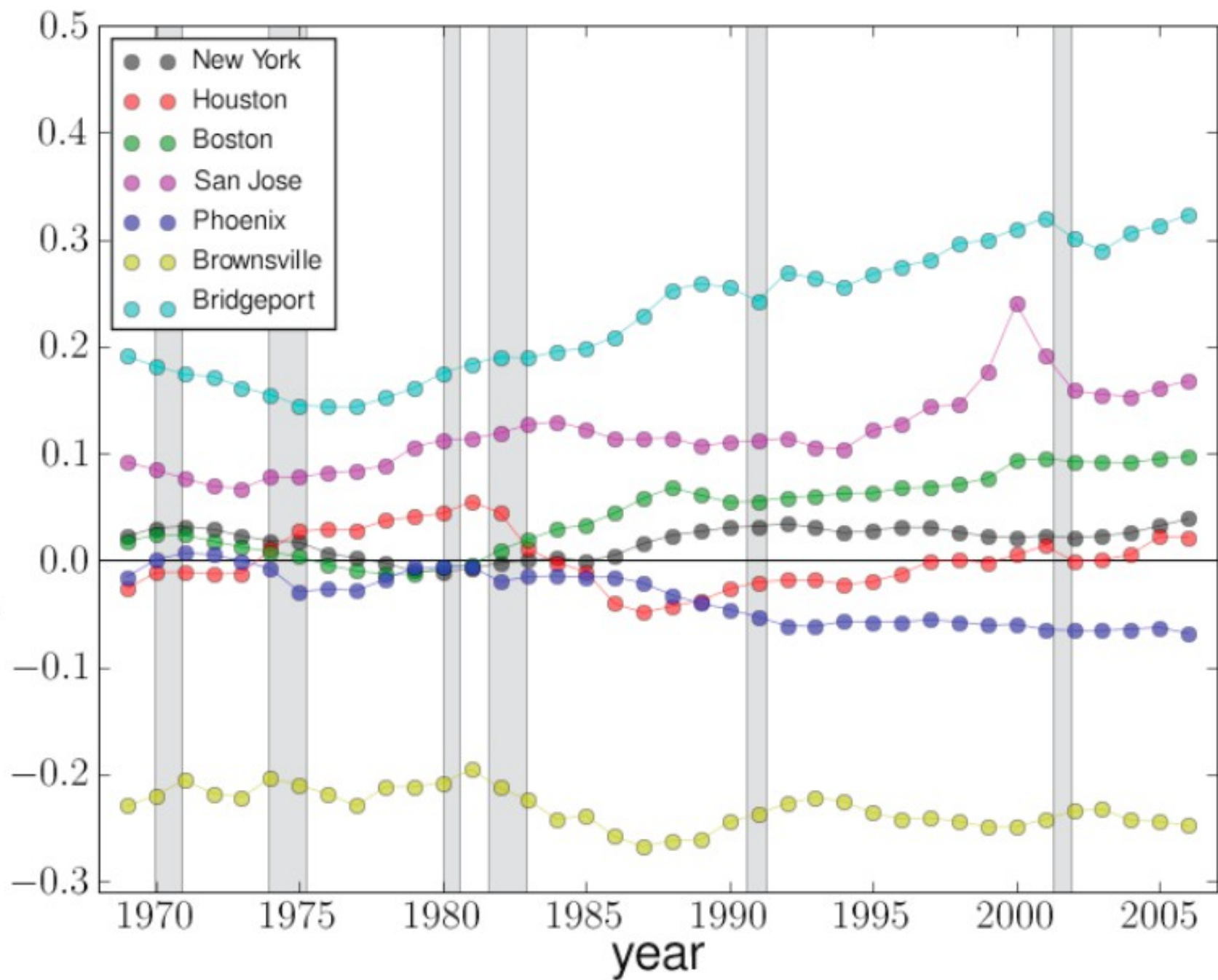
**vs.**

**Characteristics of specific cities and  
companies as measured by their deviations  
from scaling laws representing their  
individuality and local environment and  
conditions**

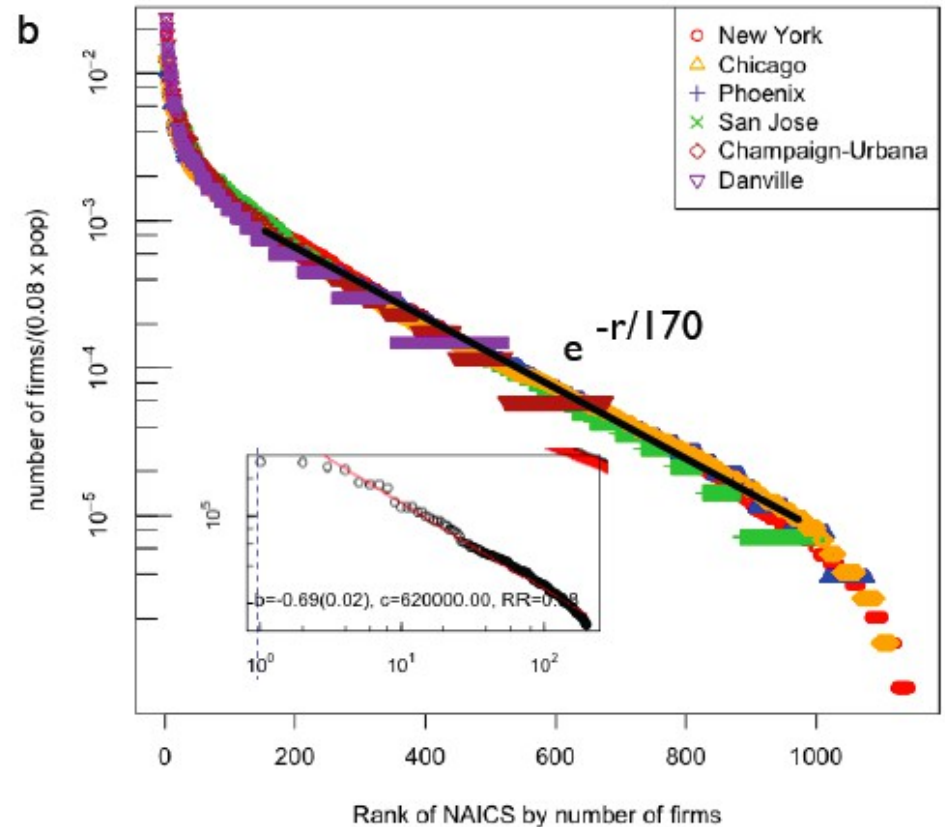
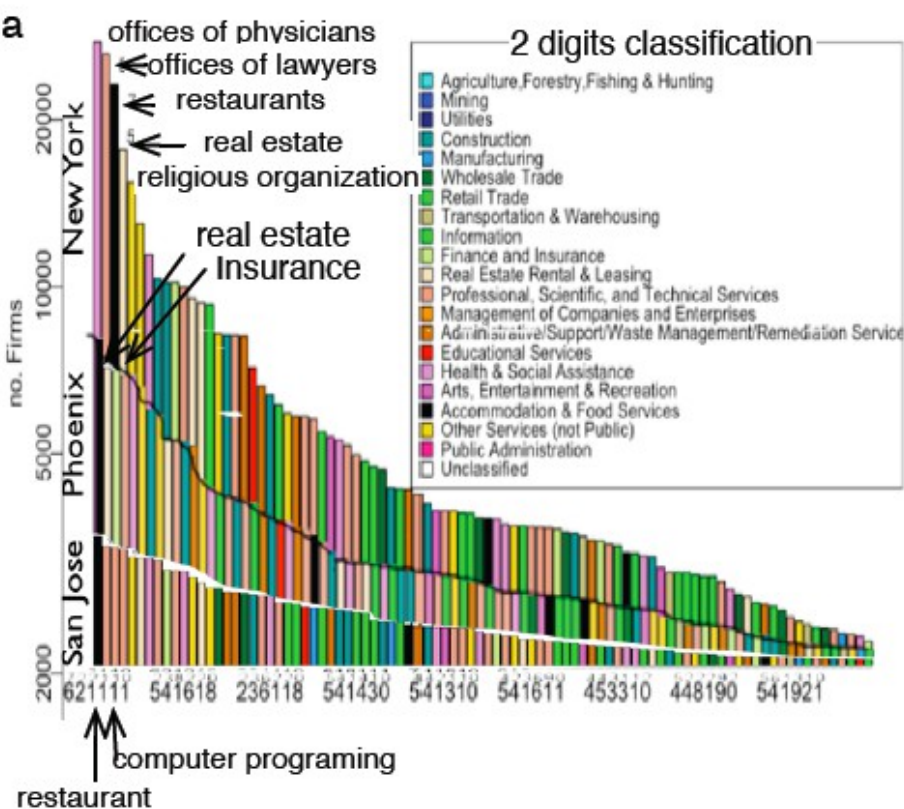
# 2003 Patenting Rankings



