

# An evolutionary view of technological progress

Complex systems summer school  
July 2, 2014

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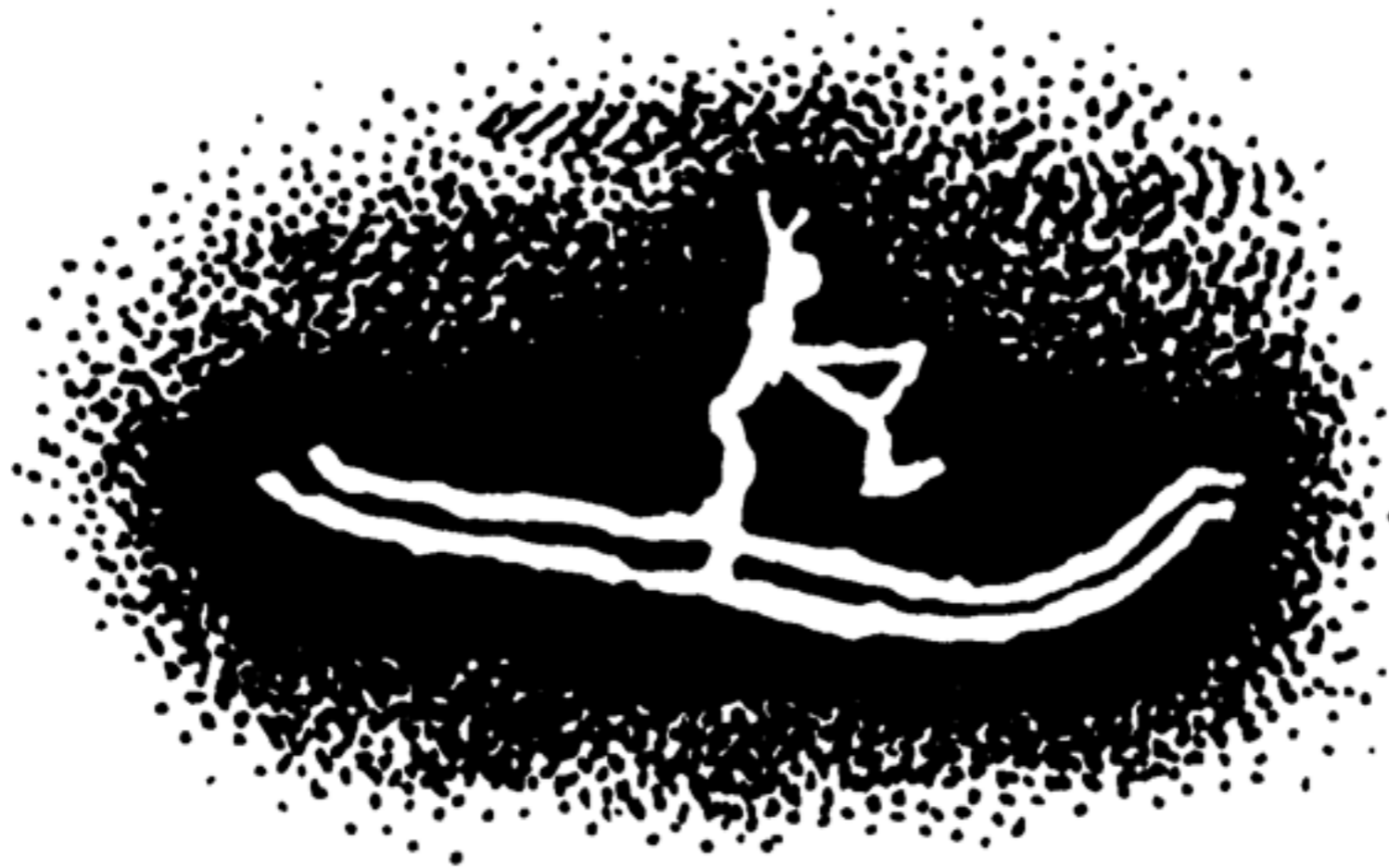
External Professor, Santa Fe Institute



# Outline of talk

- Motivation
  - technology as evolutionary process
  - understanding economic growth
  - economic mitigation of global warming
- Empirical study of rates of technological change
- Empirical laws for technological improvement
  - Wright's law
  - Moore's law
- Recipe model for Wright's law
- Analogy to autocatalytic networks
- Evolutionary extension of Leontief model
- Patent history as fossil record

Technology is an  
evolutionary process

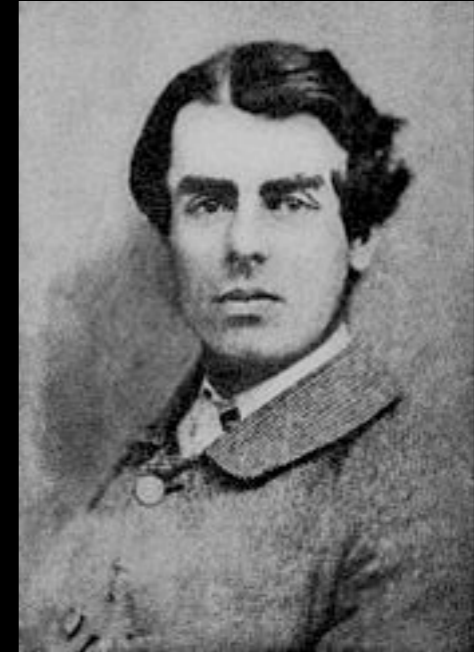


Man on skis. Stone Age rock carving, north Norway

Technology co-evolved with genus homo  
First tool use, homo habilis, 2.3M yrs ago.

# Darwin among the machines

(Samuel Butler, 1863)

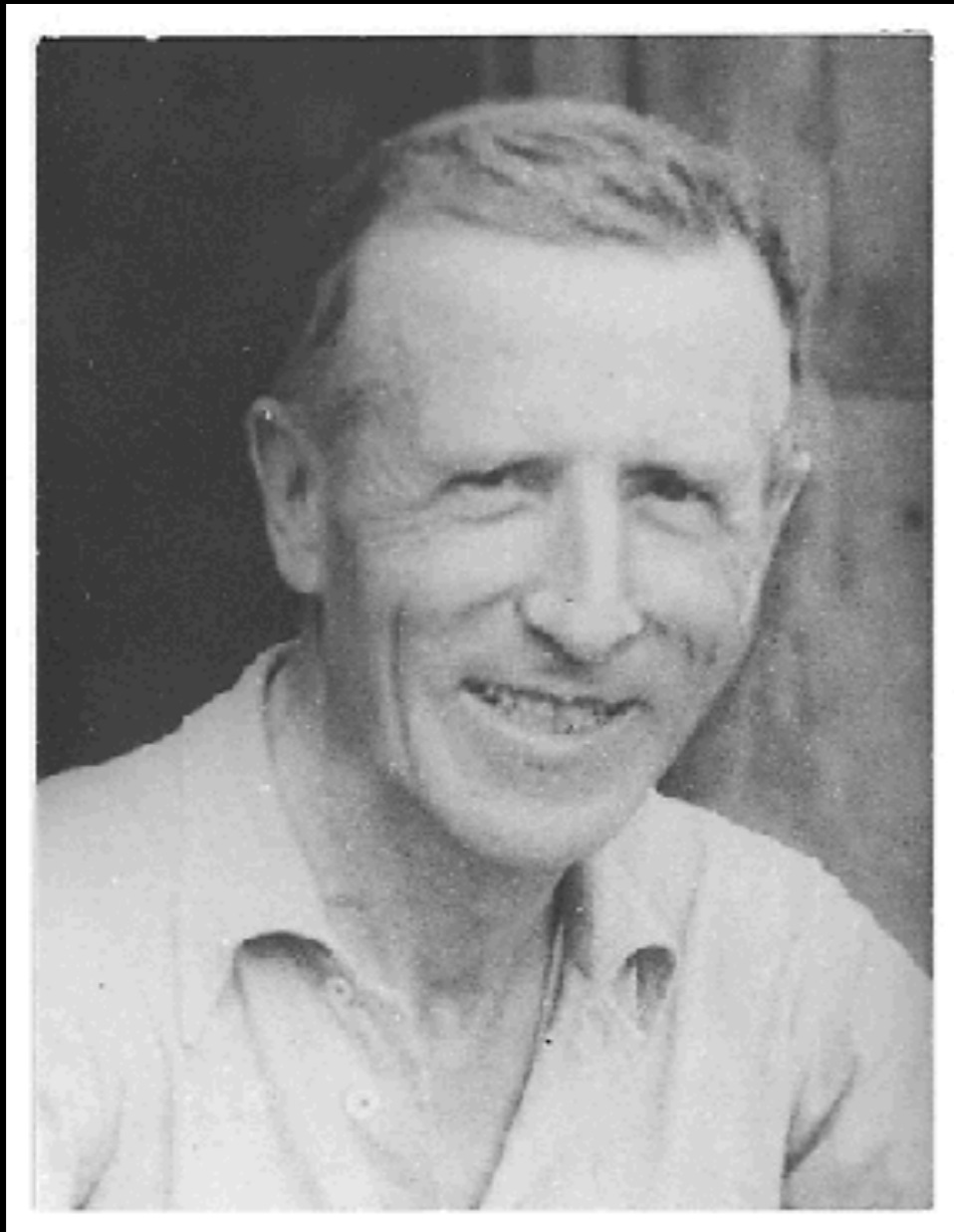


- Evolution in the Darwinian sense:
  - descent with modification and selection
  - Erewhon (1872)

Butler imagined technology  
competing with humanity:

“In the course of the ages we shall  
find ourselves the inferior race”

# The noosphere



Pierre Teilhard de Chardin

- de Chardin envisioned biology, technology and culture co-evolving to form a greater whole, the “noosphere”.

# Biological vs. technological evolution

(see Sole et al., Complexity, 2012)

- Similarities

- Both driven by selection
- Both result in diversity
- Incremental variation
- Temporal progression
- Purposeful function of units

- Differences

- Self-reproduction vs. artificial manufacture
- Random variation vs. conscious design
- Microscopic vs. macroscopic scale of organization
- Innovation in technology analogous to horizontal gene transfer (like bacteria)
- Developmental process of technology is highly distributed

# What drives economic growth?

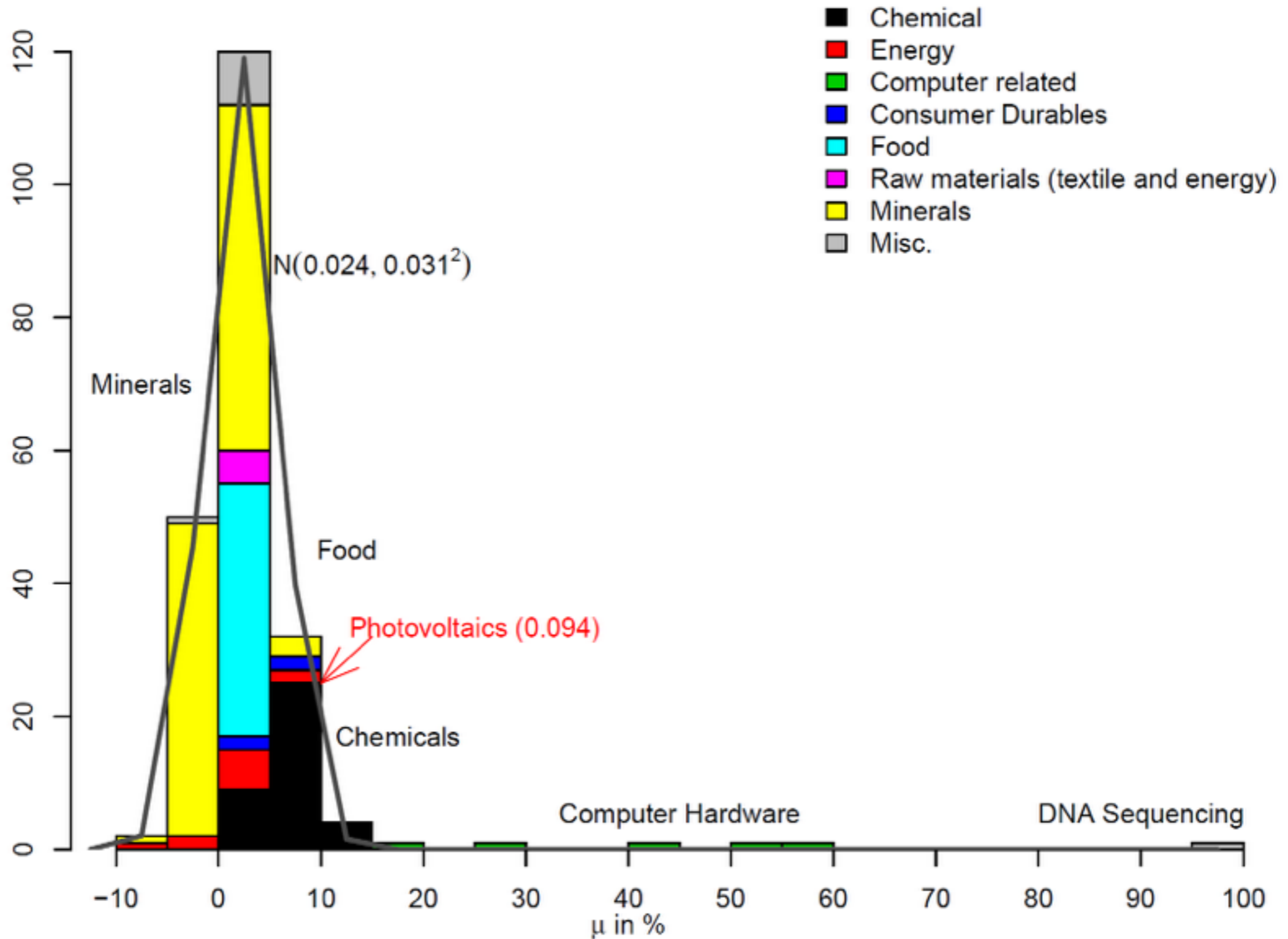
- Solow (1956): Investment can't explain it — technological progress is dominant cause
  - technology is just a scalar “ $A(t)$ ”.
- Rosenberg: Must get inside the black box.
- Arthur: Emphasizes role of combination

Need a predictive theory



Technologies improve  
at very different rates

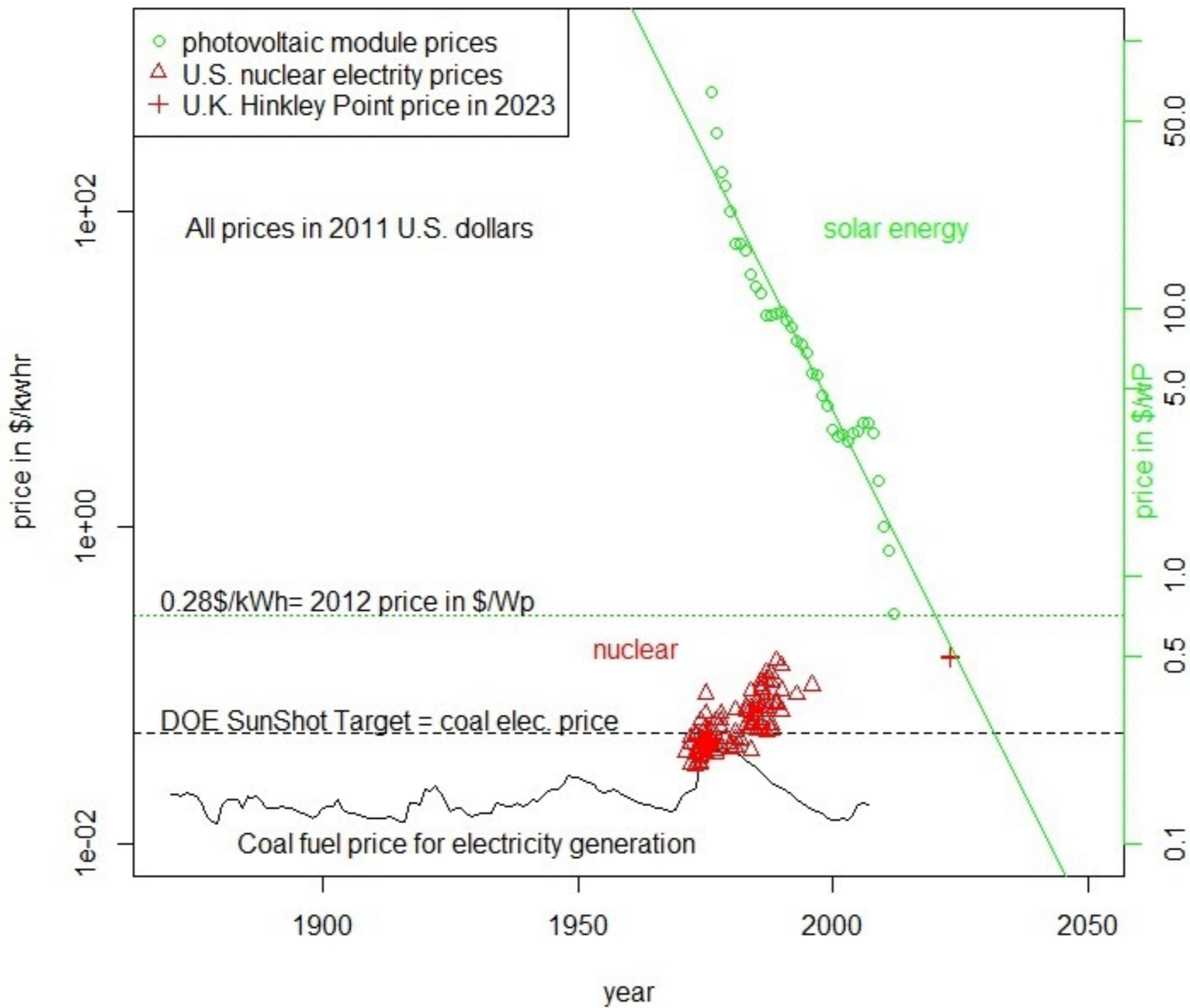
# Distribution of technological progress rates



# Consequences for public investment

It is essential that we take the dramatic differences in rates of technological evolution into account when we consider public investment in technology R&D.

# Price trends of coal, photovoltaic and nuclear electricity



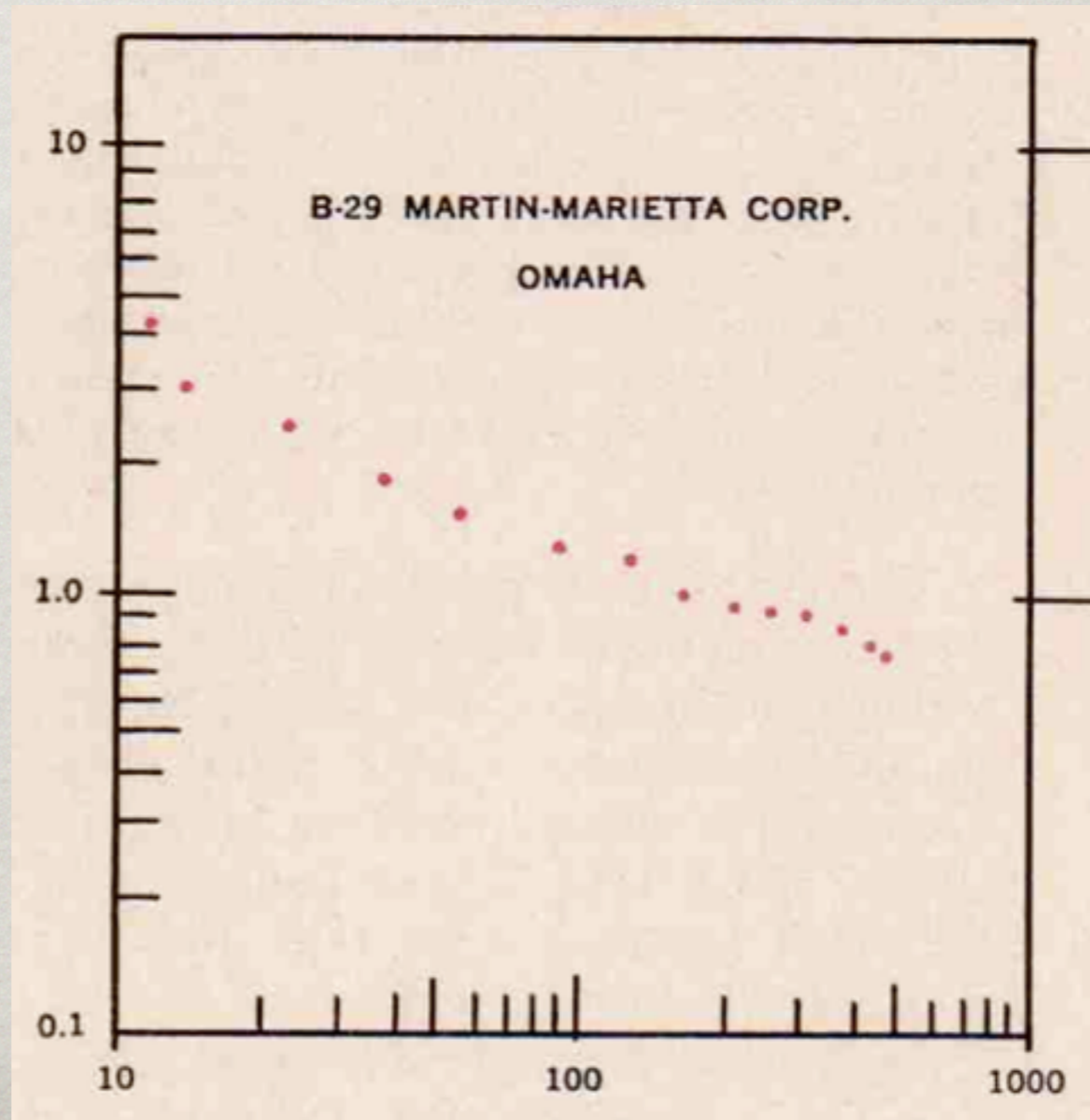
# Empirical laws for technological improvement

# WRIGHT'S LAW (1936)

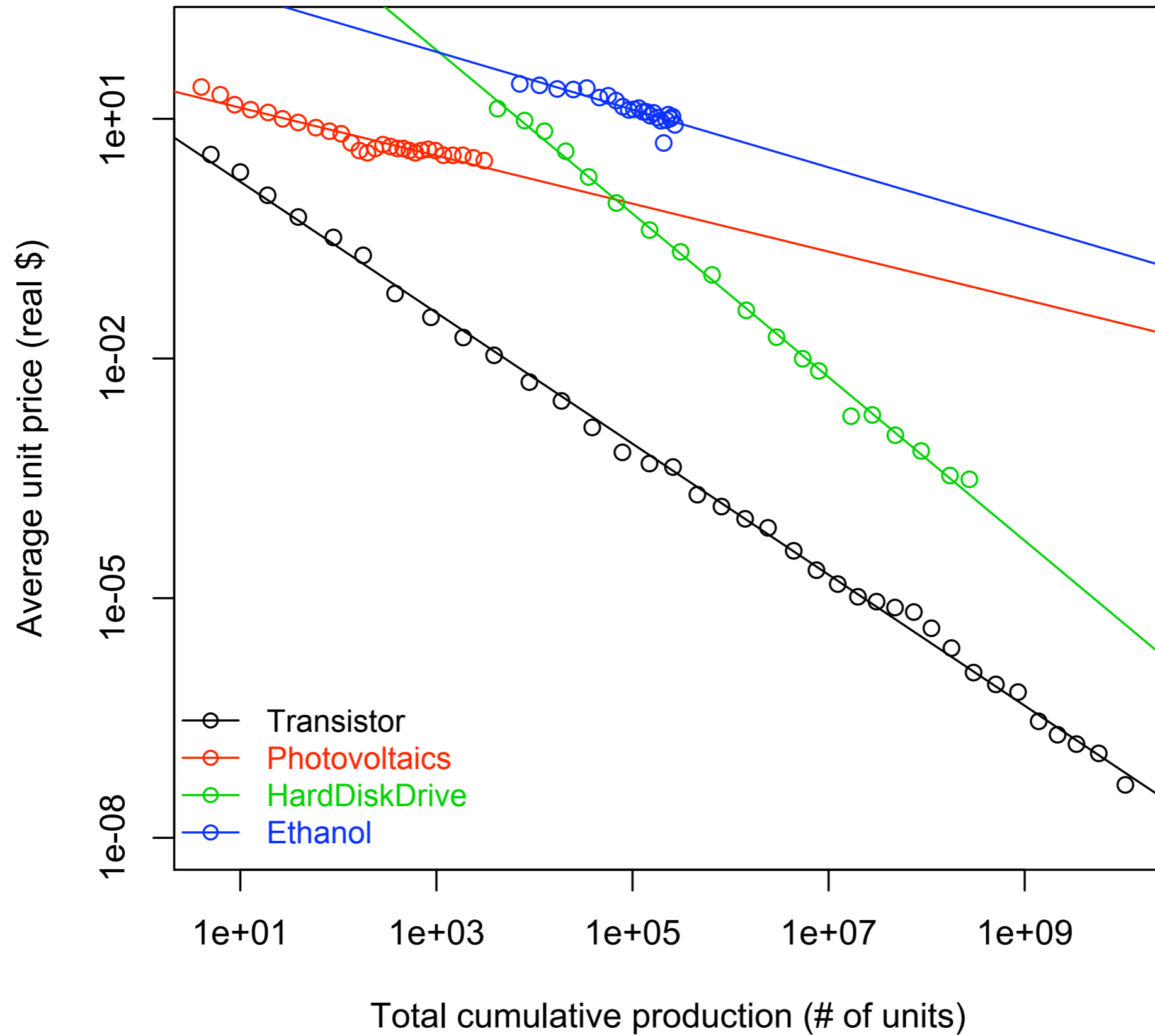
Cost vs. cumulative production = power law  $y = x^{-\alpha}$



Theodore Paul Wright

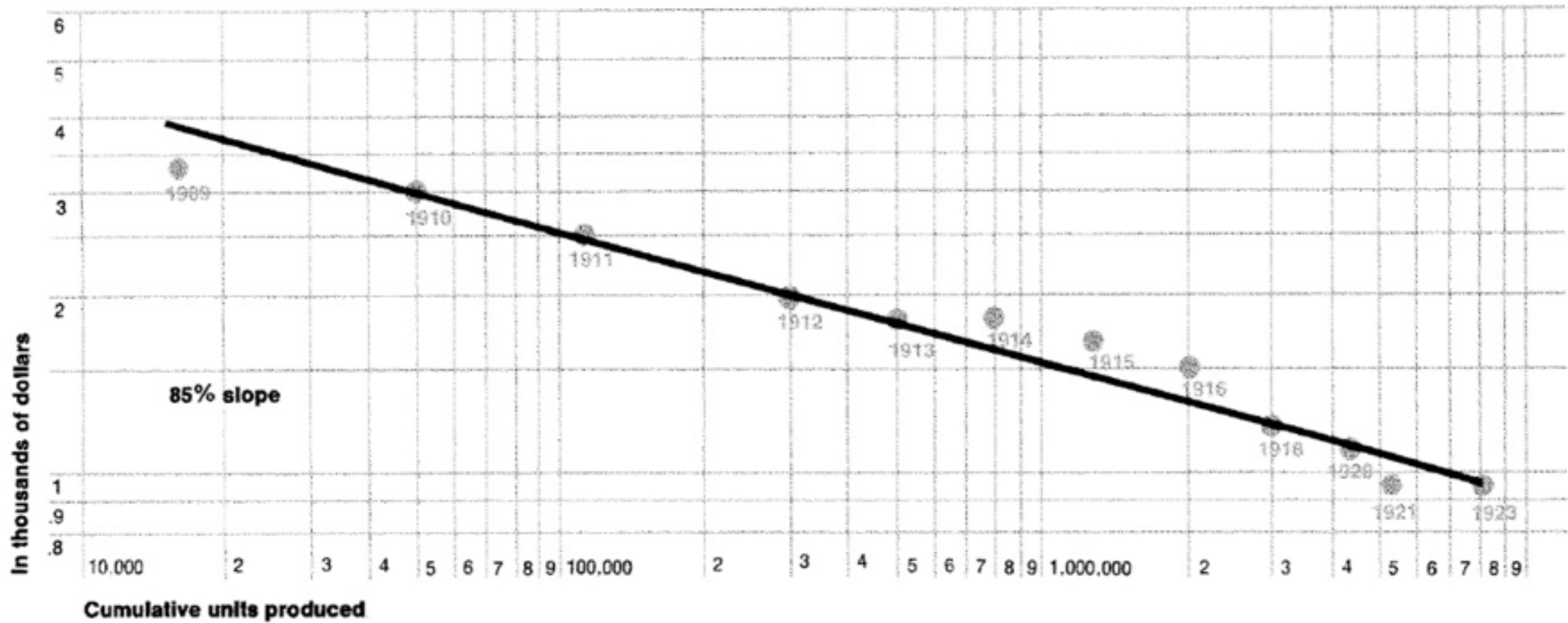


# Wright



# FORD'S MODEL T

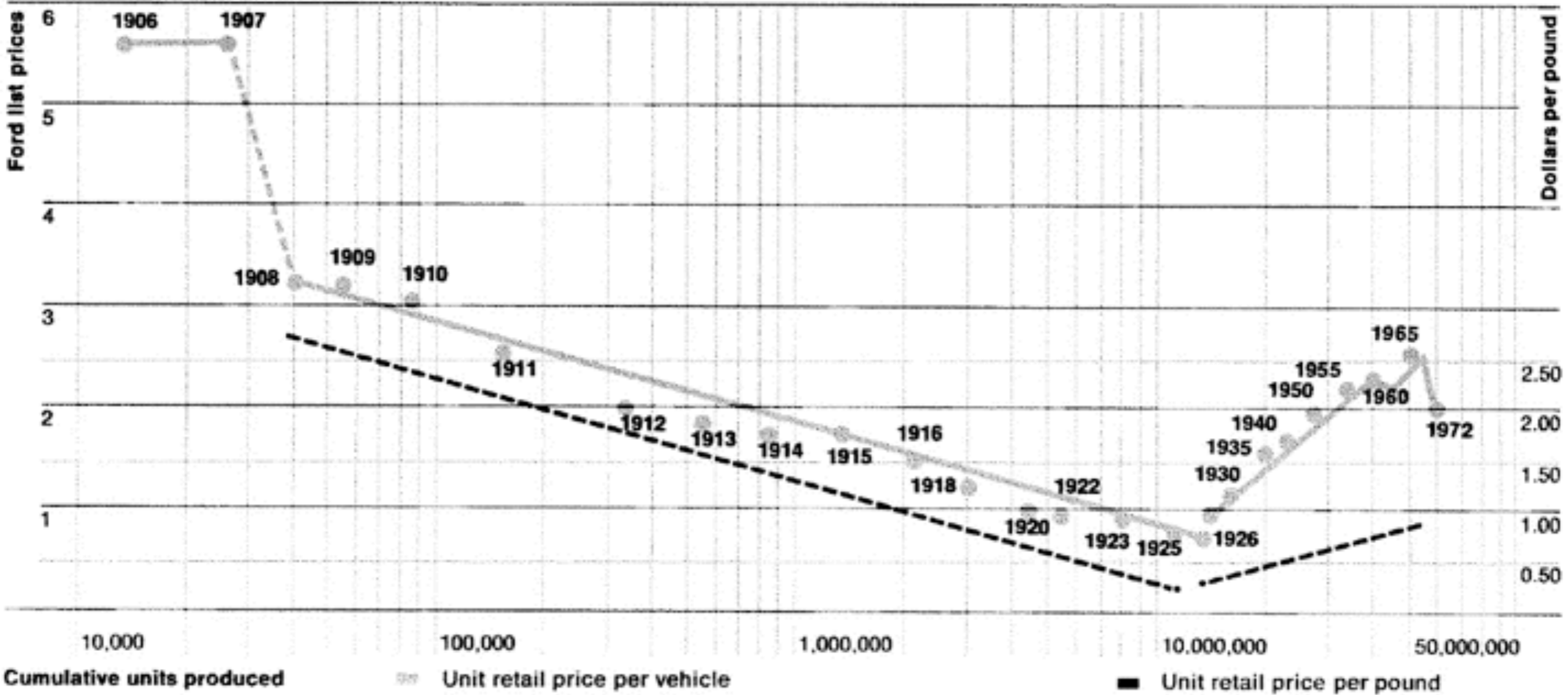
**Exhibit I**  
Price of Model T, 1909-1923 (Average list price in 1958 dollars)



Wright's law only works when reducing cost is main objective

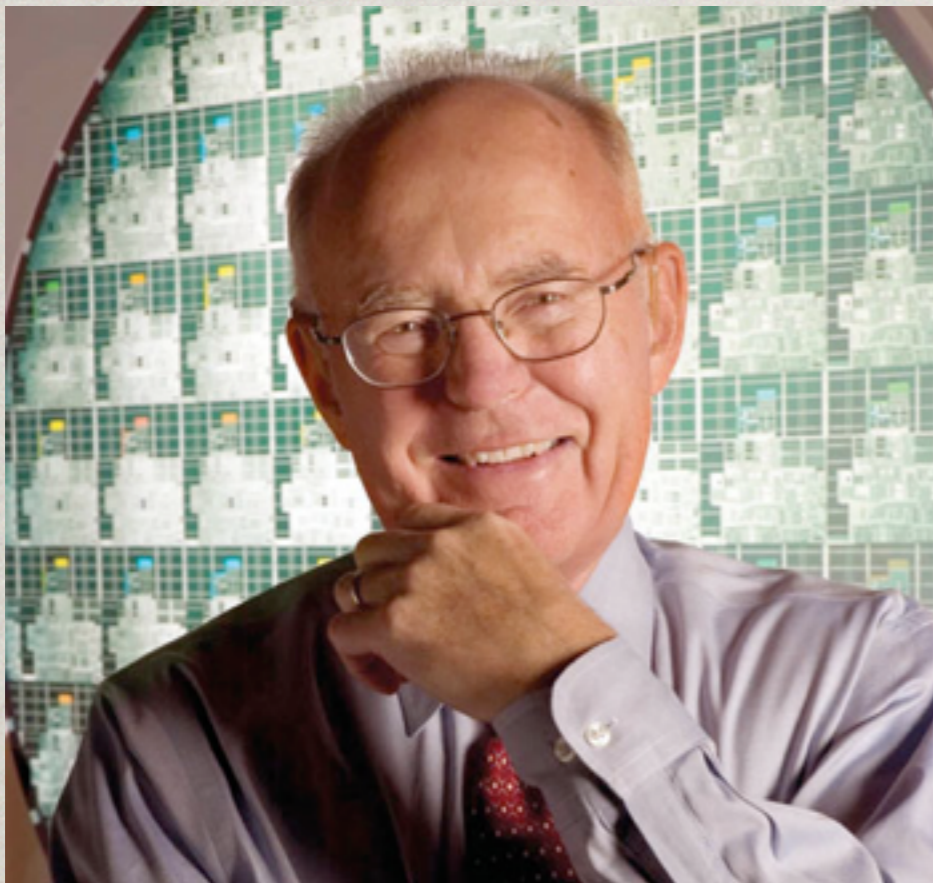


<b>Models</b>	ABCNRSK	T	A	Annual model changes
<b>Engines (H.P.)</b>	2 (15 & 50)	1 (20)	1 (24)	2 or more (50 & more)
<b>Wheel bases</b>	2	1	1	2 or more
<b>Weights</b>	Up to 1800	1100-1820	2312 (average)	2335 and up (average)



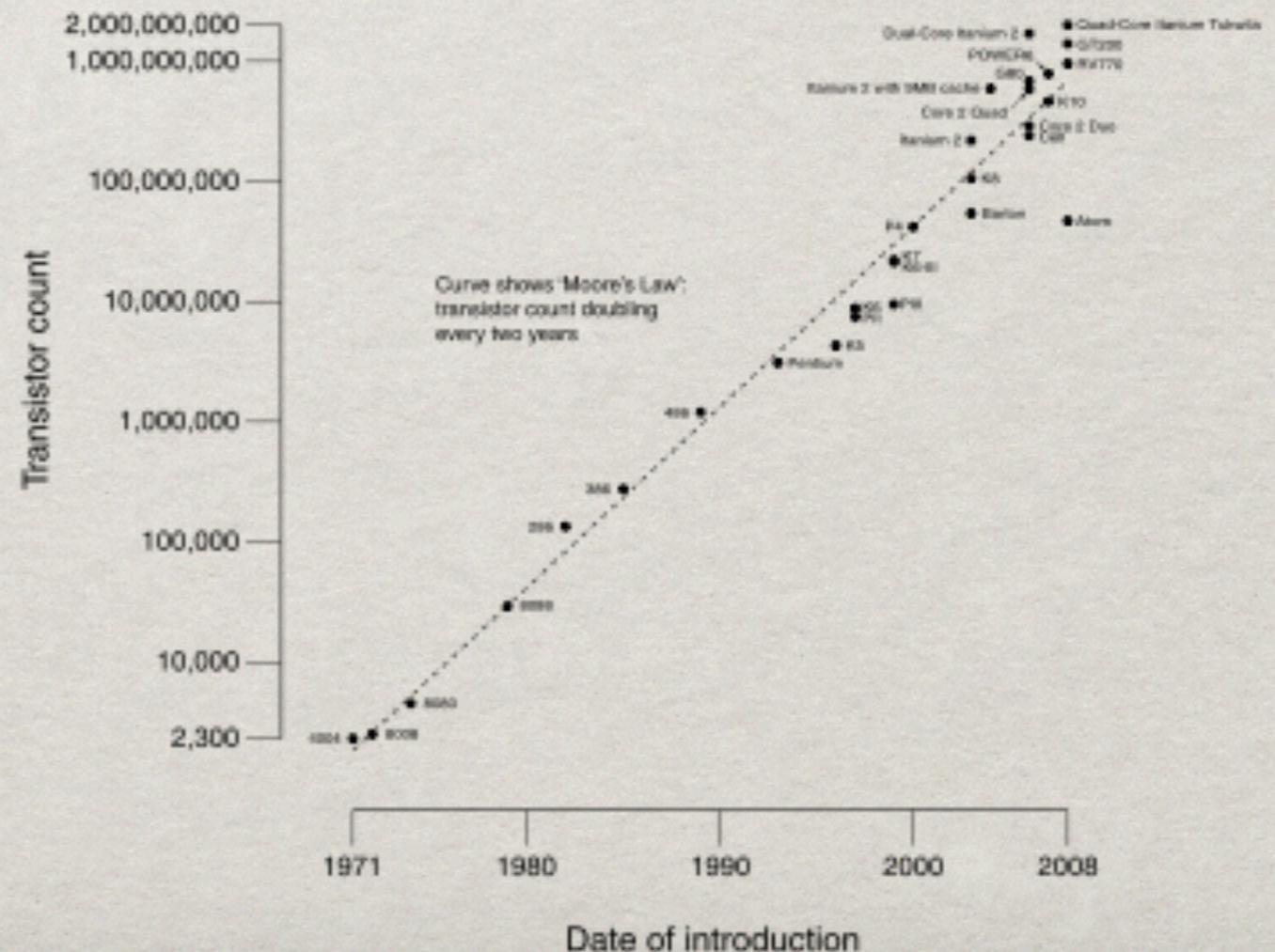
# MOORE'S LAW (1965)

Originally a statement about density of transistors  
We will use to refer to the hypothesis that technological  
performance improves exponentially with time

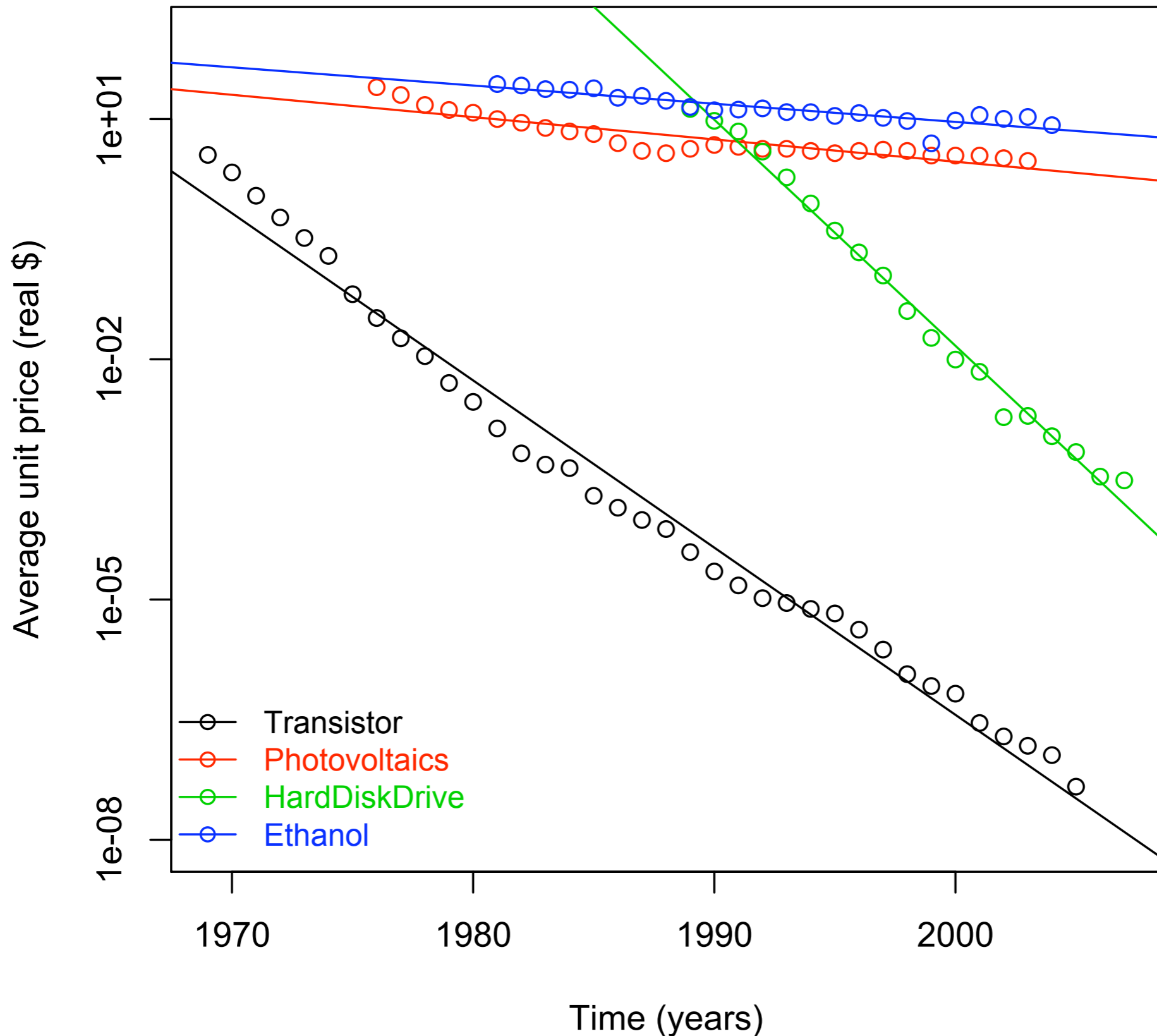


Gordon Moore

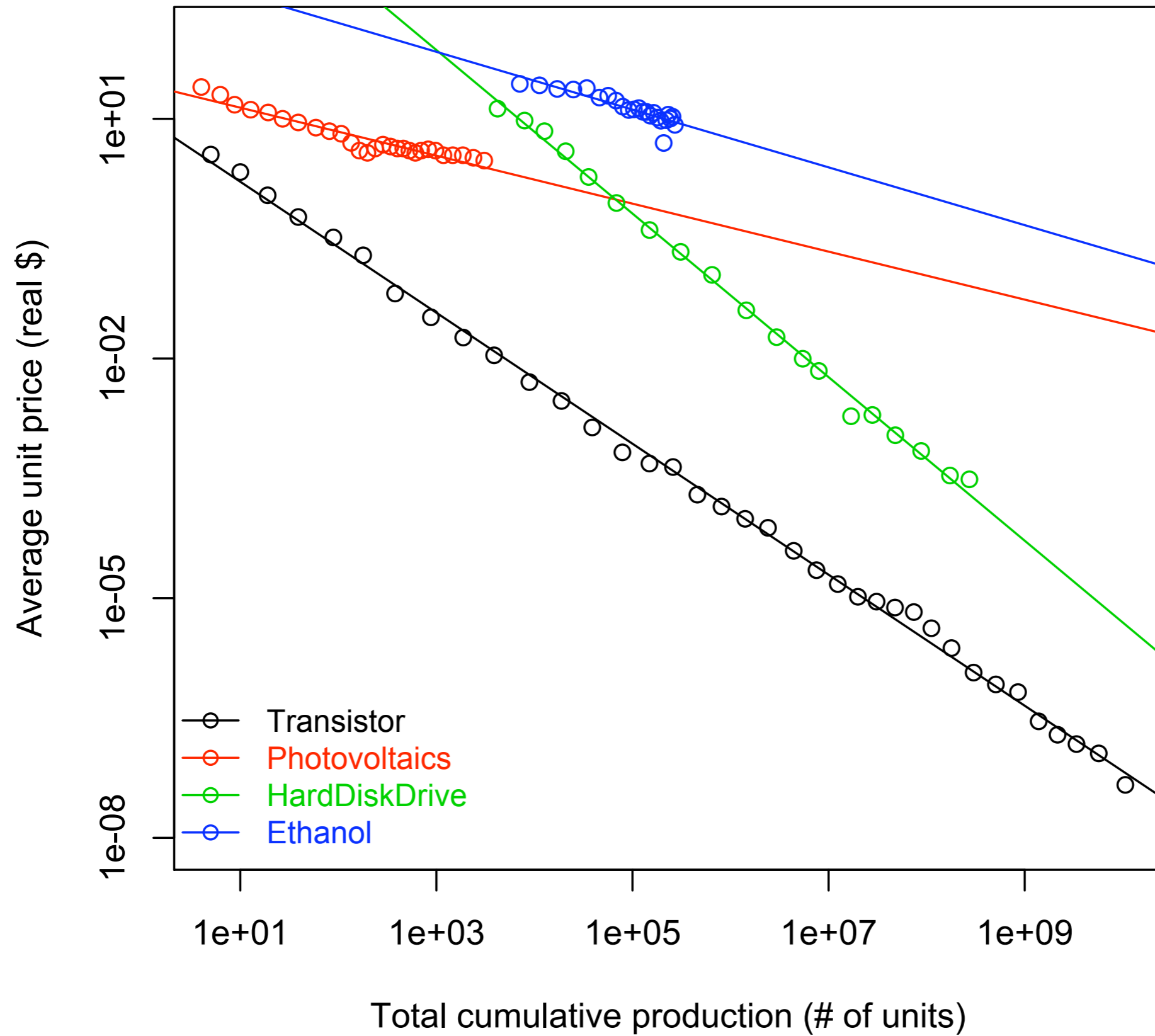
CPU Transistor Counts 1971-2008 & Moore's Law