Scale Adjusted Urban Indicators



# DIVERSITY OF FIRMS AND OCCUPATIONS



***GDP INCREASES  
EXPONENTIALLY WITH DIVERSITY***

***AND***

***AS A POWER LAW WITH POPULATION SIZE***

***SINCE GDP SCALES WITH SIZE AS***

$$G(N) = G_0 N^b \quad (b \sim 1.15)$$

***THEN:***

$$G(N) = G_1 e^{D/x} \quad (X_0 \sim 211)$$

$$\text{WHERE } G_1 = G_0 N_0^b$$

**IF NUMBER OF ESTABLISHMENTS OF TYPE  $j$   
SCALES AS**

$$n_j \propto N^{\alpha_j}$$

**THEN ITS RANKING SCALES AS**

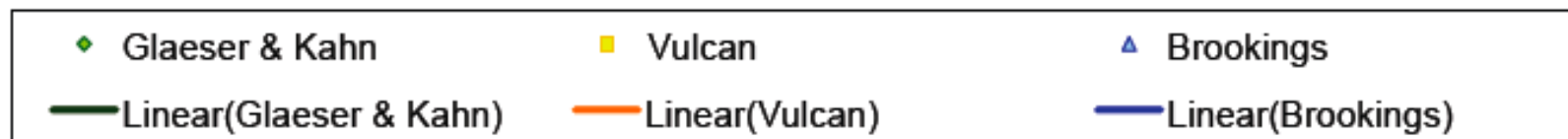
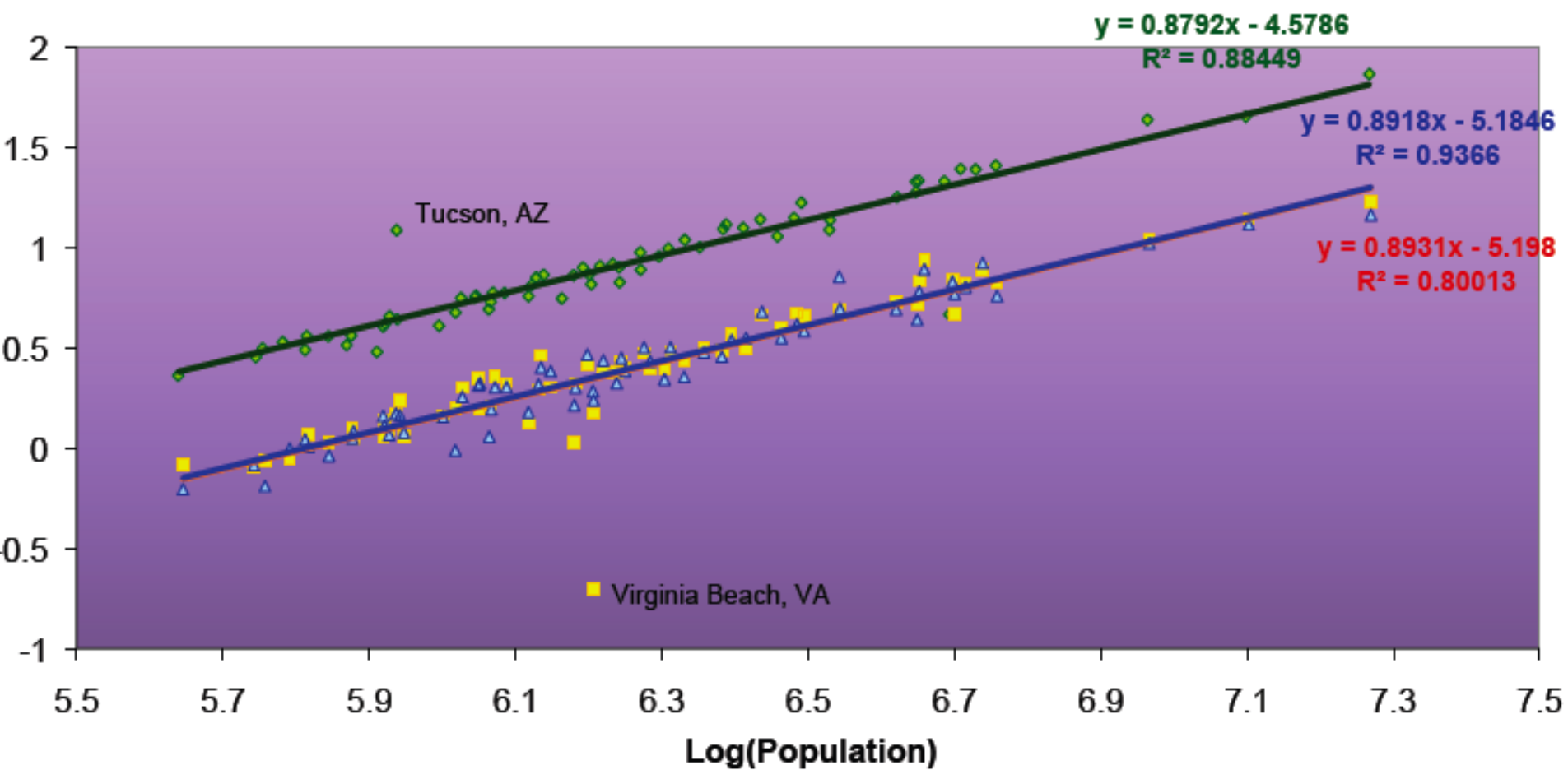
$$x_j \propto N^{(1 - 2\alpha_j)/B}$$

$$x_j \propto (1 - 2\alpha_j) \ln N$$

$$x_j >$$

$$x_0$$

**SO BUSINESS TYPES WHOSE ABUNDANCES  
SCALE SUPER-LINEARLY (PROFESSIONAL,  
SERVICE,.....e.g. LAWYERS, DOCTORS) INCREASE  
IN RANK WITH INCREASING CITY SIZE WHEREAS**



***SOCIO-ECONOMIC QUANTITIES DEPEND ON  
“TWO-BODY” INTERACTIONS (INFORMATION  
EXCHANGE) AND THEREFORE NUMBER AND  
DENSITY OF SOCIAL INTERACTIONS:***

$$Y(N) \propto N_{\text{int}}$$

***[UNLIKE BIOLOGY WHERE  $Y(N) \sim N$ ]***

***IF EVERYONE INTERACTED WITH  
EVERYONE ELSE, THEN***

$$Y(N) \propto N_{\text{int}} \sim N^2$$

***EFFECTIVE INTERACTION SPATIAL AREA  
FOR AVERAGE INDIVIDUAL =  $\varepsilon^2$***

**EACH INDIVIDUAL INTERACTS WITH  $\Delta N$  OTHERS:**

$$\Delta N \approx \rho \varepsilon^2$$

**TOTAL NUMBER OF INTERACTIONS  $\approx N\Delta N \approx N\rho \varepsilon^2$**

**SOCIO-ECONOMIC METRICS**

$$Y(N) \propto (N\Delta N)Y_0 \sim (N\rho\varepsilon^2)Y_0 \sim \frac{N^2}{A}(\varepsilon^2 Y_0)$$

$$Y(N) \sim \left(\frac{\varepsilon^2}{A}\right)N^2 Y_0$$

**IF ROADS, CABLES, ETC ARE SPACE-FILLING  
(THEY SERVICE EVERYONE) WITH TOTAL LENGTH  
 $L$ , THEN**

**AREA**       $A \sim L\varepsilon$

$$\square \quad \frac{\varphi L}{\phi N} \propto \frac{\varphi Y}{\phi N \varepsilon} \propto \int Y_0 \quad \text{INVARIANT!}$$