# Moore



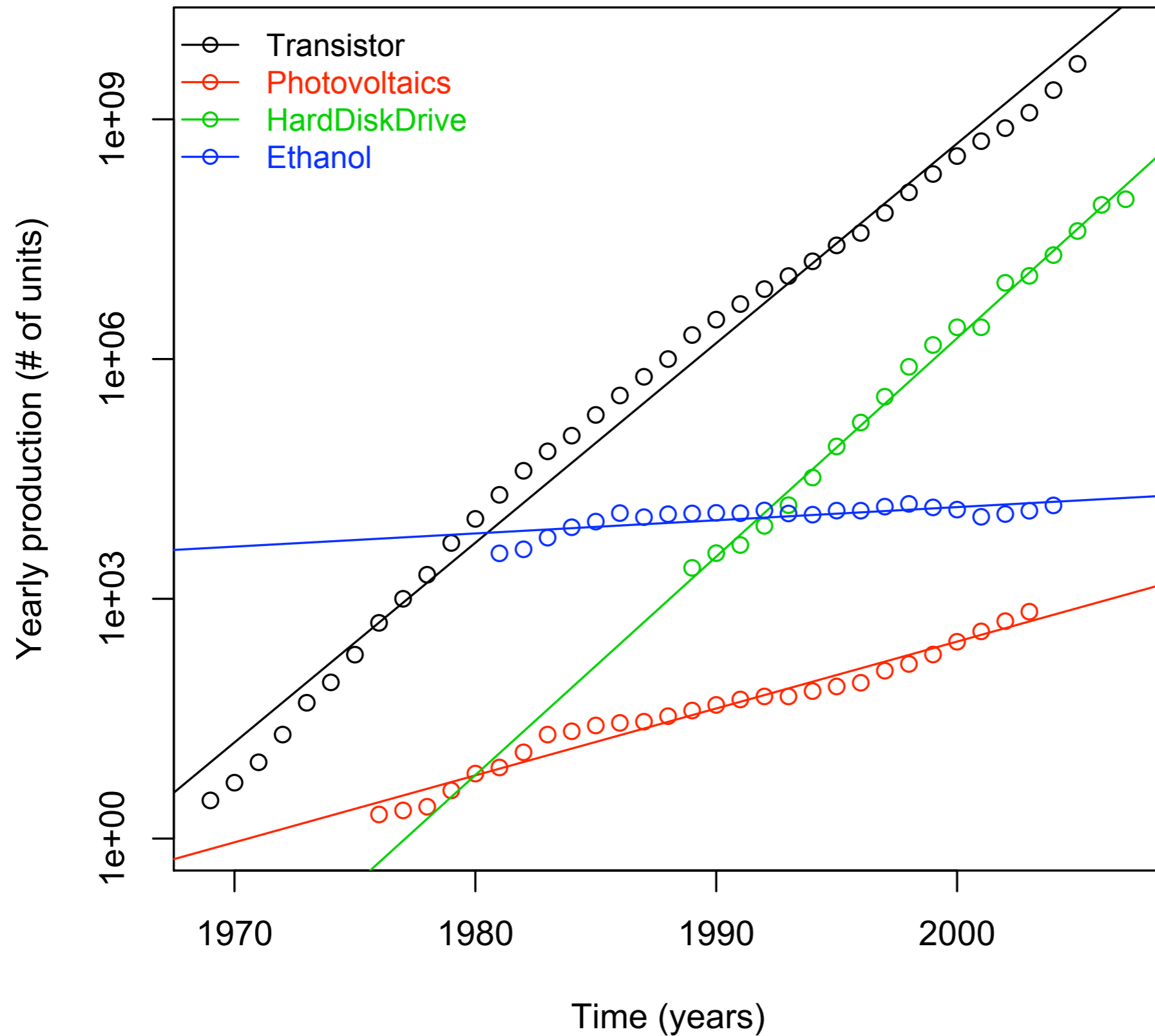
# Wright



# PRODUCTION VS. TIME

- For technologies in this sample, also reasonable to postulate that production increases exponentially with time

# Production volume



(Nagy, Farmer, Bui, Trancik, PlosOne, 2013)

# COMPATIBILITY OF WRIGHT AND MOORE (SAHAL, 1987)

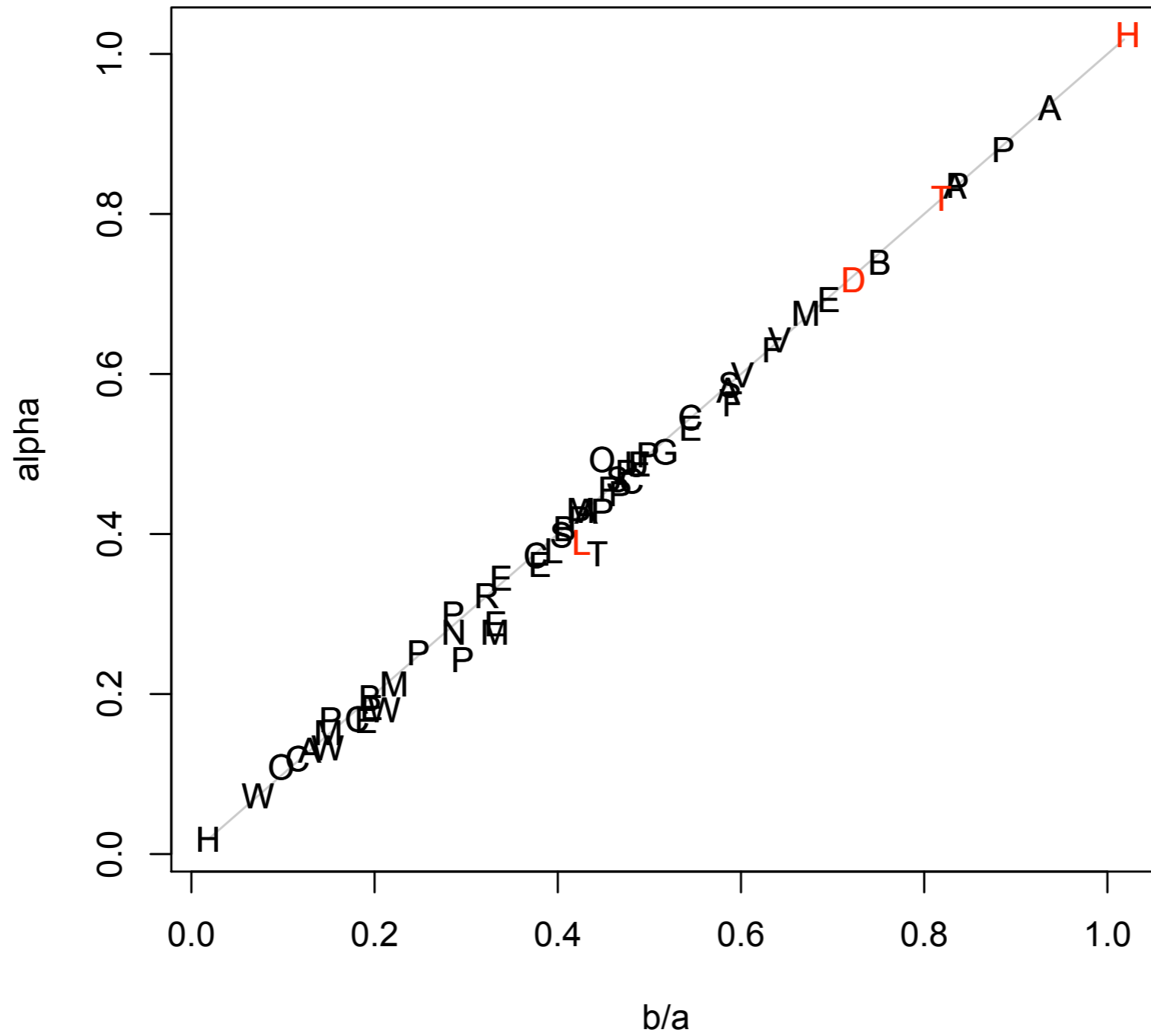
If production expands exponentially and costs drop exponentially, Wright's law will hold.

$$x(t) = \exp(at)$$

$$y(t) = \exp(-bt)$$

$$y(x) = x^{-b/a}$$

**Empirical validation of Sahal's identity:  $\alpha = b/a$**



(Nagy, Farmer, Bui, Trancik, PlosOne, 2013)



Bela Nagy



# Key hypothesis

- All technologies obey same random process
  - parameters vary across technologies

# Time series models

Moore's law as a random walk with drift

$$y_{t+1} - y_t = \mu + Kn_t$$

Change in log(cost)

Drift

Noise

Wright's law as random walk with drift  
dependent on cumulative production

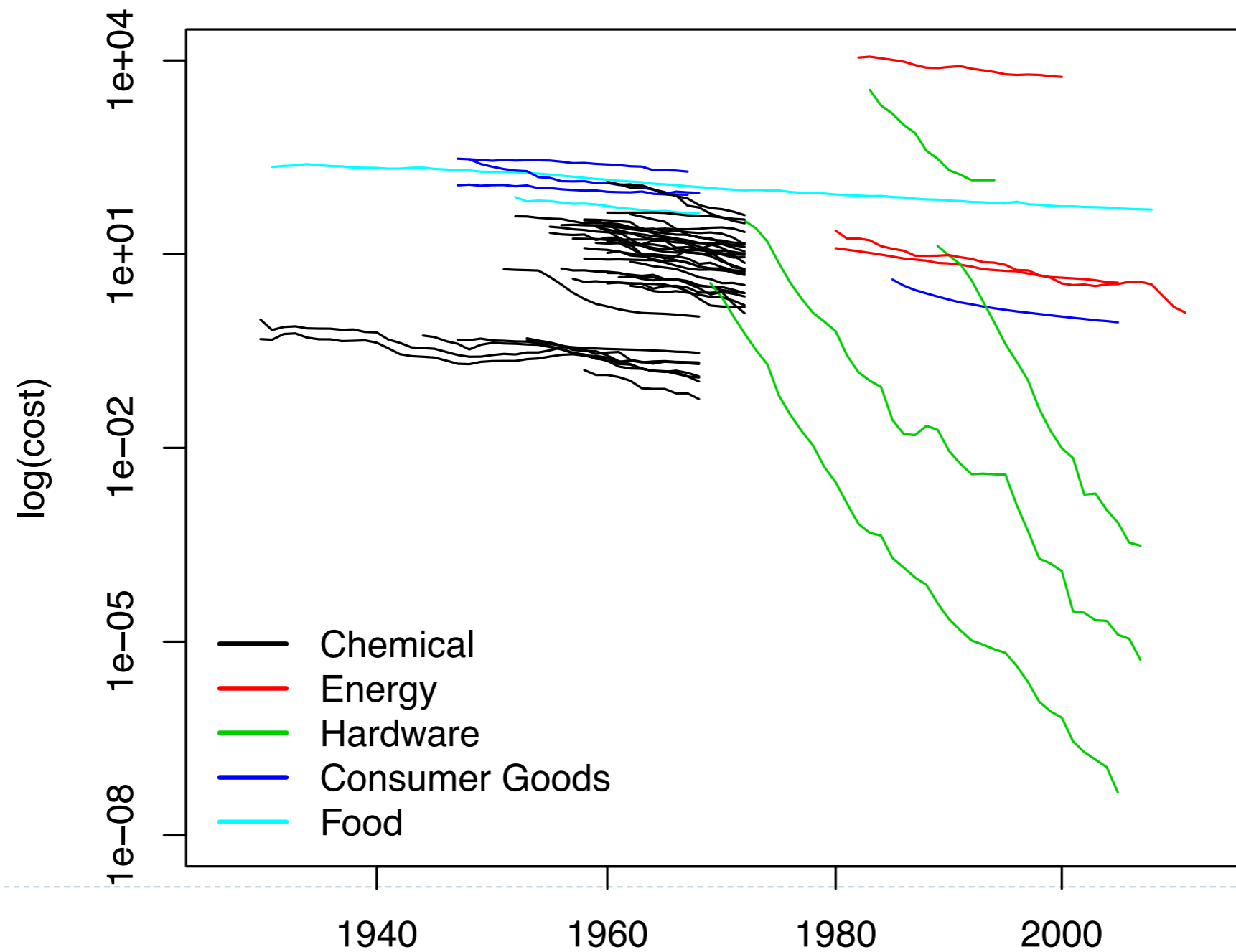
$$y_{t+1} - y_t = \omega \log \left( \frac{x_{t+1}}{x_t} \right) + Sn_t$$

# TESTING FOR PREDICTABILITY THROUGH HIND CASTING

(WITH AIMEE BAILEY, JAN BAKKER, FRANCOIS LAFOND,  
PATRICK MCSHARRY, DYLAN REBOIS,

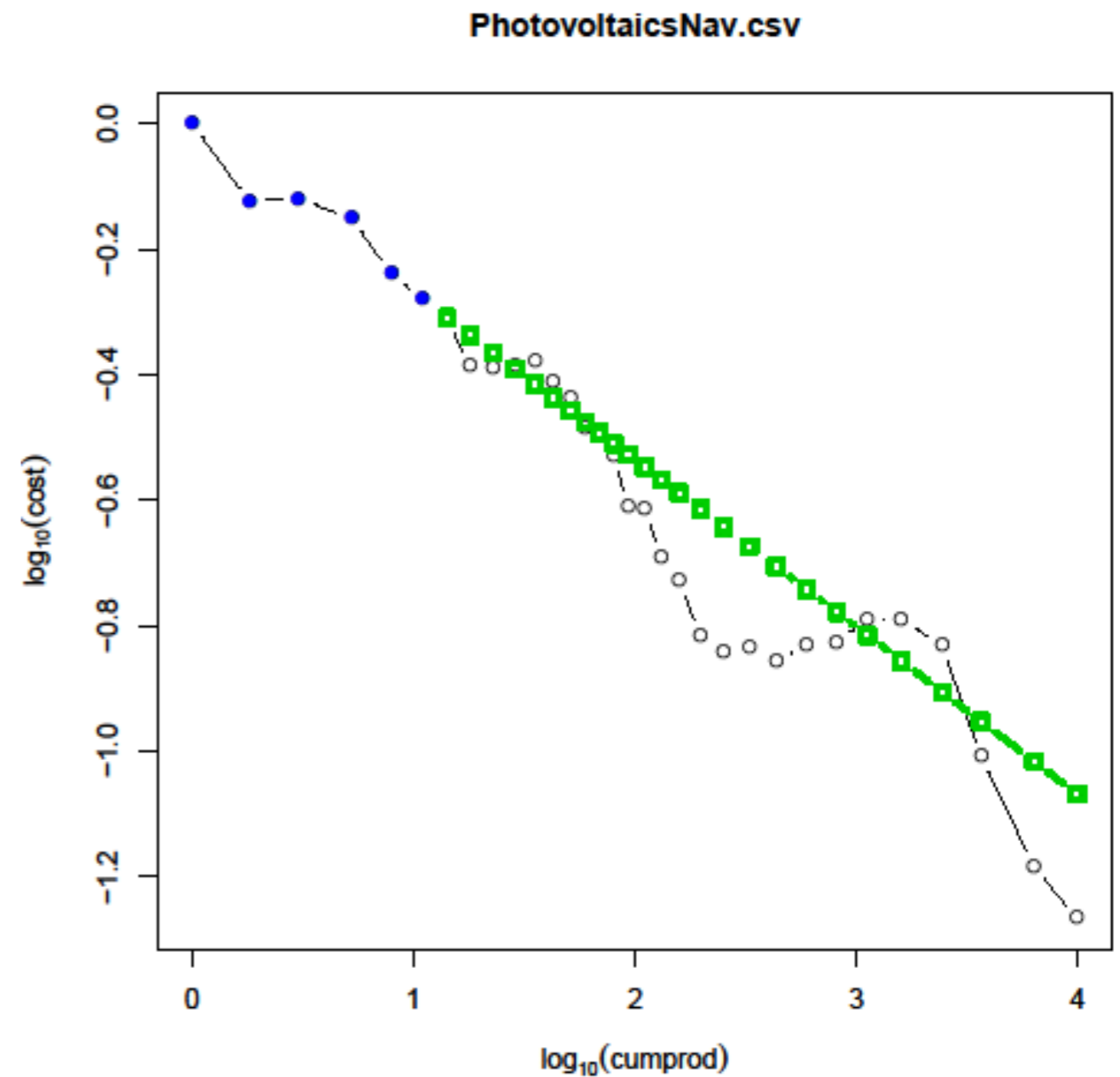
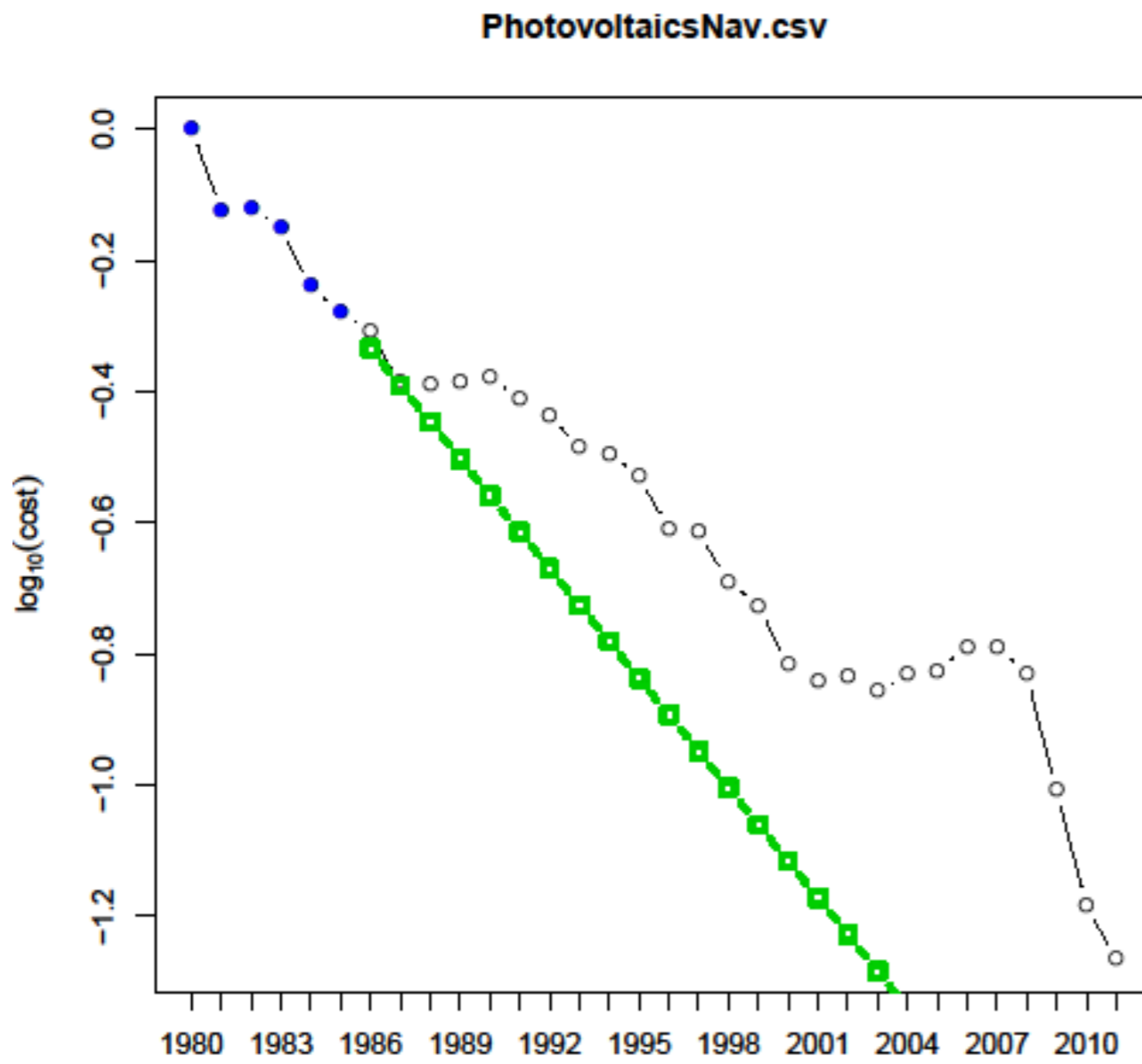
- Pretend to be at a given time in the past
- Use given method to forecast each future year
- Repeat for all past dates
- Score methods based on forecasting errors
- Make hypothesis that improvement process is the same for all technologies, except for parameters.

# Data for 48 different technologies



# Moore

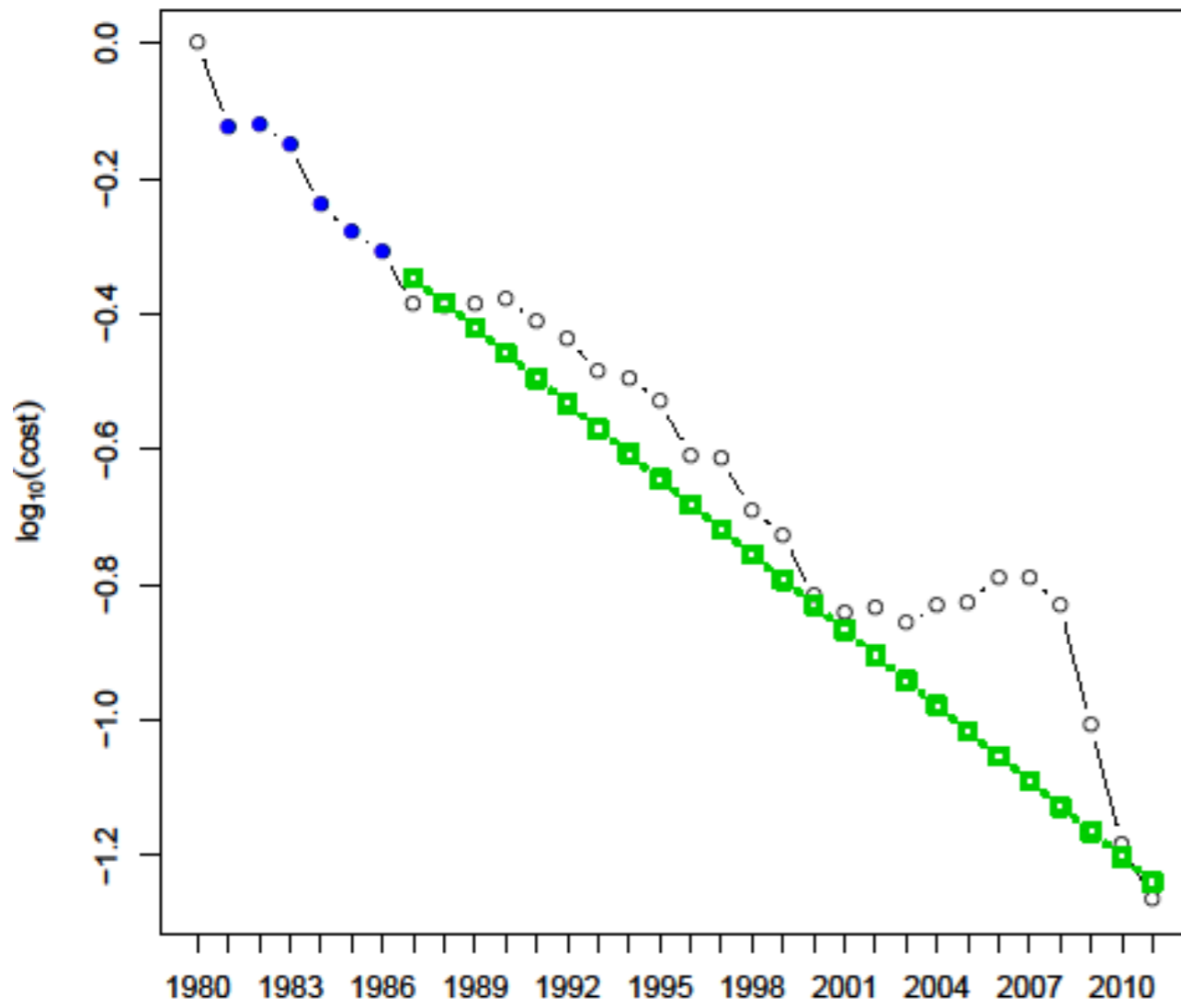
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(Data are normalized by initial value; Learning Window = 6 years)

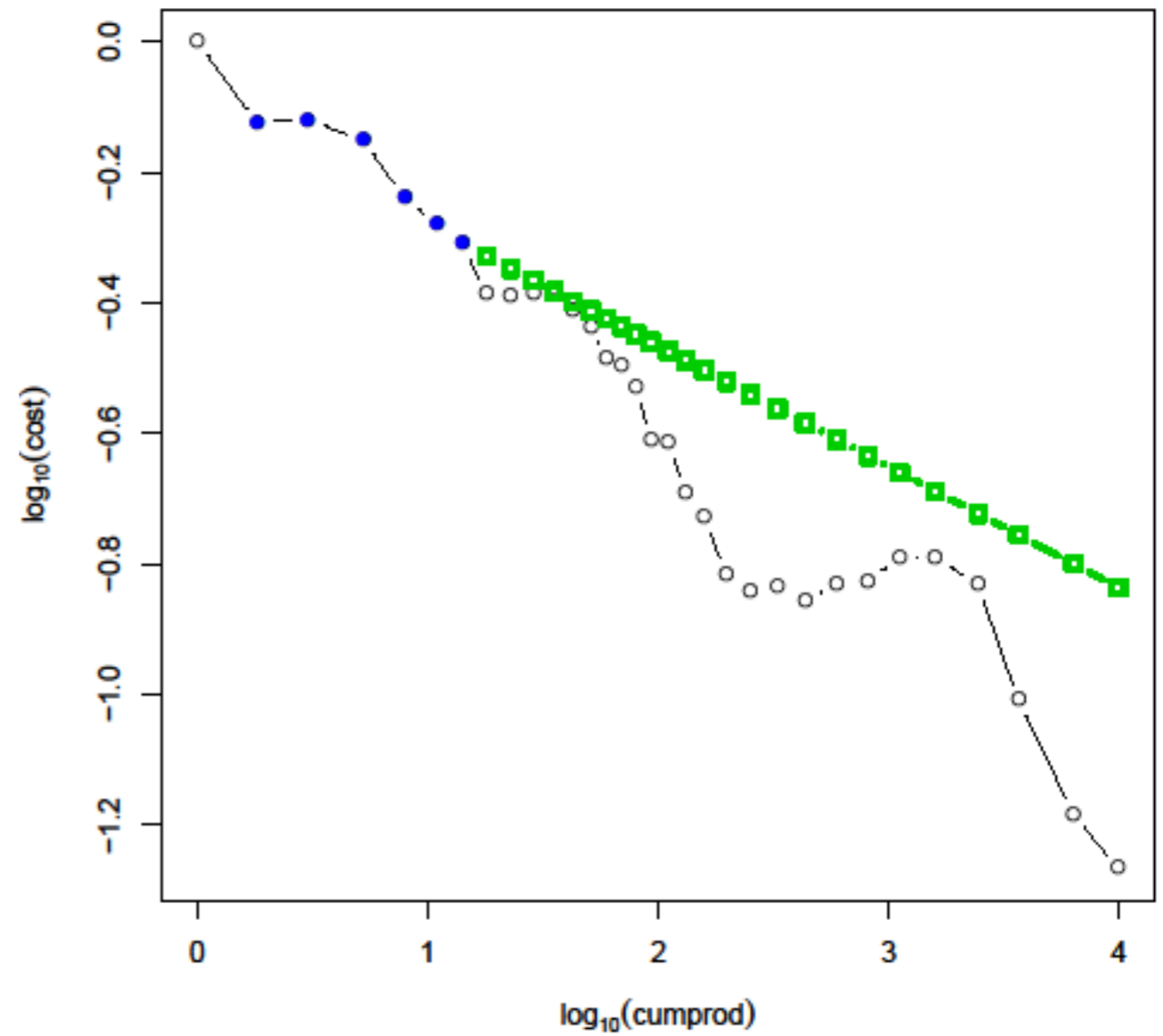
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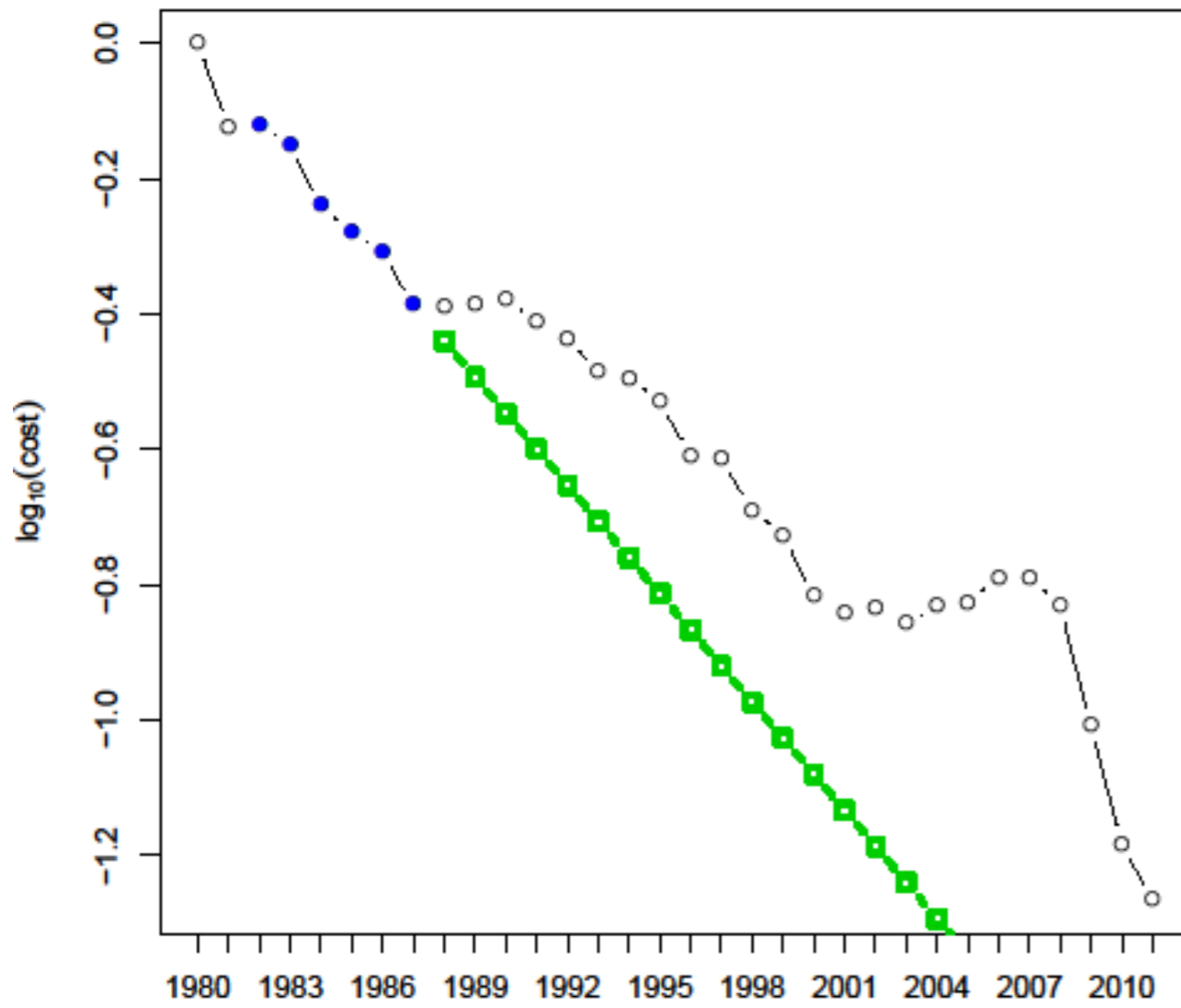
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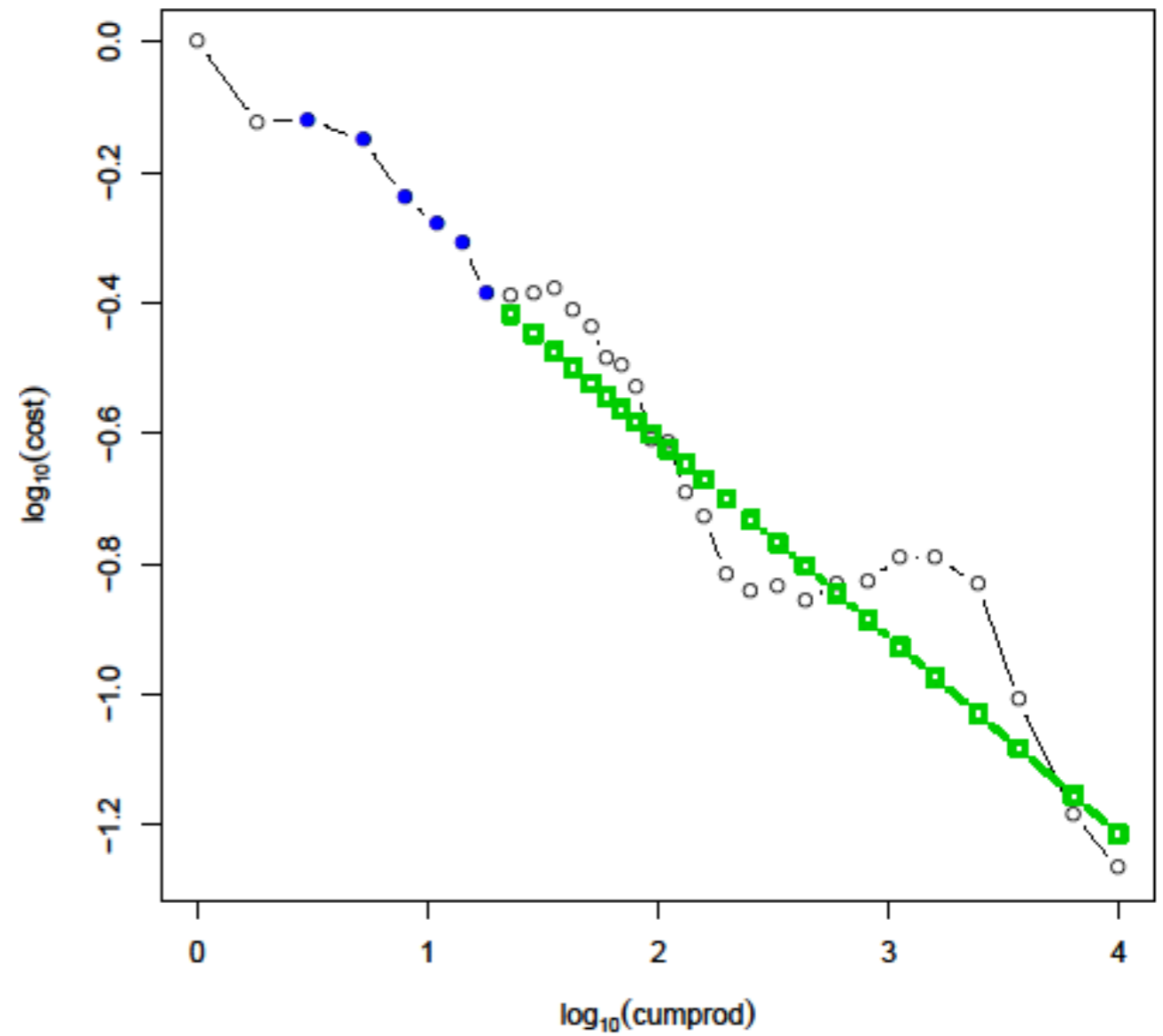
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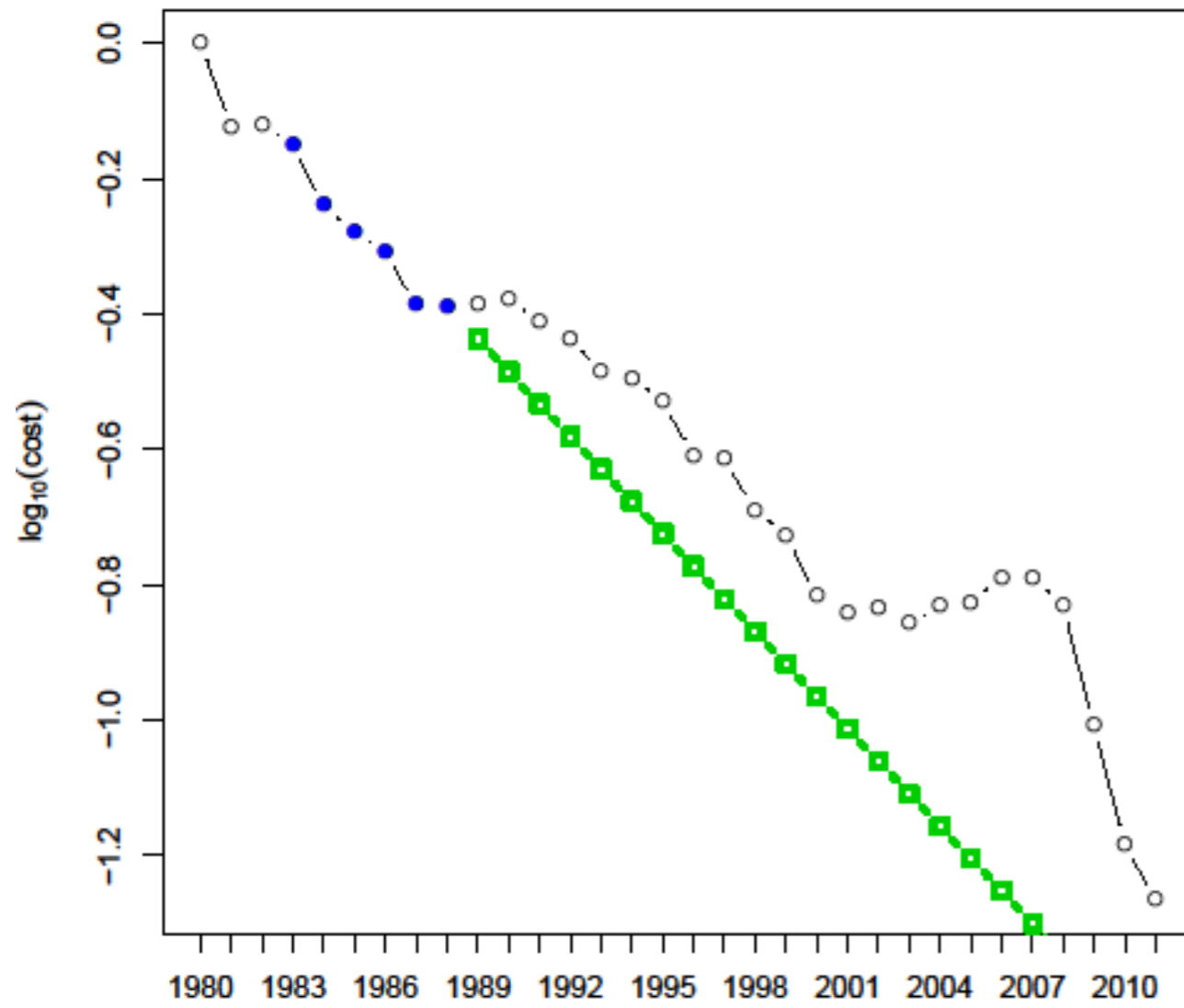
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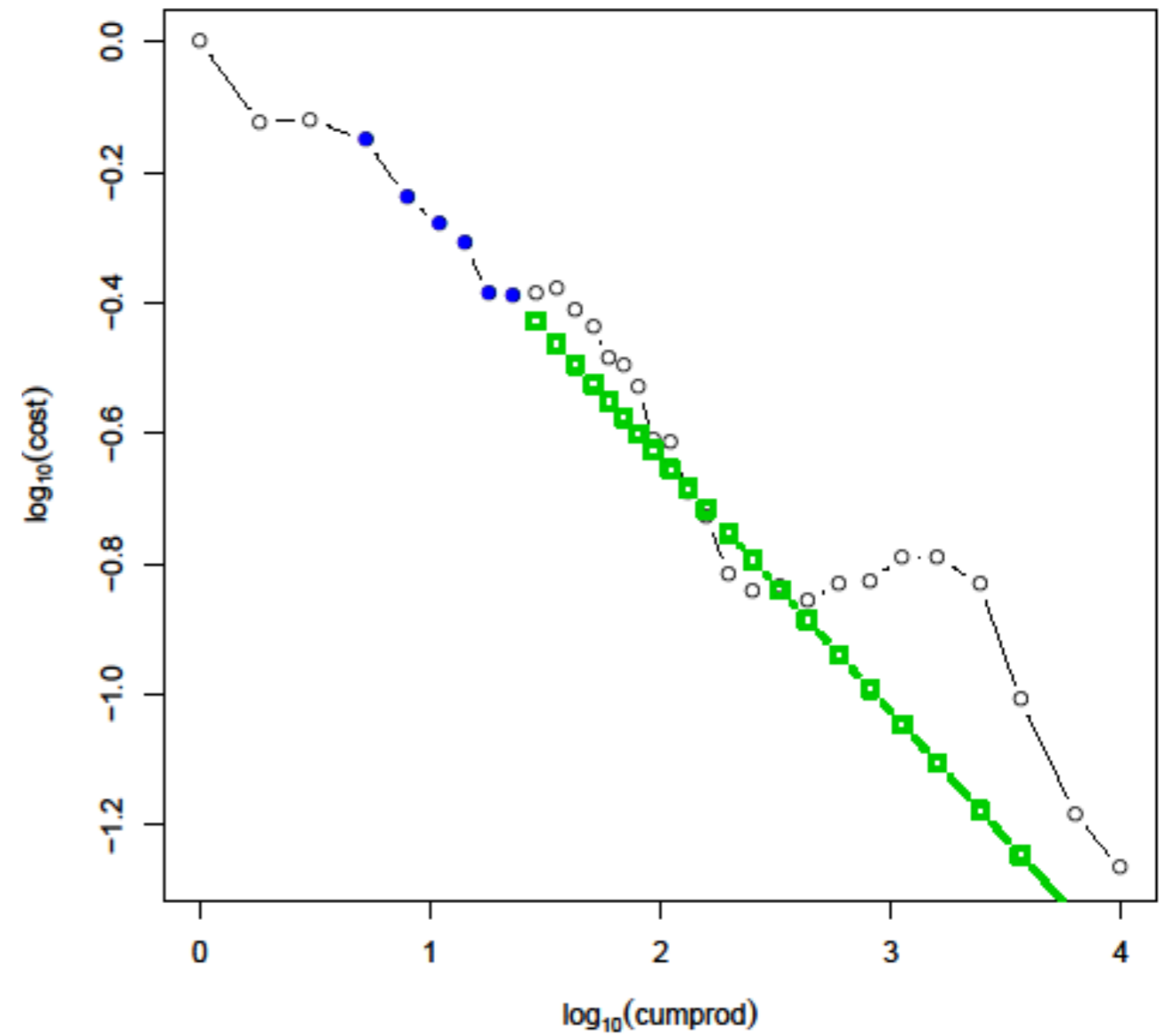
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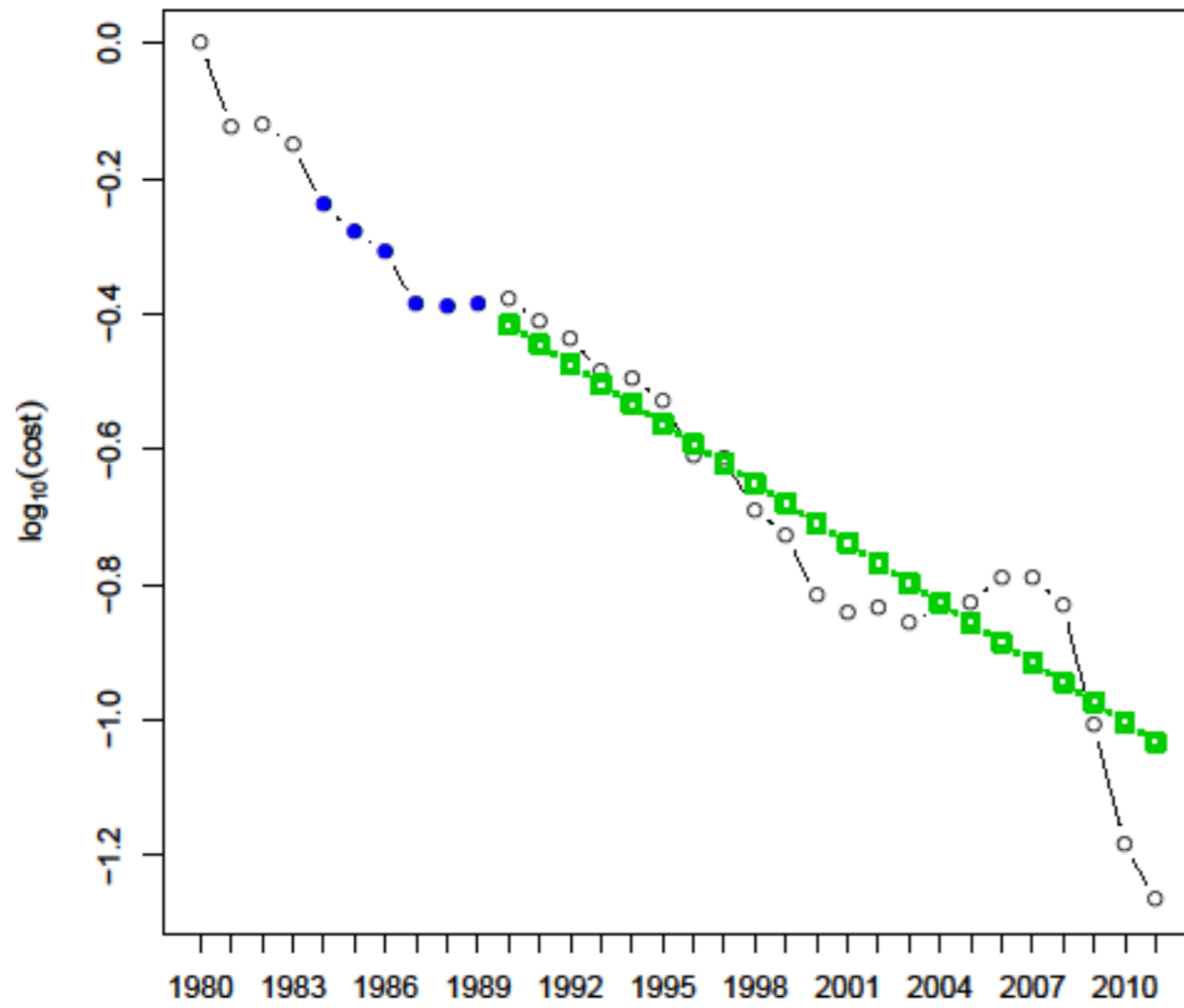
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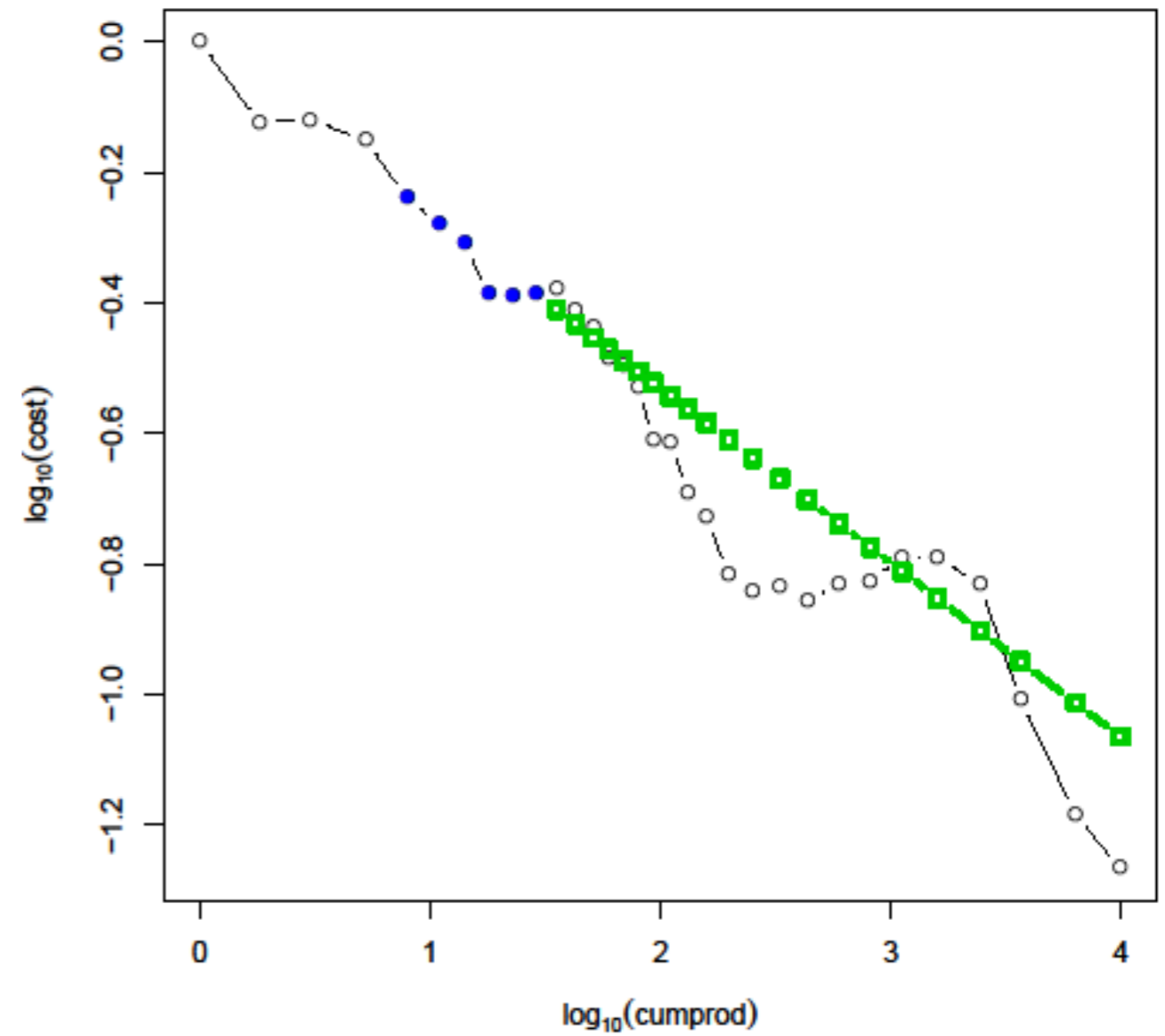
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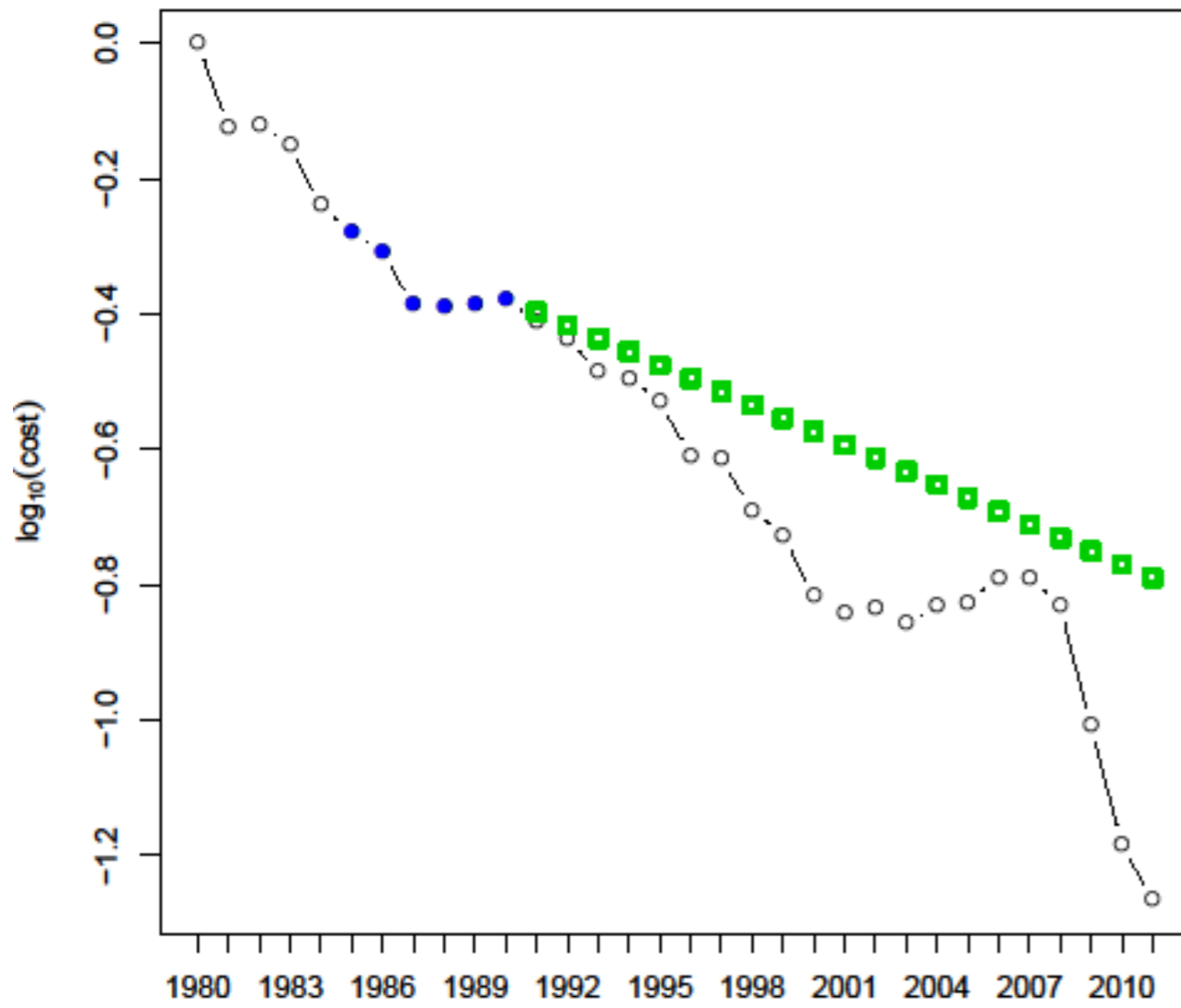
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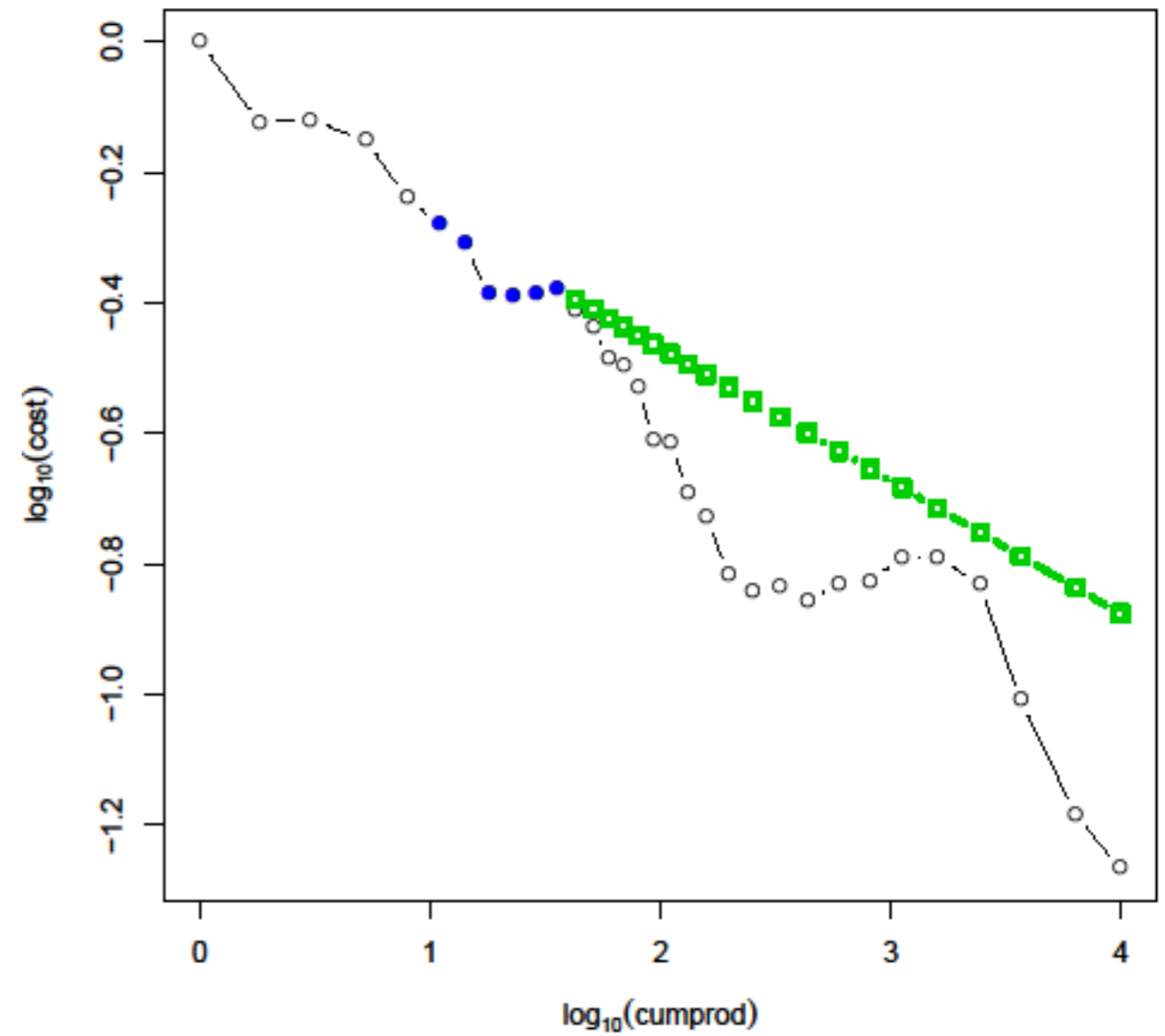
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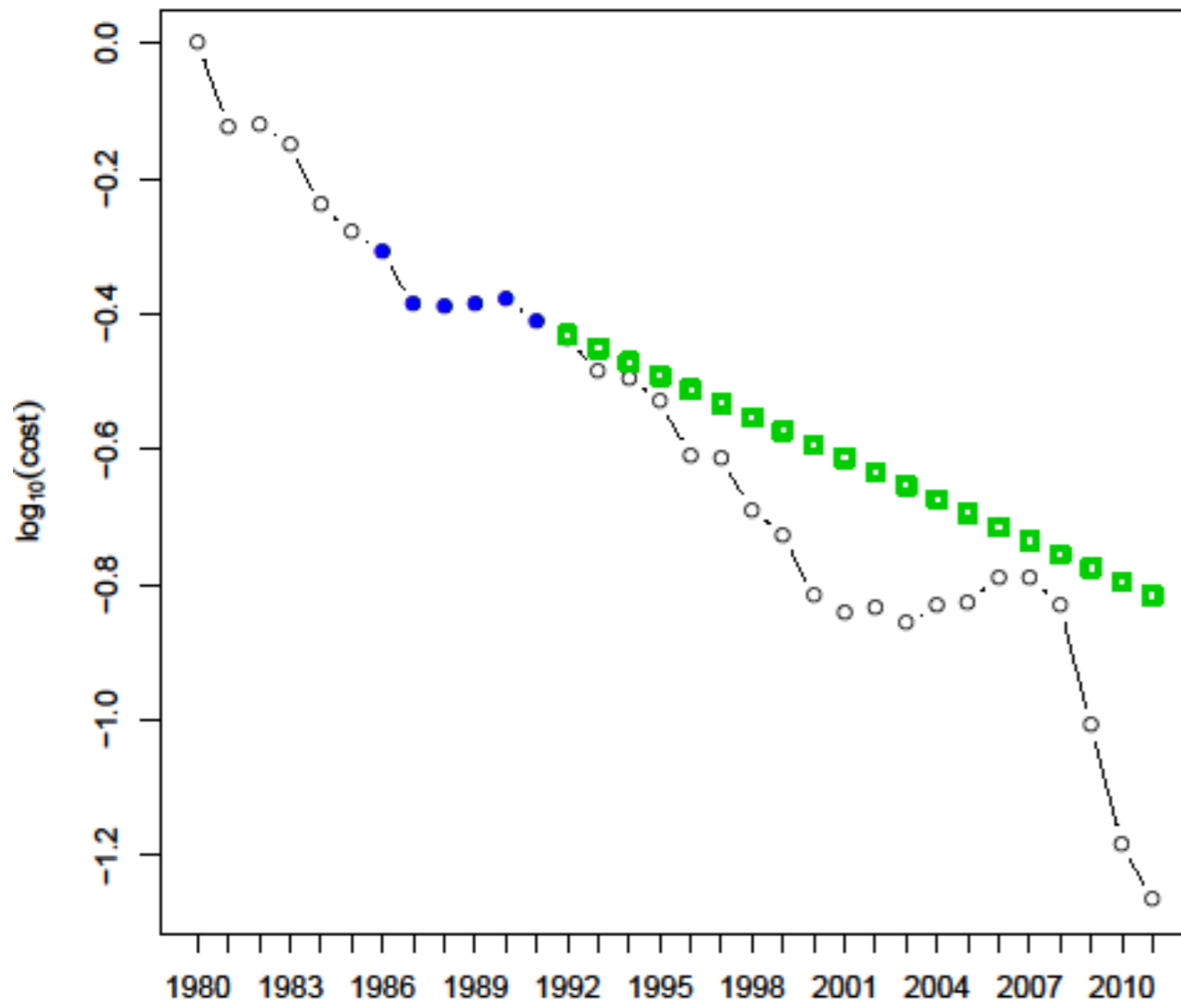
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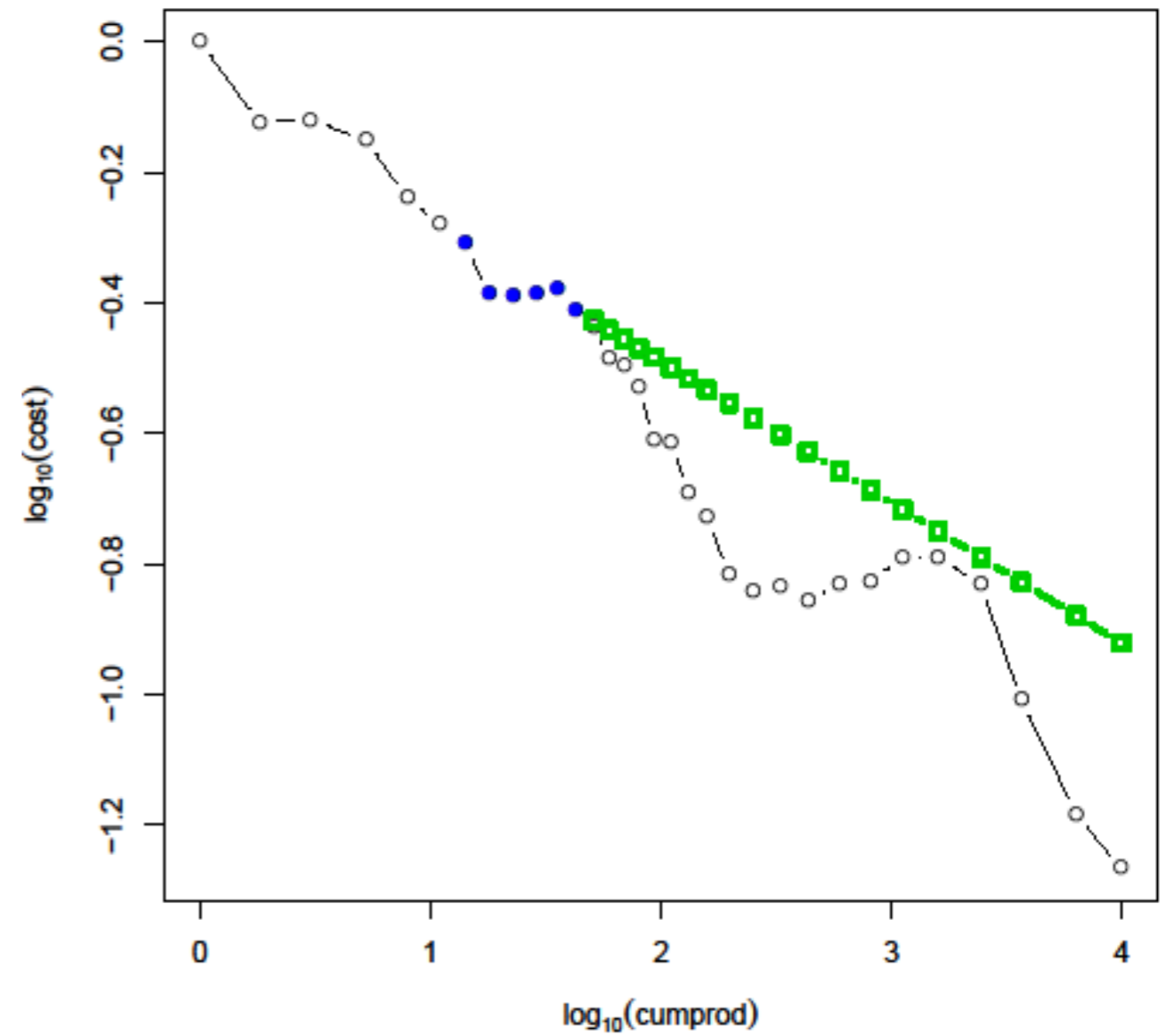
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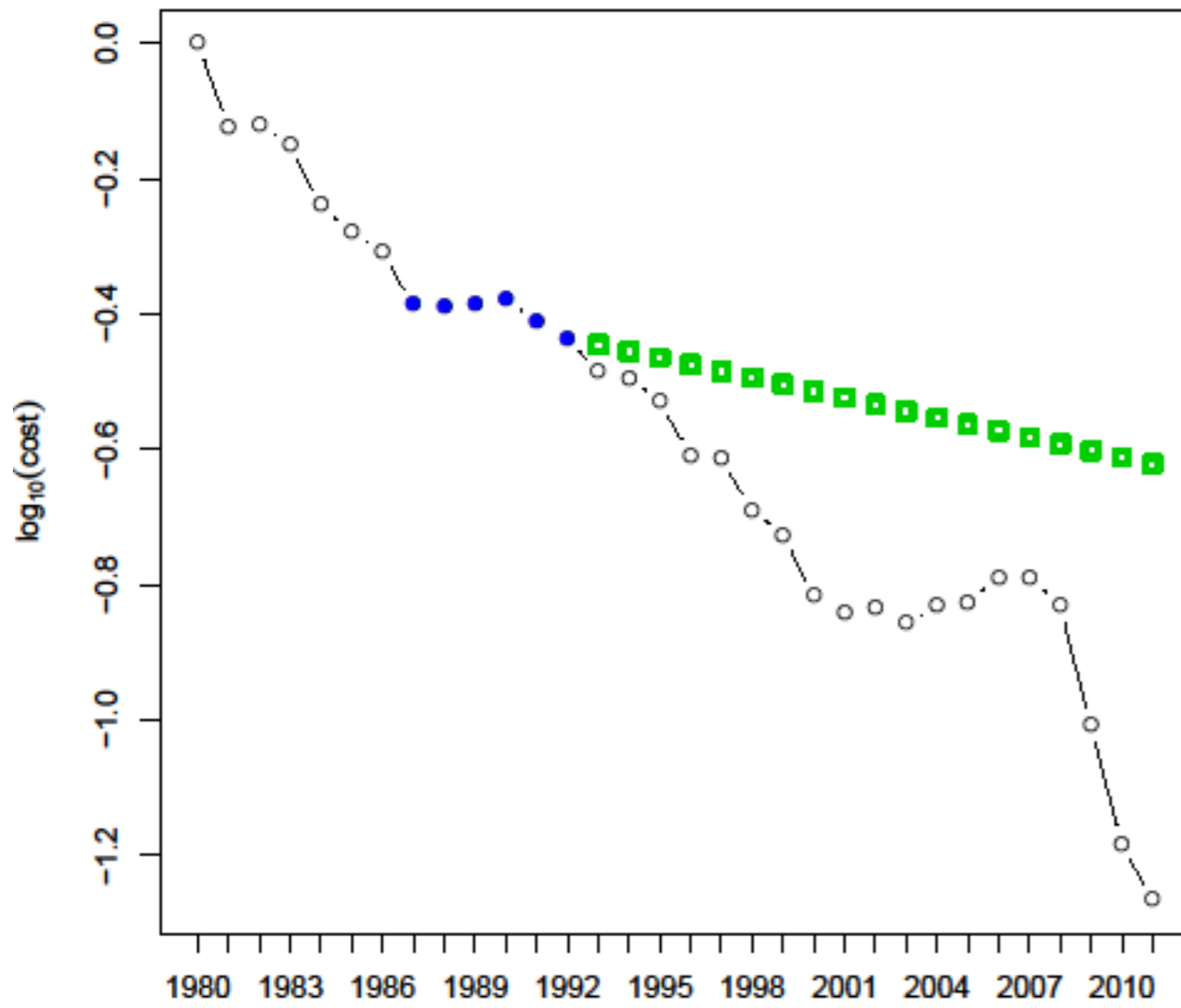
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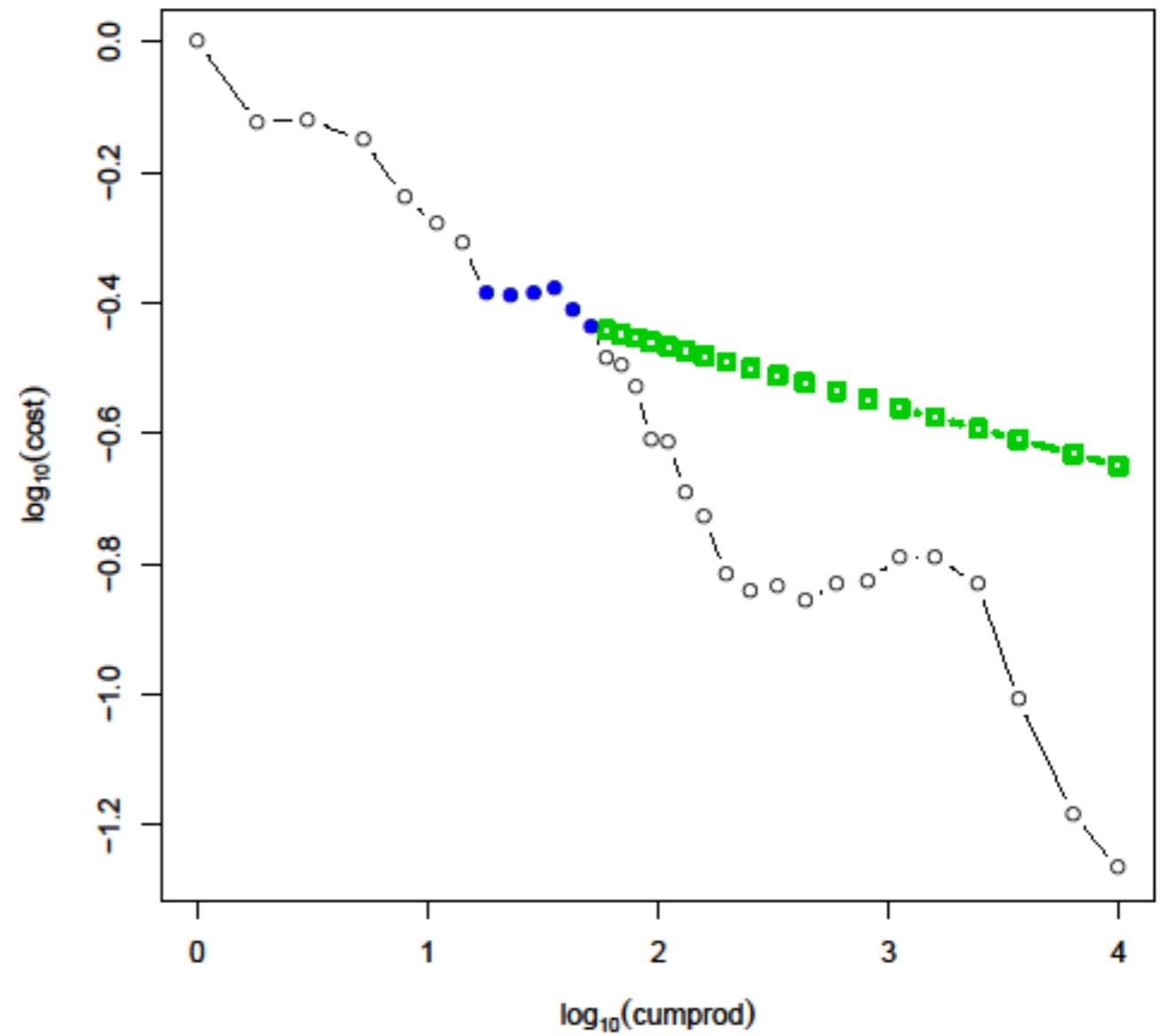
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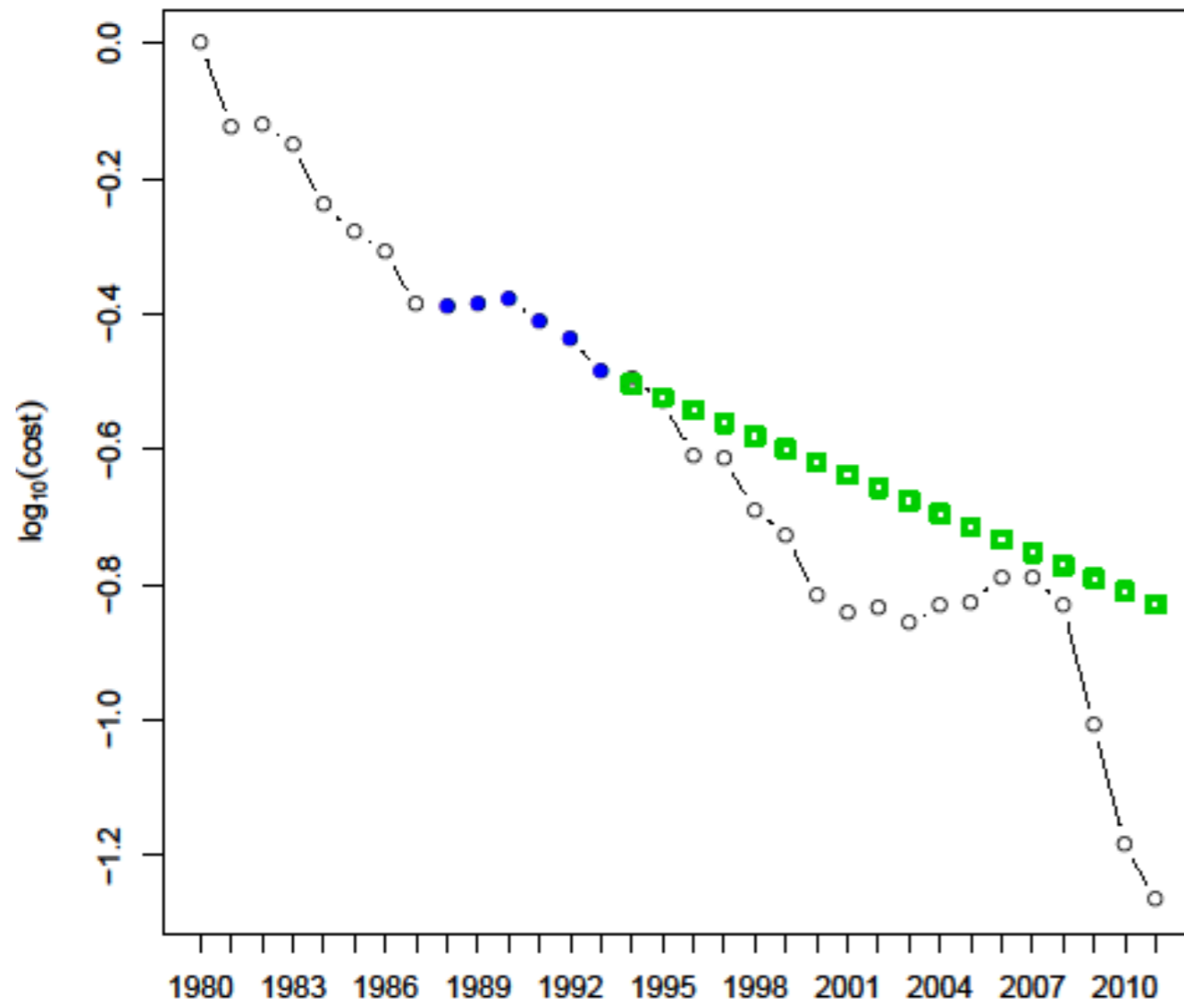
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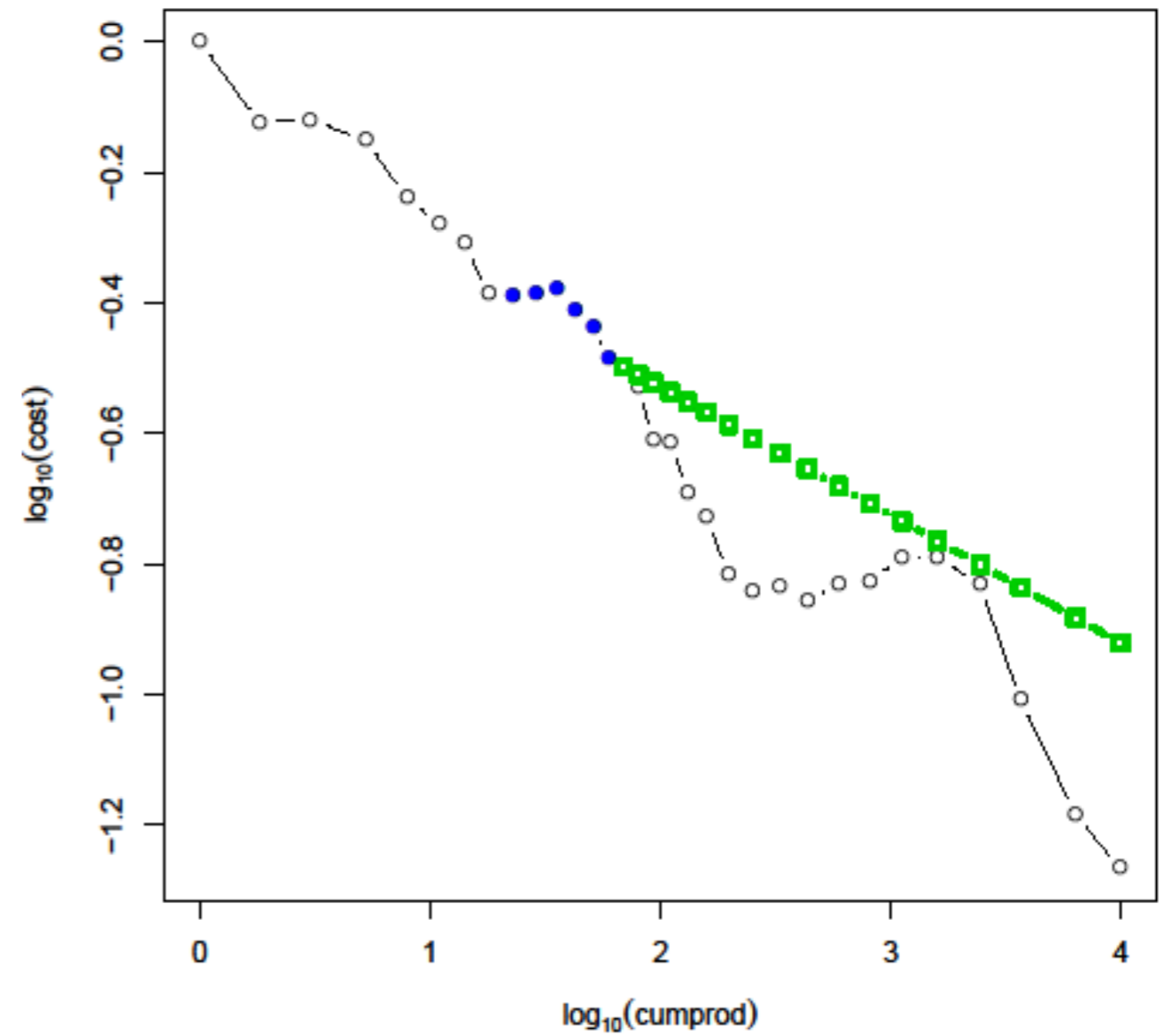
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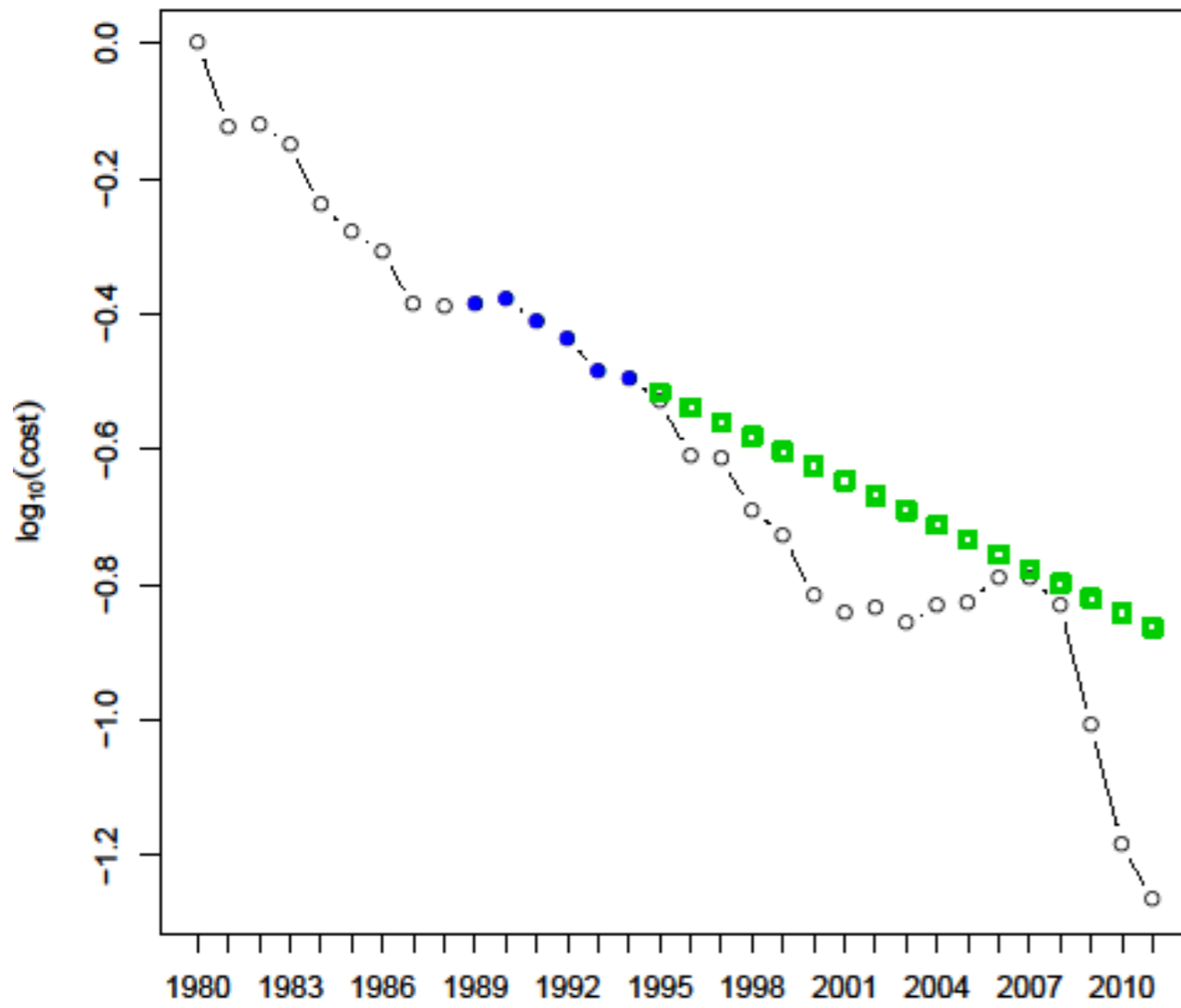
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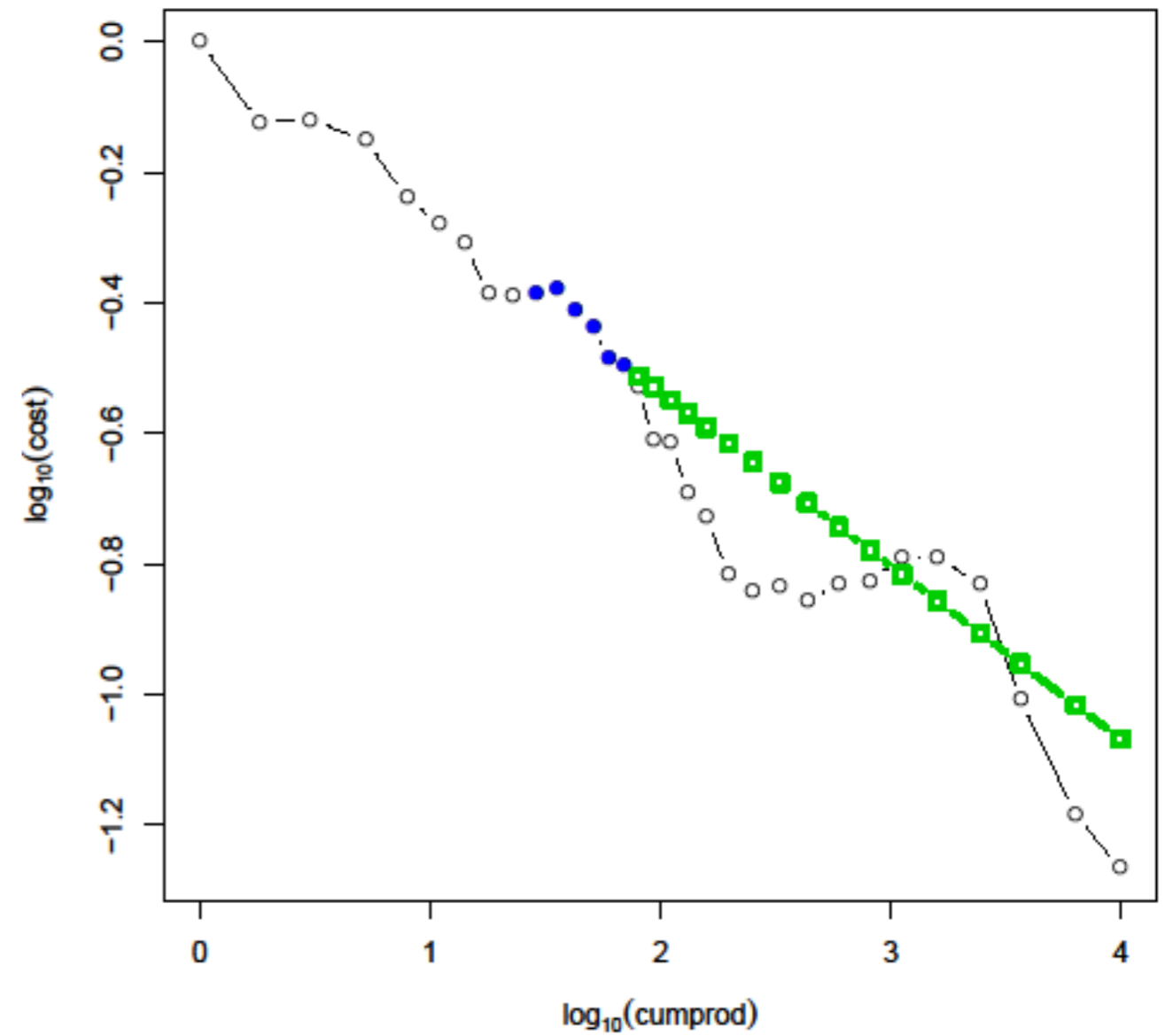
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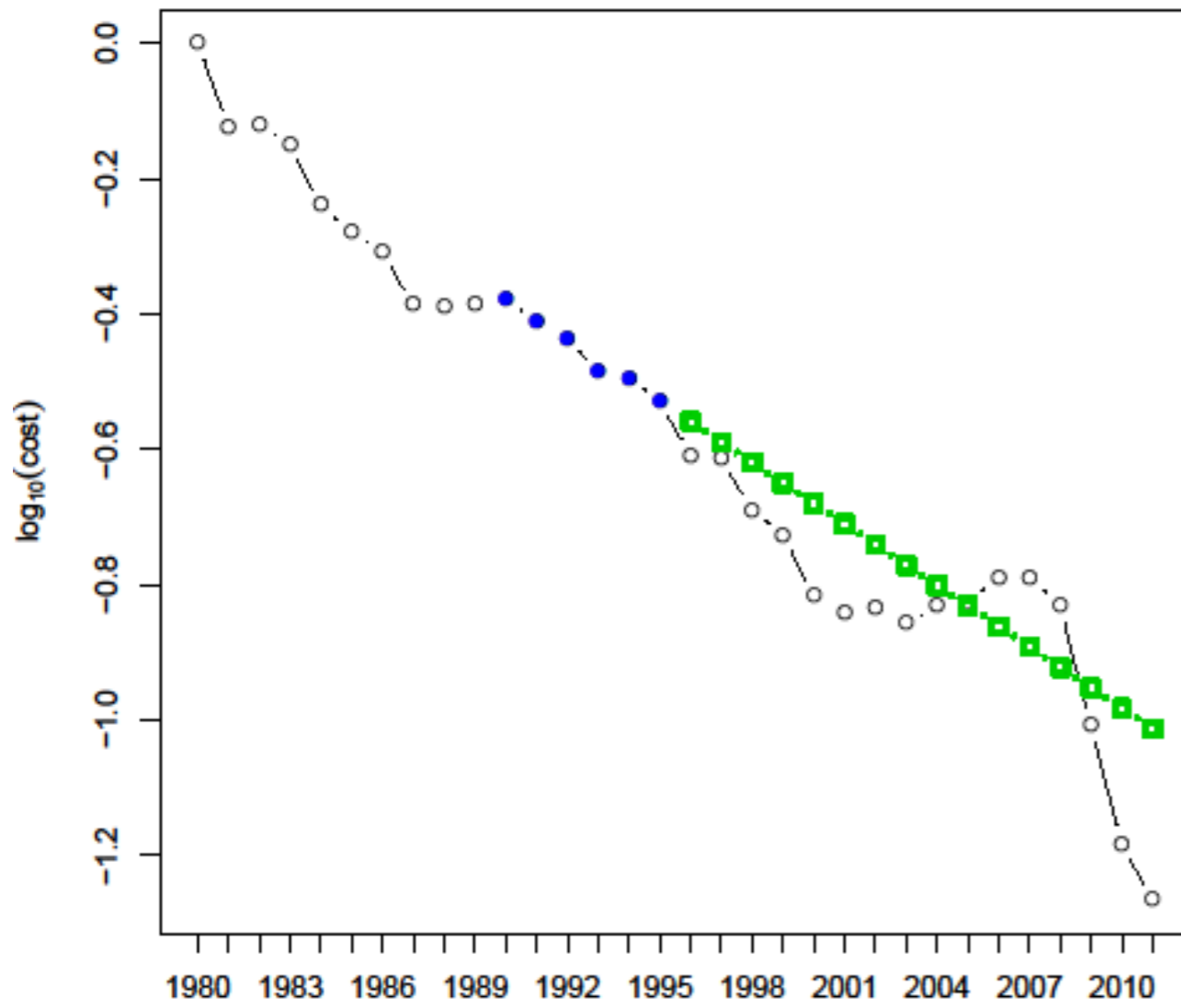
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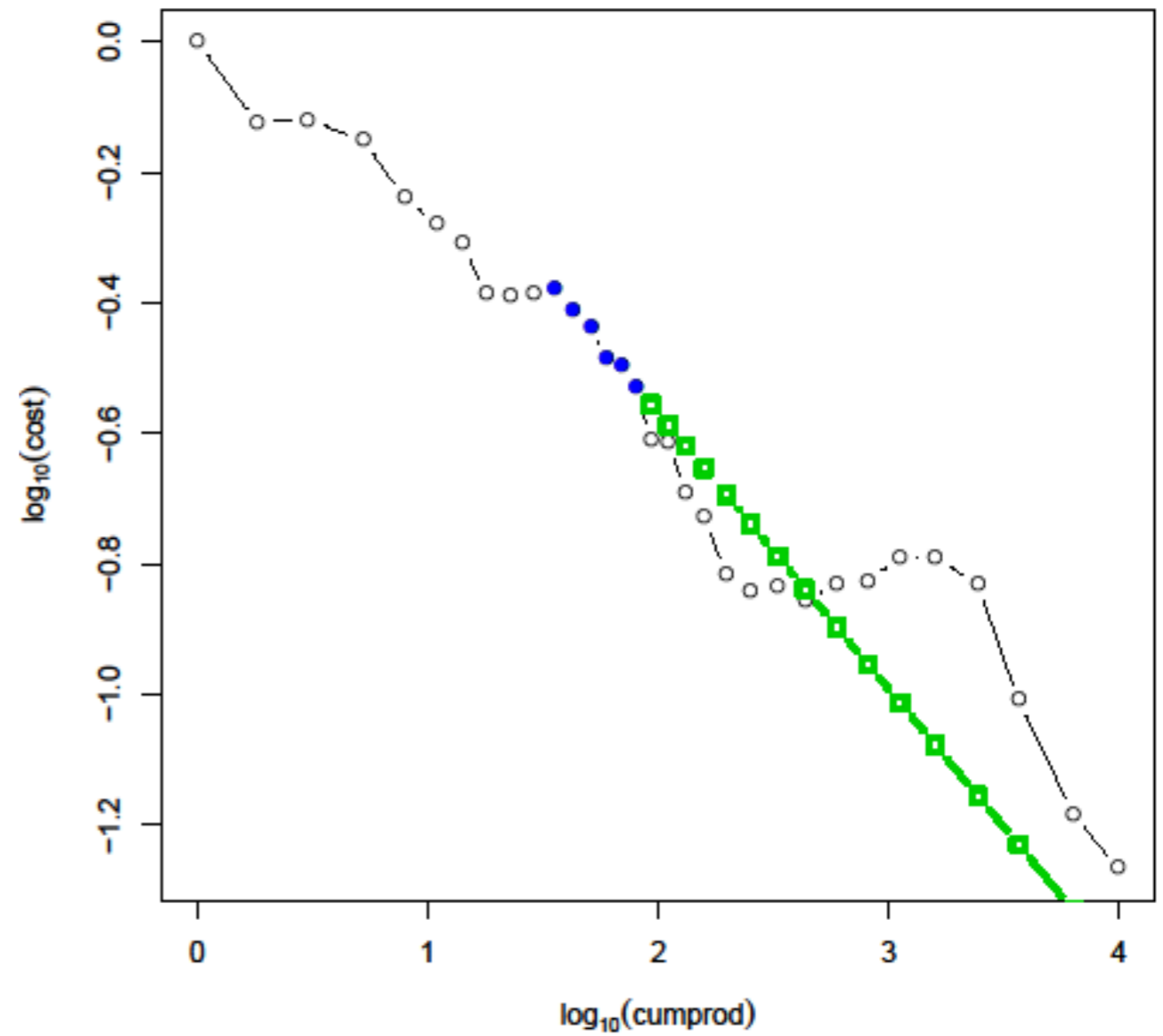
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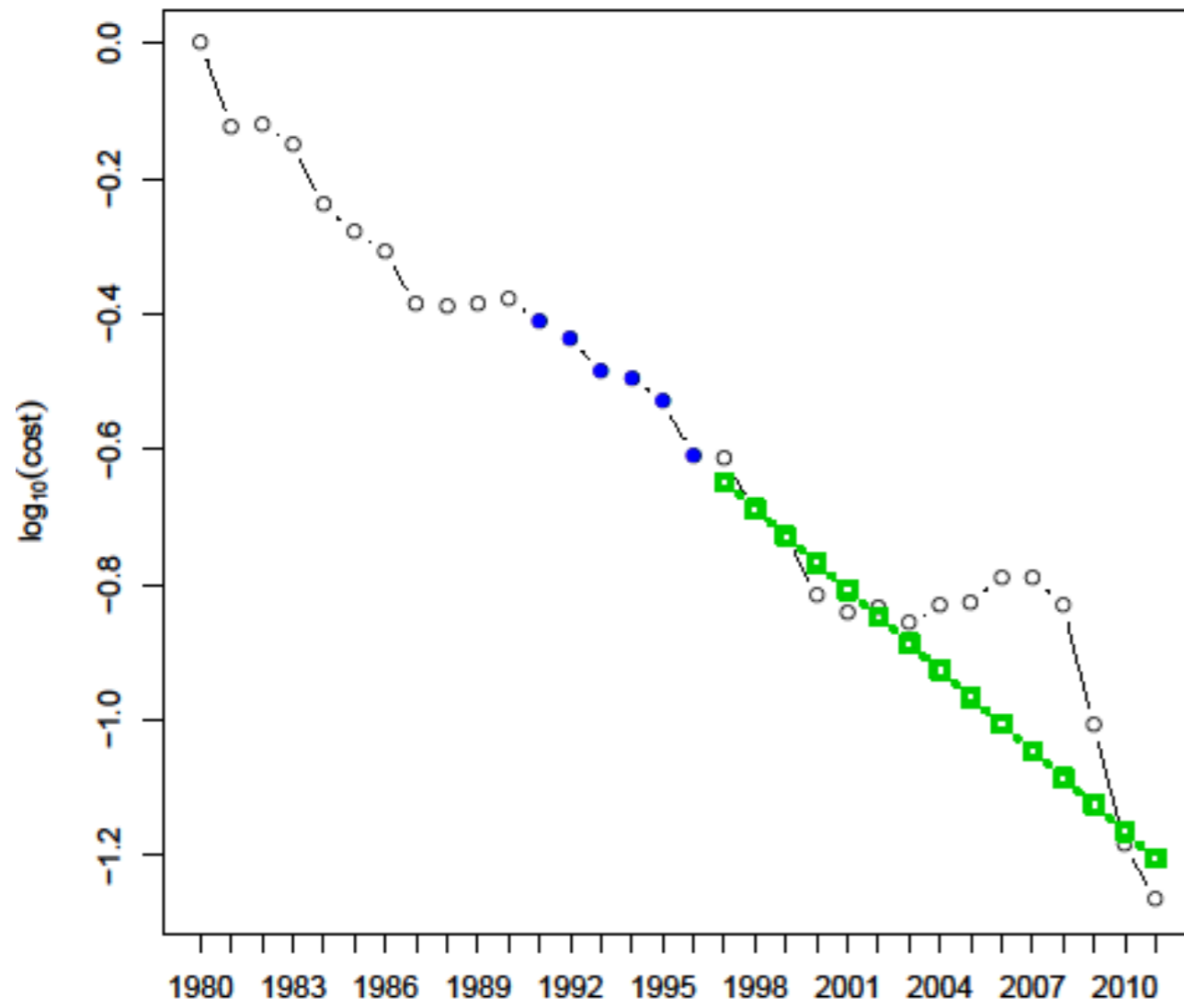
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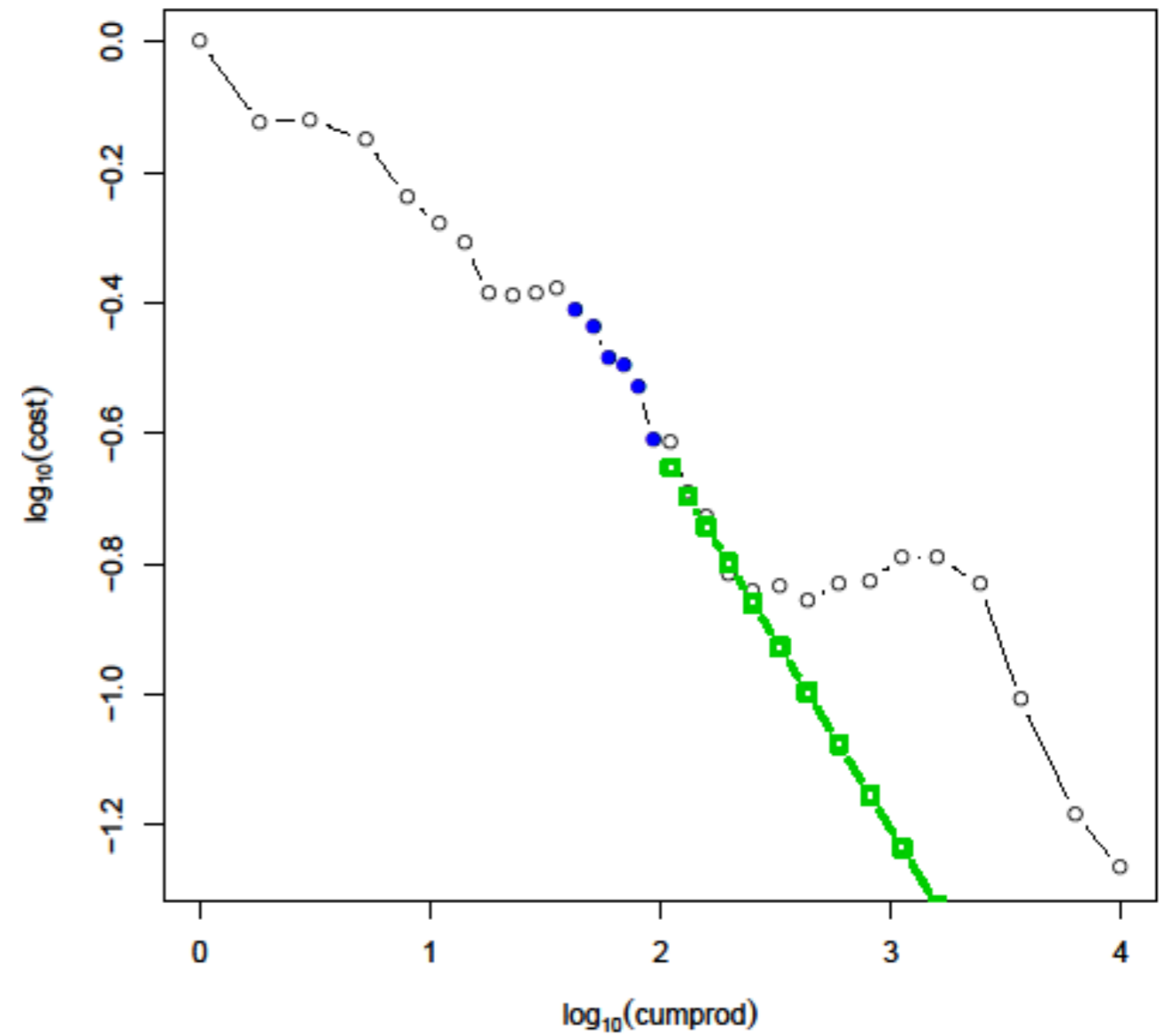
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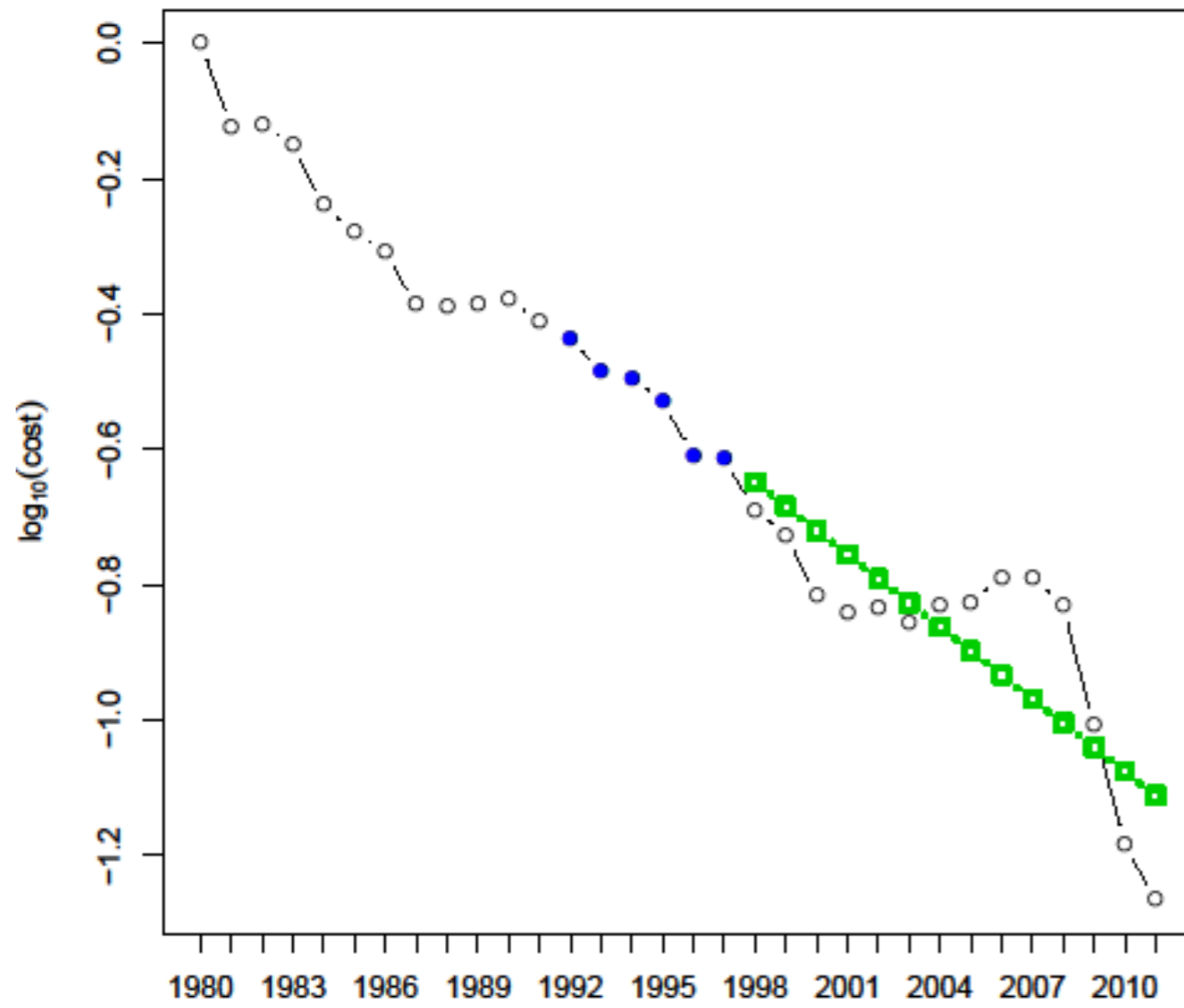
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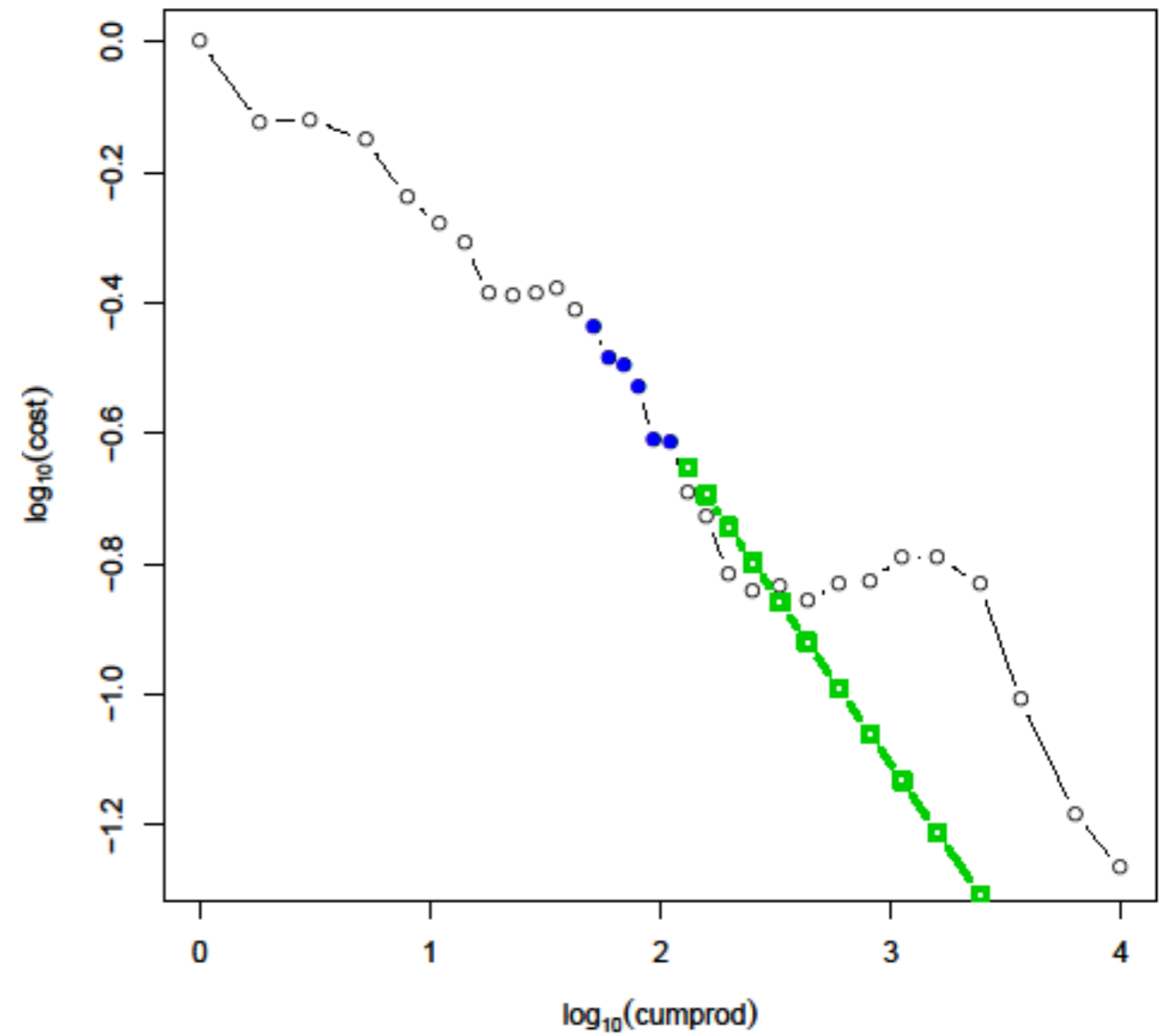
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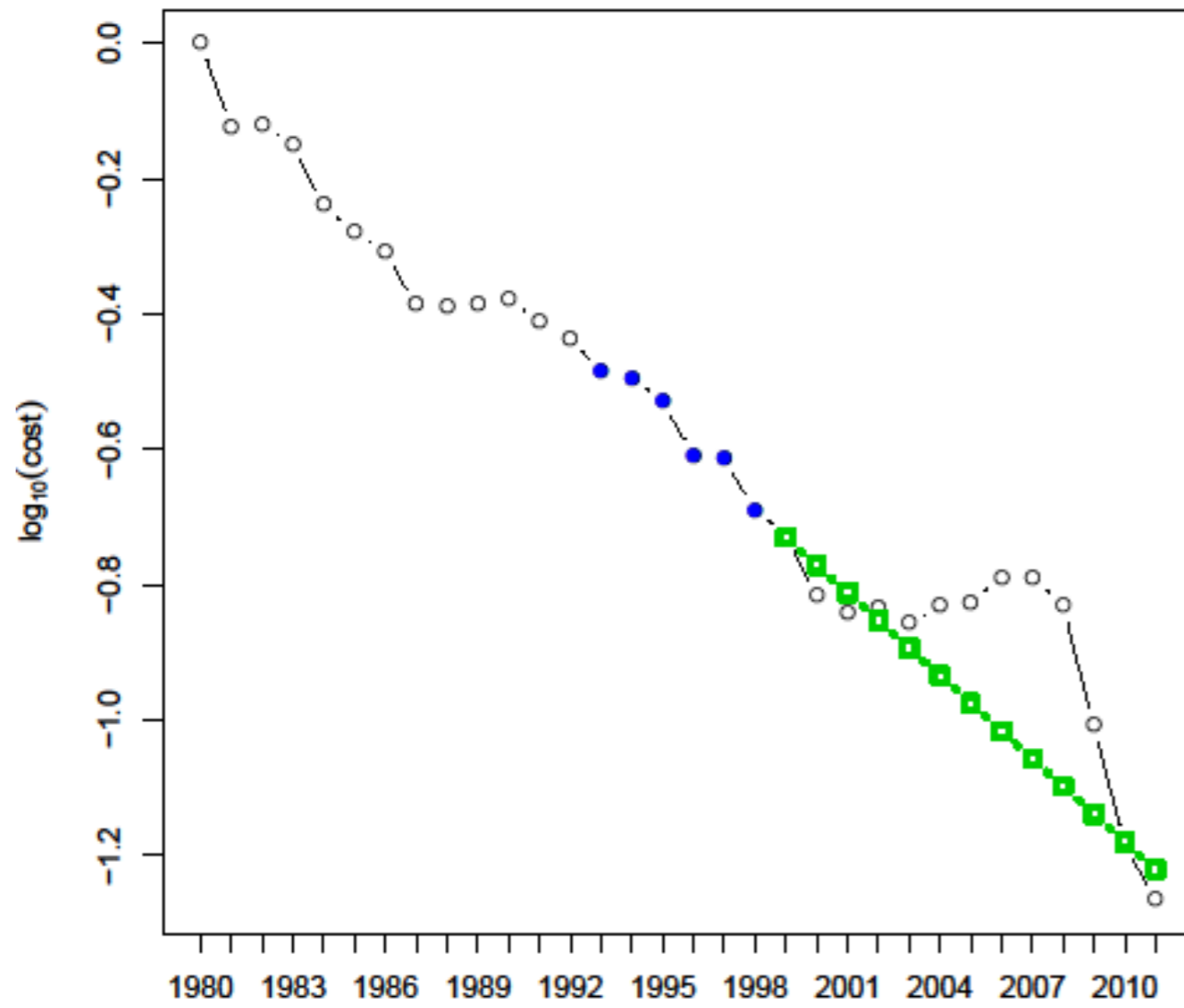
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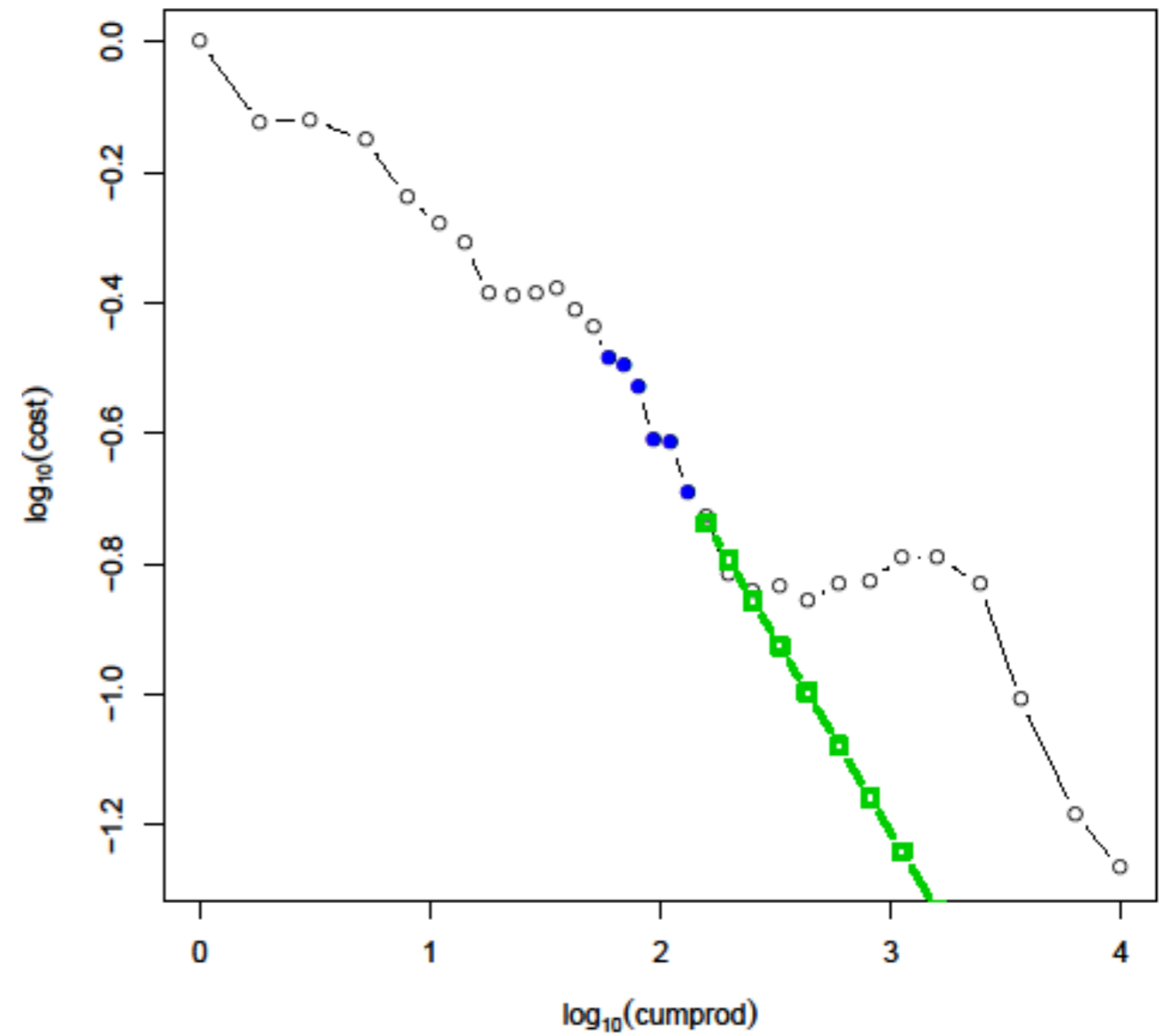
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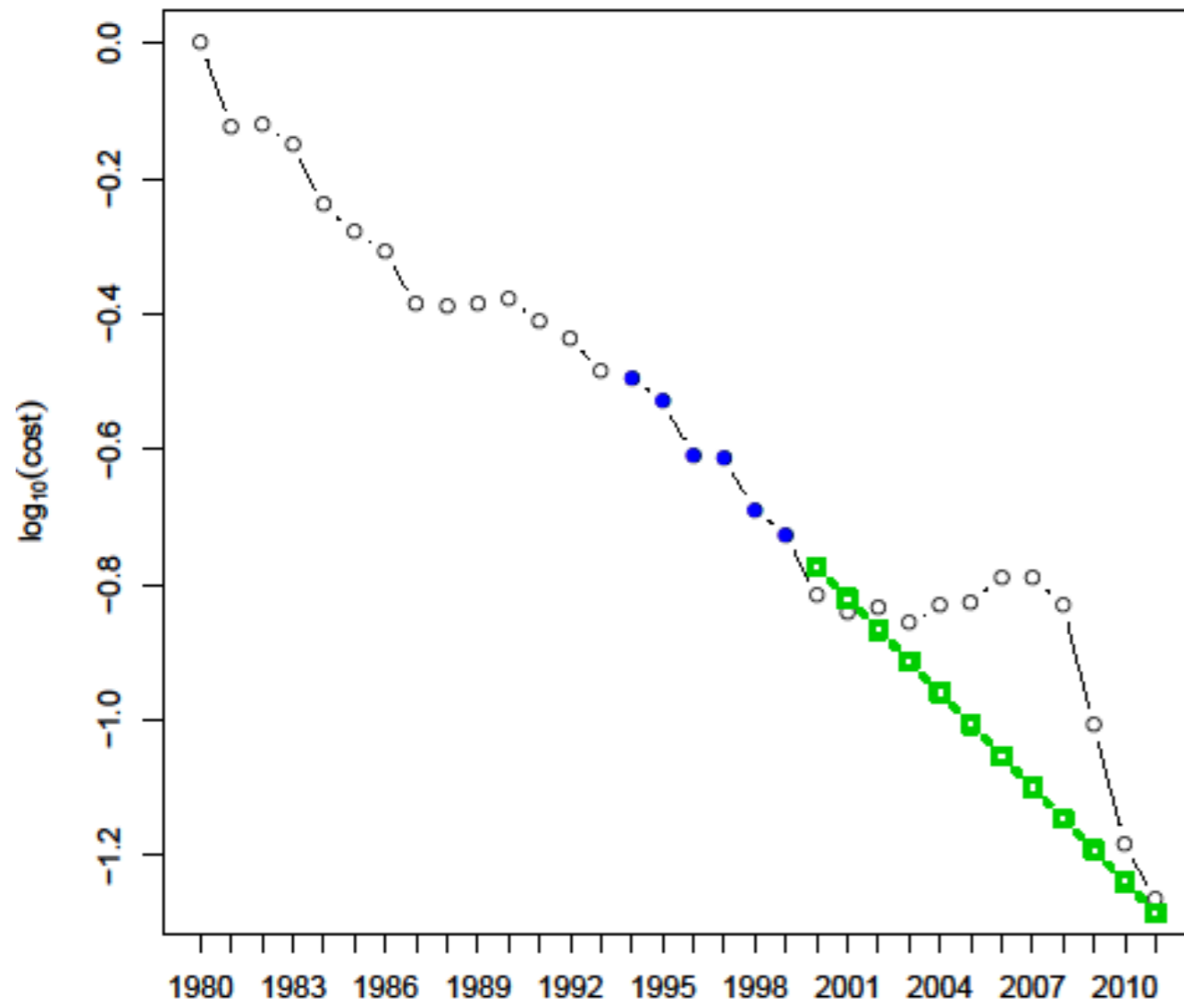
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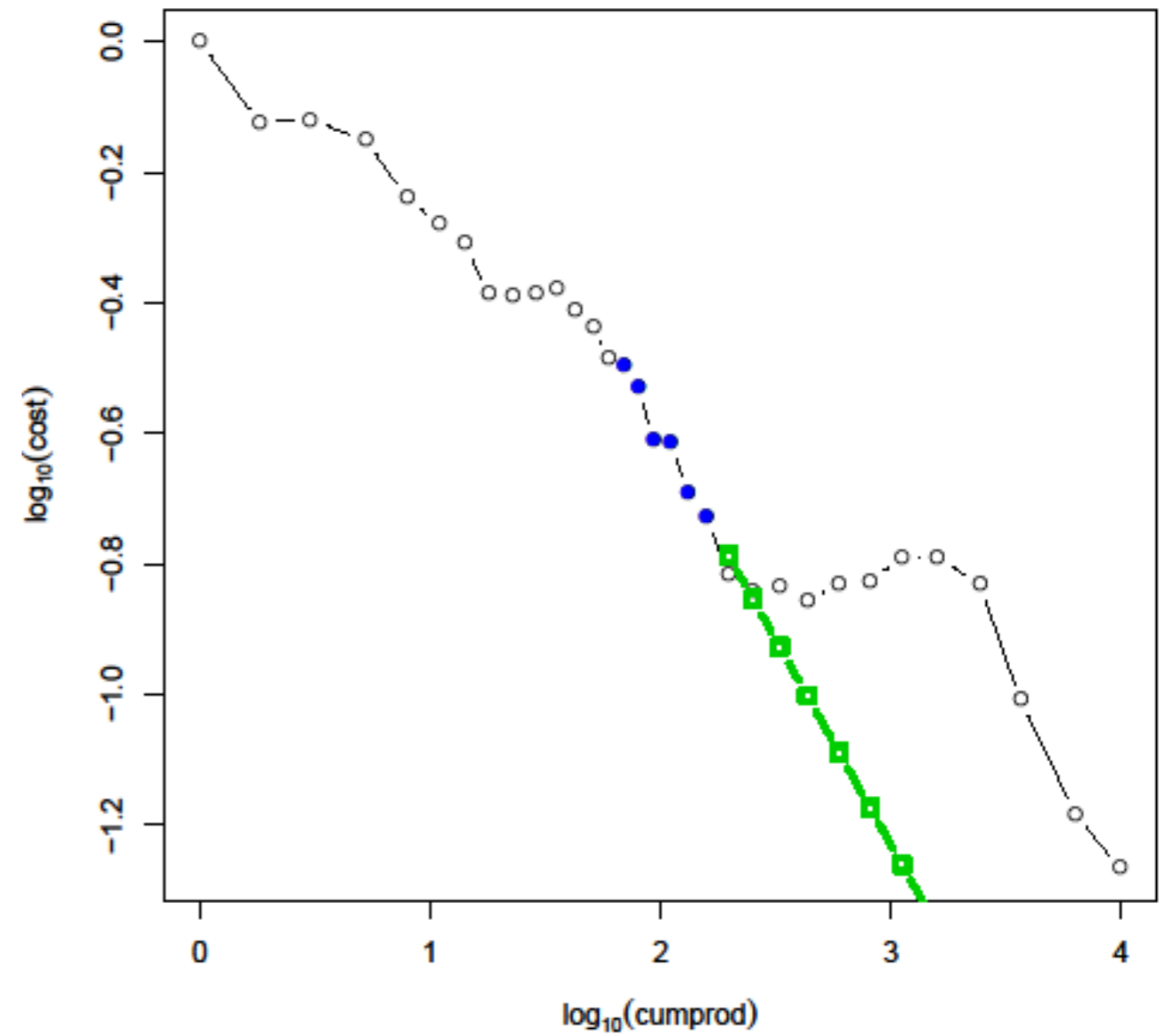
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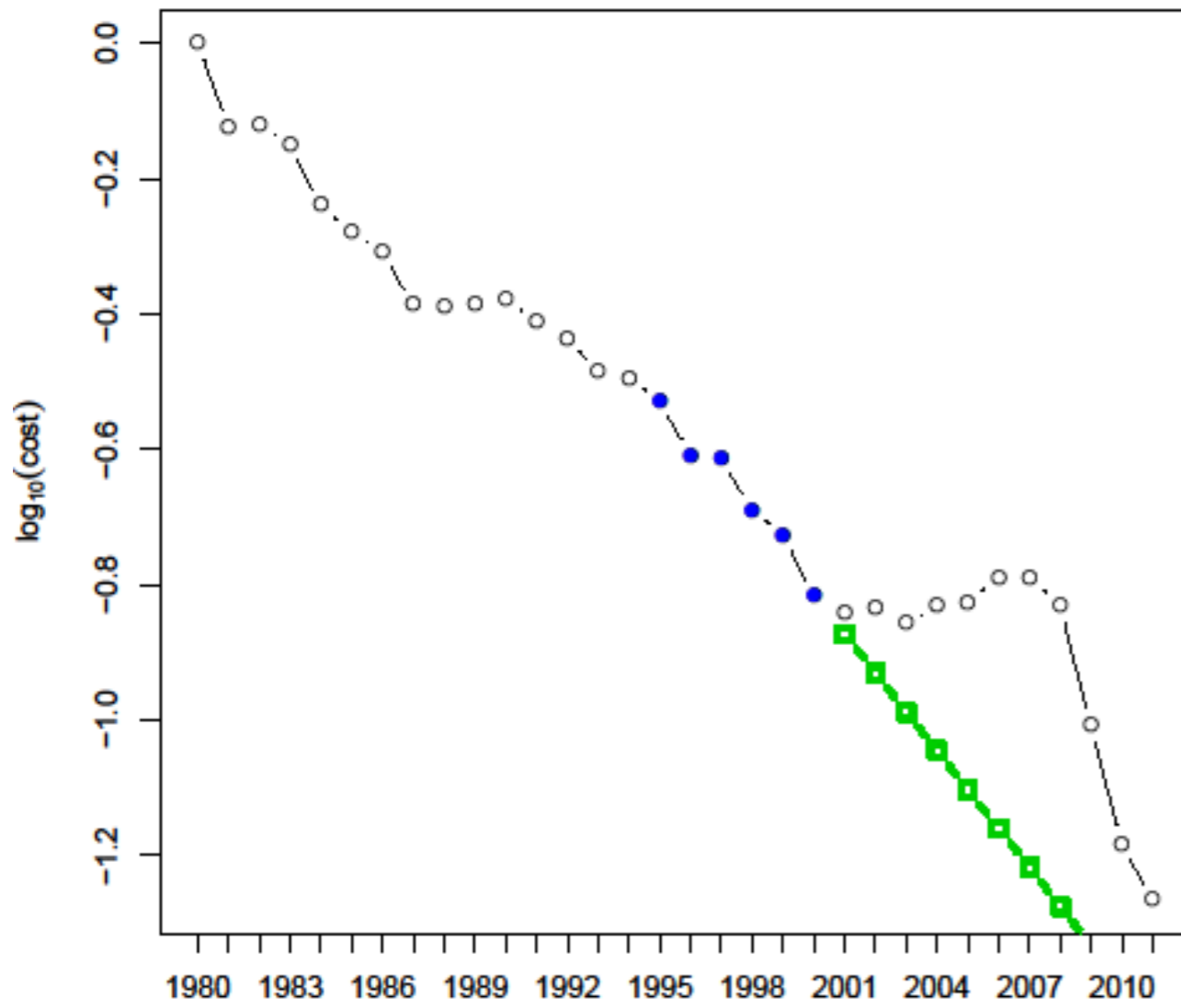
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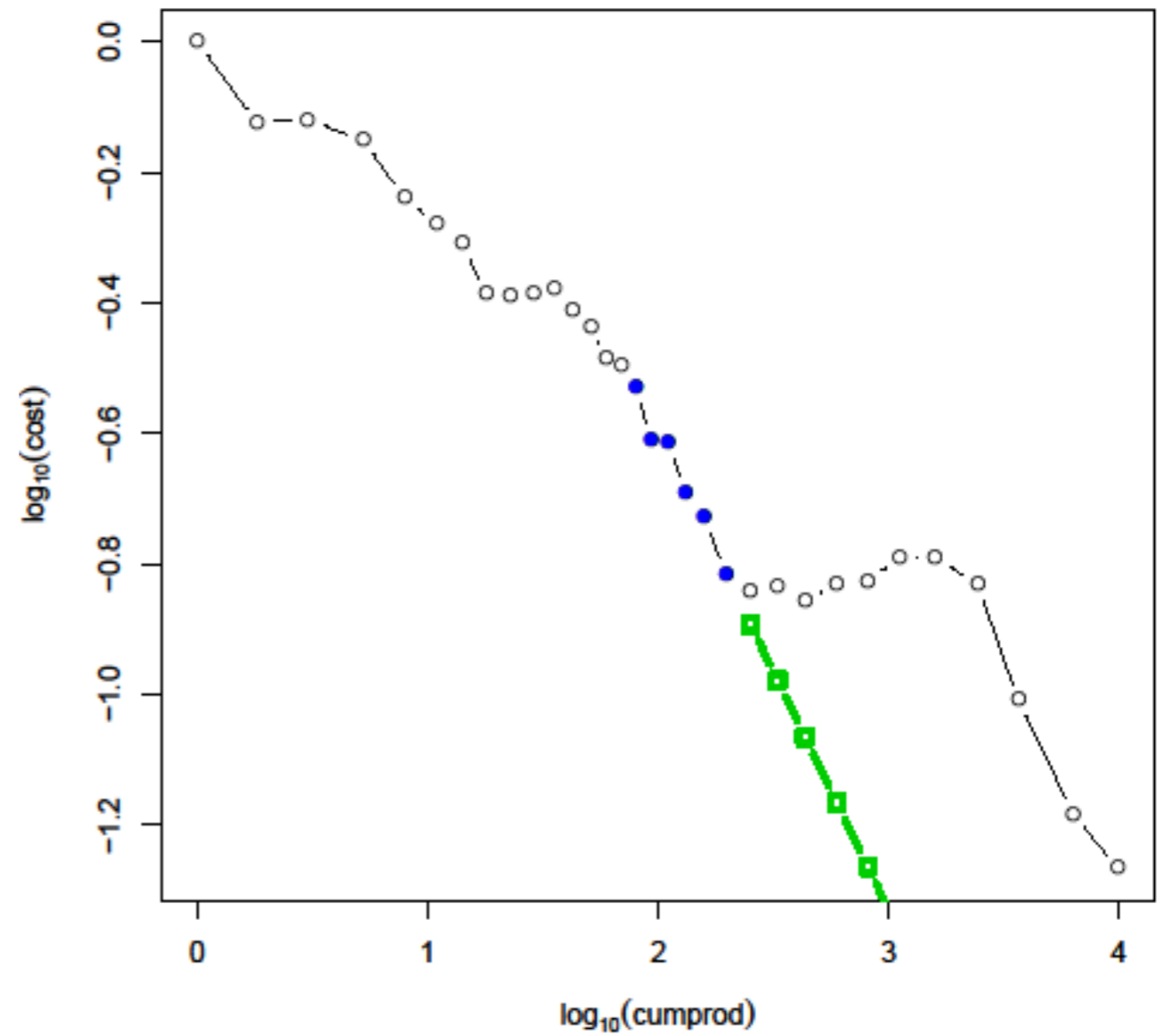
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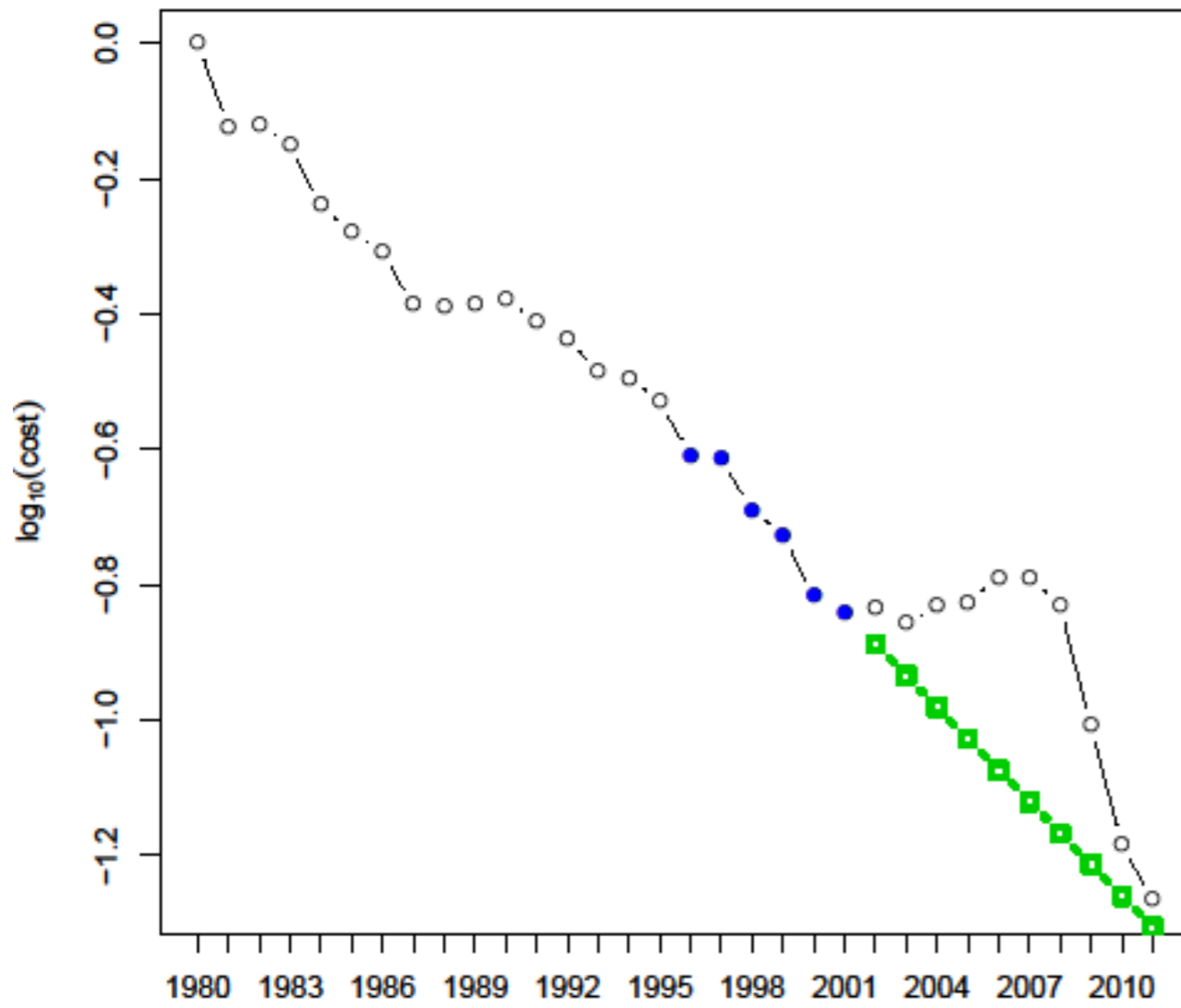
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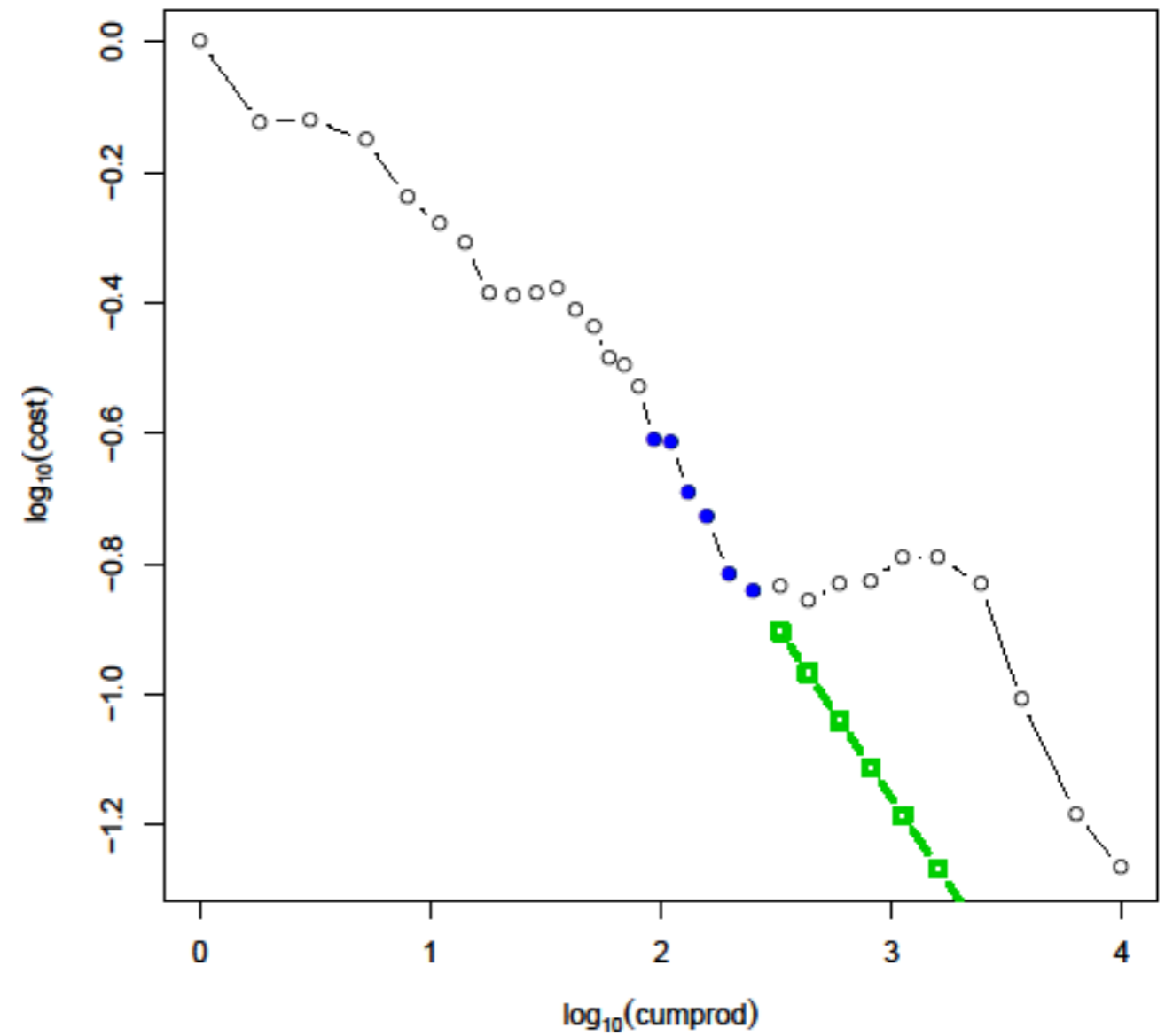
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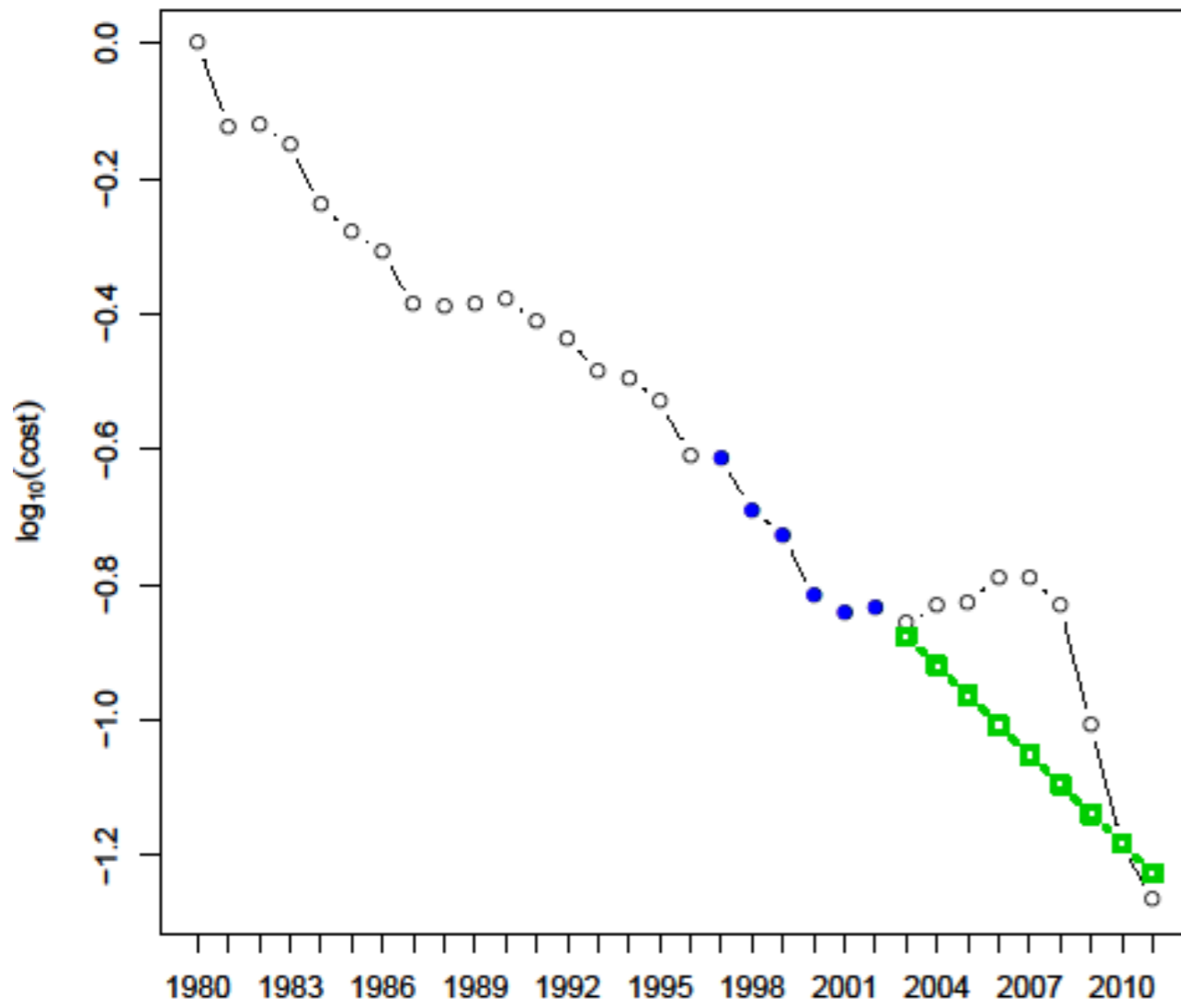