**IF**

$$L \propto L_0 N^{\gamma_I}$$

$$R \propto R_0 N^{\gamma_{SE}}$$

**WITH**

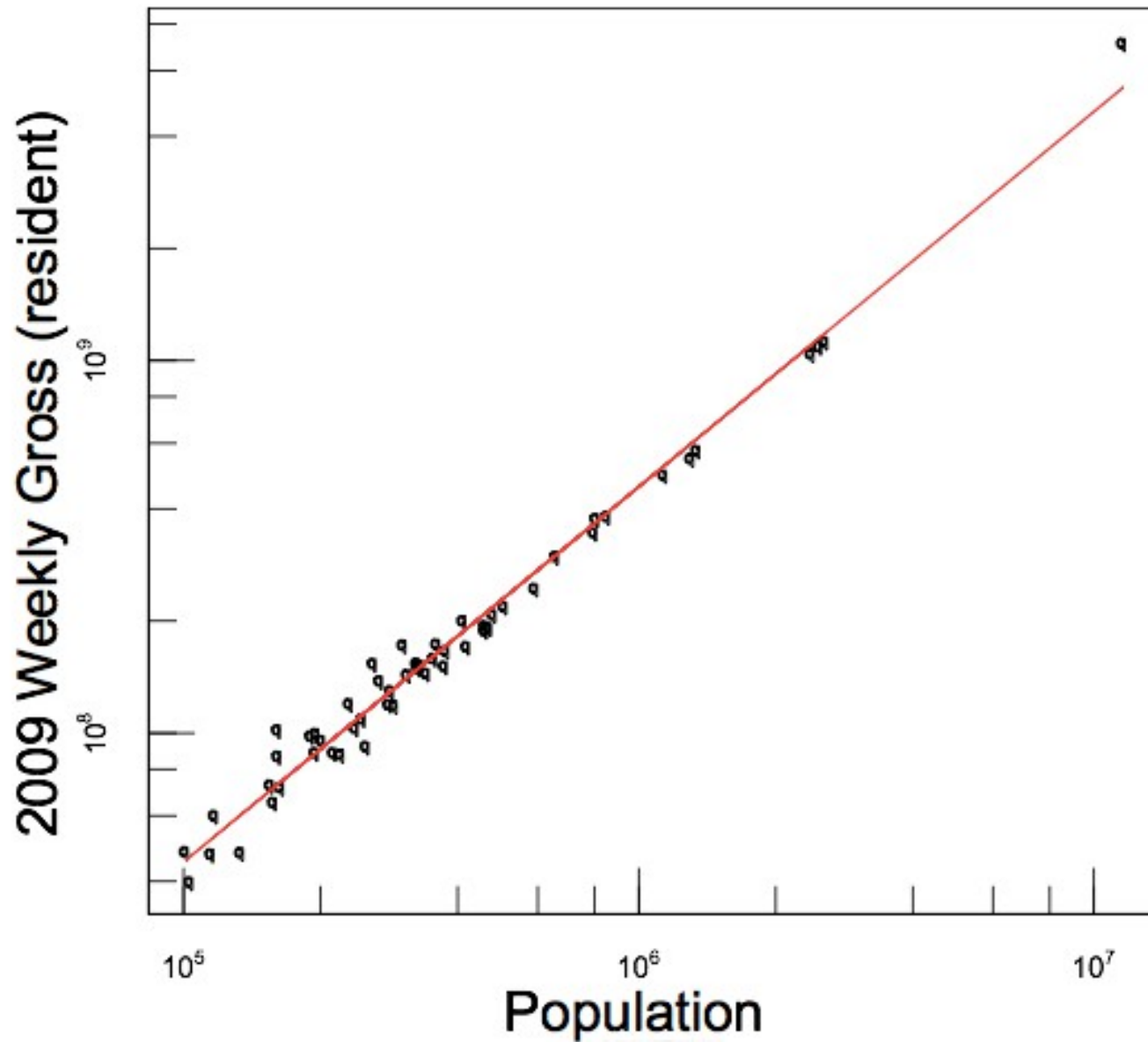
$$\beta_I = 1 + \varepsilon_I$$

$$\beta_{SE} = 1 + \varepsilon_{SE}$$

**THEN**

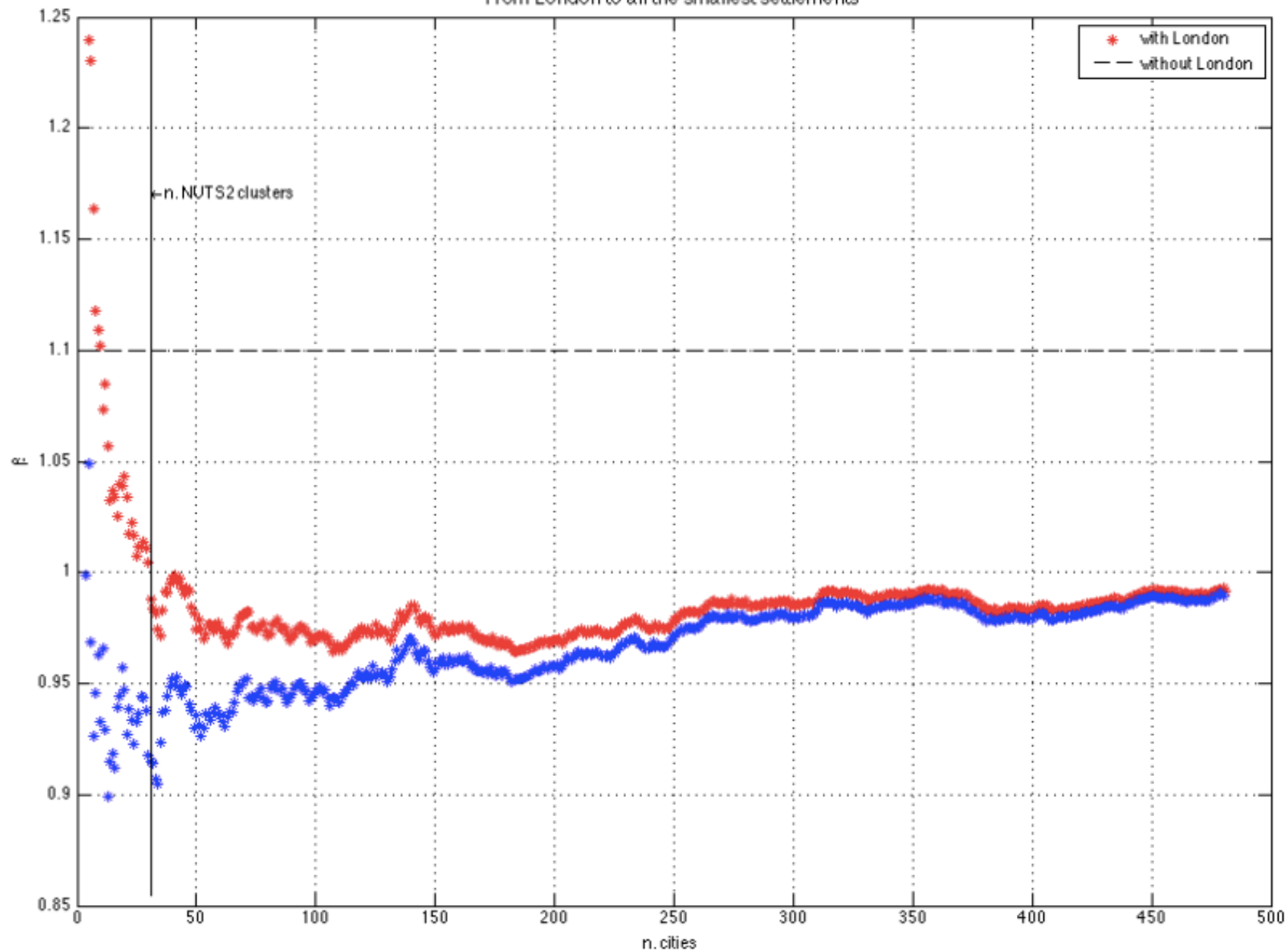
$$\varepsilon_I = \varepsilon_{SE} \quad (\sim 0.15)$$

**→ CAN DETERMINE THE SOCIAL INFORMATION NETWORK SCALING FROM**



E. Arcaute, E. Hatna, P. Ferguson, H. Youn, A. Joahnason & M. Batty  
(submitted)

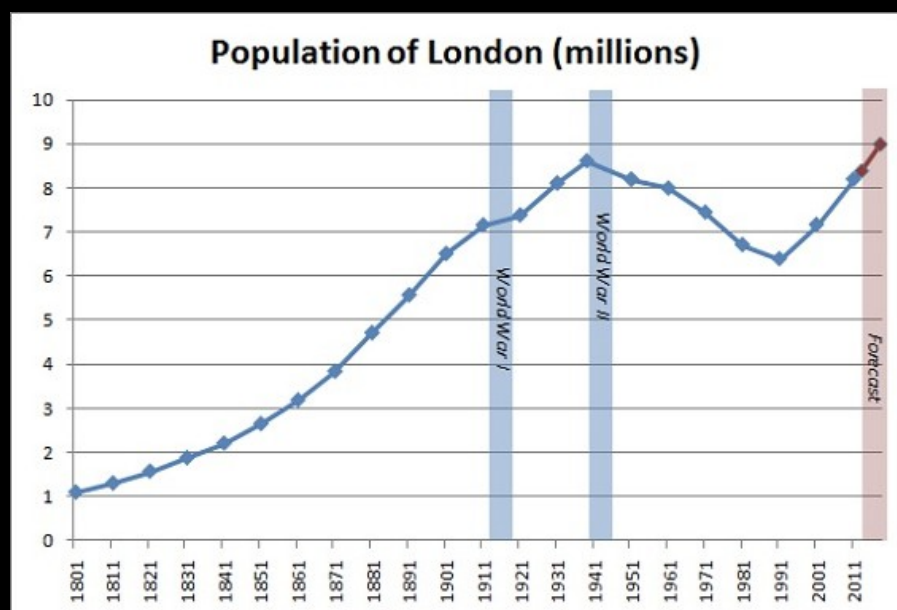
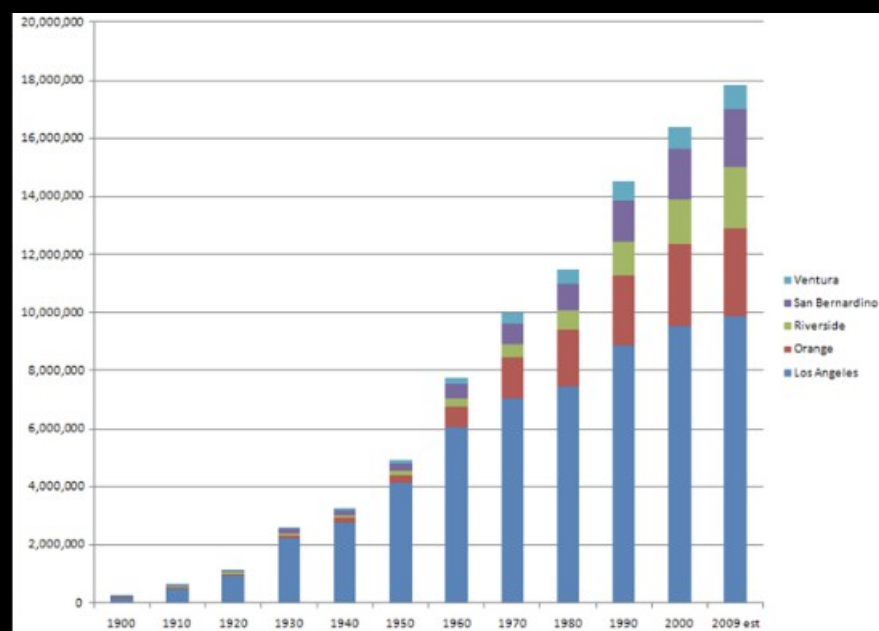
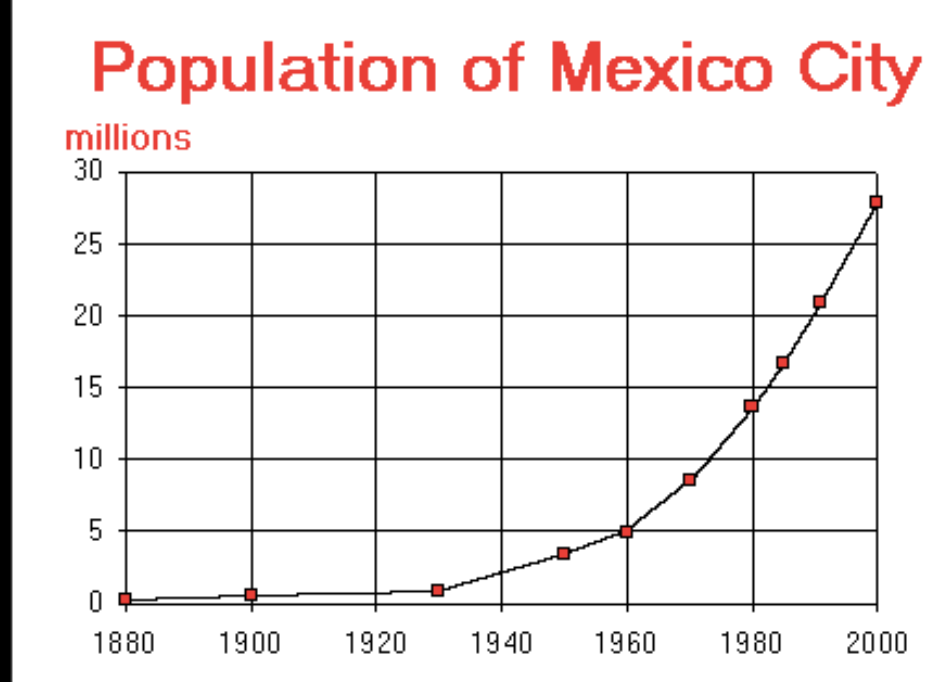
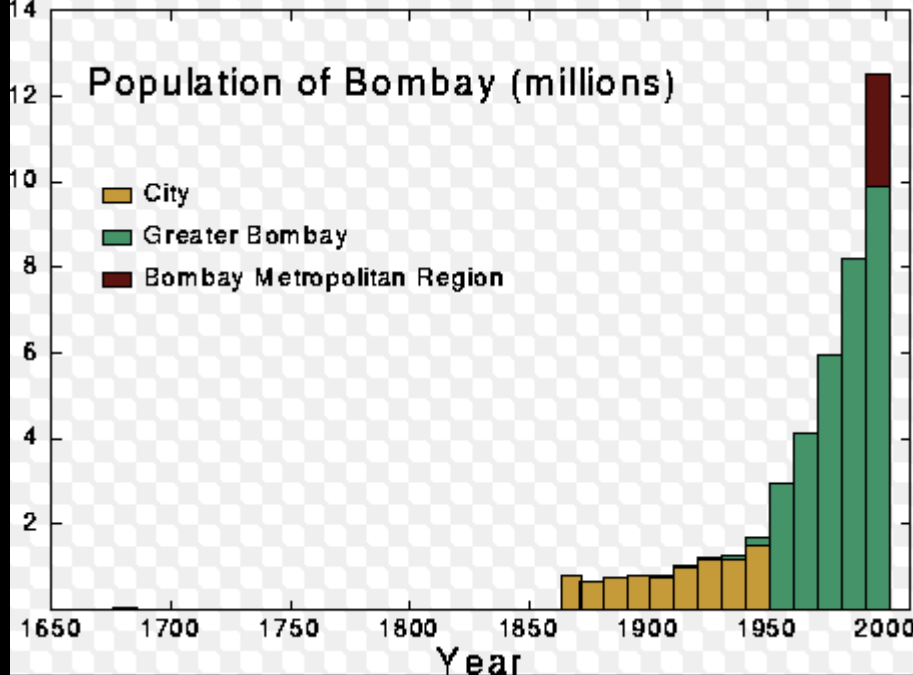
From London to all the smallest settlements