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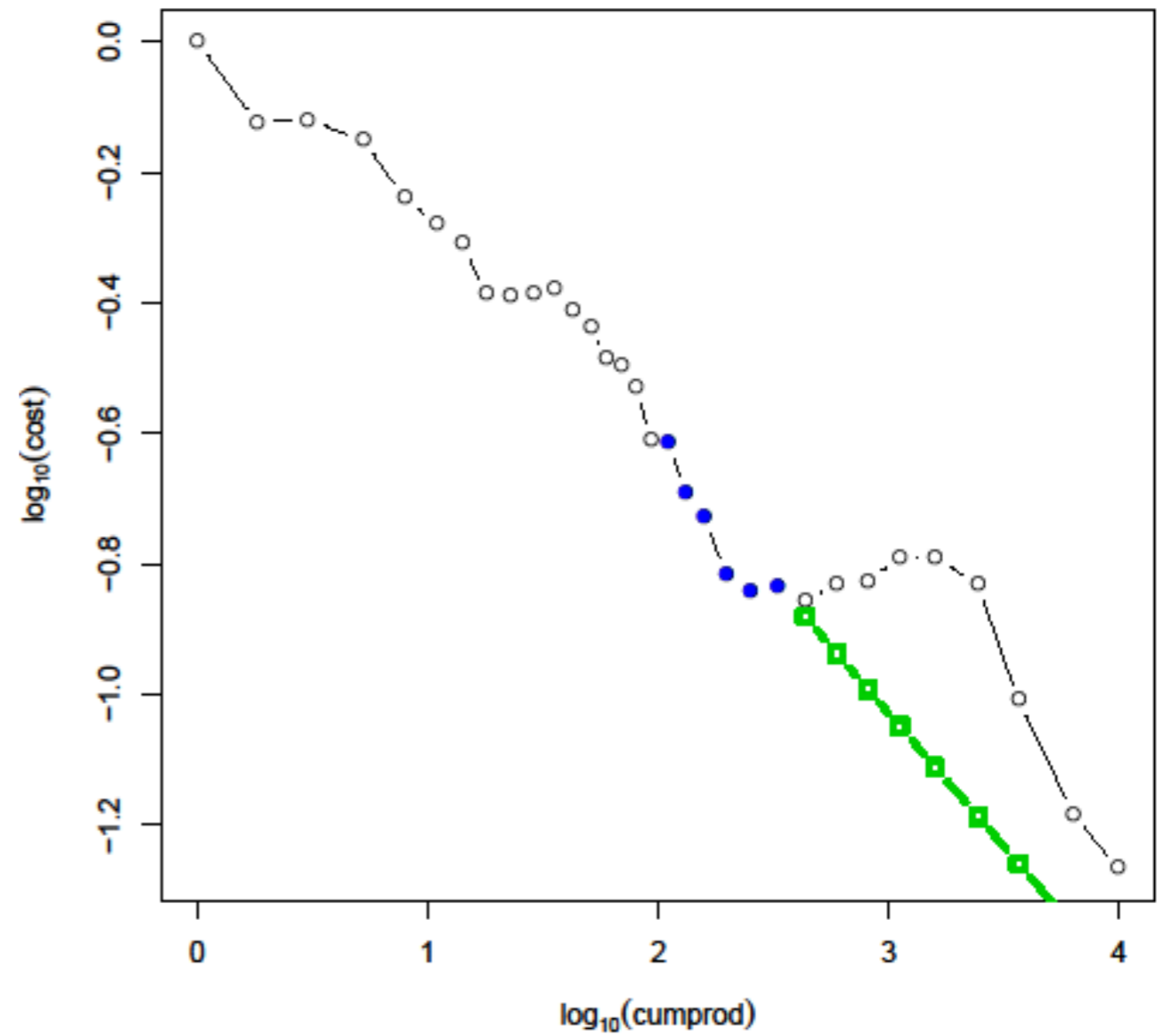
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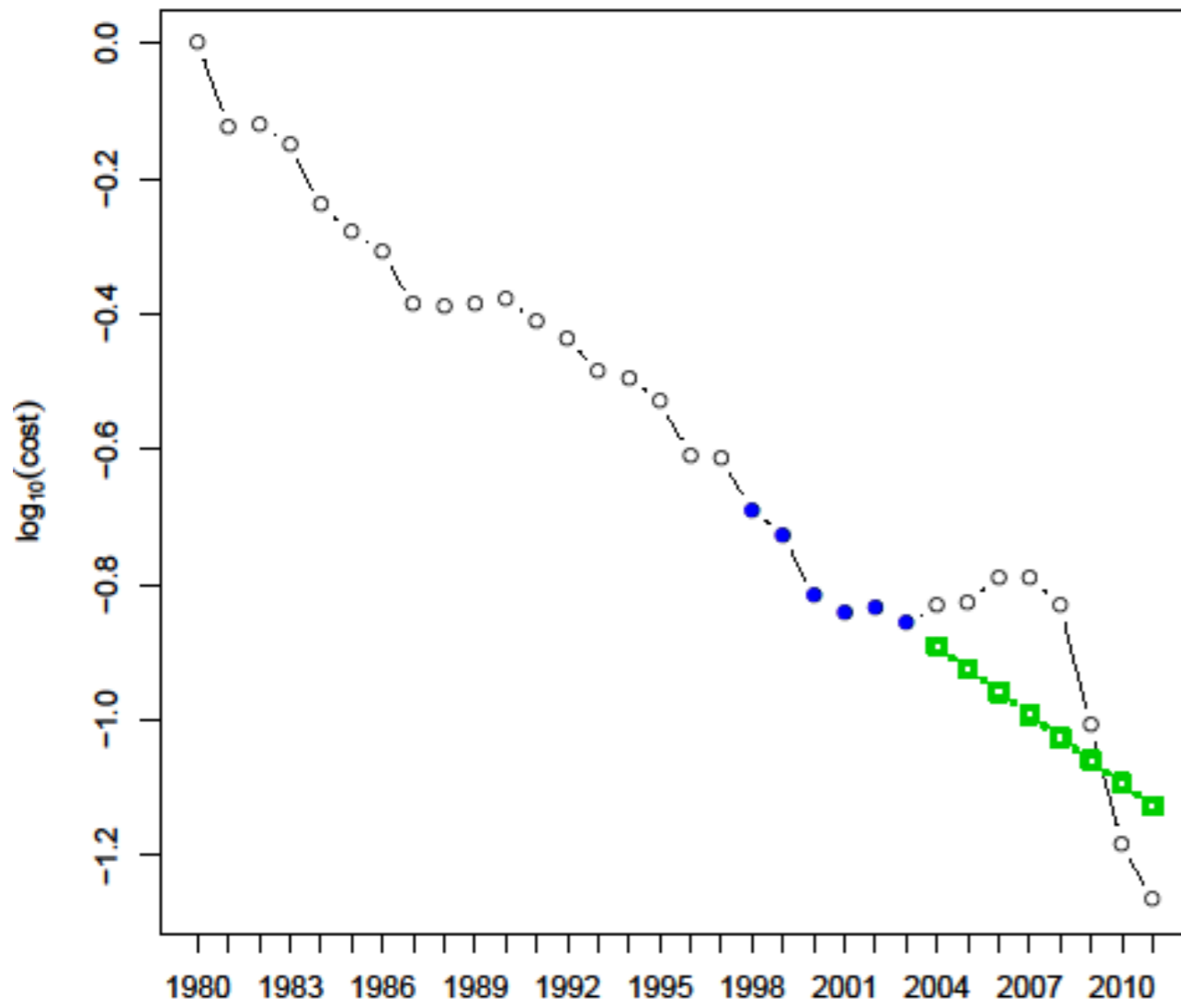
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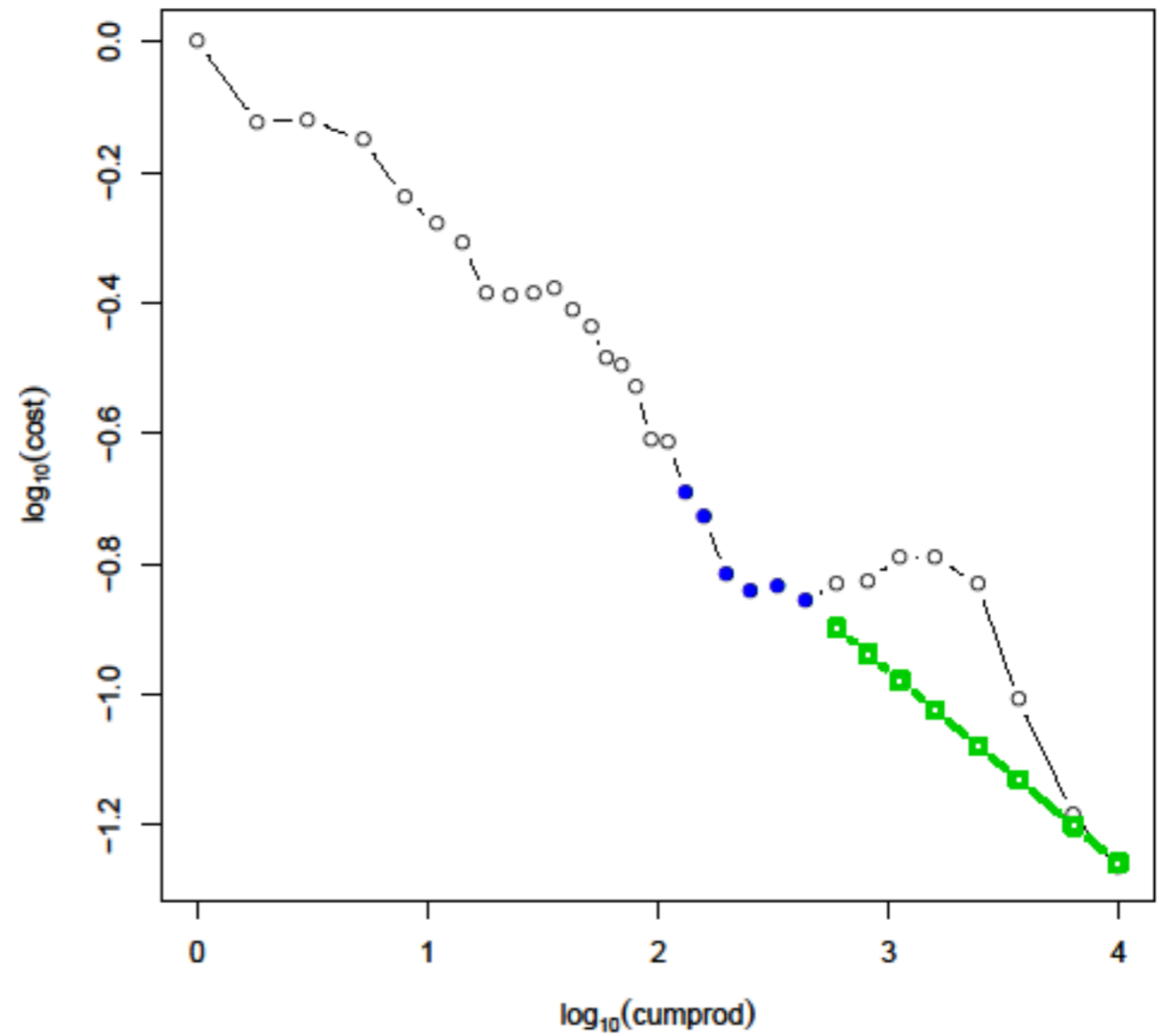
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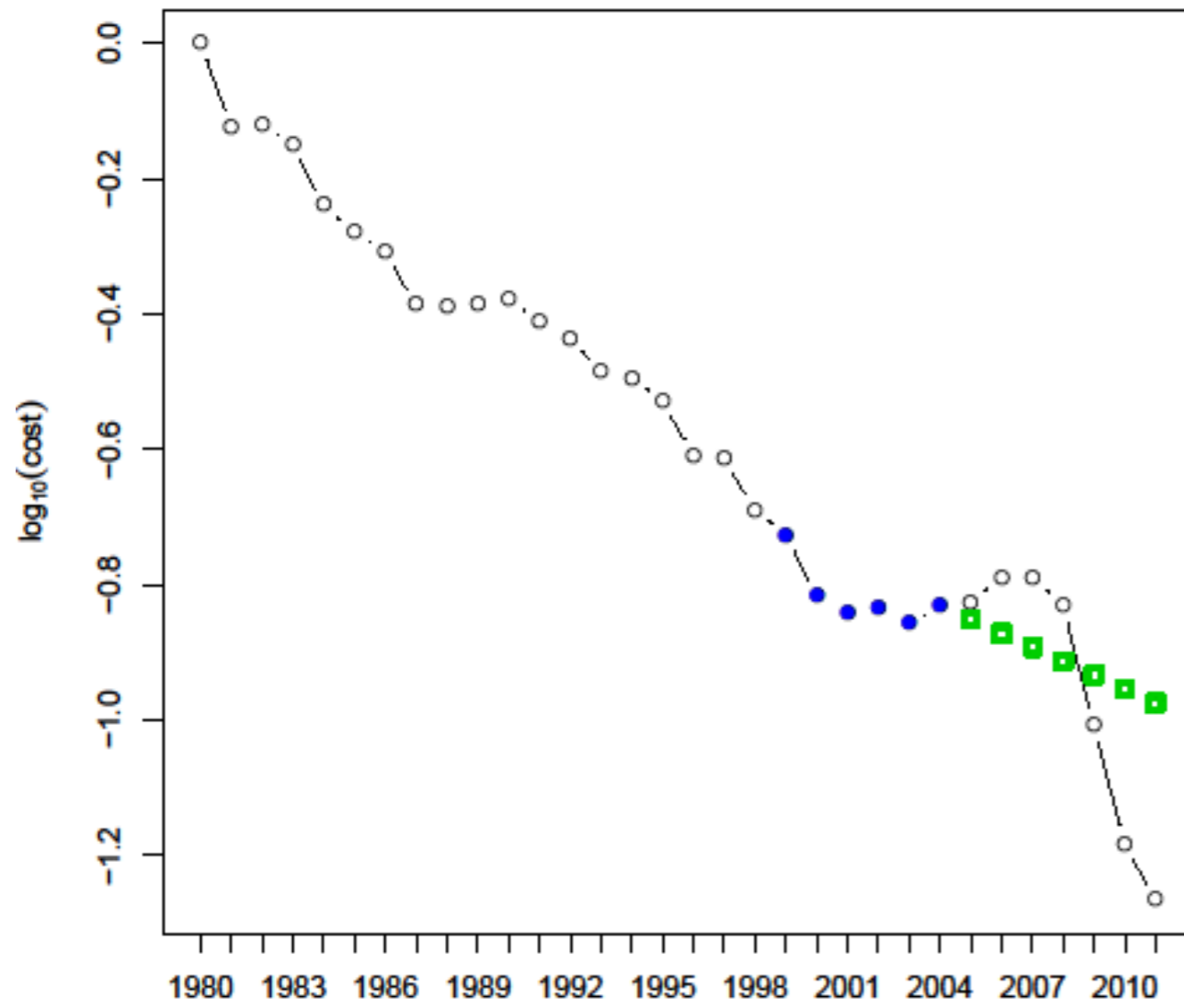


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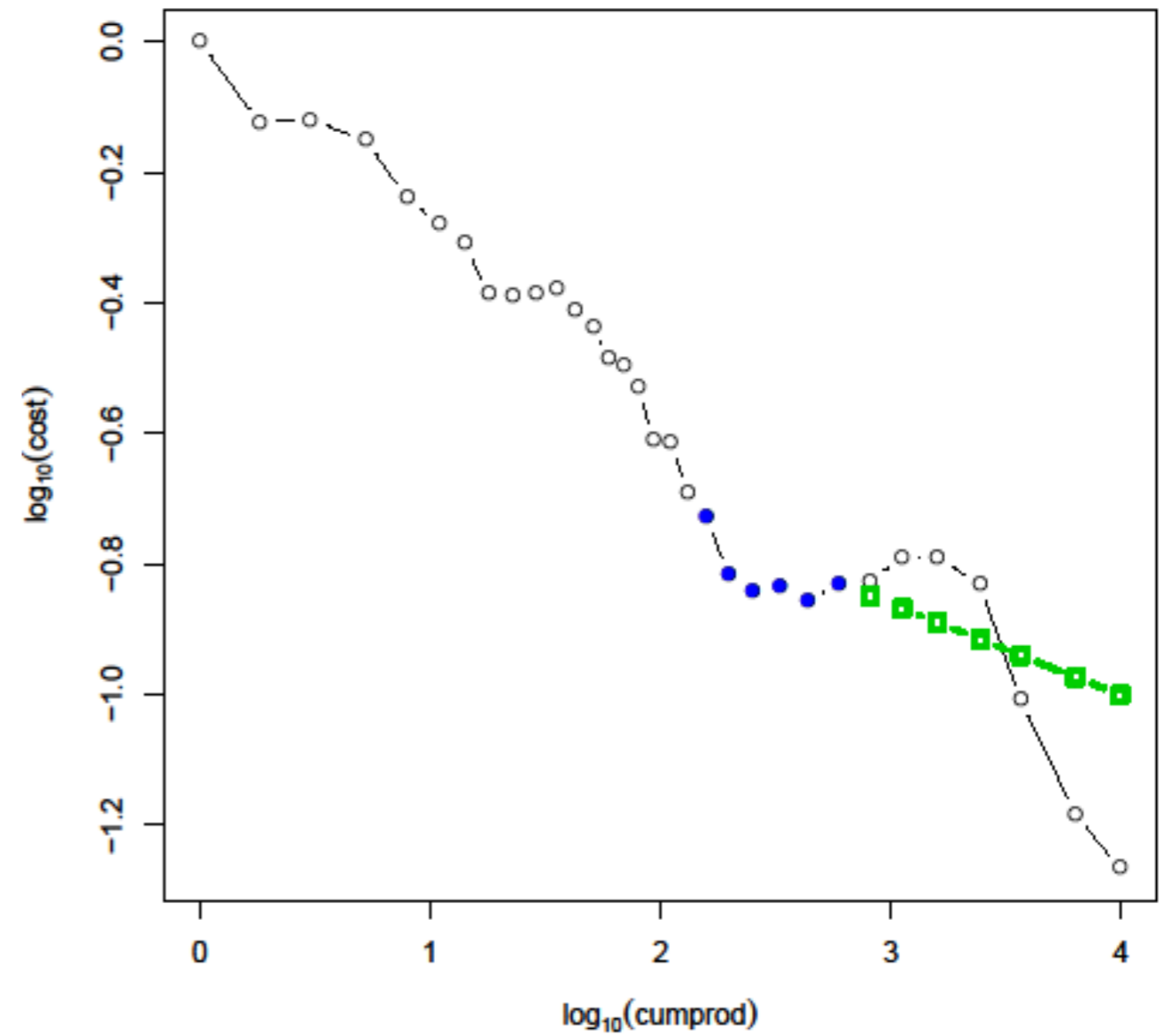
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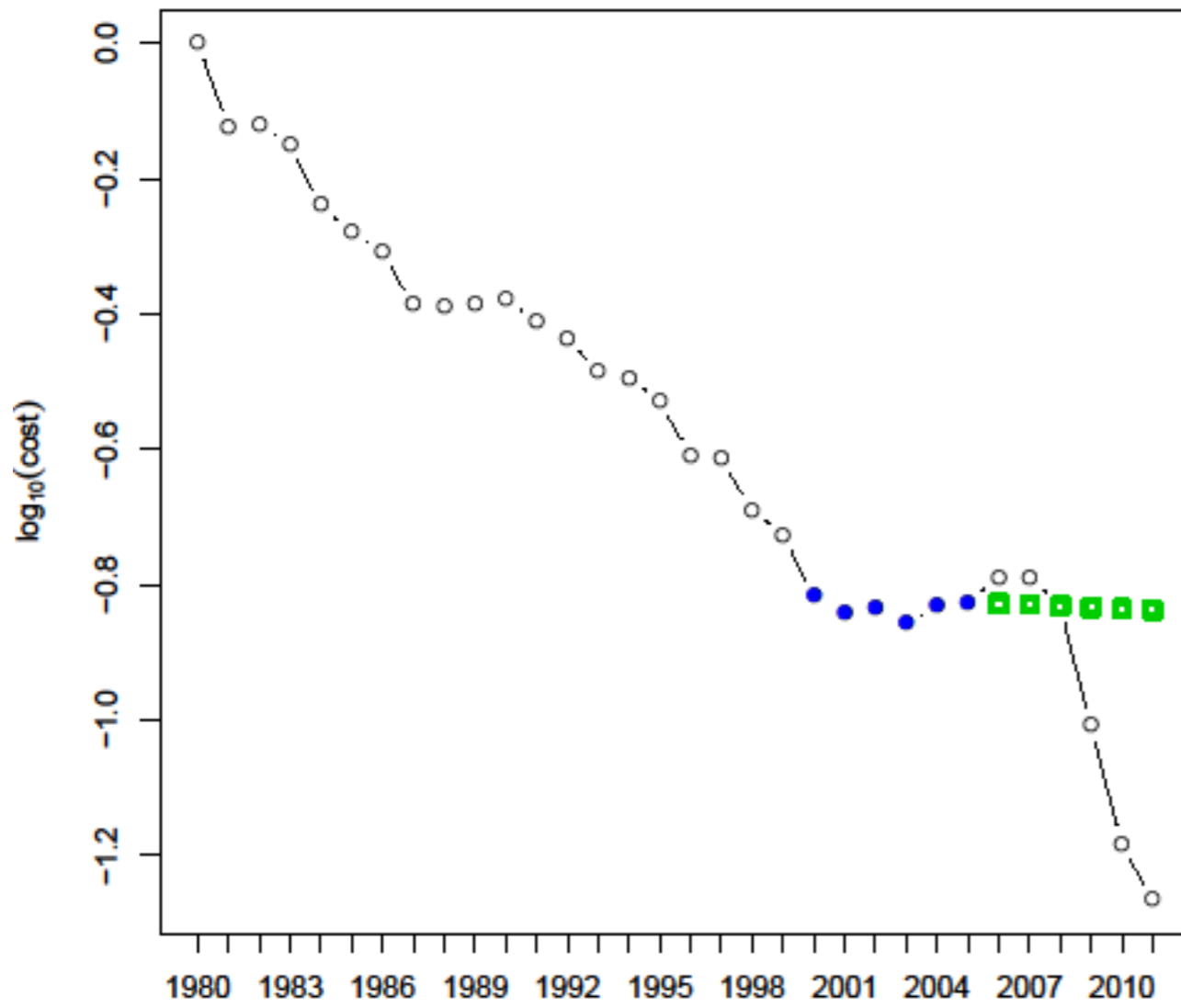
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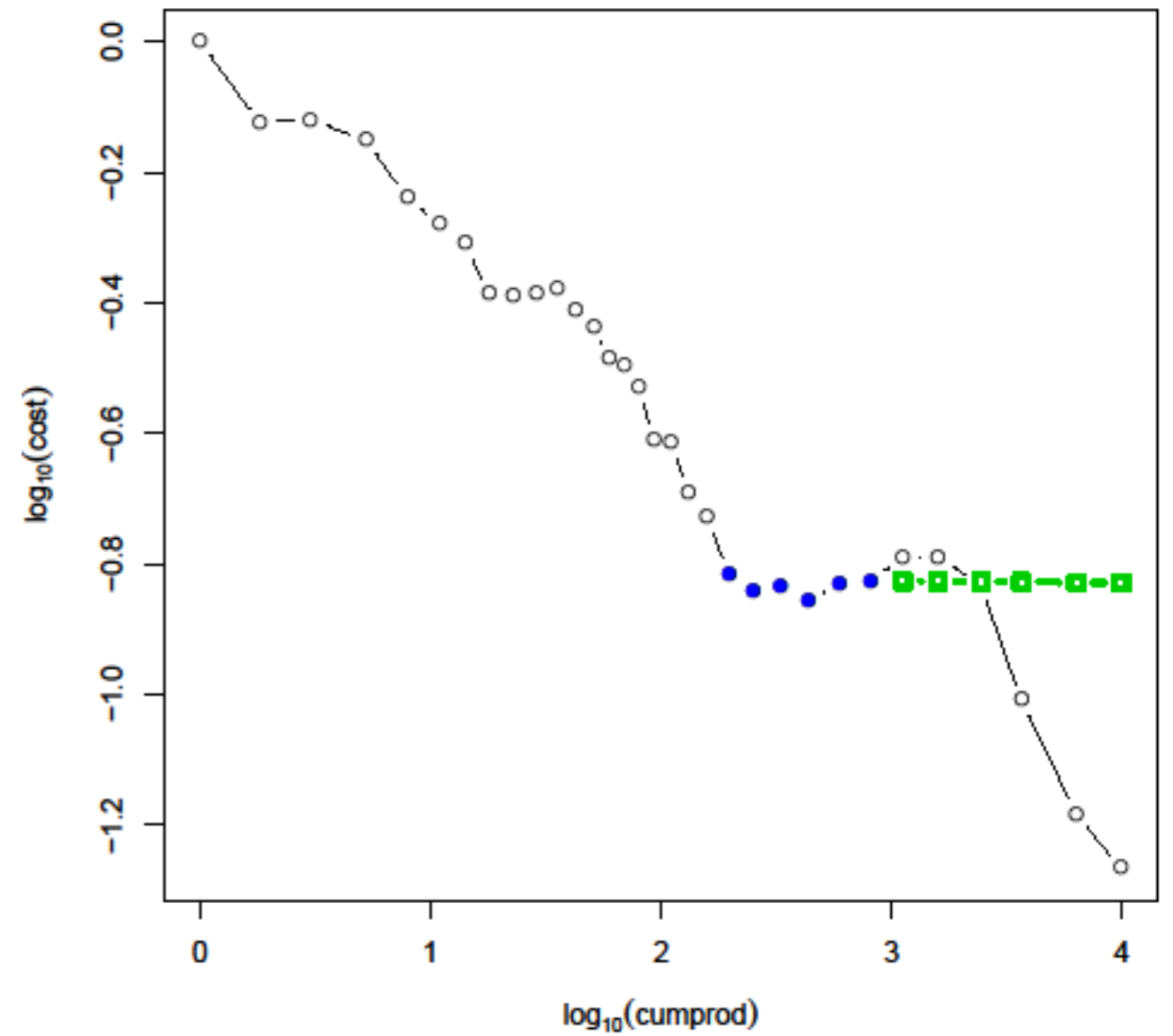
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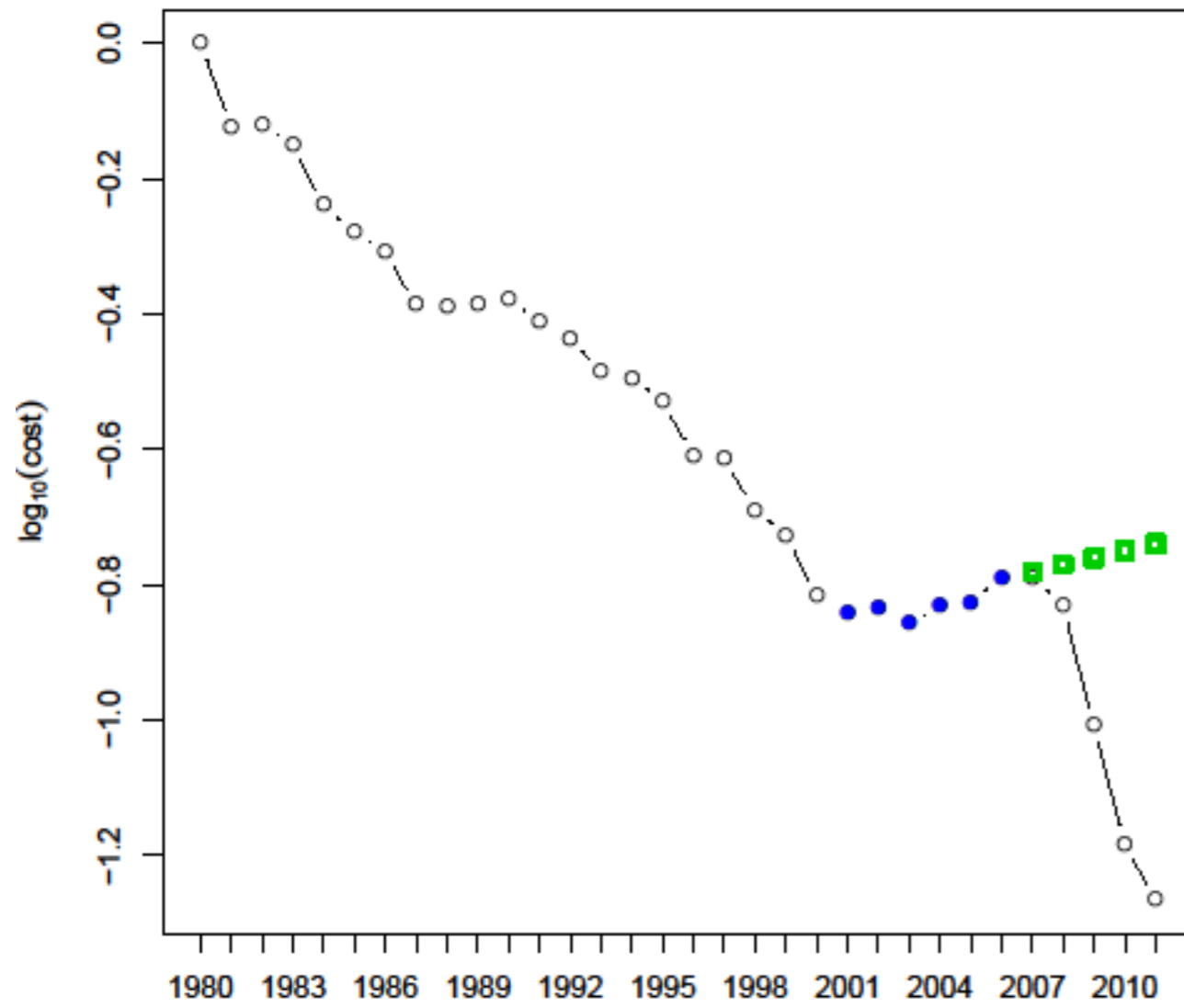
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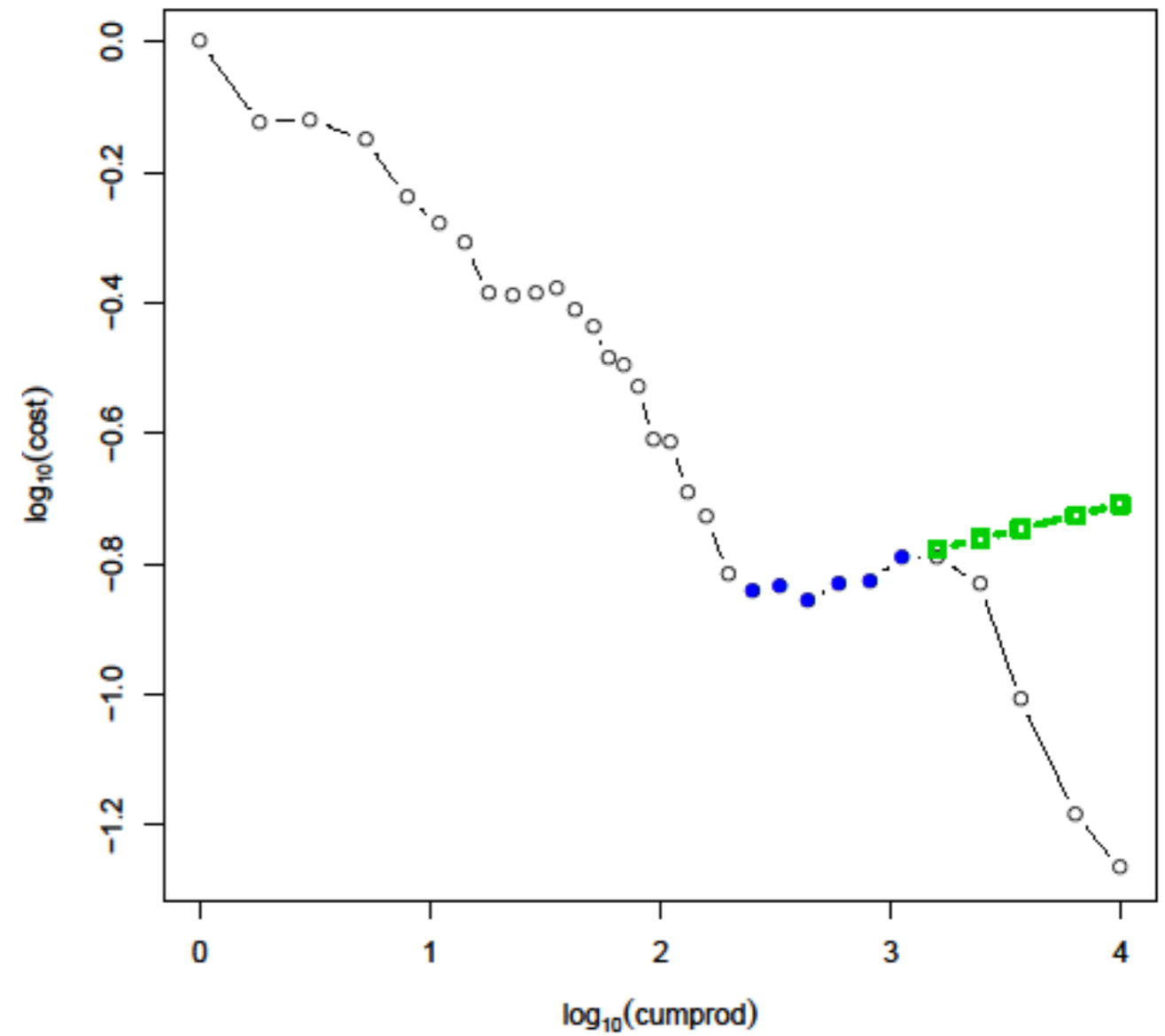
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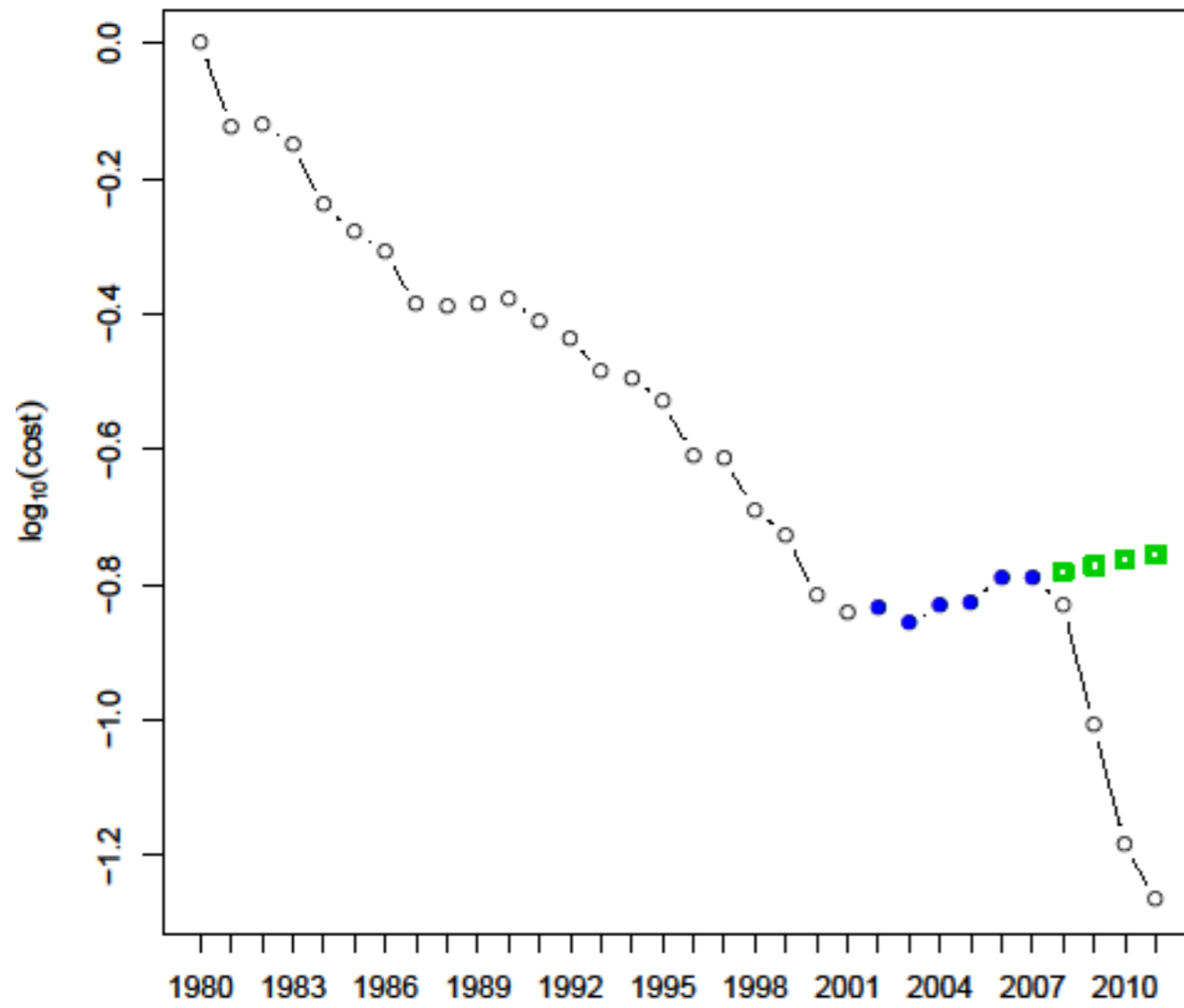
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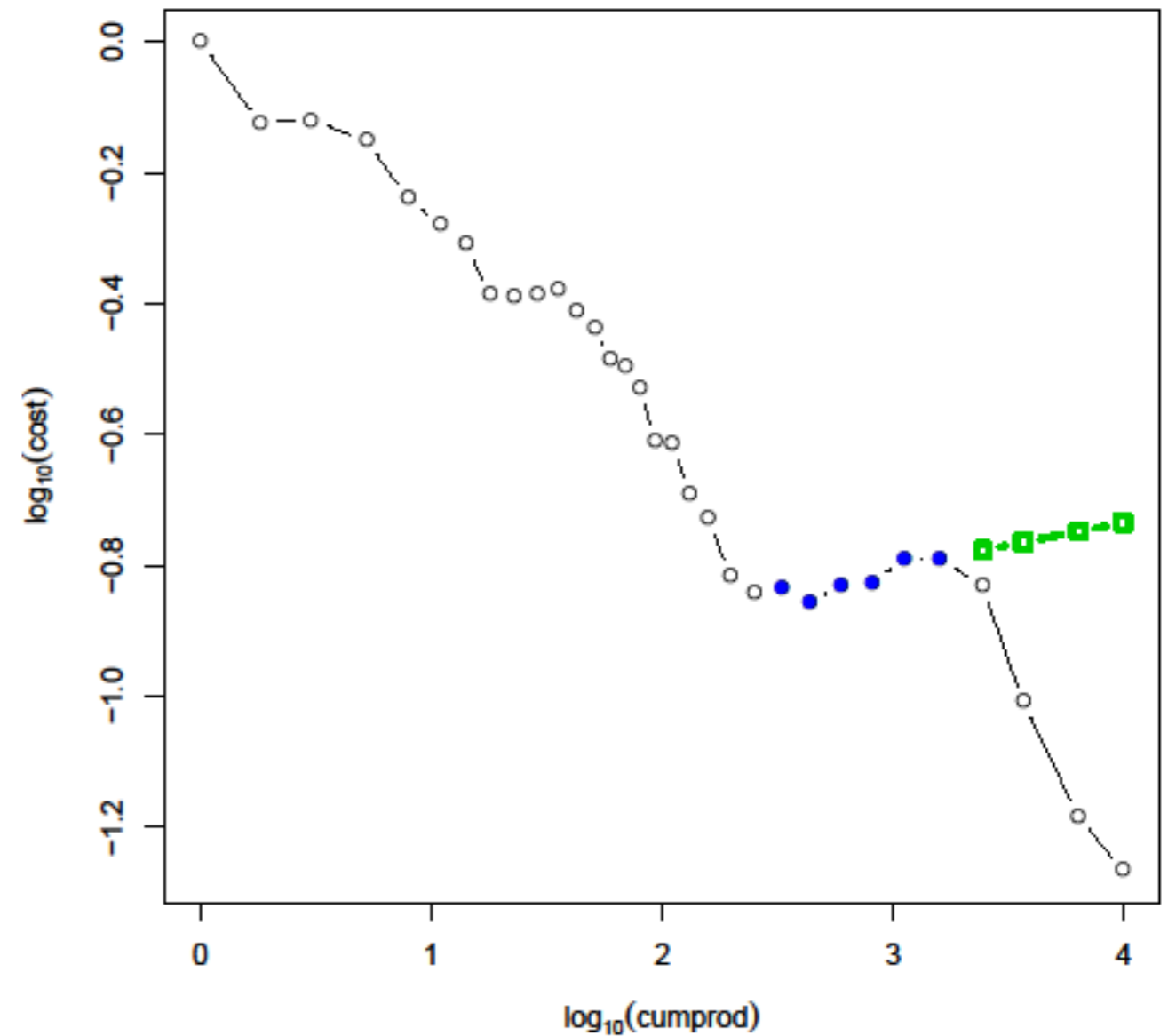
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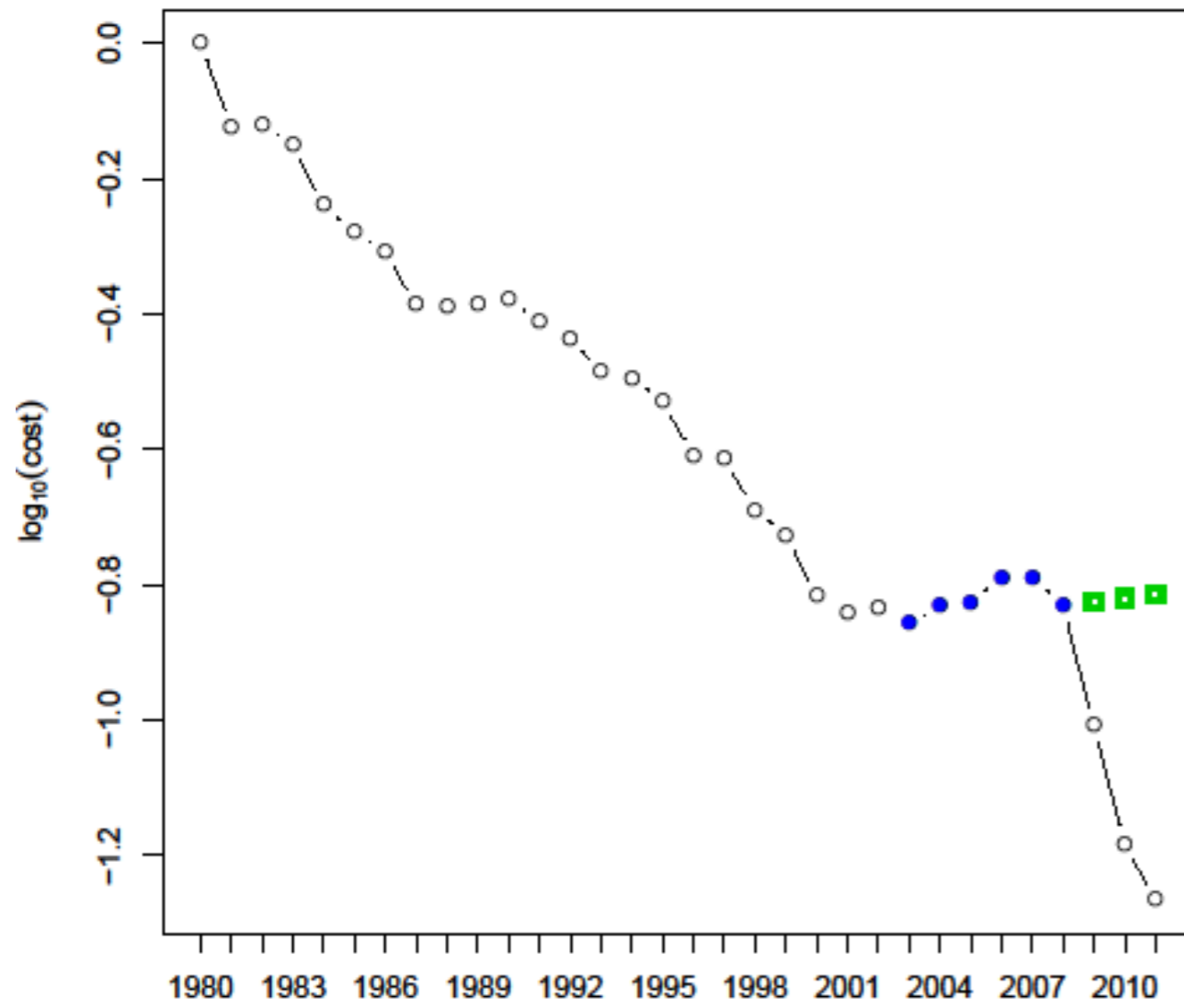
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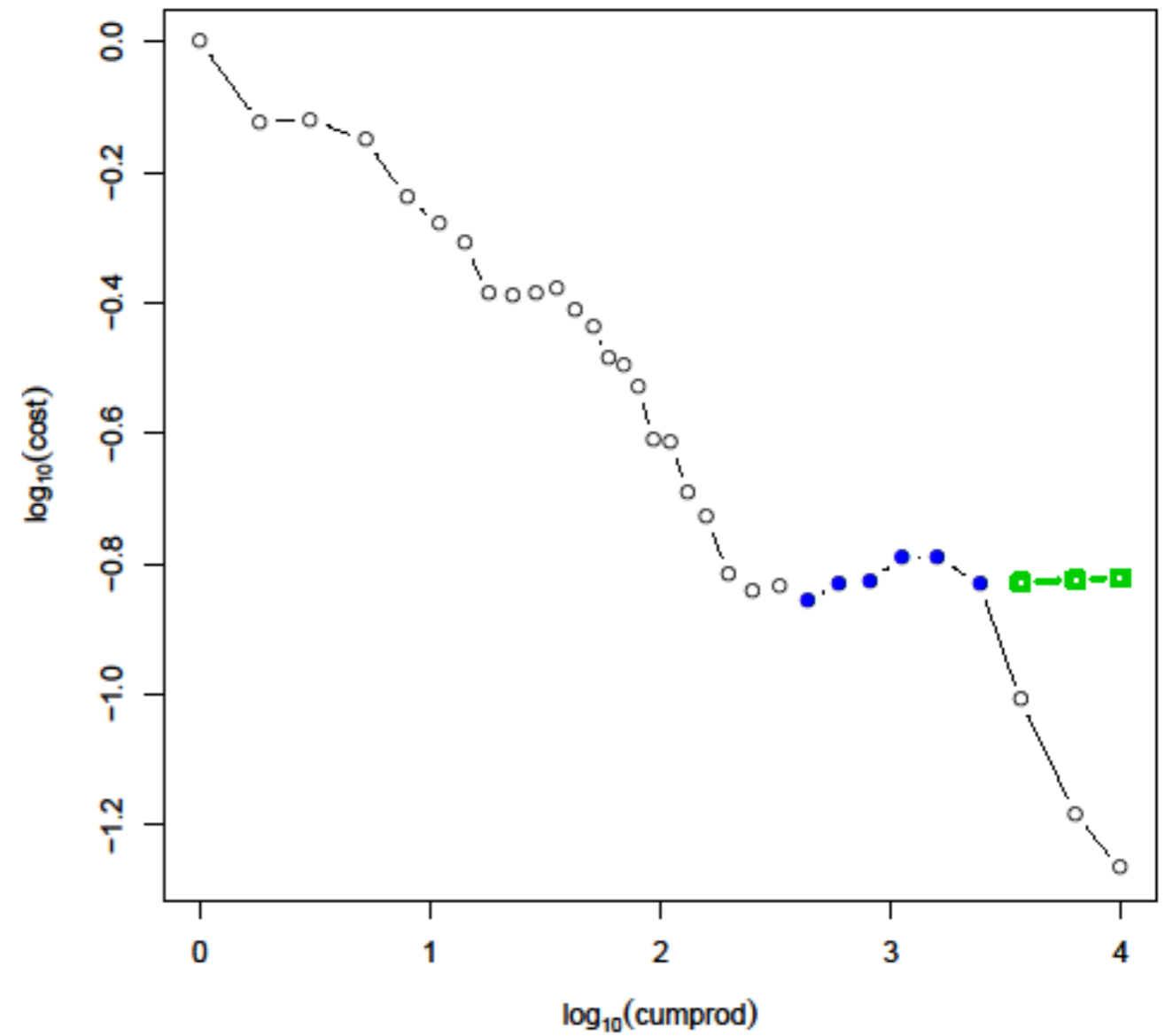
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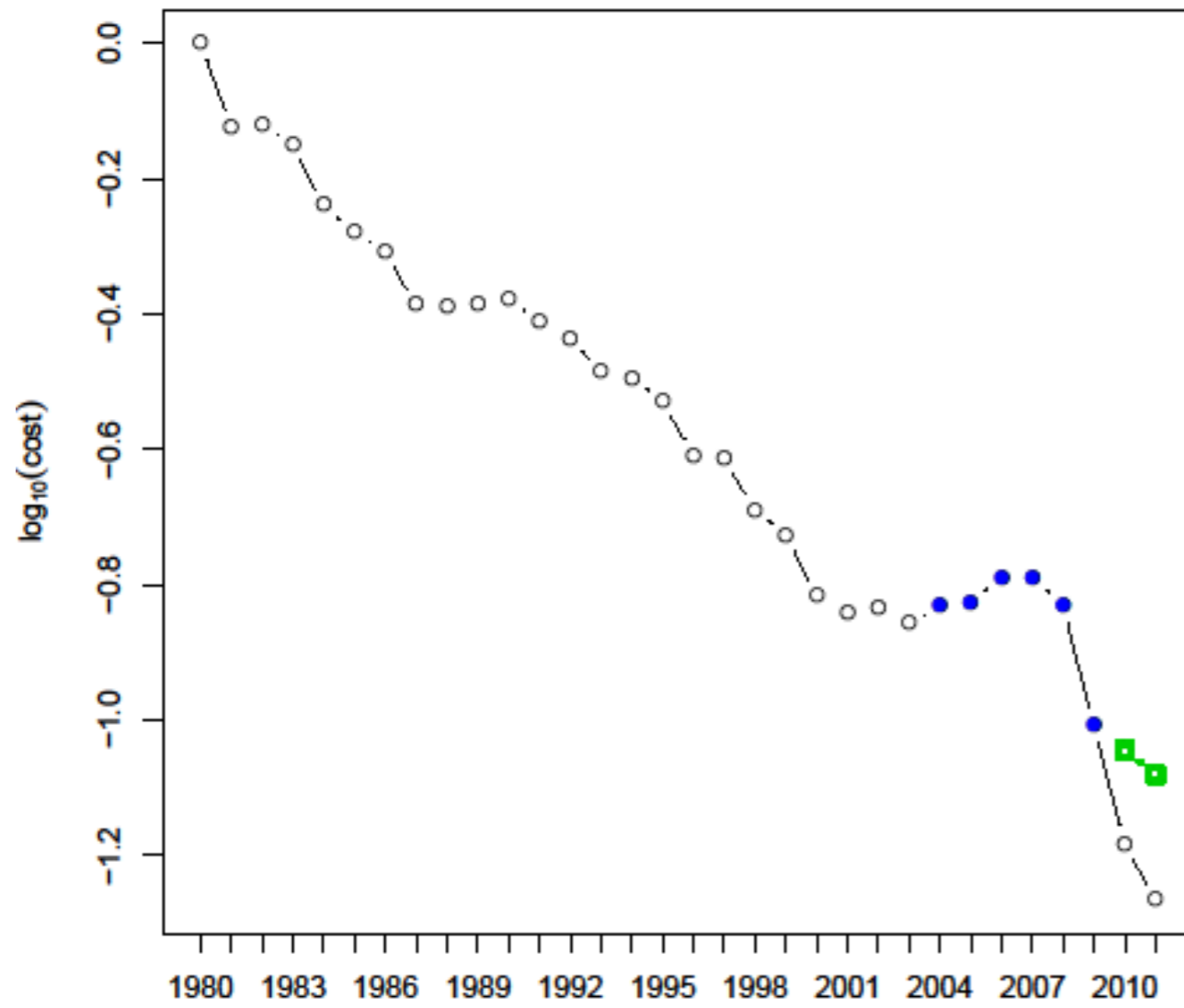
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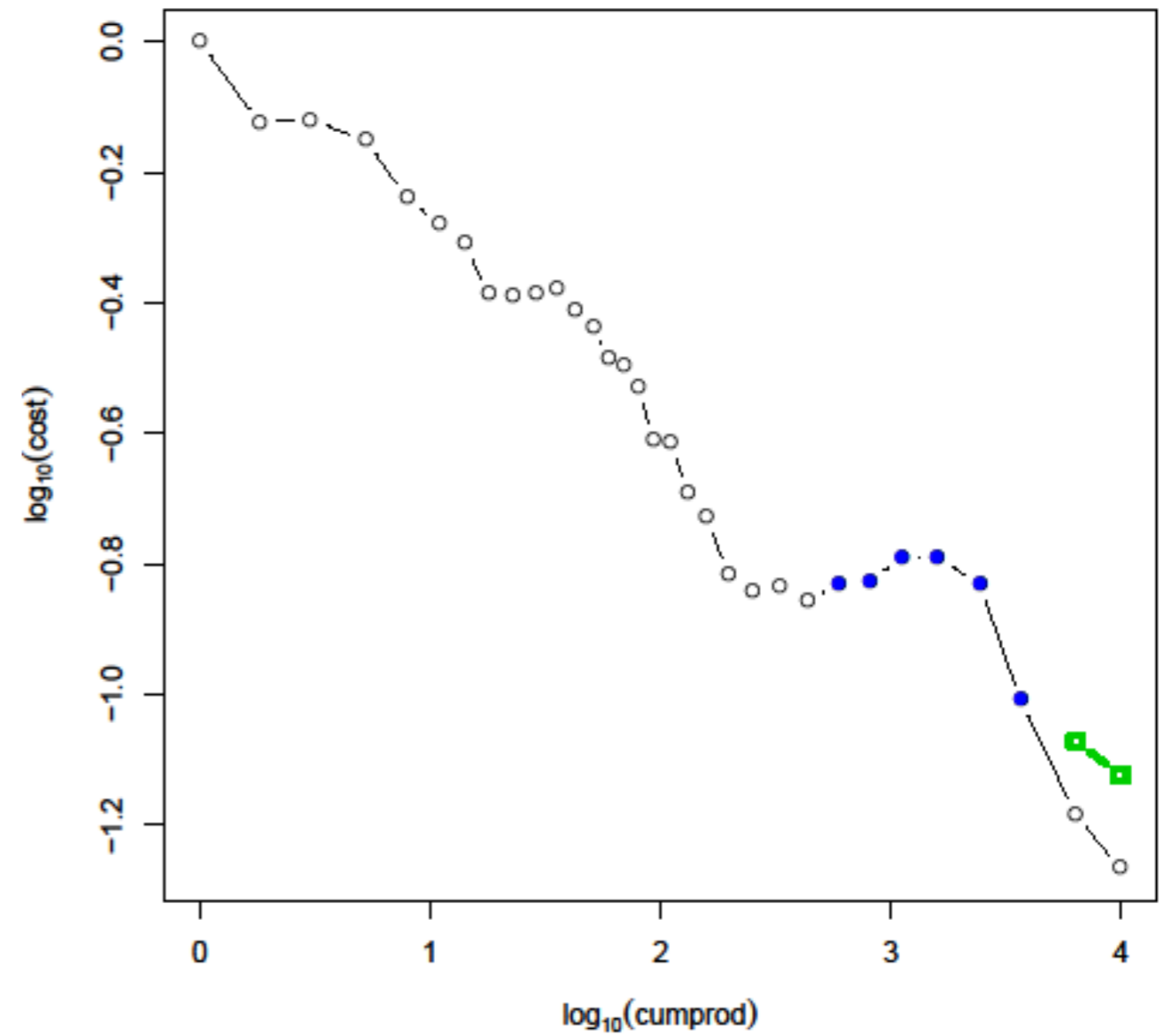
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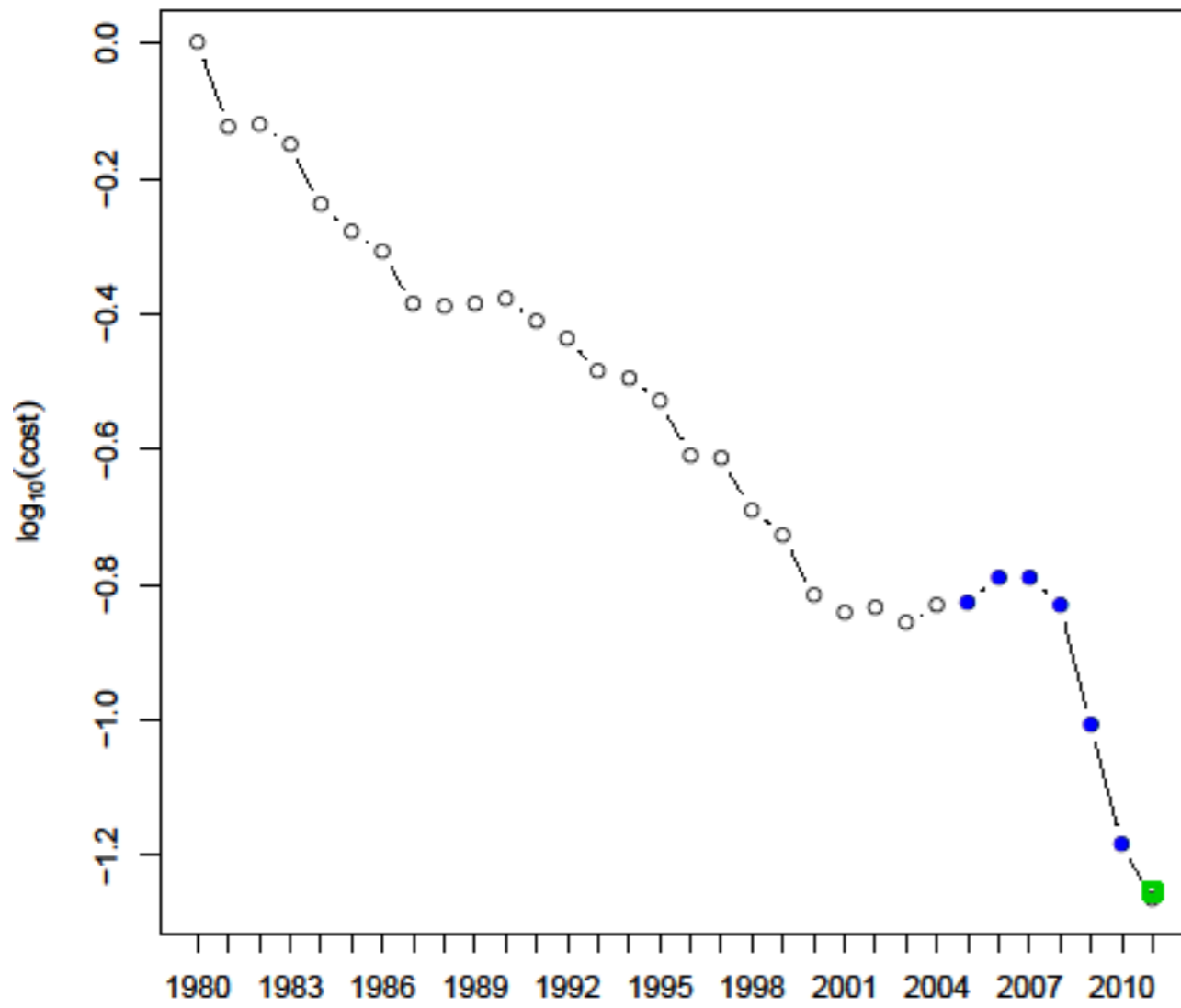
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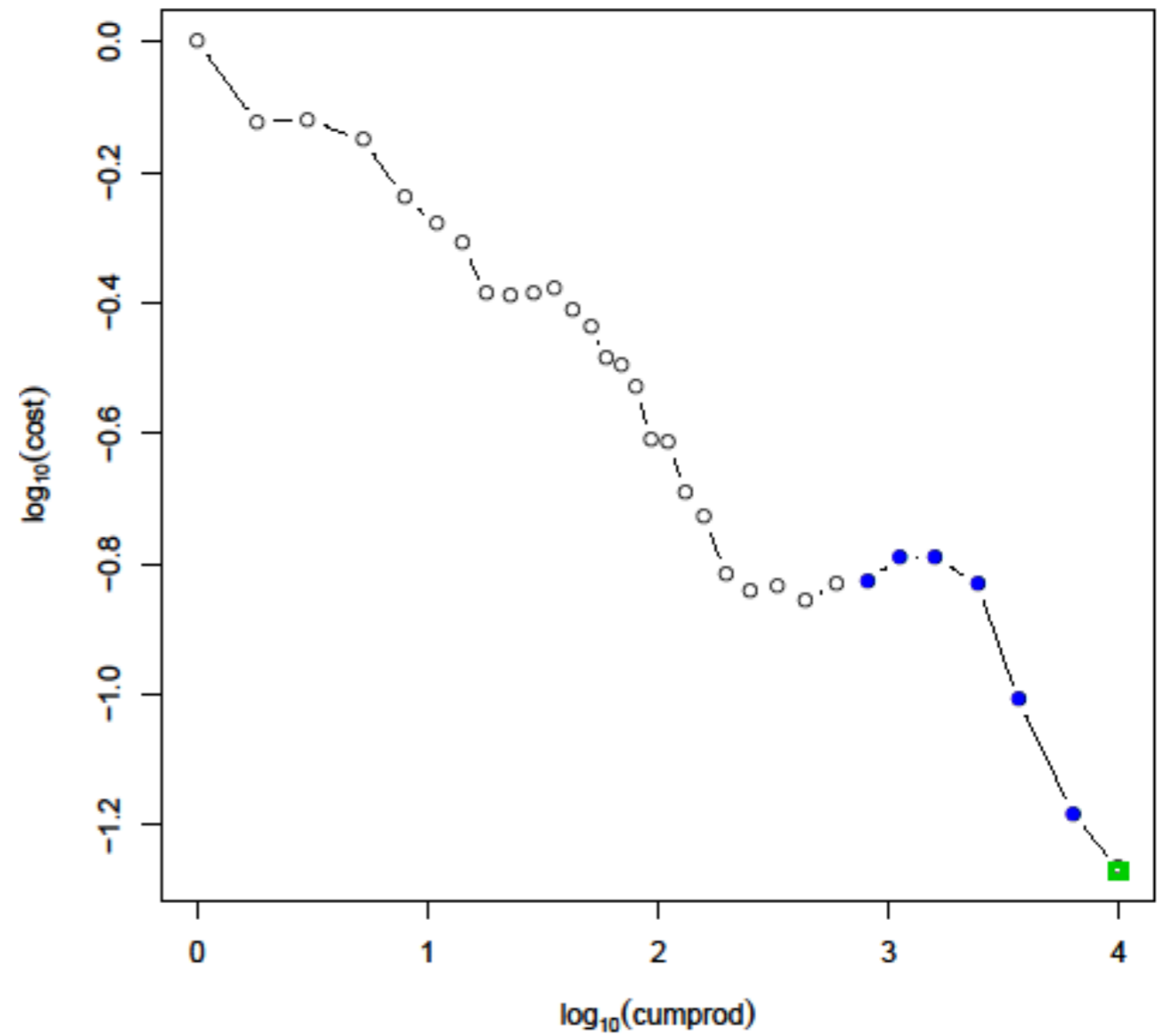
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# Wright