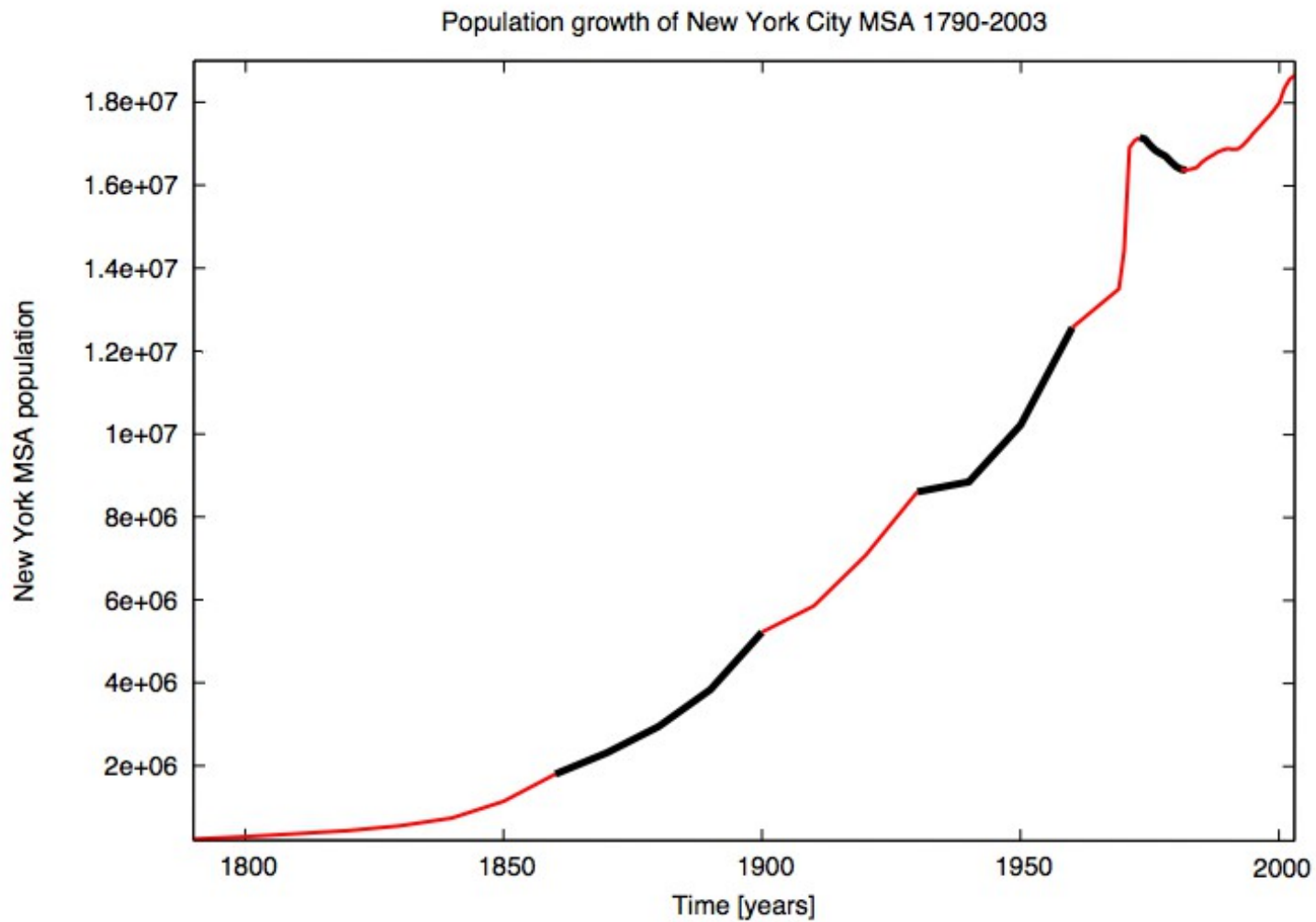
# The Editors: Is London's success causing the UK a problem?

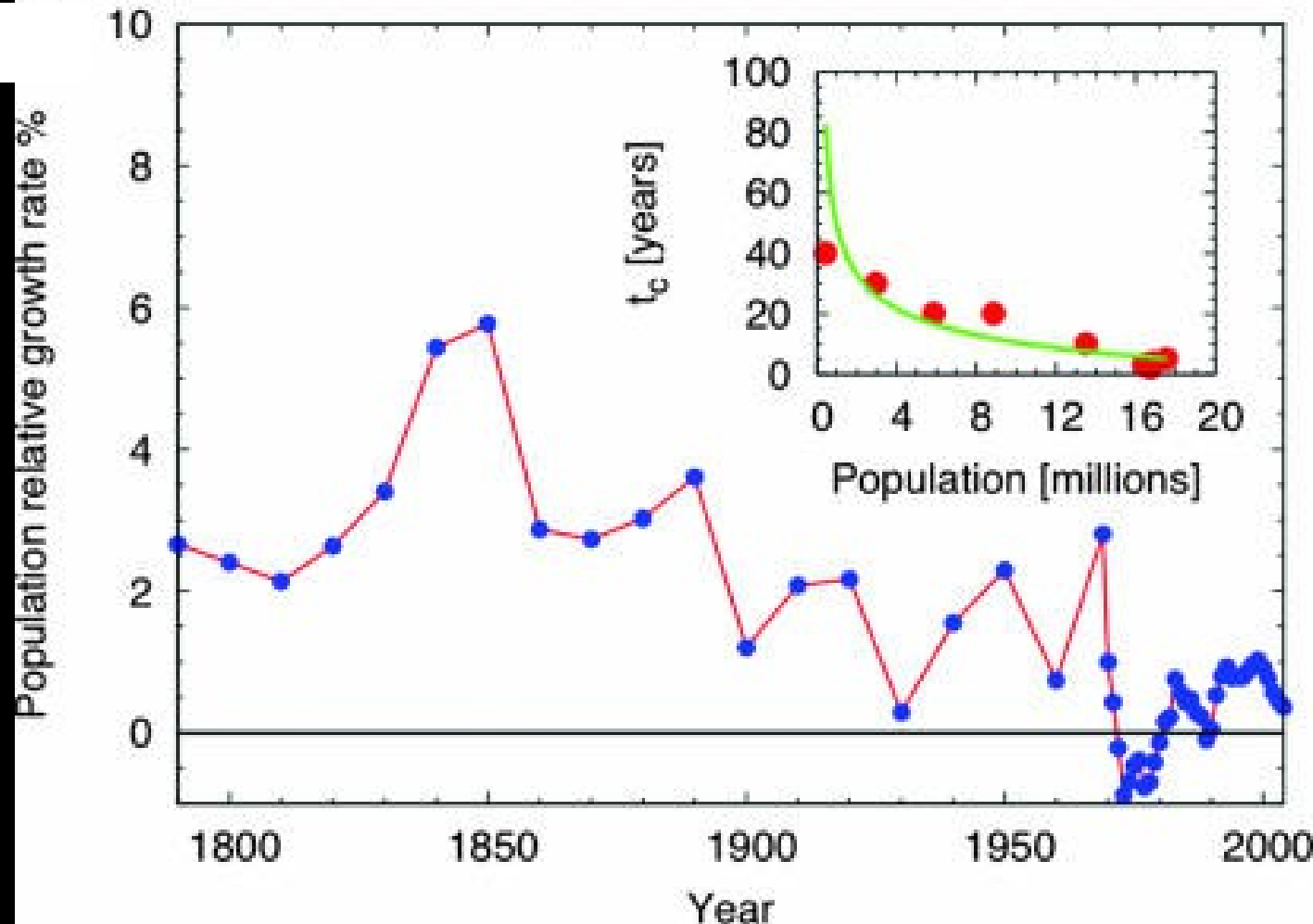
The BBC's Economics Editor, Stephanie Flanders, visits London, Birmingham and Manchester and discovers wide discrepancies between the capital's economy and the rest of the country.

She says London's ebullient economy is subsidising other parts of the country but there is a lot of resentment in other big provincial cities.



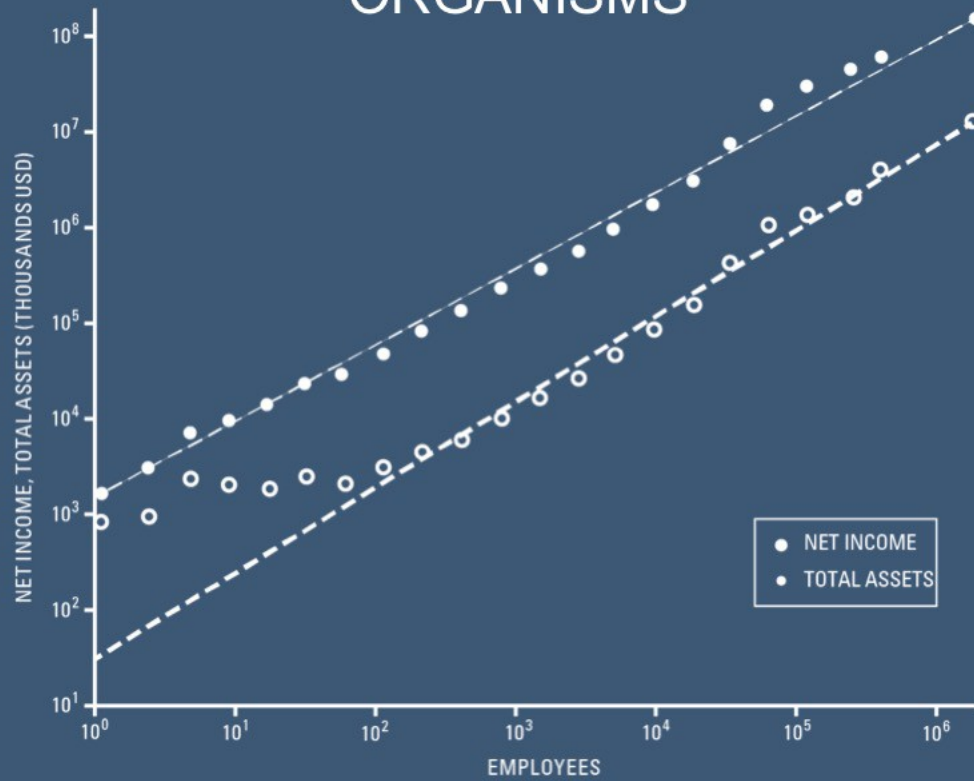
# 1790 - 2003

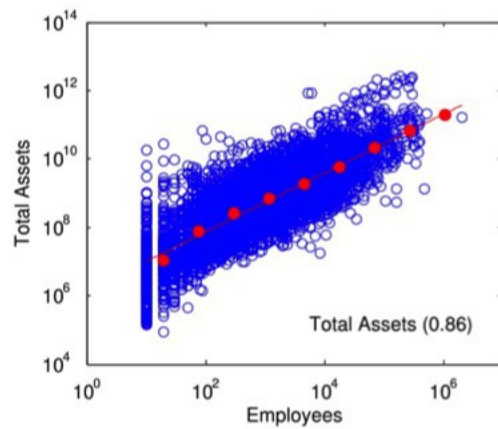
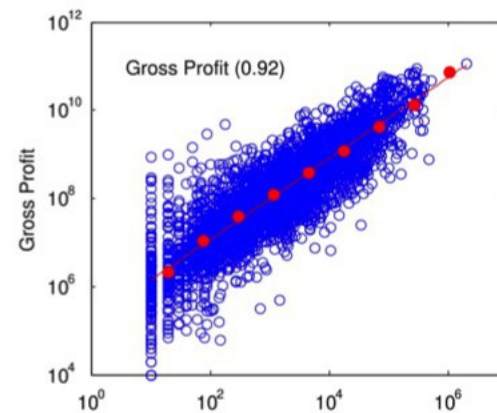
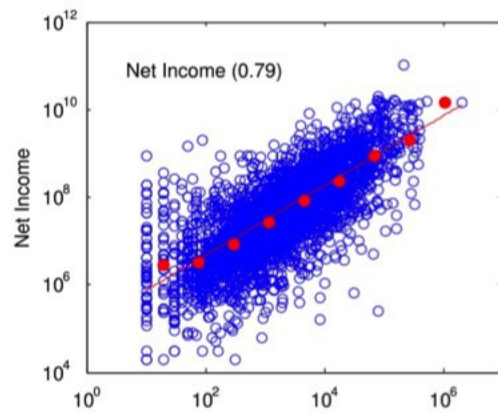