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# How good are the forecasts?

Forecasts without error bars  
are not very useful.

# Time series models

Moore's law as a random walk with drift

$$y_{t+1} - y_t = \mu + Kn_t$$

Change in log(cost)

Drift

Noise

Wright's law as random walk with drift  
dependent on cumulative production

$$y_{t+1} - y_t = \omega \log \left( \frac{x_{t+1}}{x_t} \right) + Sn_t$$



# Predicted forecasting error for Moore's law assuming normally distributed IID noise

$$\mathcal{E} = y_{t+\tau} - \hat{y}_{t+\tau}$$

$$\frac{1}{\sqrt{A}} \left( \frac{\mathcal{E}}{\hat{K}} \right) \sim t(m-1)$$

$$A = \tau(1 + \tau/m)$$

$\hat{y}_{t+\tau}$  = prediction for time  $t + \tau$

$\hat{K}$  = estimated noise amplitude

$m$  = number of points used to estimate  $\mu$

$t$  = Student's "t" distribution

This works surprisingly well

However, it is possible to do better by taking correlations into account

# Random process with correlated noise

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$$y_{t+1} - y_t = \mu + v_t + \theta v_{t-1}$$

$v_t$  = noise at time  $t$

$\theta$  = parameter describing correlation

$$\frac{1}{\sqrt{A^*}} \begin{pmatrix} \mathcal{E} \\ \hat{K} \end{pmatrix} \sim t(m-1)$$

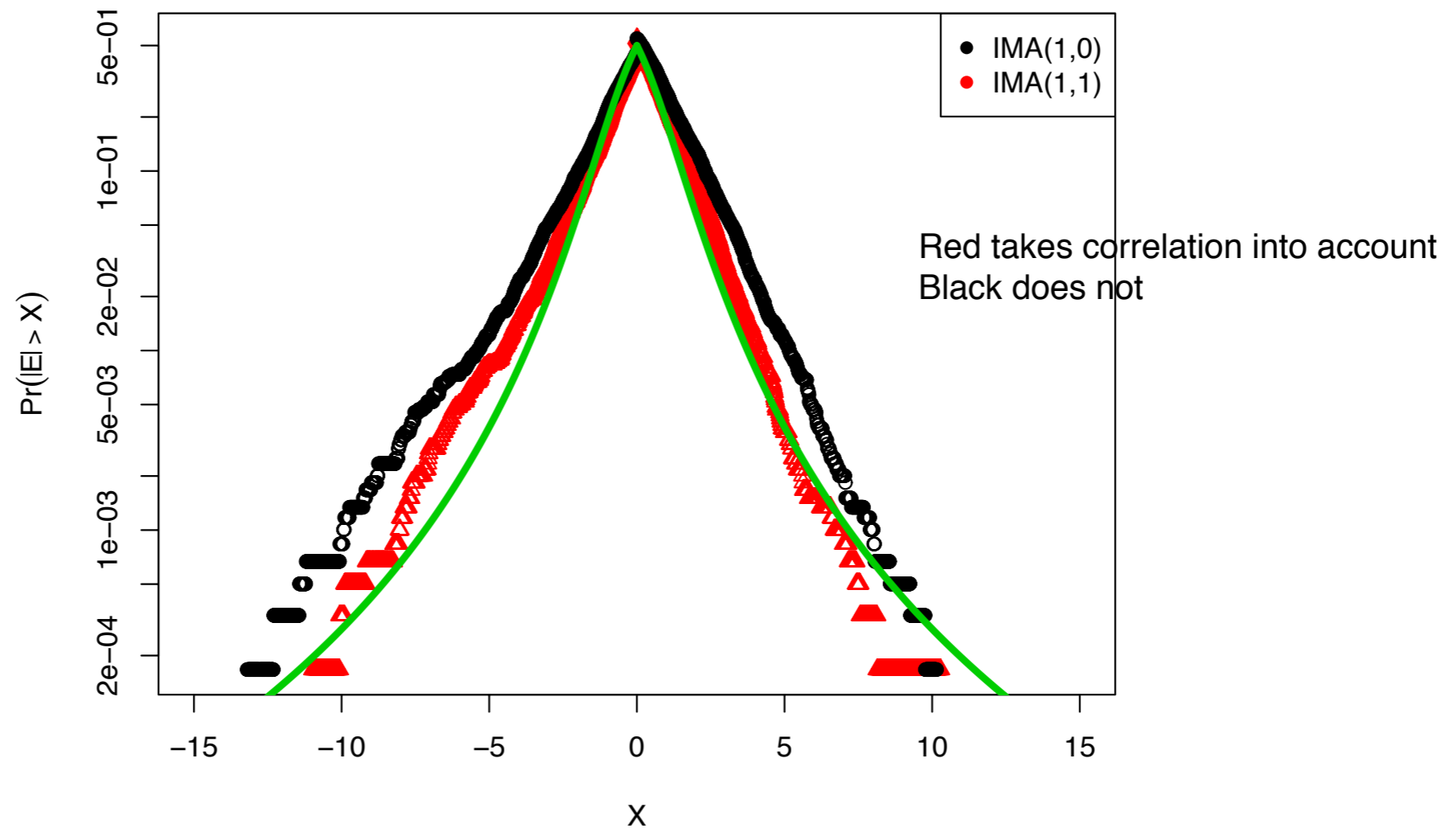
$A^*$  =  $2^{nd}$  degree polynomial in  $\tau$

whose coefficients depend on  $\theta$  and  $m$

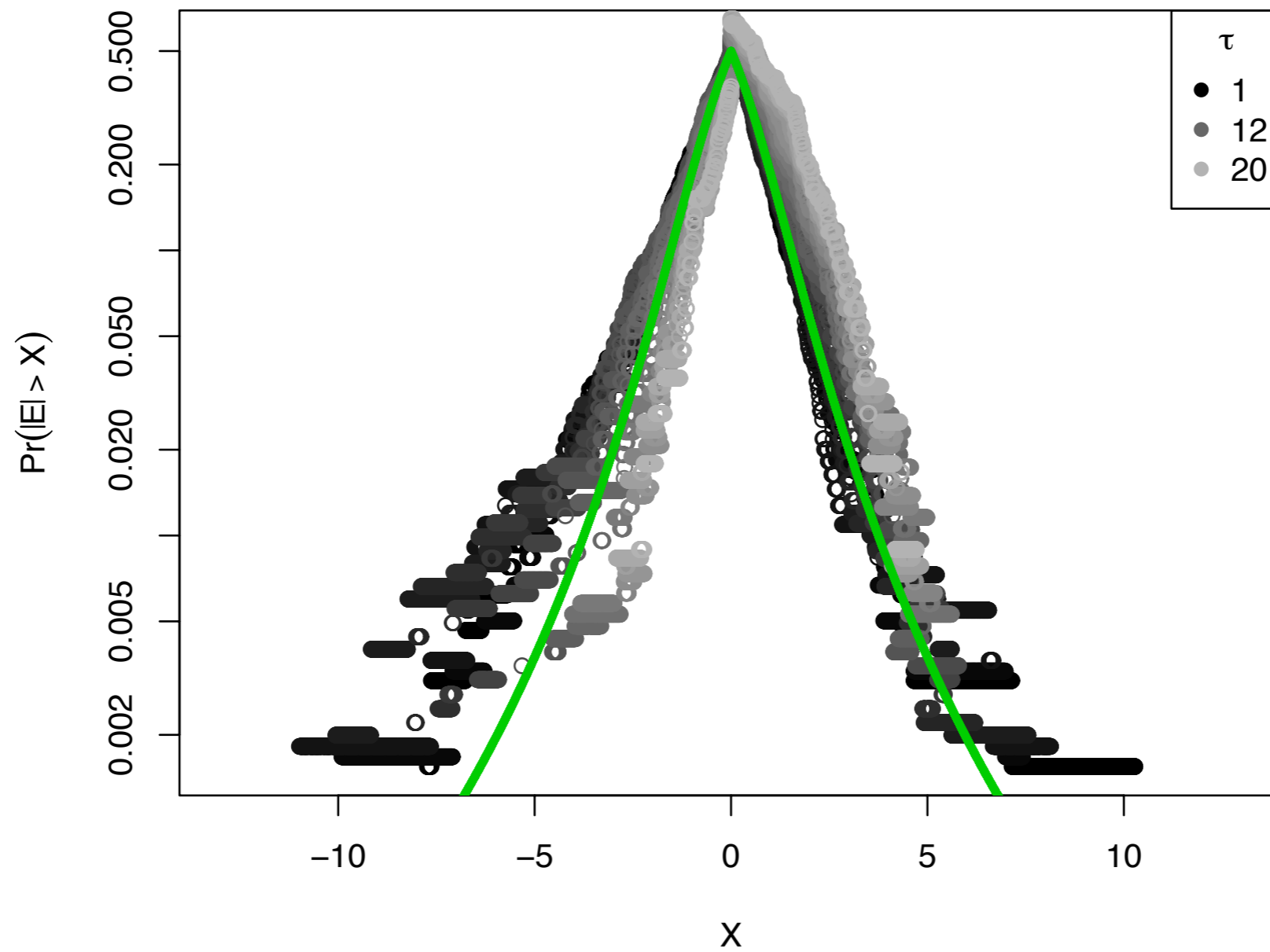
# Comparison to empirical data for 48 different technologies

5,973 annual forecasts, all  $\tau$  with  $\tau < 20$

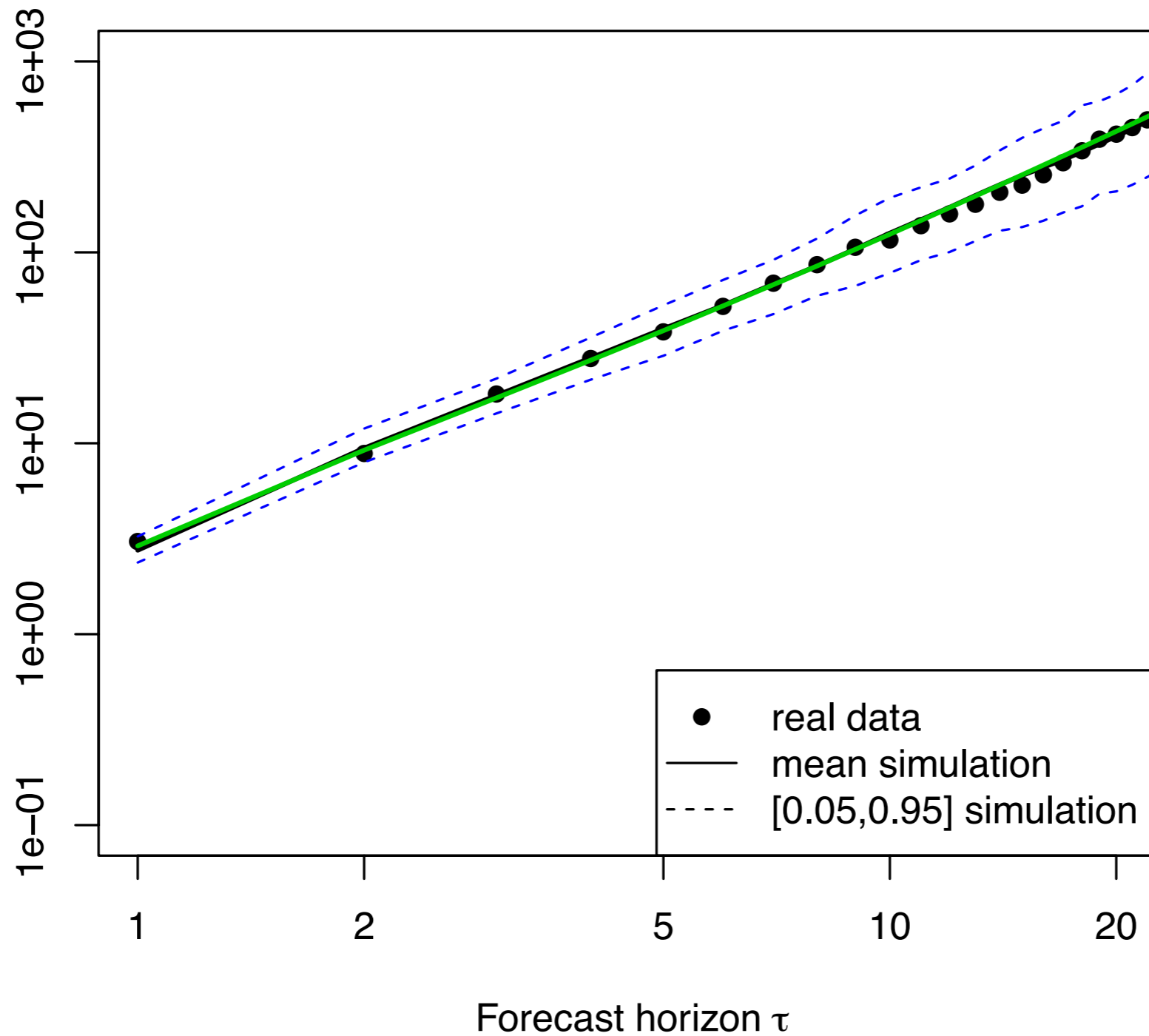
Cumulative distributions for positive and negative errors plotted separately



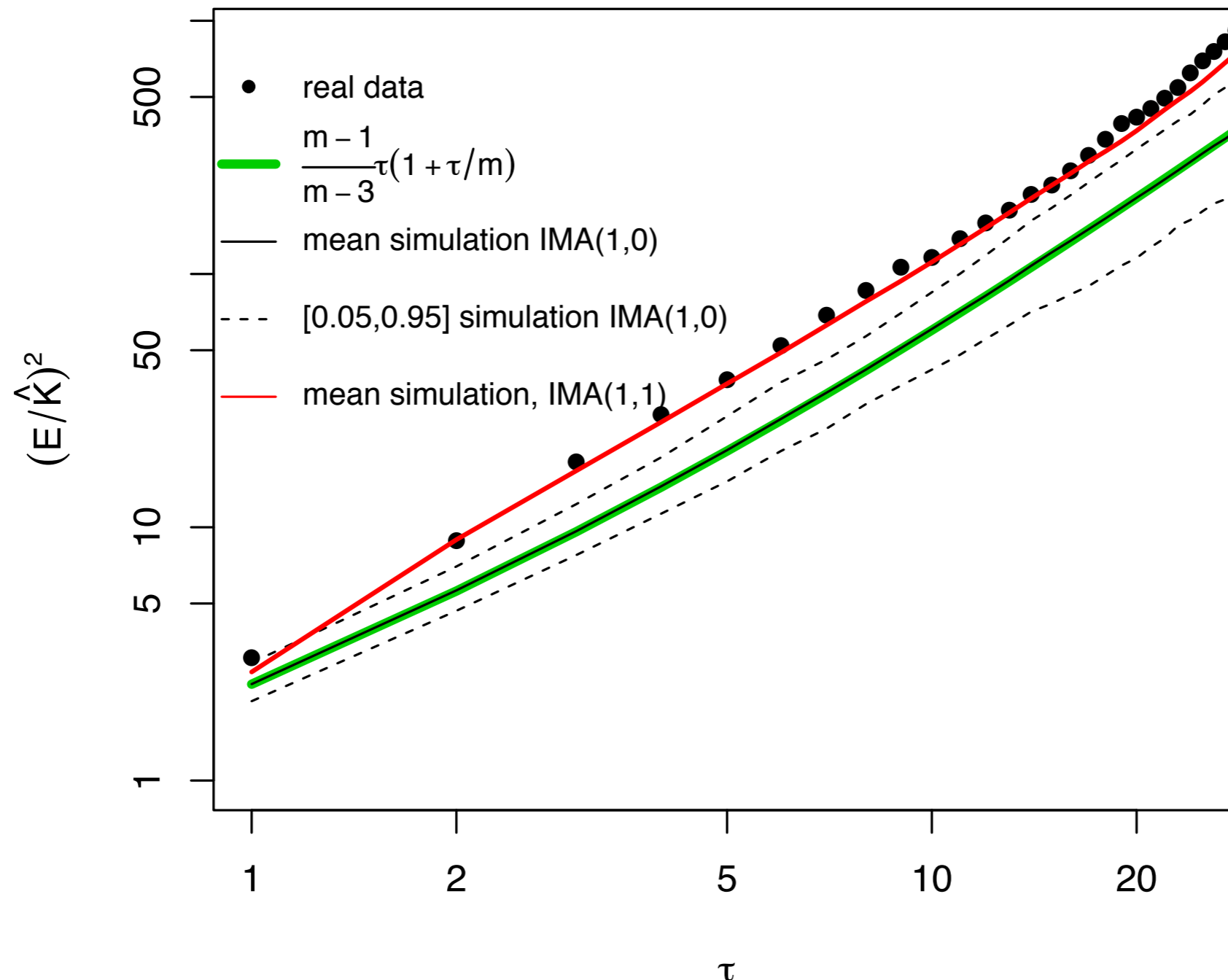
Comparison to empirical distribution for 48 technologies