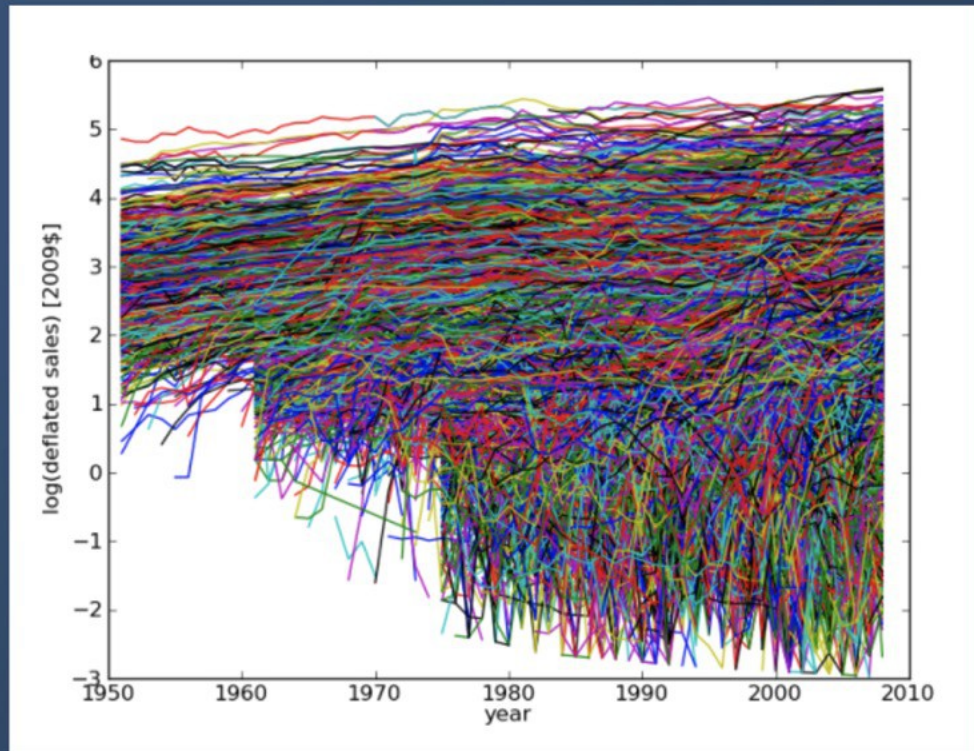
**Successive cycles of superlinear innovation reset the singularity and postpone instability and subsequent collapse.** The relative population growth rate of New York City over time reveals periods of accelerated (super-exponential) growth. Successive shorter periods of super exponential growth appear, separated by brief periods of deceleration. (Inset)  $t_c$  for each of these periods vs. population at the onset of the cycle. Observations are well fit with  $\beta = 1.00$  (green line)

# COMPANIES SCALE SUB-LINEARLY ANALOGOUS TO ORGANISMS

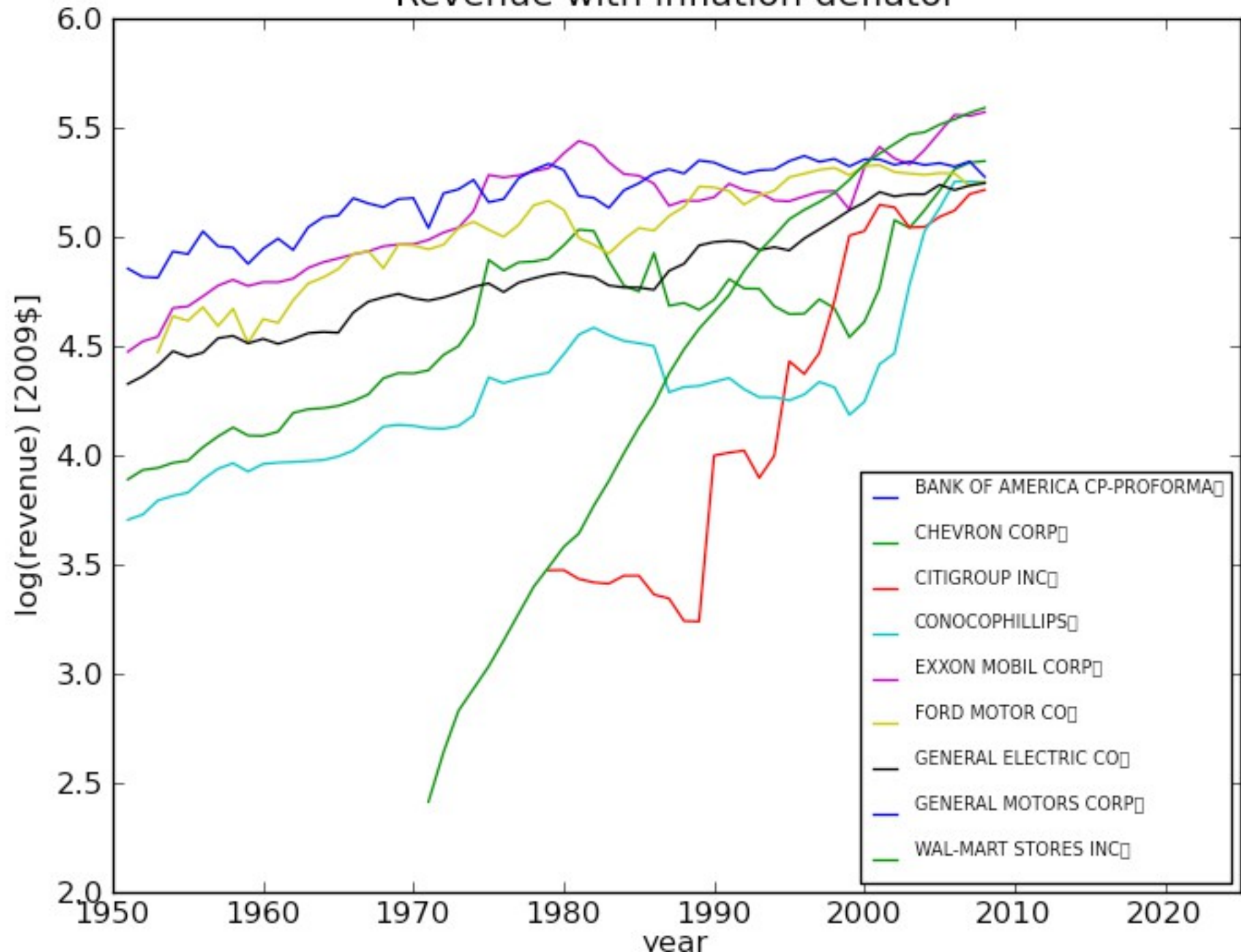




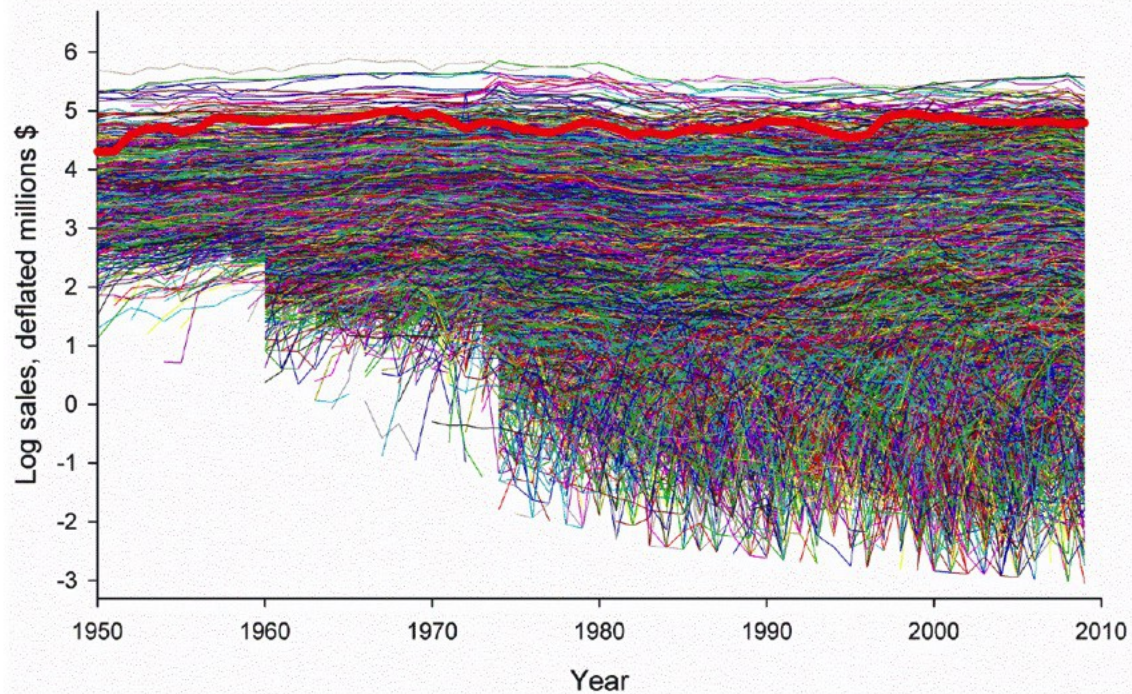
## GROWTH OF FIRMS



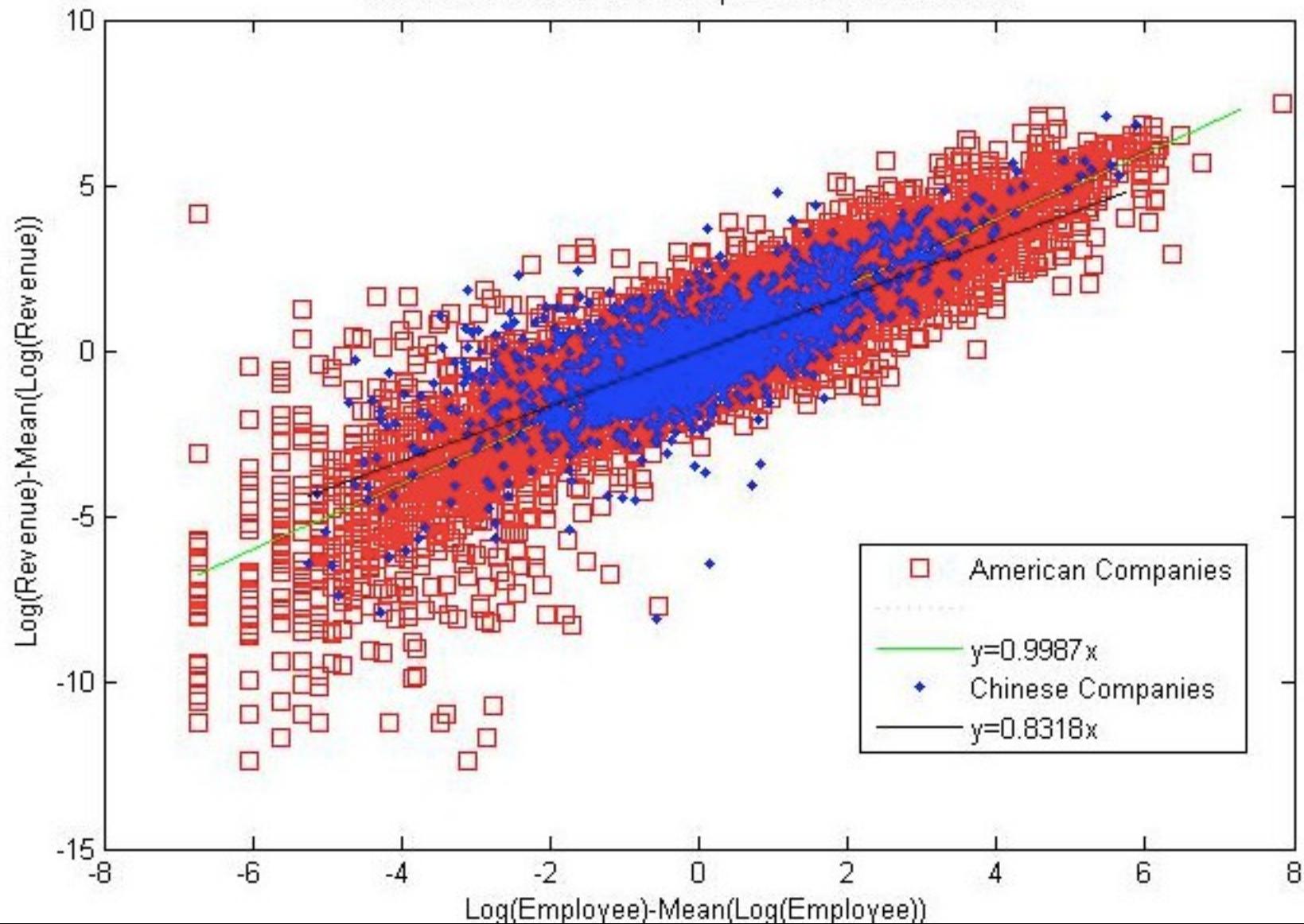
# Revenue with inflation deflator



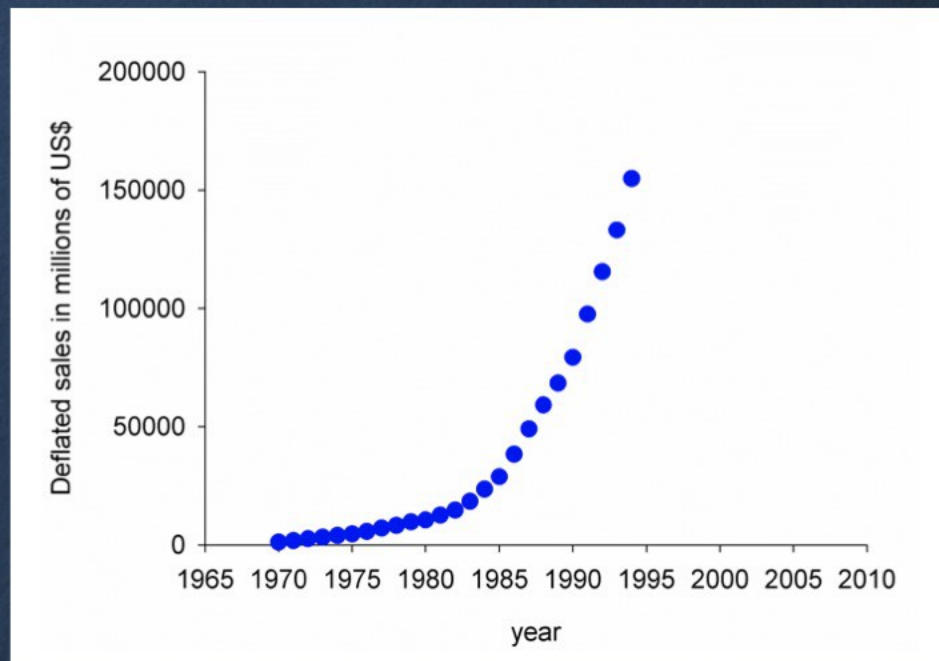
## GROWTH OF FIRMS RELATIVE TO GDP



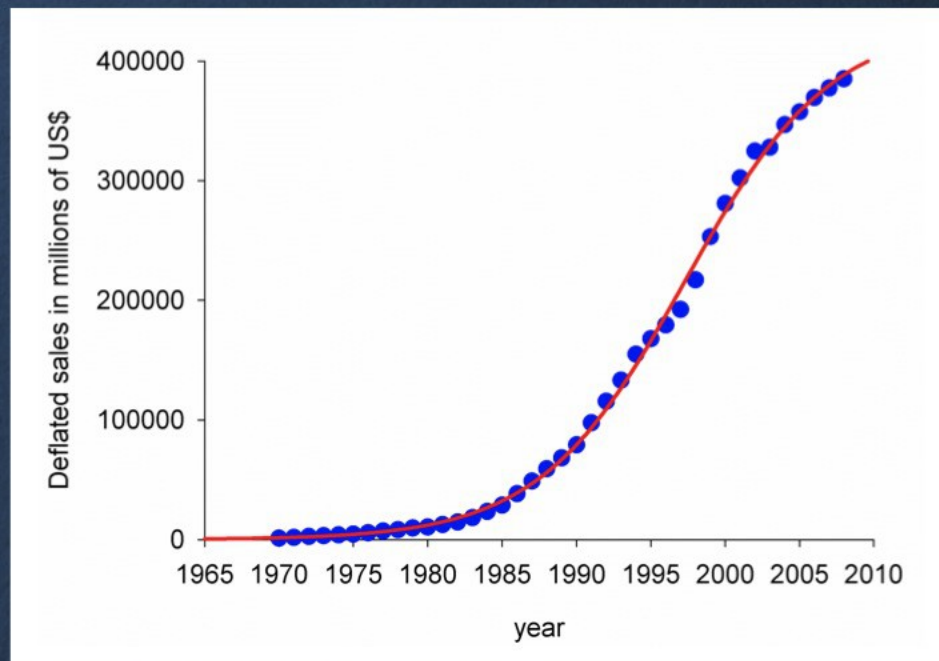
American and Chinese Companies Normalized Data



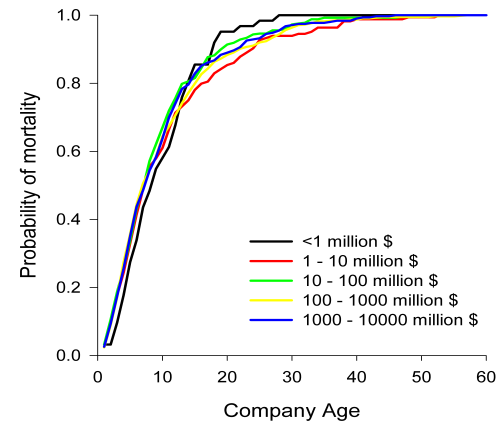
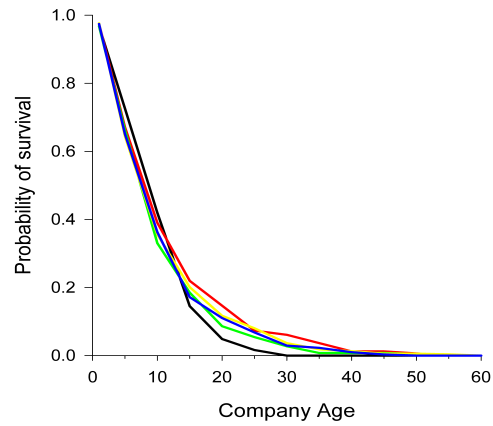
## WALMART, 1970-1994



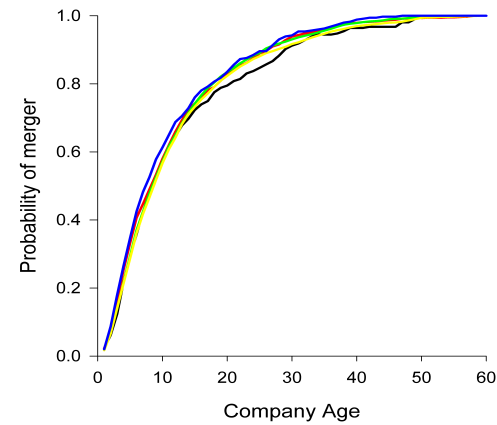
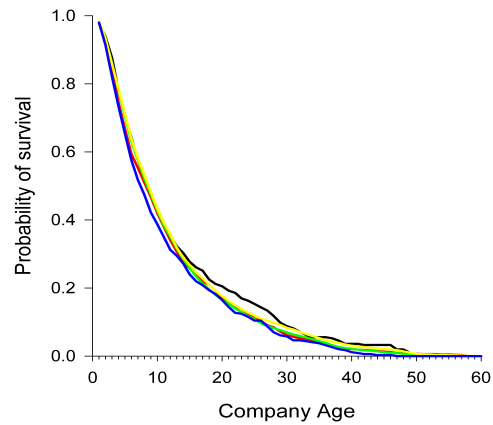
## WALMART, 1970-2008