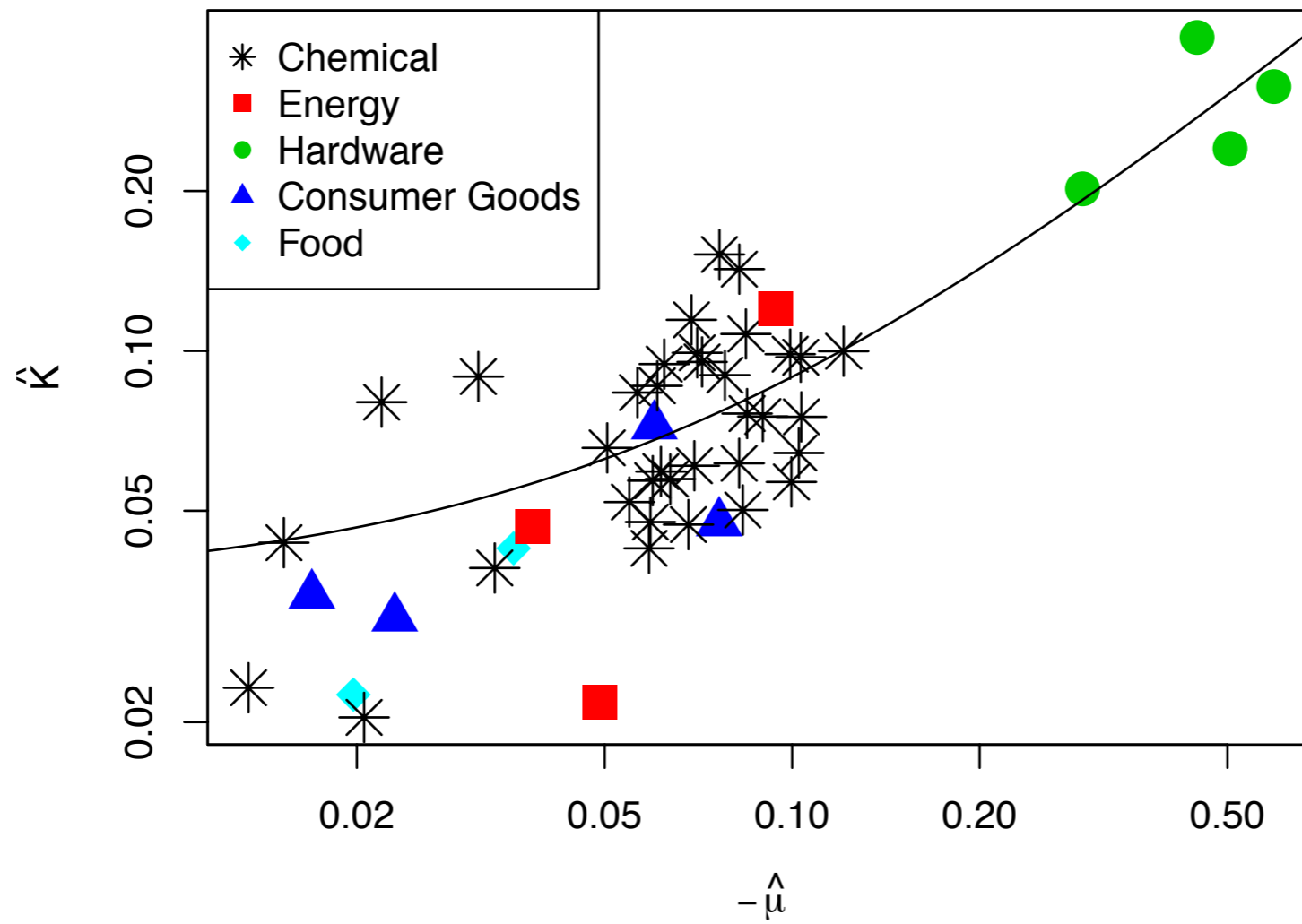
# Errors vs. forecast horizon



# Comparison of errors vs forecast horizon

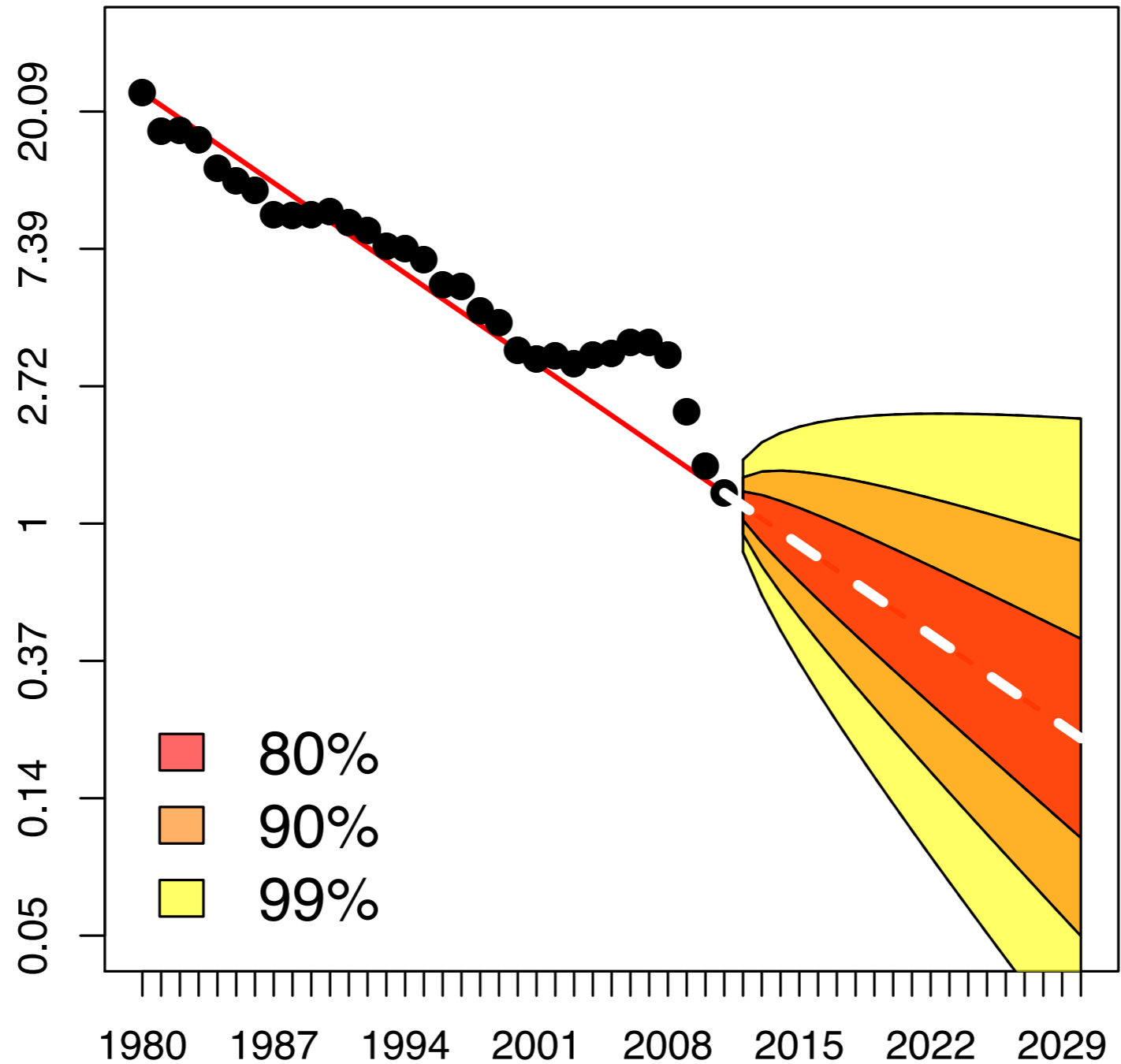


# Innovation noise amplitude vs. improvement rate

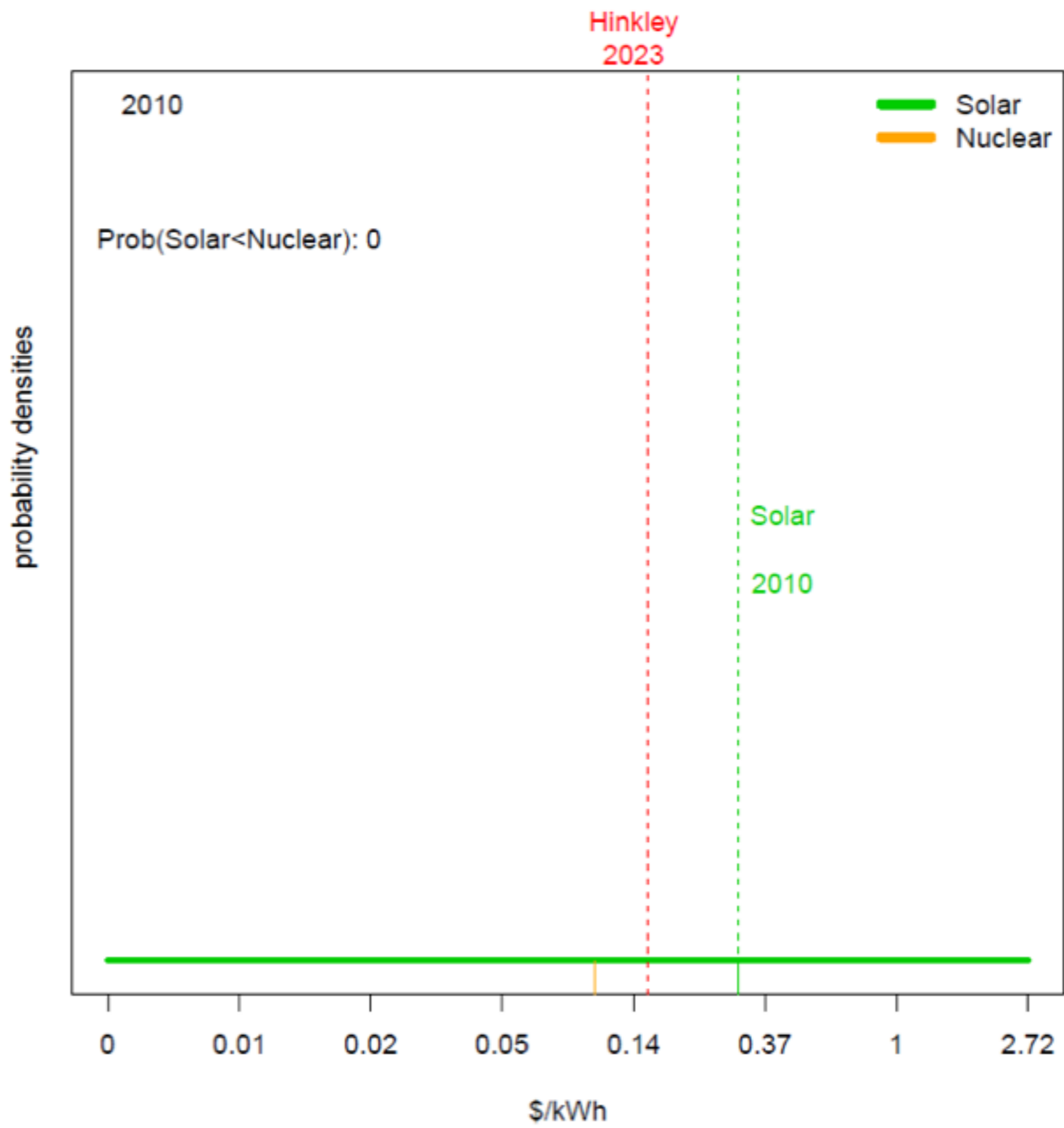


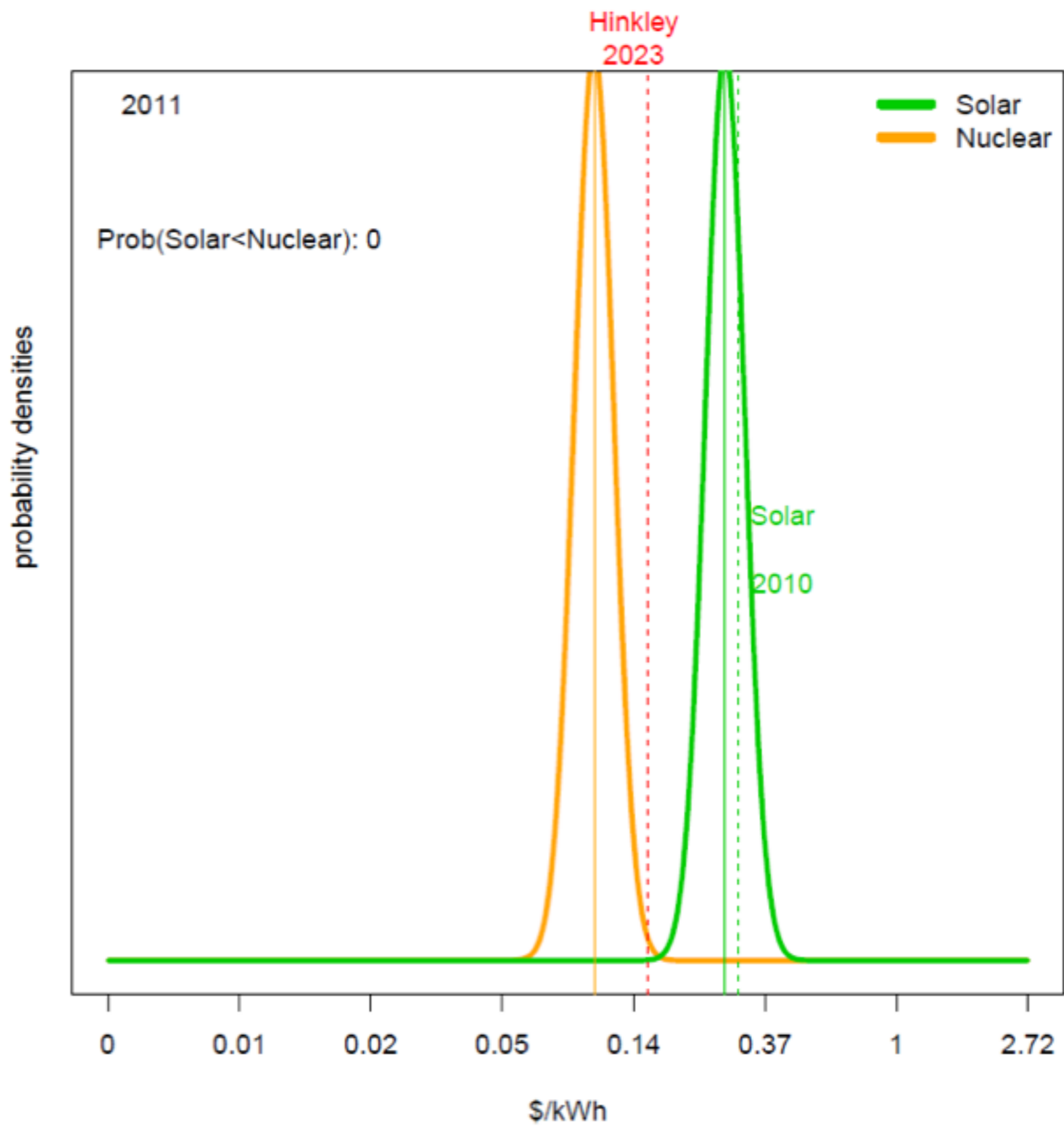


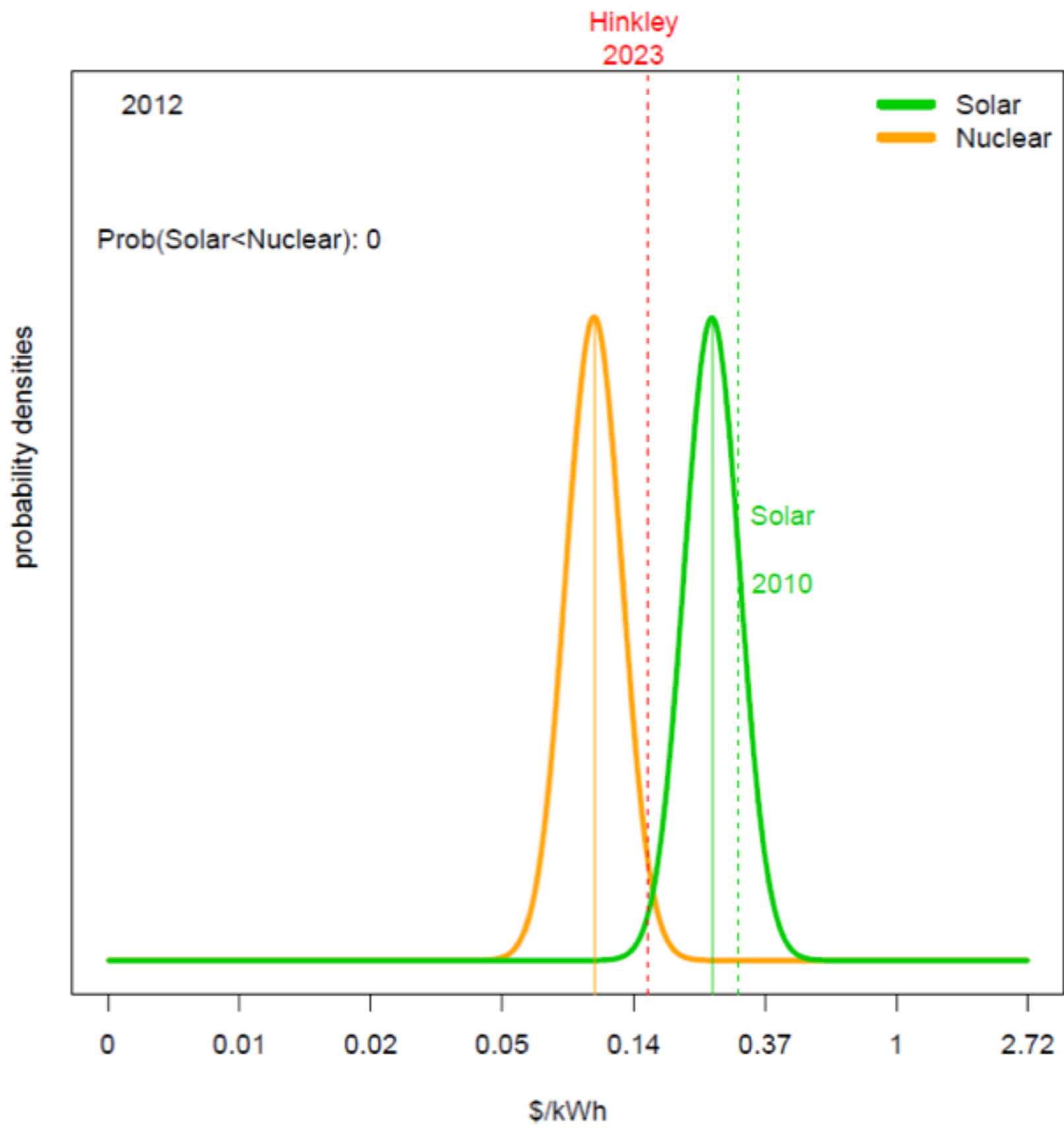
# Distributional forecast of solar PV assuming business as usual

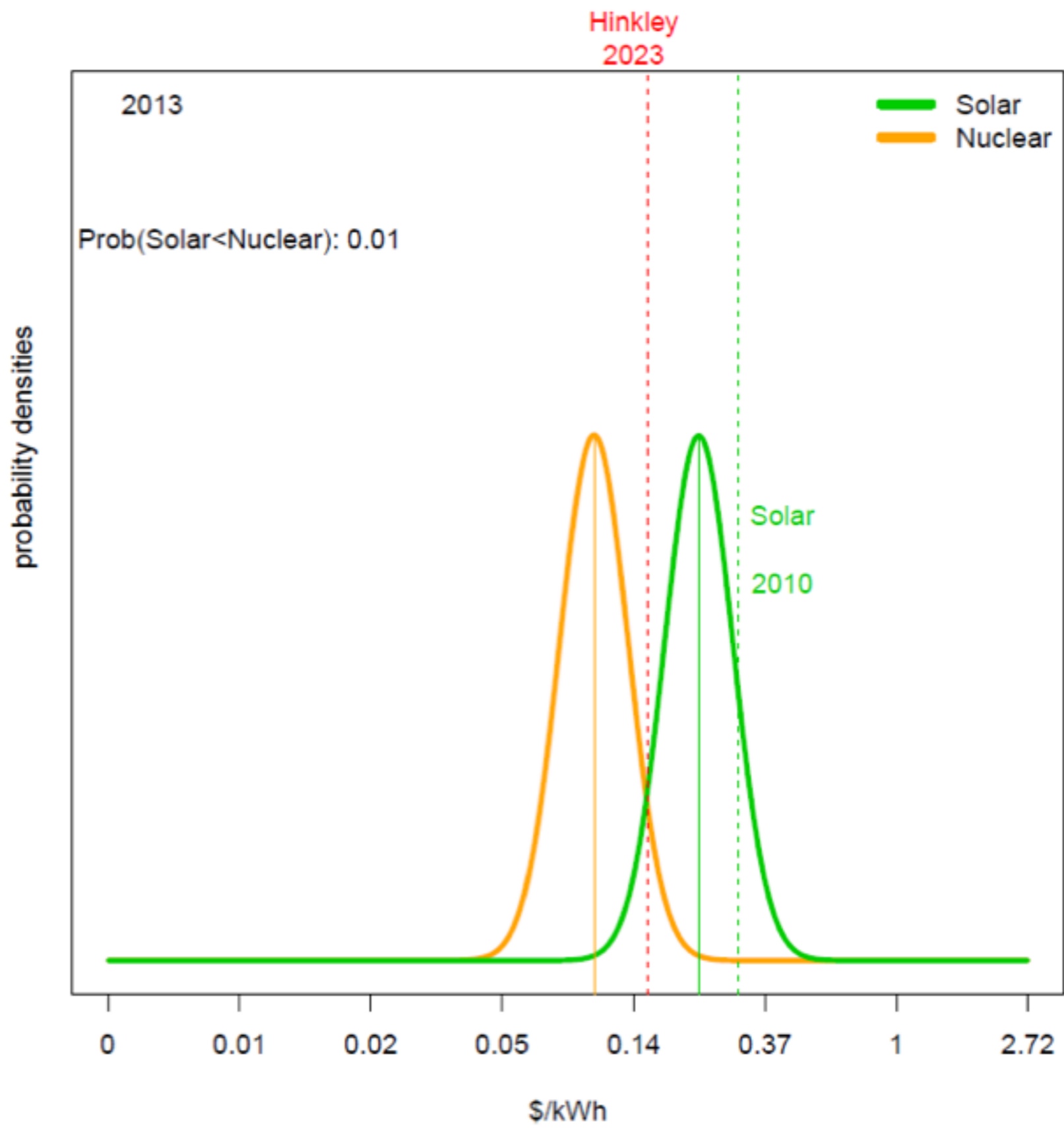


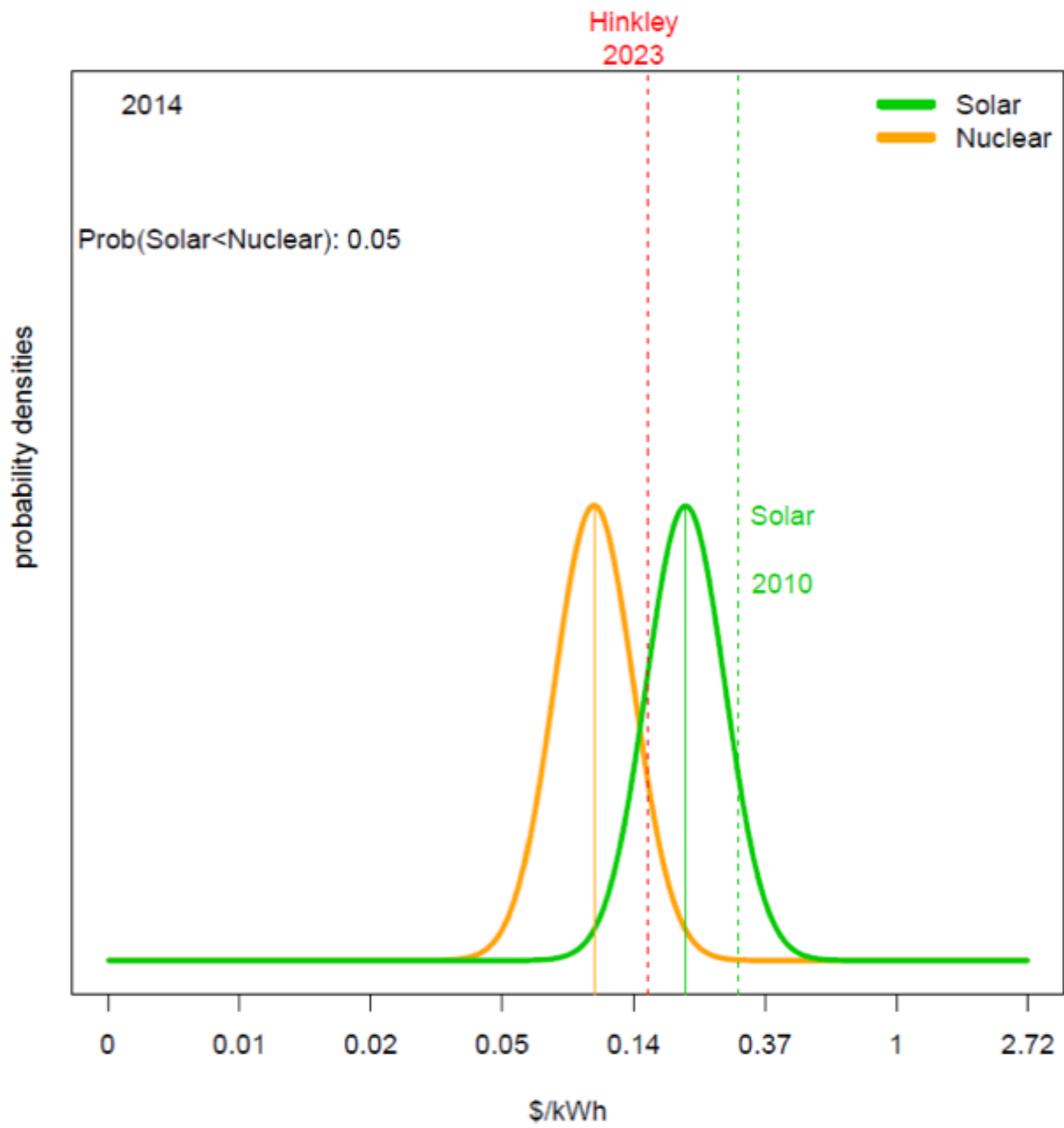
What is the probability that  
solar PV will be cheaper  
than nuclear power?