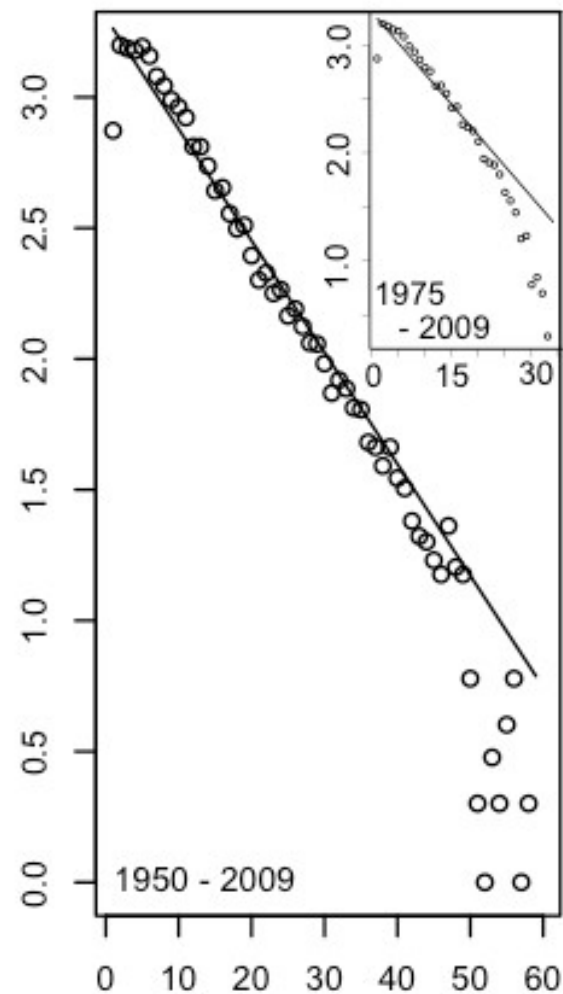
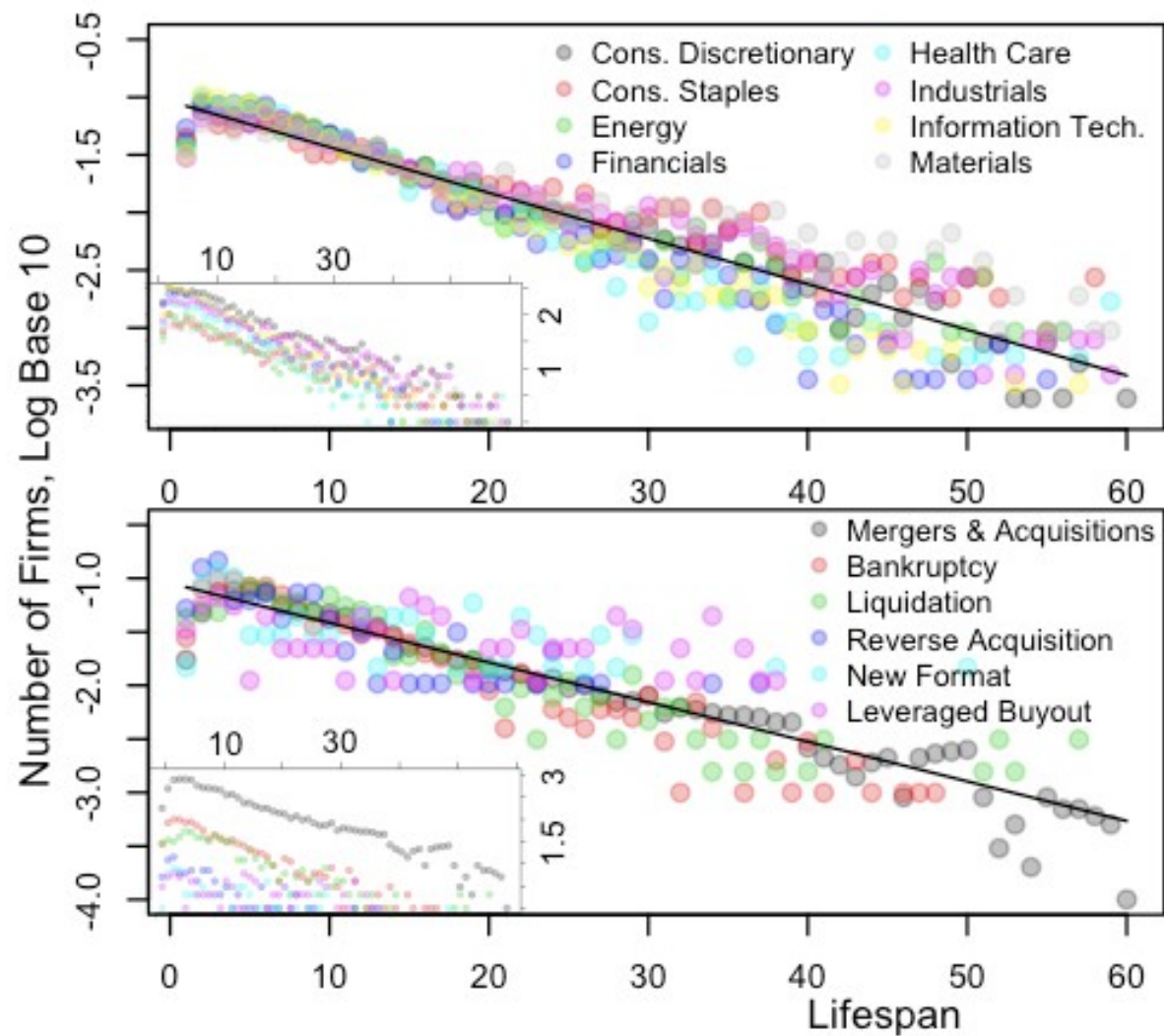


Bankrupt or liquidation



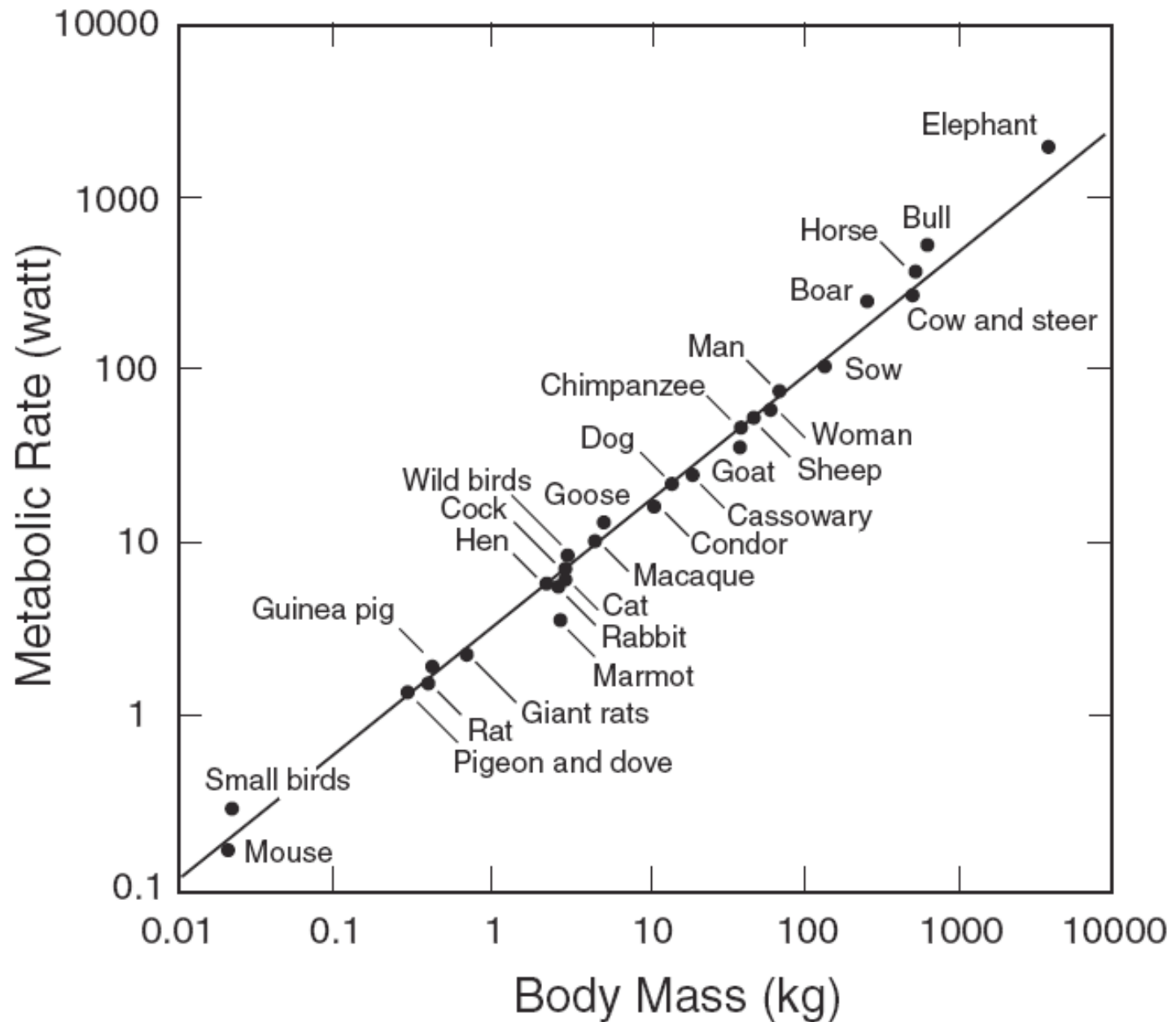
Acquisition or merger



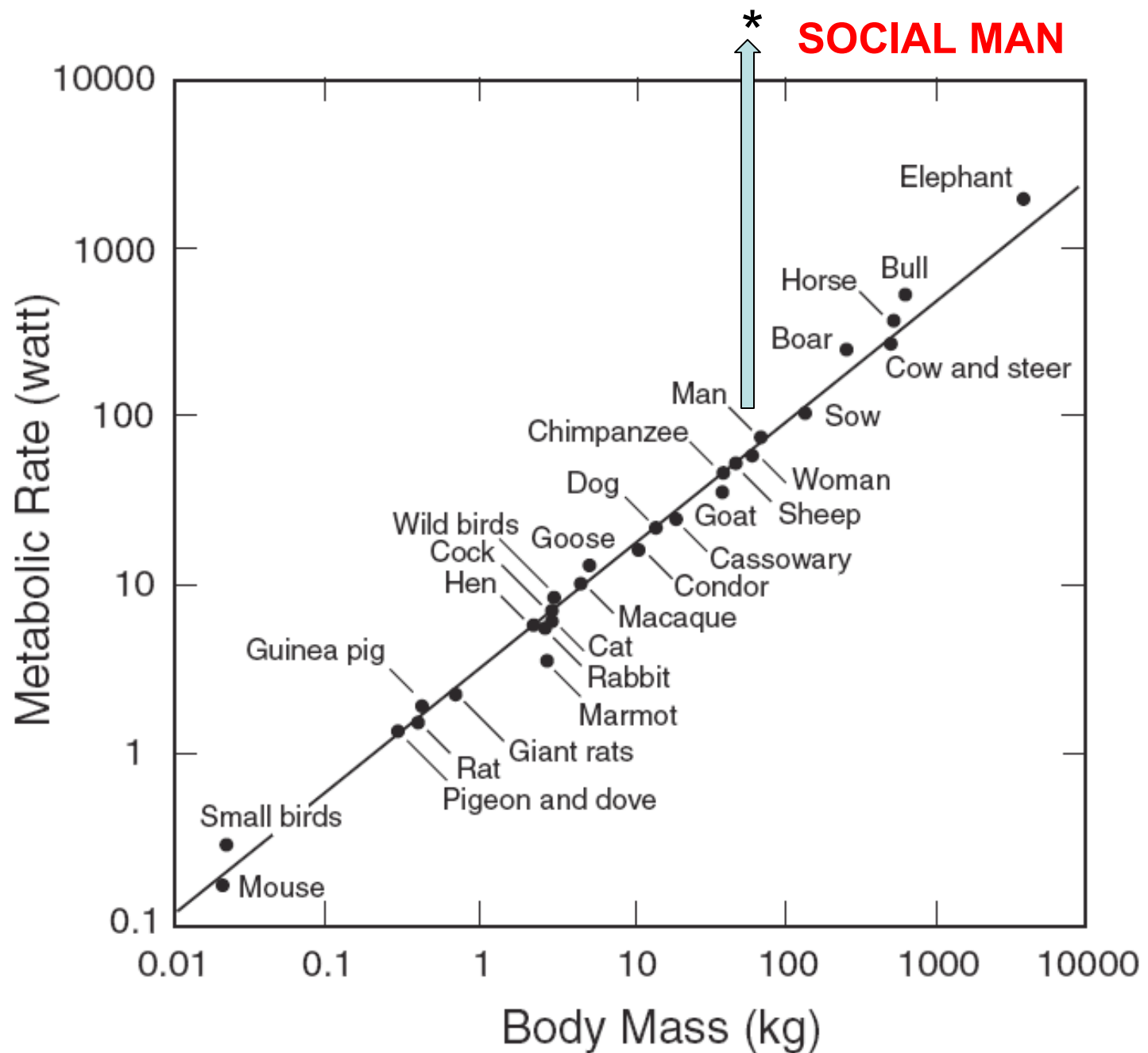


Our “natural” metabolic rate ~90 *watts*

Our social metabolic rate ~11,000  
*watts*



**SLOPE =  $\frac{3}{4} < 1$ ; SUB-LINEAR; ECONOMY OF SCALE**



**SLOPE =  $\frac{3}{4} < 1$ ; SUB-LINEAR; ECONOMY OF SCALE**

We are equivalent to a  
*30,000 kg Gorilla*

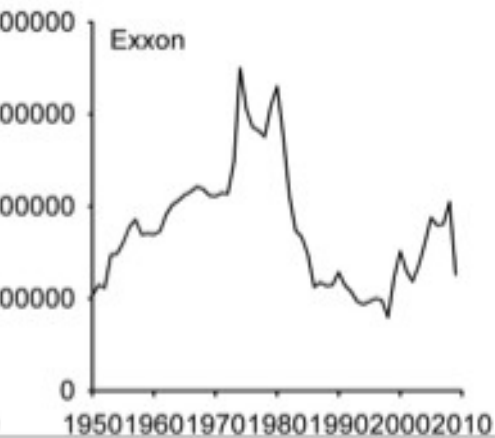
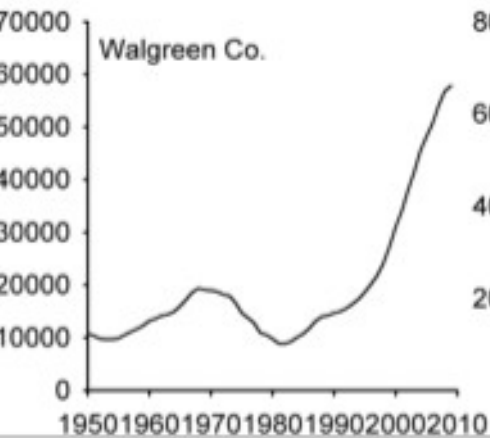
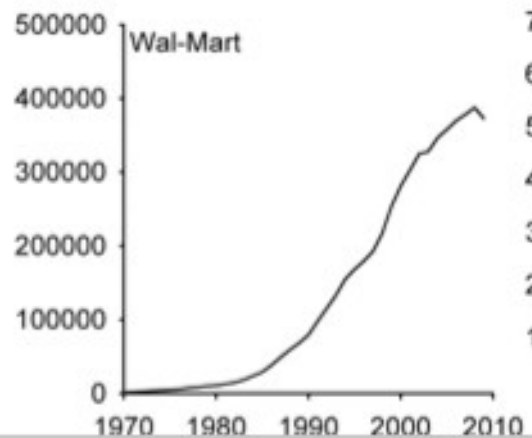
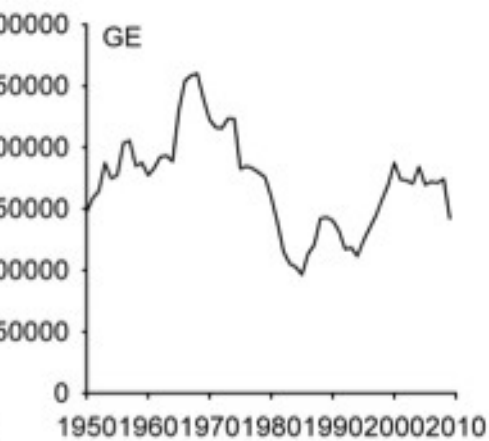
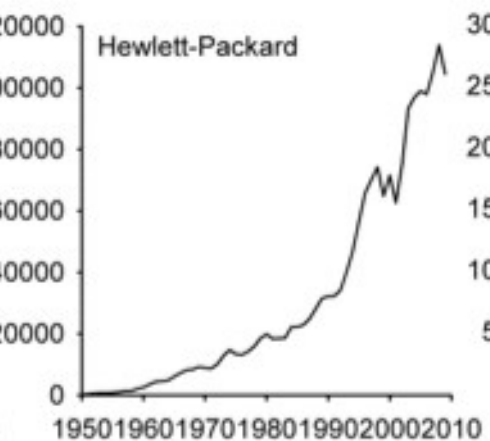
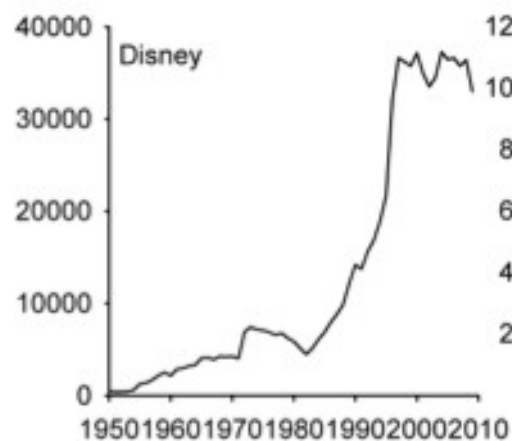
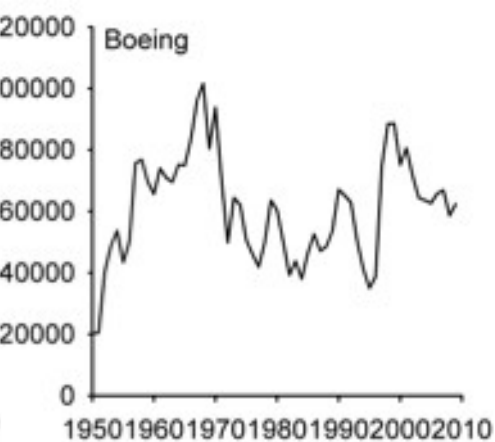
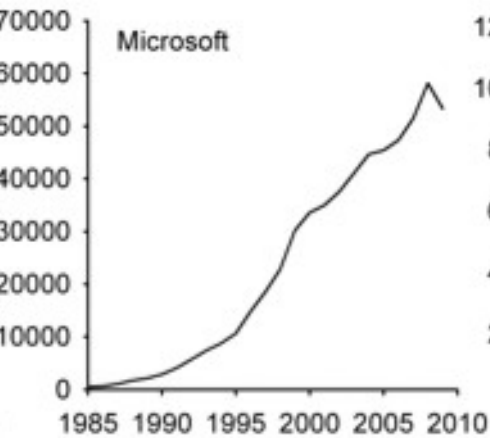
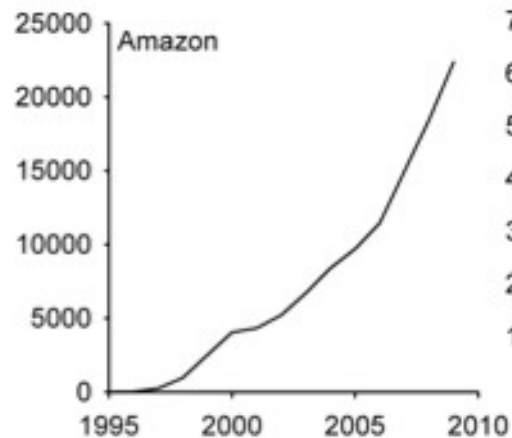




# *12 Elephants*



Sales, \$millions



***NEED A NEW PARADIGM, A NEW  
INTEGRATED CONCEPTUAL  
FRAMEWORK:***

***SYSTEMIC, HOLISTIC, QUANTITATIVE,  
MECHANISTIC, COMPUTATIONAL,  
PREDICTIVE***

***COUPLED WITH, INSPIRED BY,  
MOTIVATED BY, INSPIRING AND  
MOTIVATING,  
“BIG DATA”***

**BUT MINDLESS BIG DATA IS  
(PROBABLY) BAD AND EVEN  
DANGEROUS**

**WITHOUT SOME CONCEPTUAL  
FRAMEWORK**

**HOW MUCH, WHERE, WHEN, WHAT, WHY?**

**.....AND THERE IS NO VIRGIN  
DATA**



# Big Data Needs a Big Theory to Go with It

Just as the industrial age produced the laws of thermodynamics, we need universal laws of complexity to solve our seemingly intractable problems

By Geoffrey West

As the world becomes increasingly complex and interconnected, some of our biggest challenges have begun to seem intractable. What should we do about uncertainty in the financial markets? How can we predict energy supply and demand? How will climate change play out? How do we cope with rapid urbanization? Our traditional approaches to these problems are often qualitative and disjointed and lead to unintended consequences. To bring scientific rigor to the challenges of our time, we need to develop a deeper understanding of complexity itself.



[Pin it](#)

Image: Eva Vazquez

# **Creative destruction is the essential fact about Capitalism**

**Joseph Schumpeter**



**Creative destruction is  
the essential fact about  
Capitalism**

**All successful people are  
standing on ground that  
is crumbling beneath  
their feet**

**Joseph Schumpeter**



# THE SINGULARITY IS NEAR!

The ever accelerating progress of technology....gives the appearance of approaching some essential singularity in the history of the race beyond which human affairs, as we know them, could not continue.



**John von  
Neumann  
(1903 - 1957)**



**Jim Brown (UNM)**



**Brian Enquist (U of Arizona)**



**Woody Woodruff (LANL)**



**Van Savage (UCLA)**



**Jamie Gillooly (U of Florida)**



**Drew Allen (MacQuarie U )**



**Chen Hou (MissouriTech)**



**Melanie Moses (UNM)**



**Alex Herman (UCSF)**



**Ric Charnov (UNM)**



**Chris Kempes (SFI)**



**Wenyun Zhuo (UNM)**



**Luis Bettencourt (Chicago)**



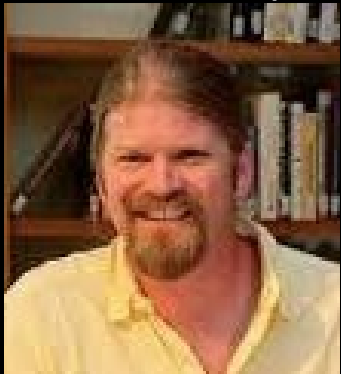
**Jose Lobo (U of Arizona)**



**Debbie Strumsky (U of  
Arizona)**



**Hyejin Youn (Oxford  
U)**



**Marcus Hamilton  
(U of Missouri)**



**Madeleine Daep (MIT)**



**Markus Schlapfer (ETH Zurich)**



**Carlo Ratti (MIT)**



**David Lane (U of Reggio)**



**Sander van der Leeuw  
(U of Arizona)**



**Denise Pumain (U  
of Paris)**



**Dirk Helbing (ETH Zurich)**