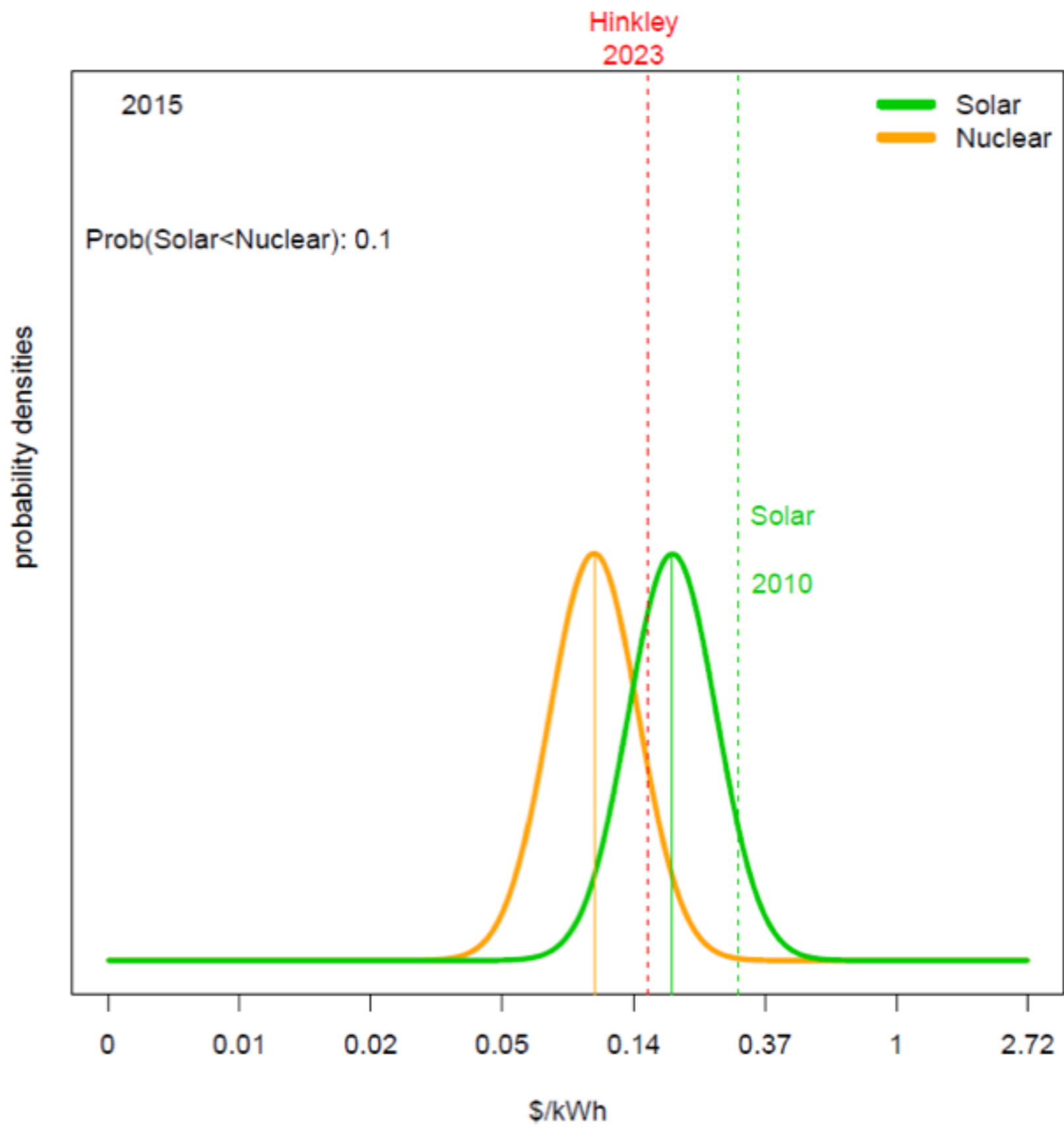




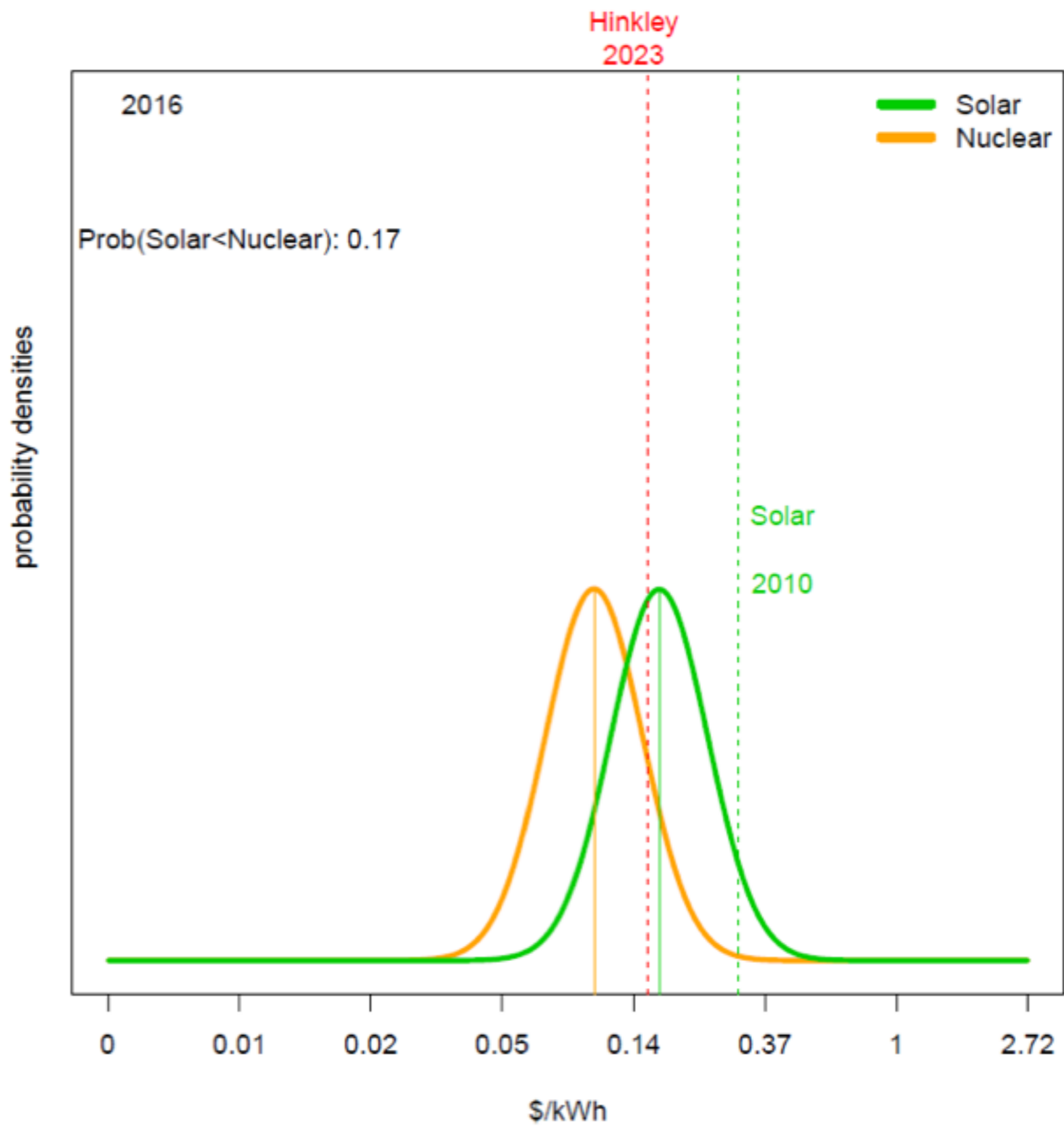


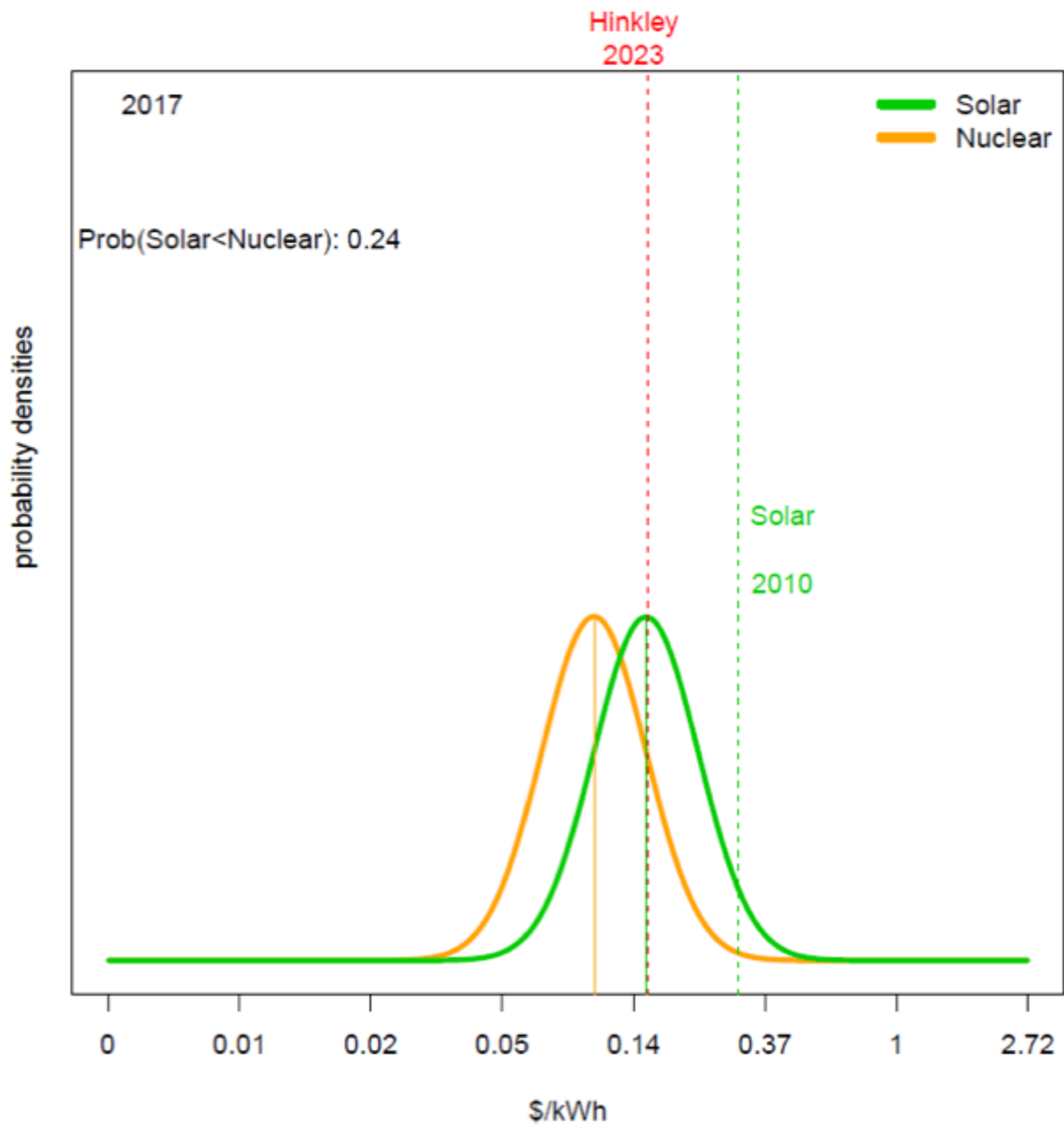


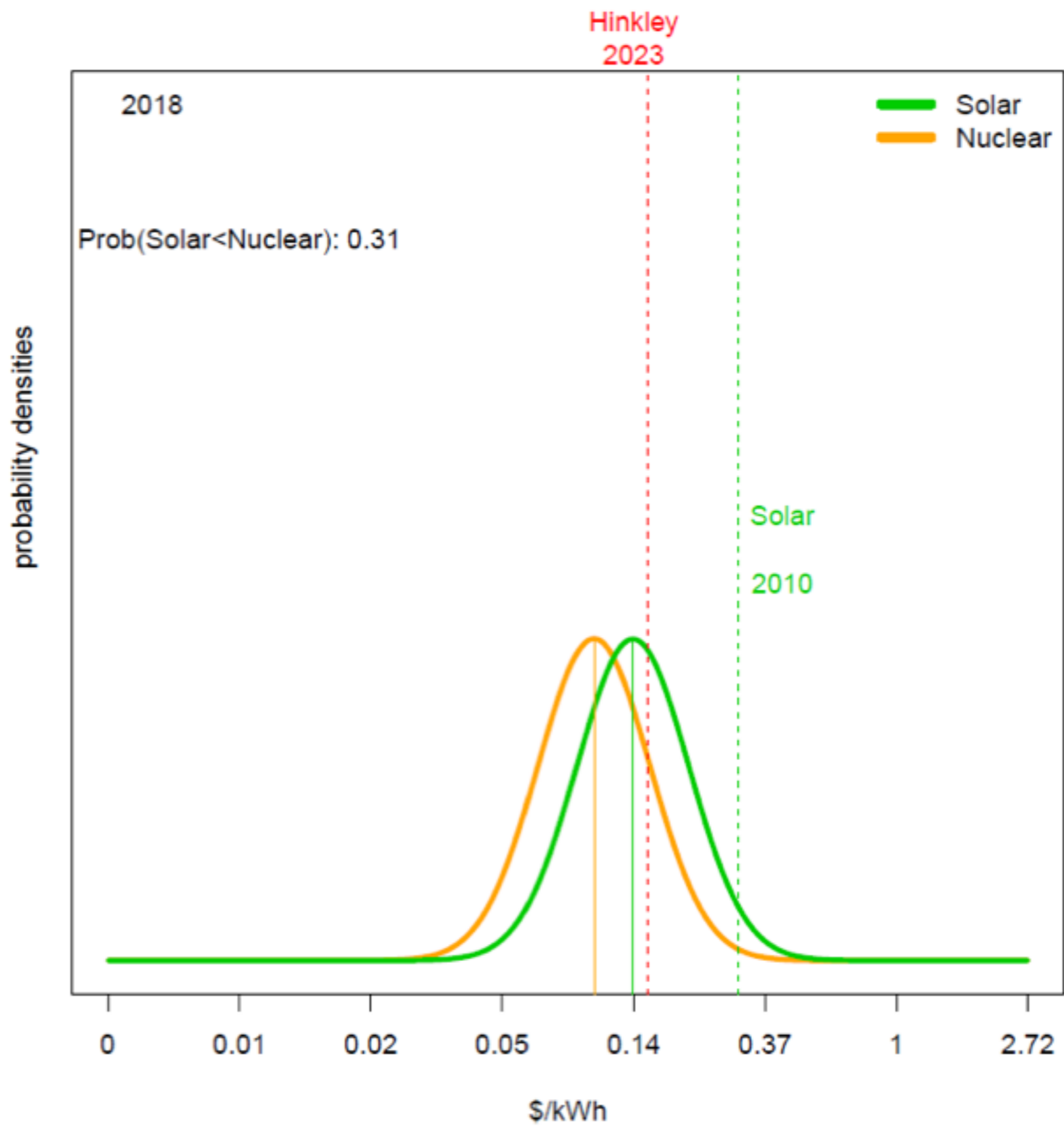


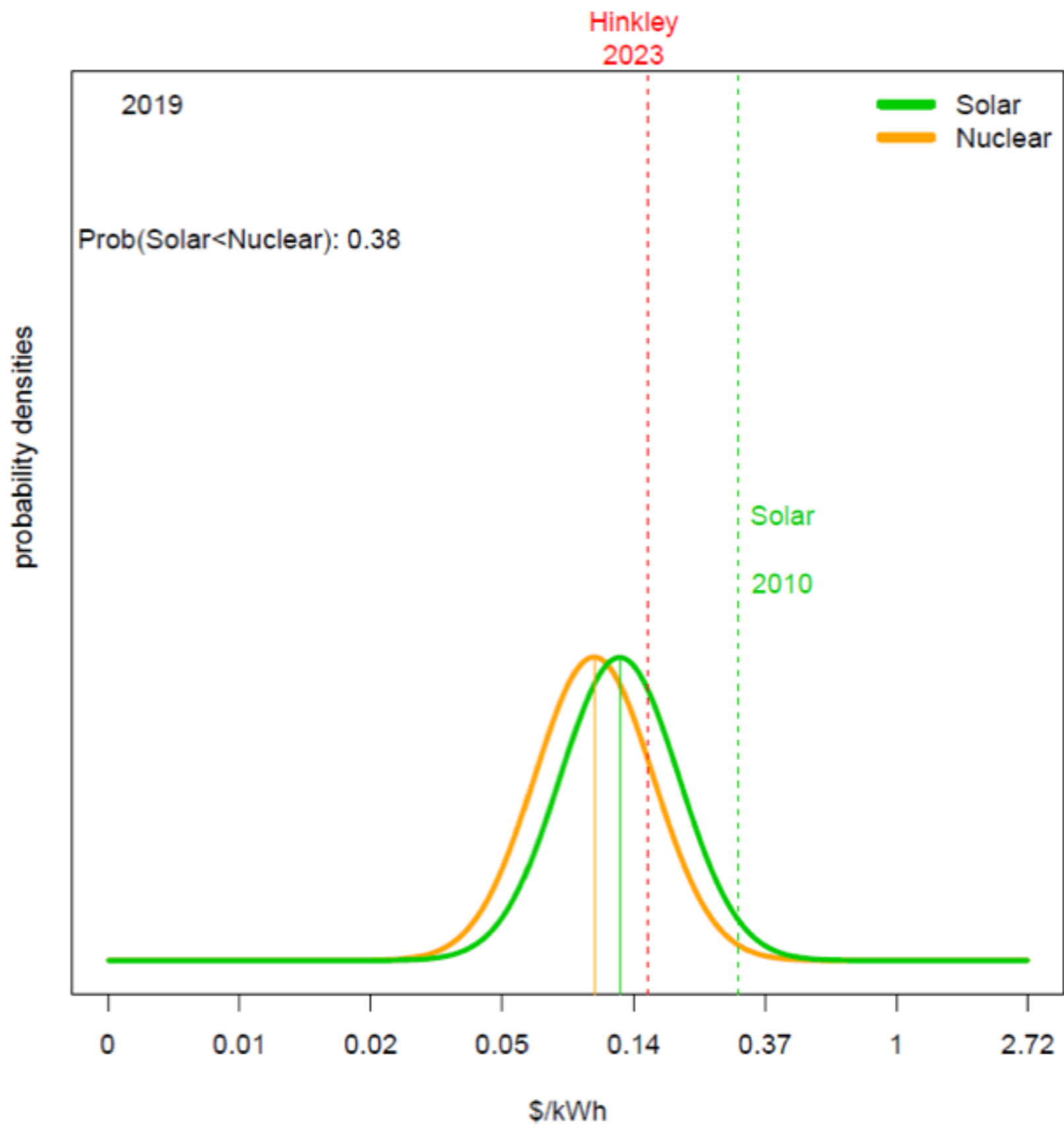


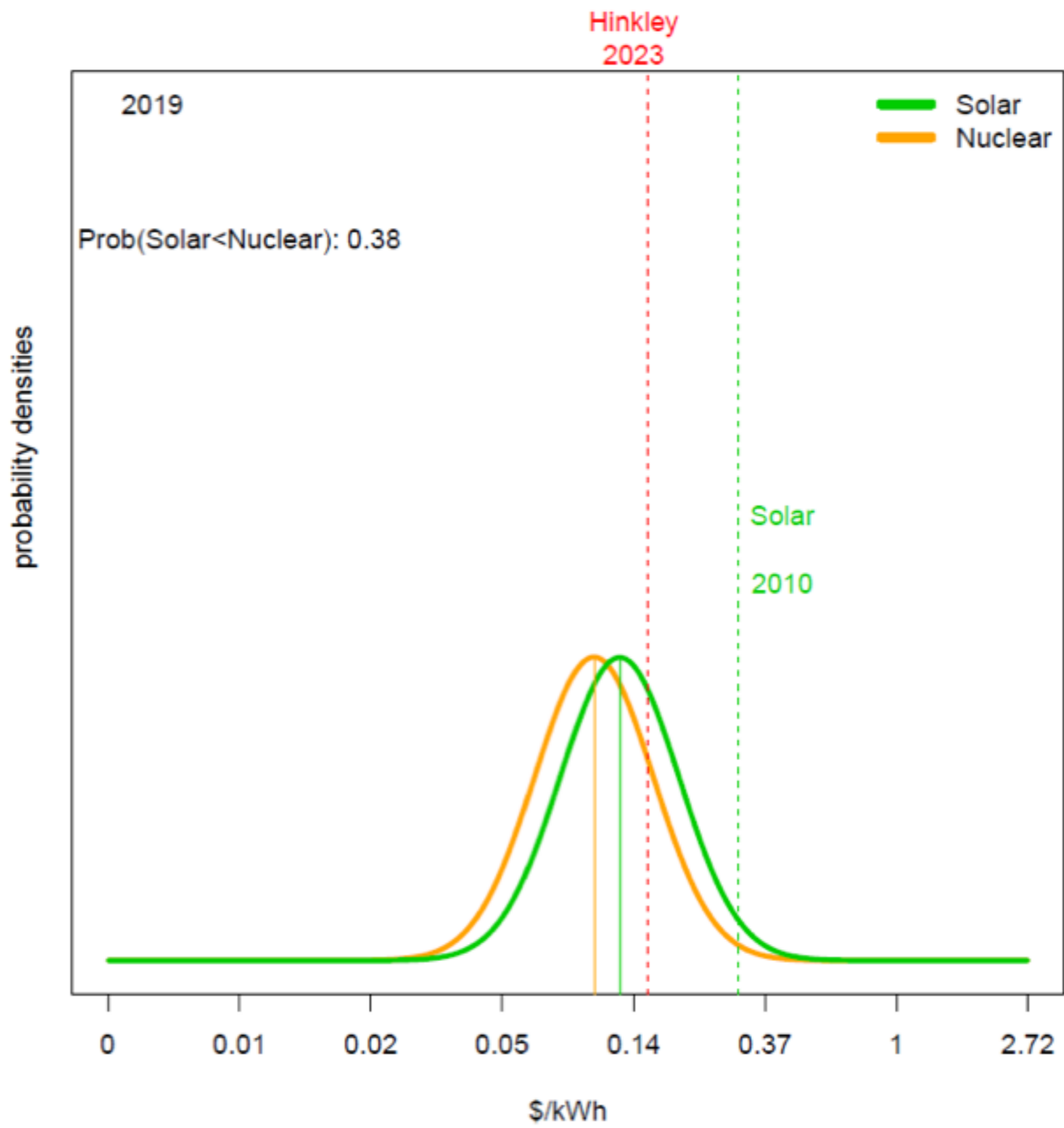


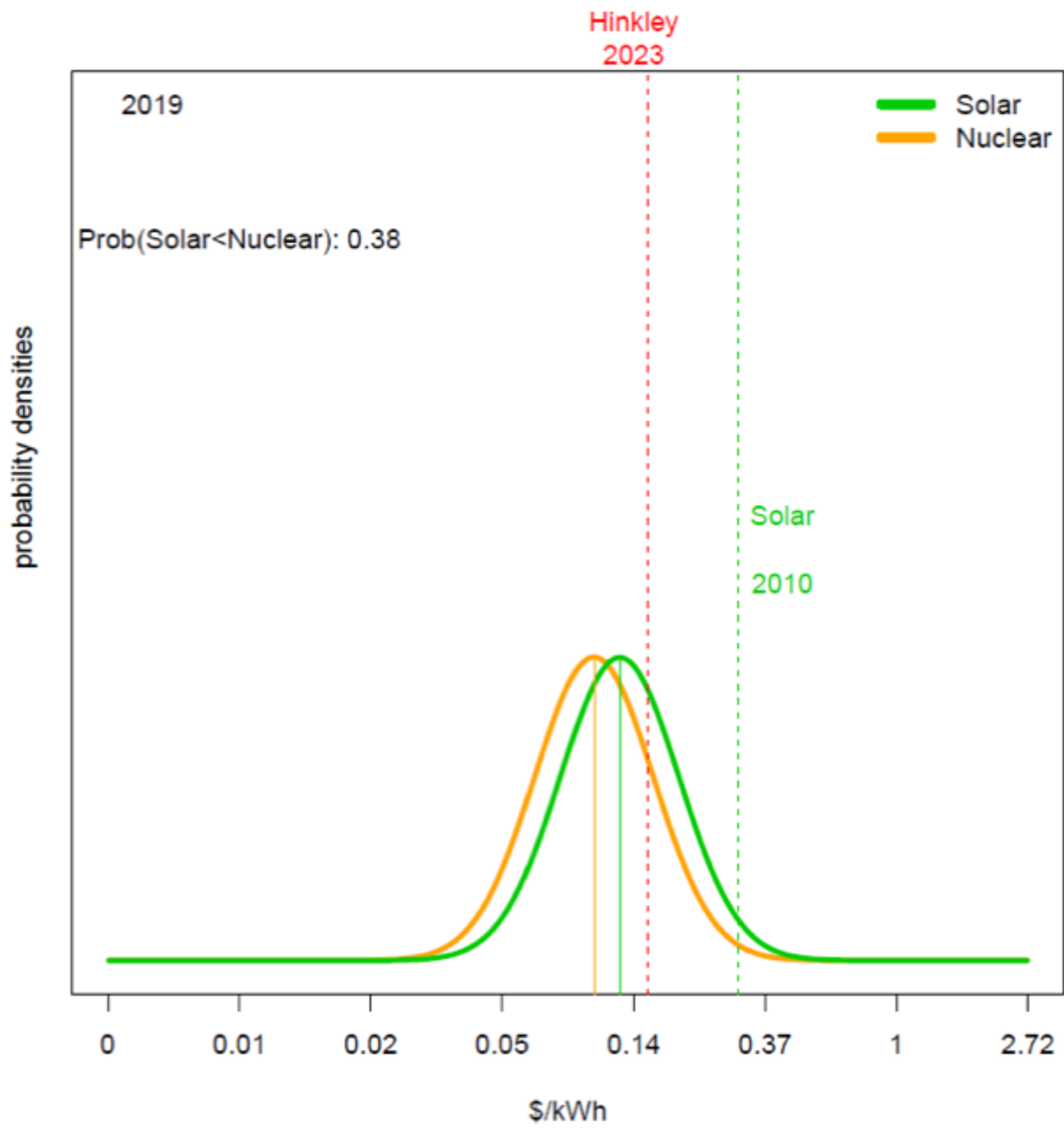


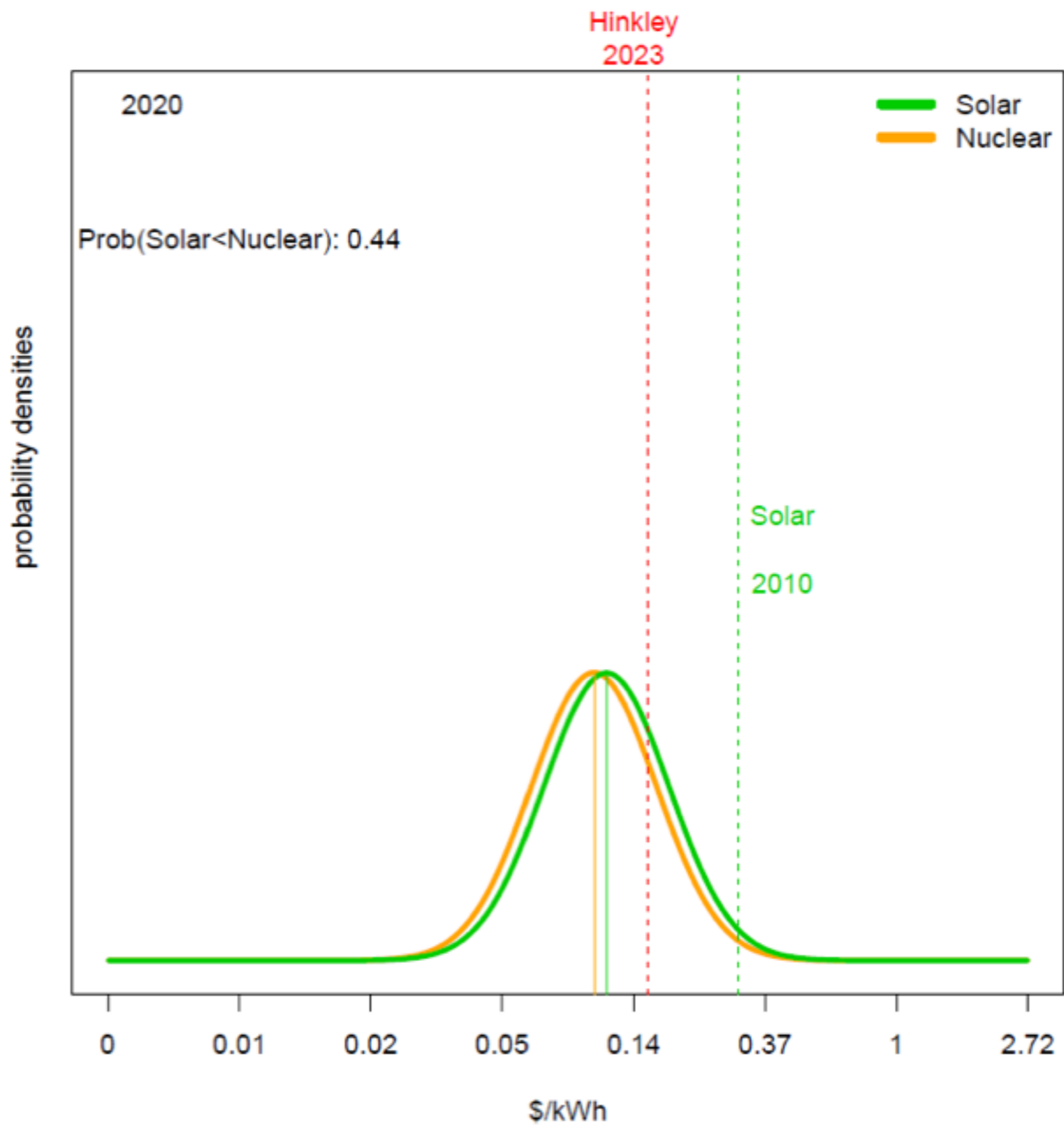


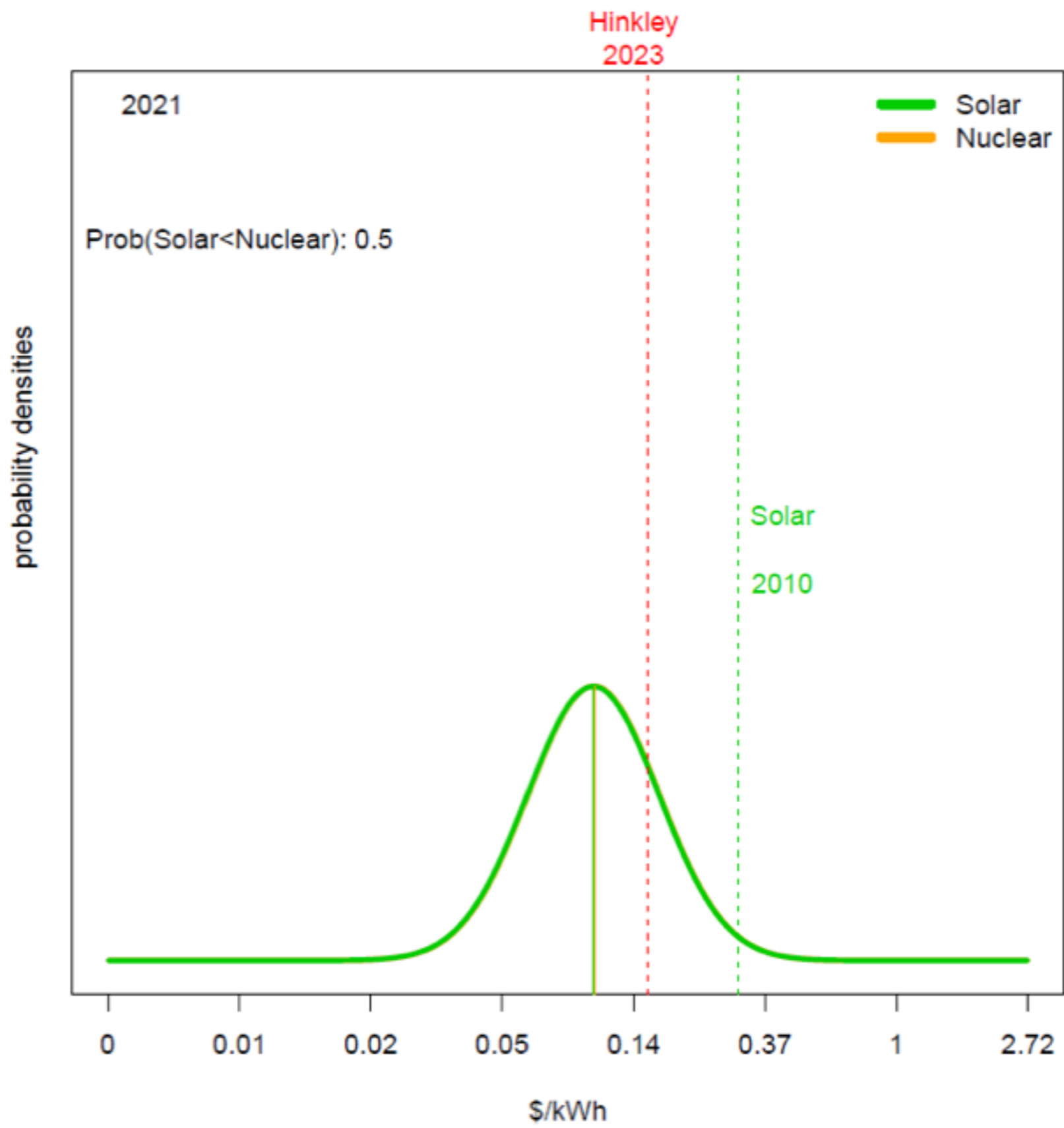




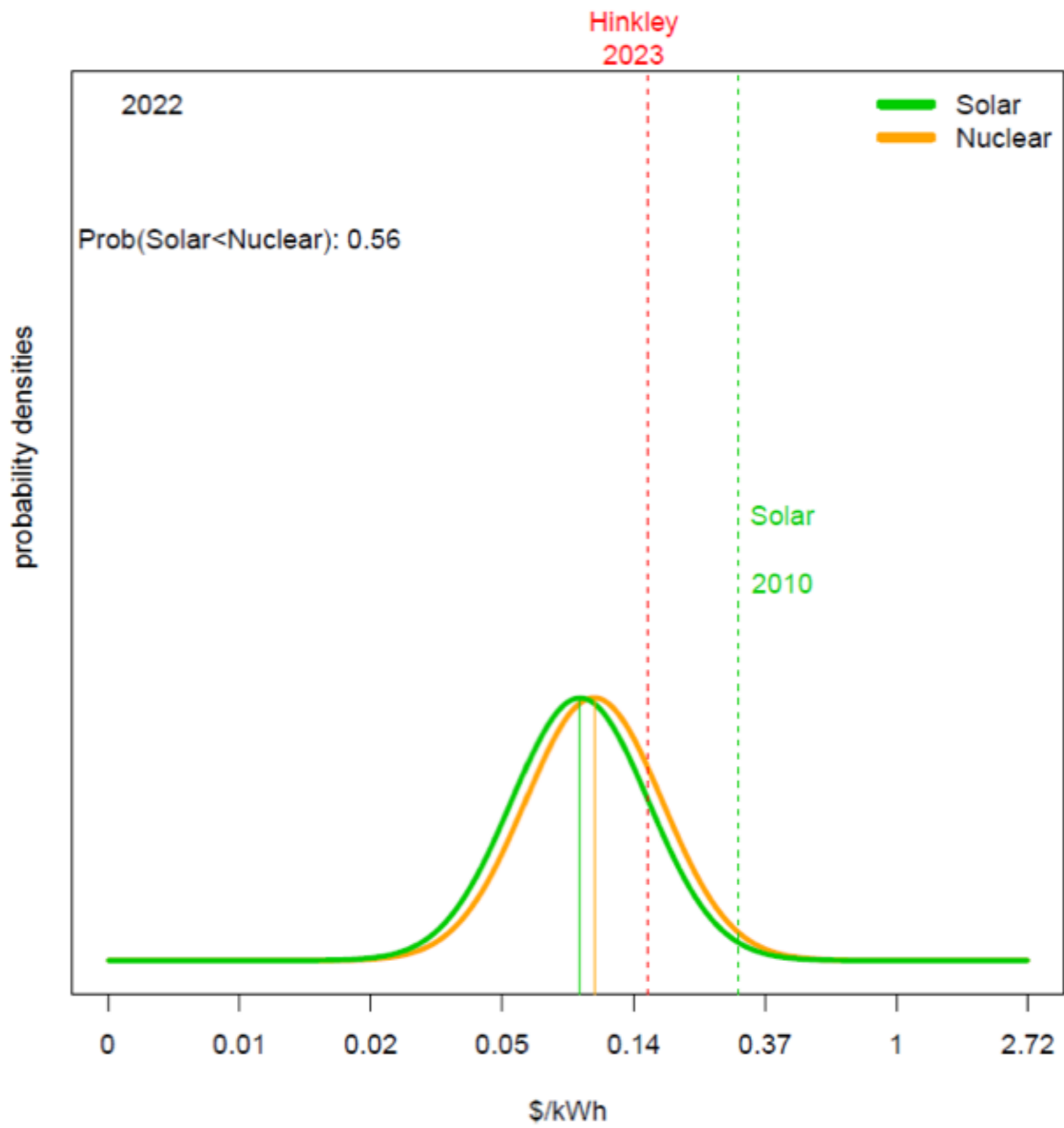


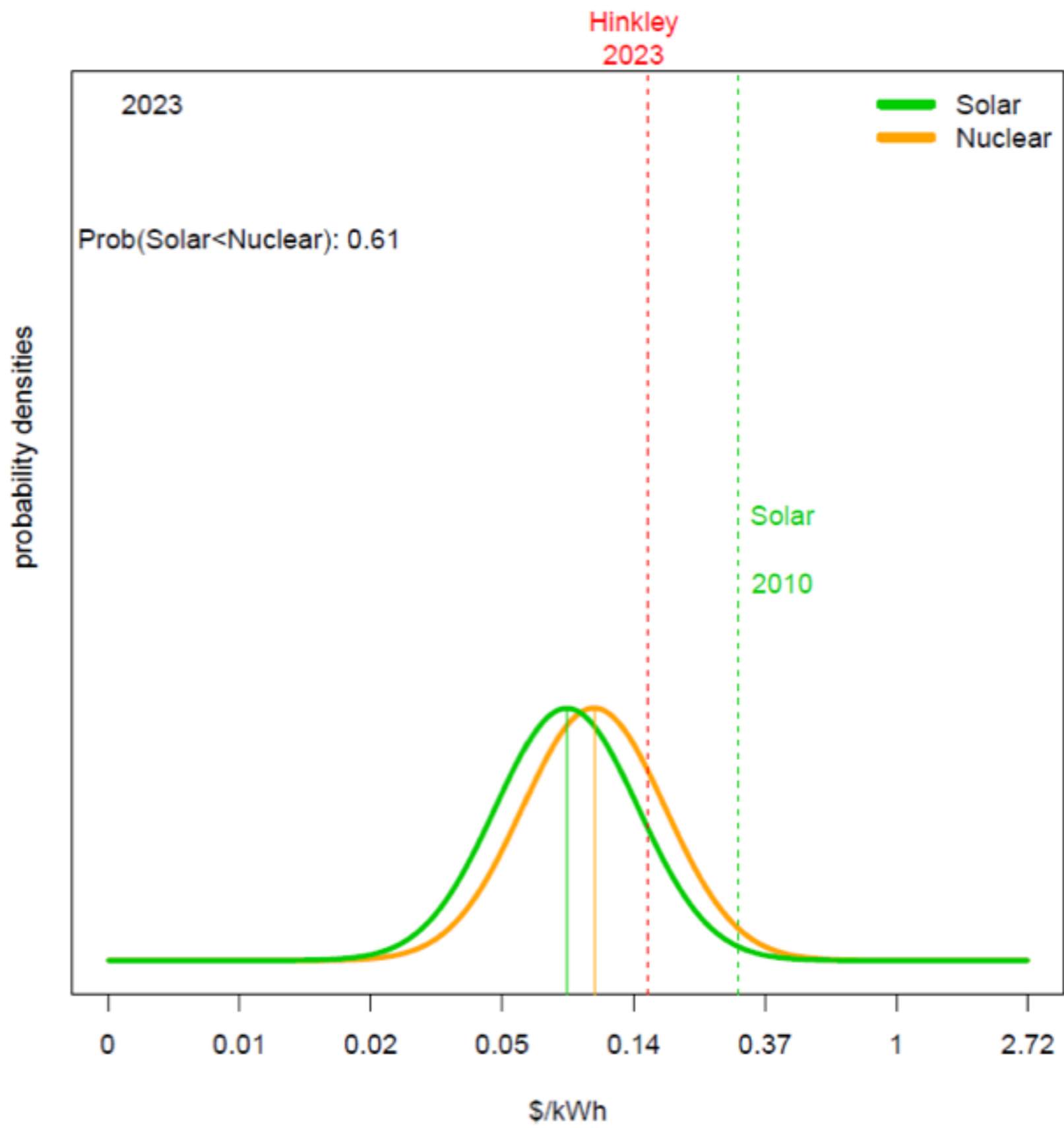


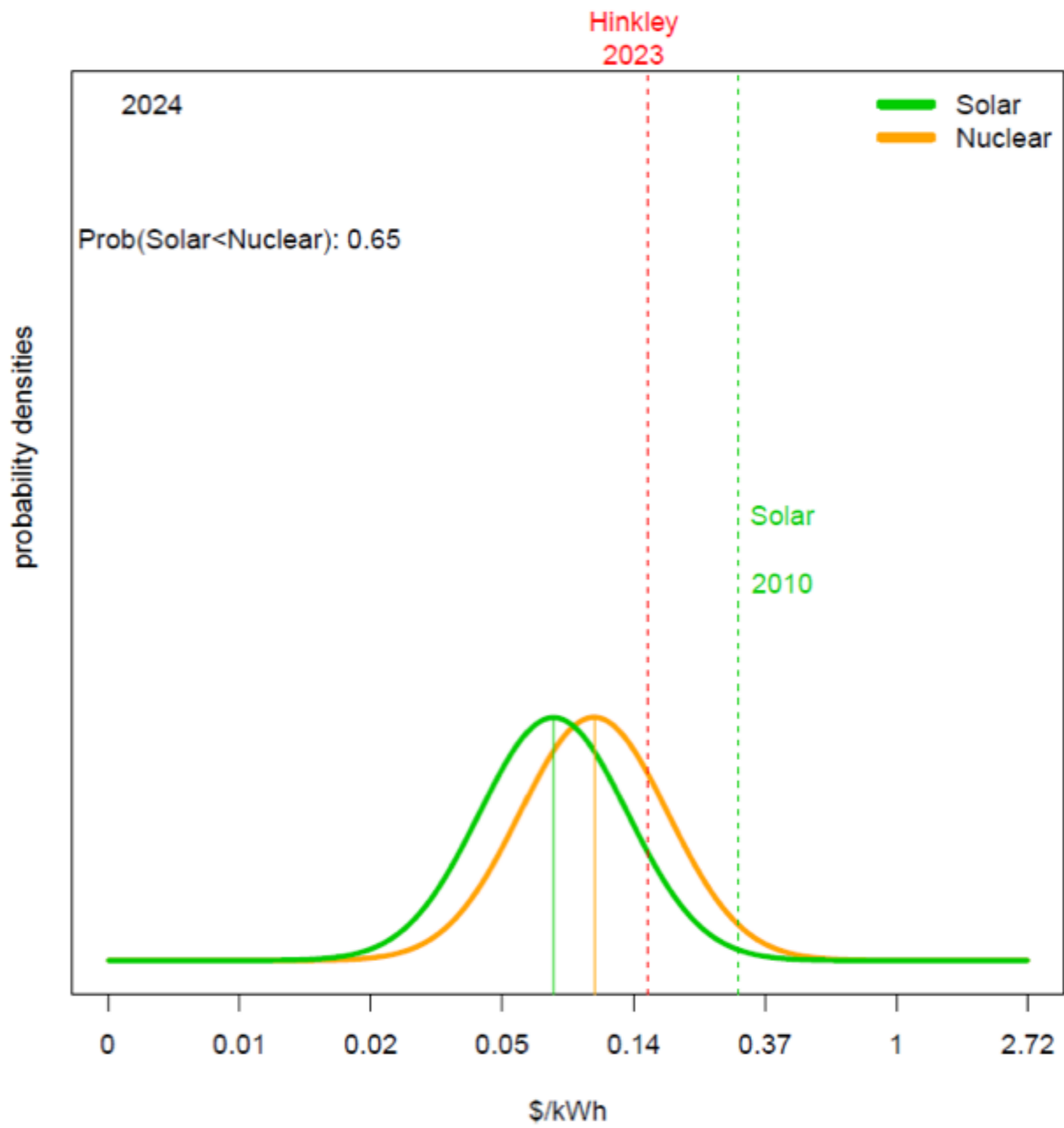


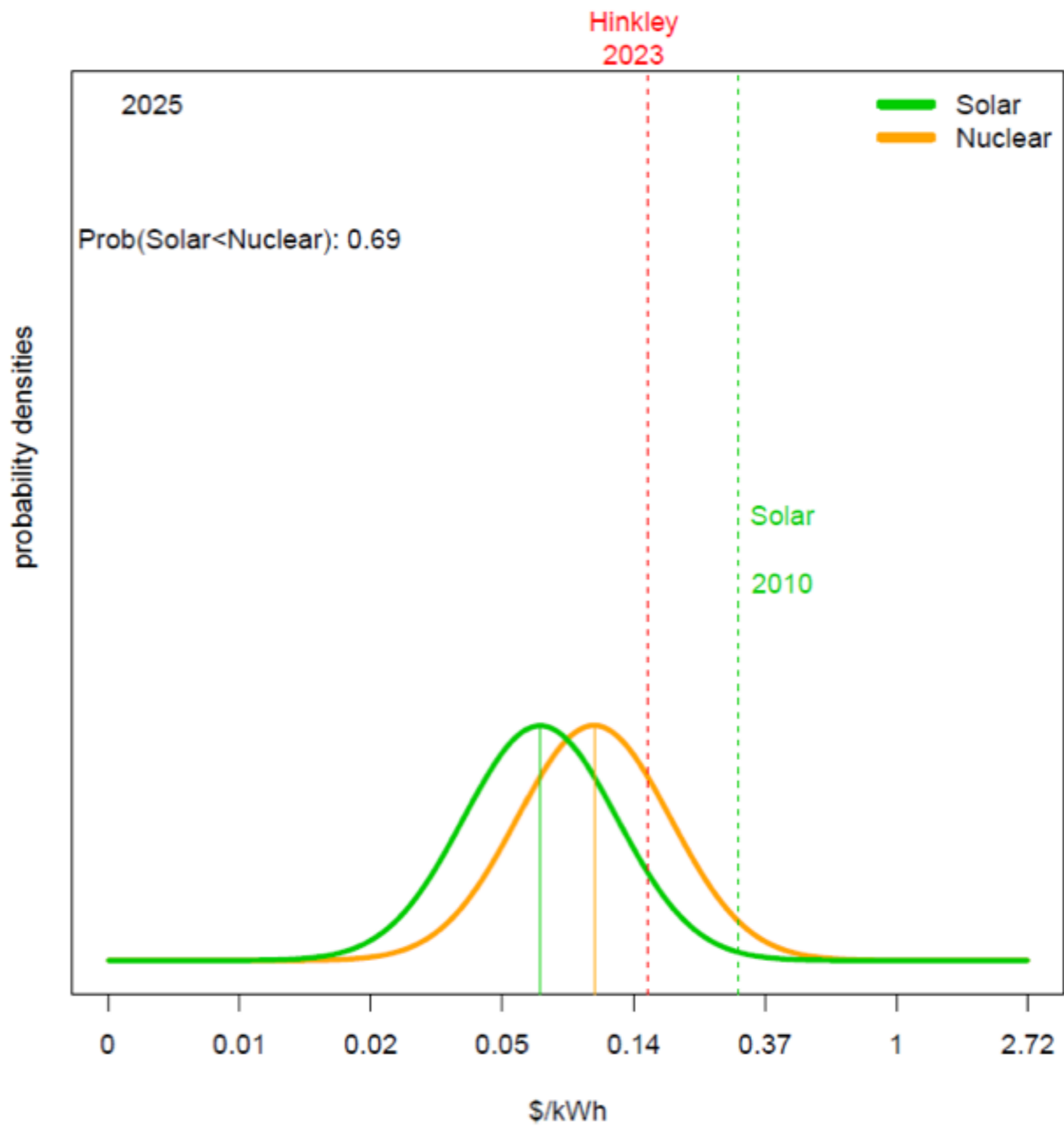


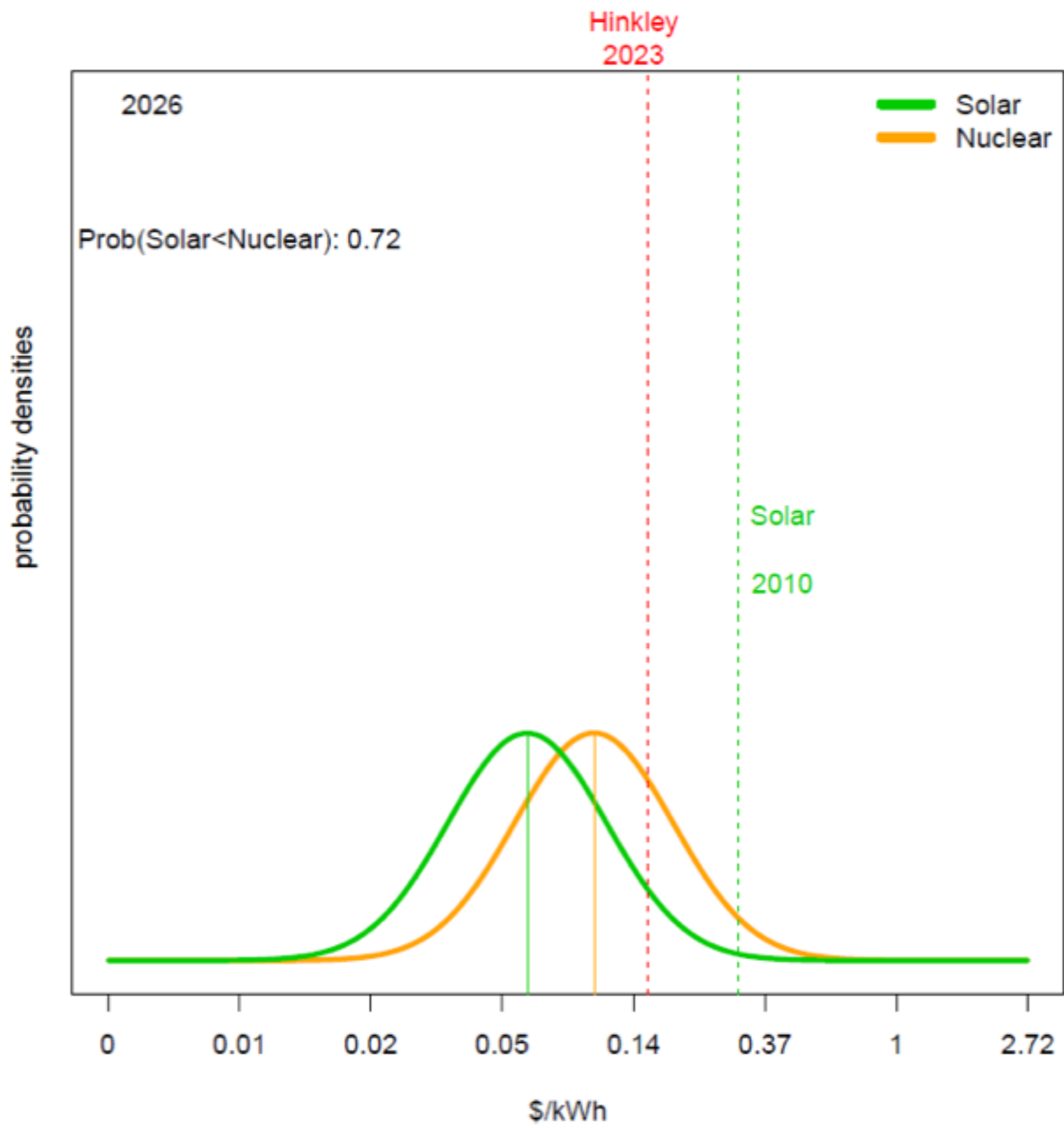


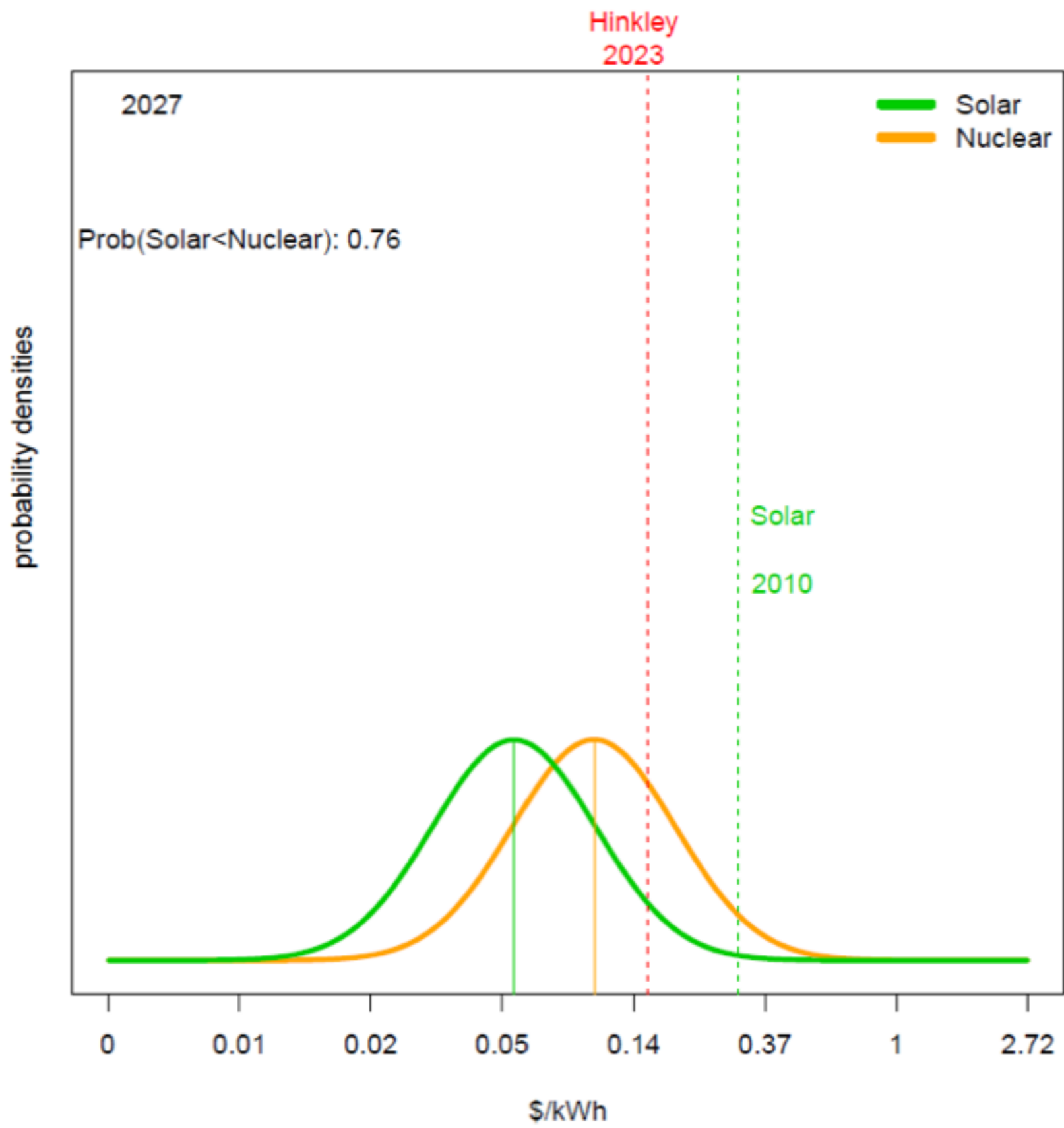


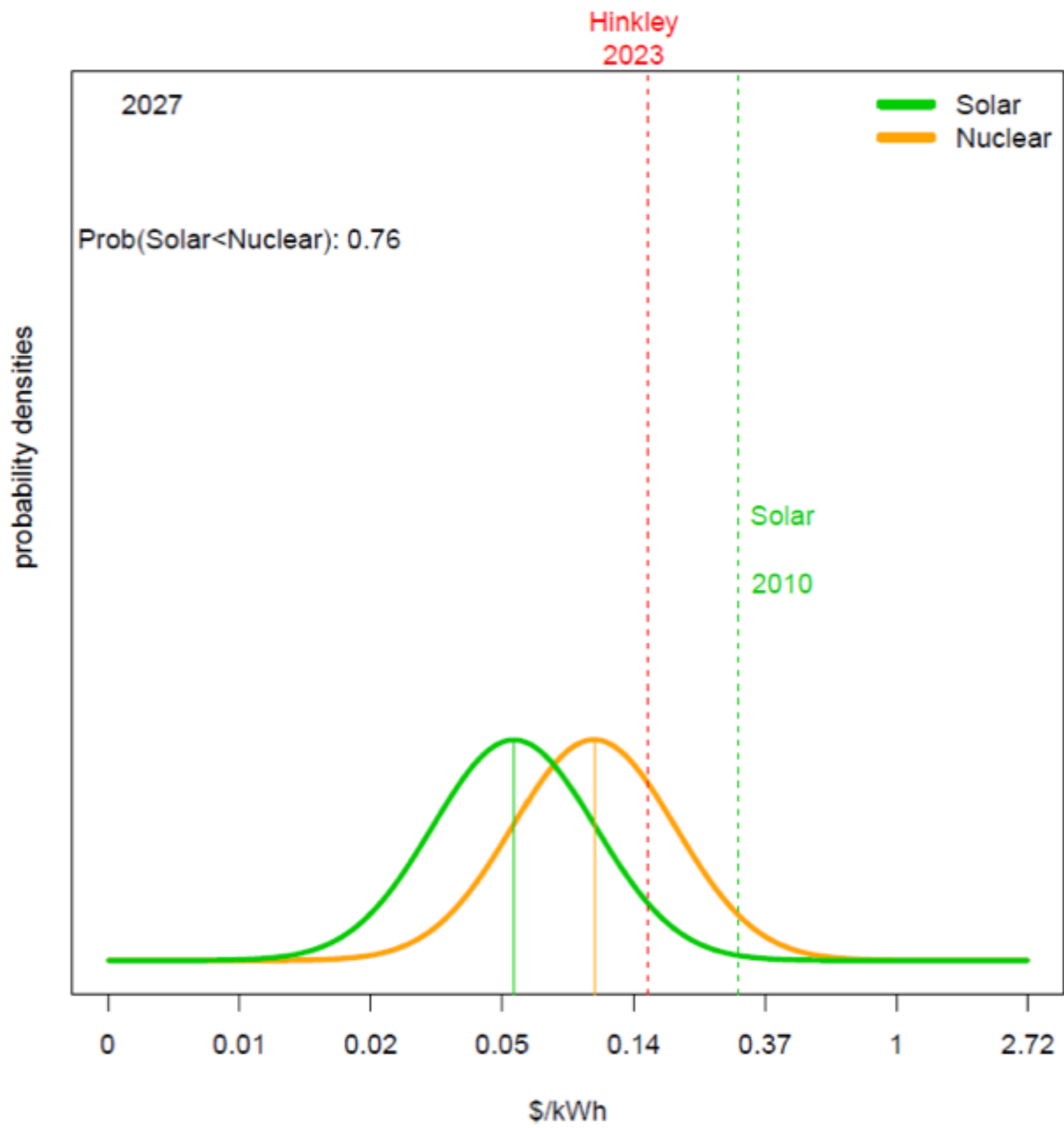


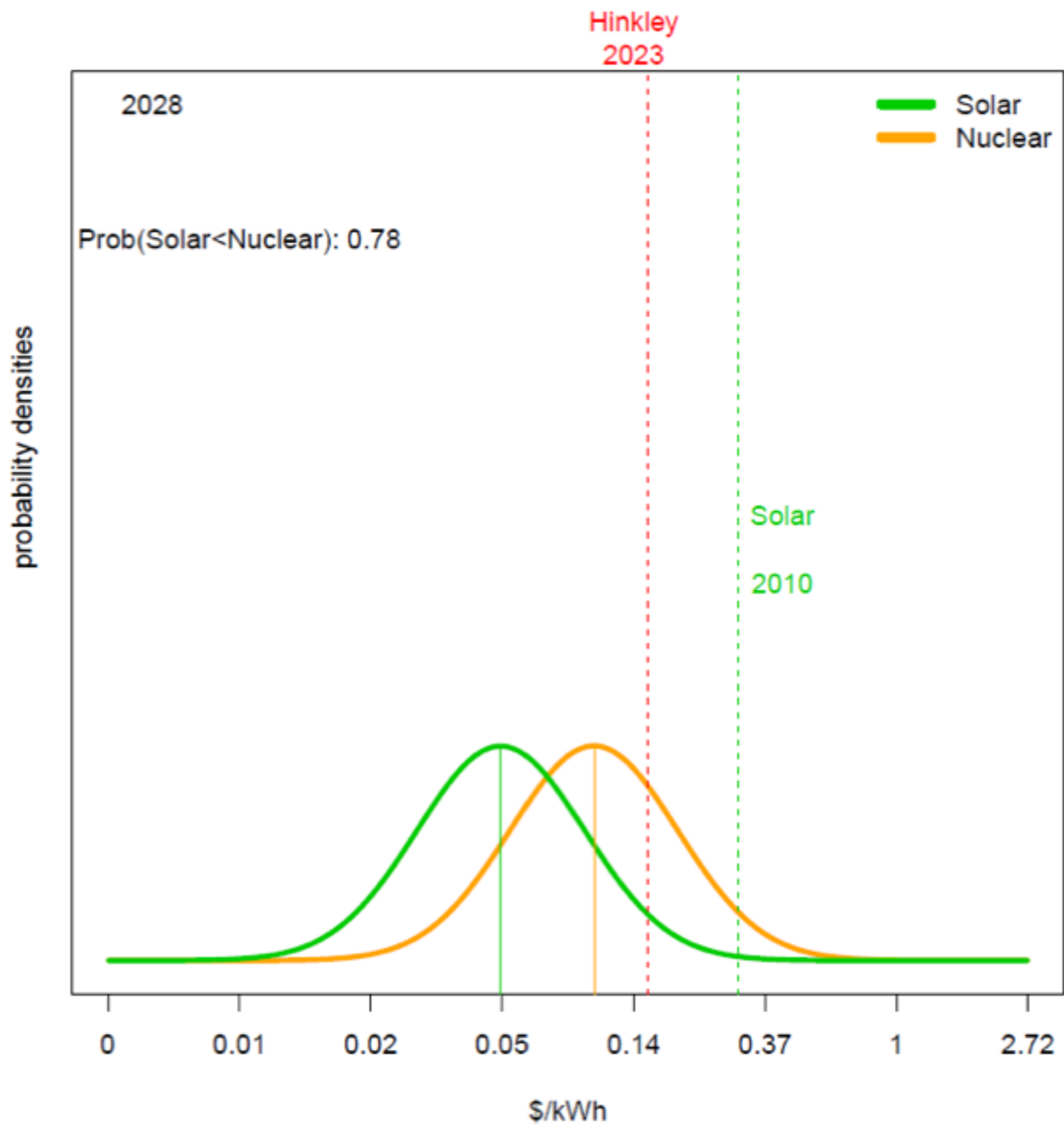




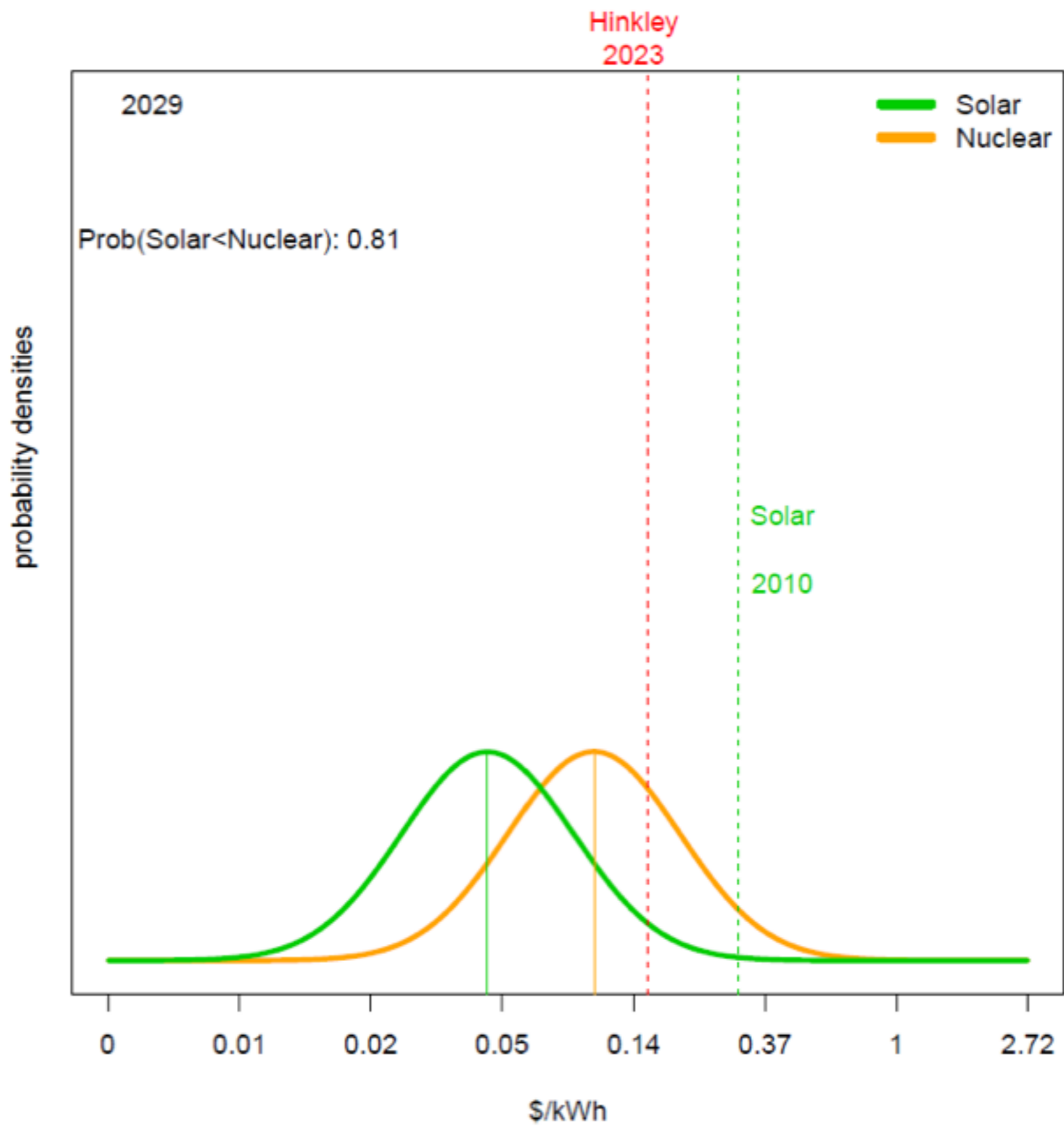


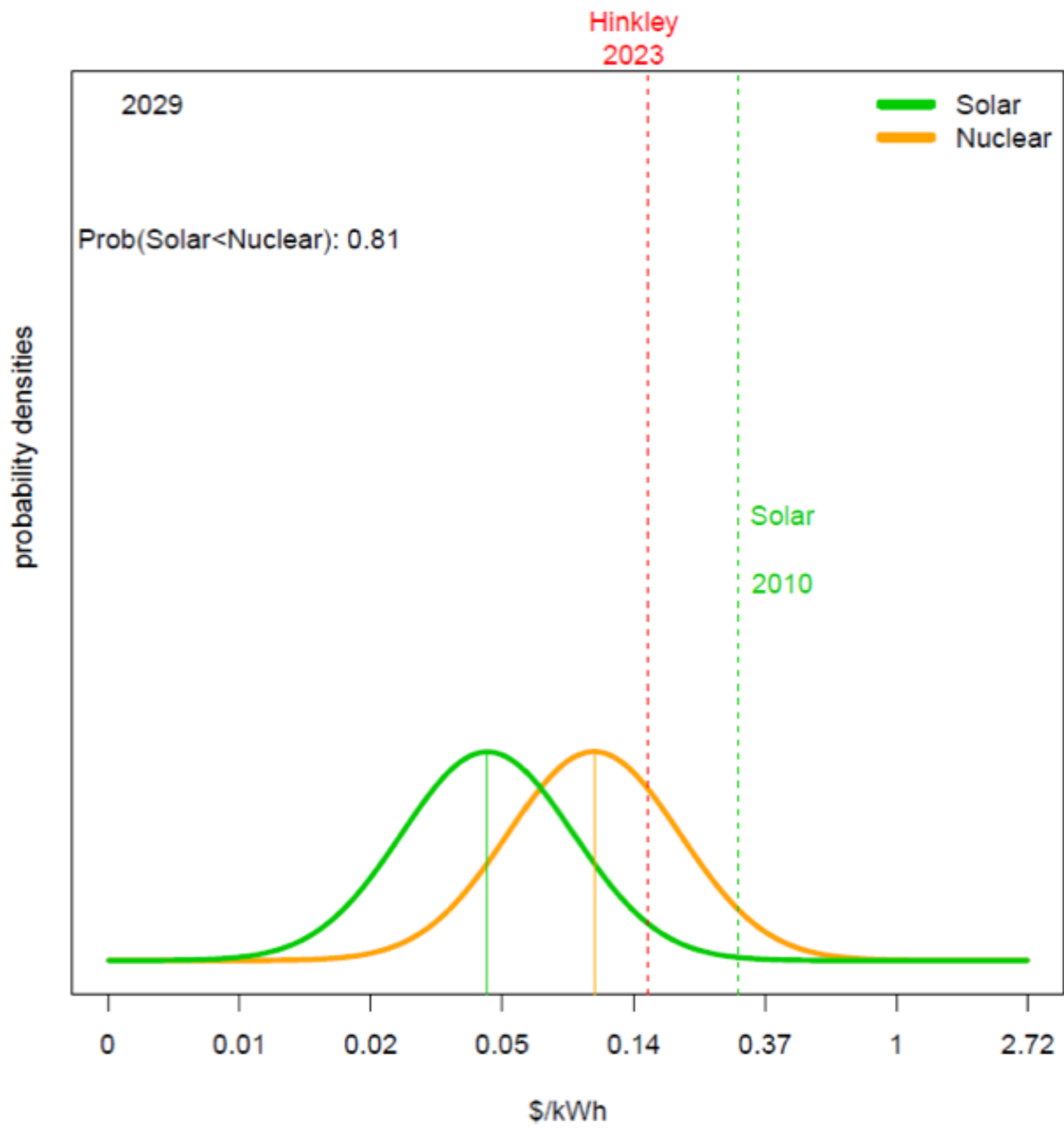


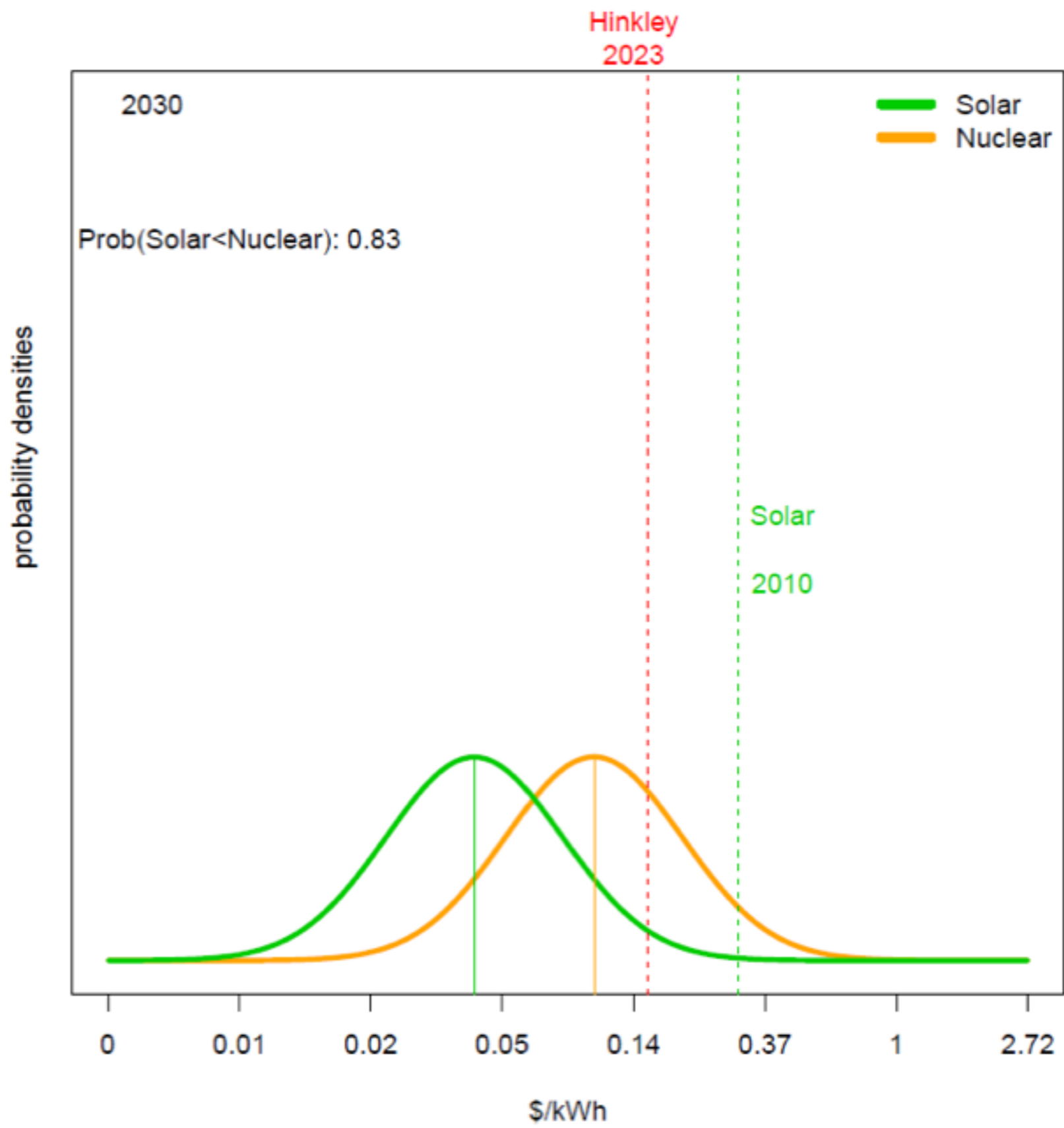




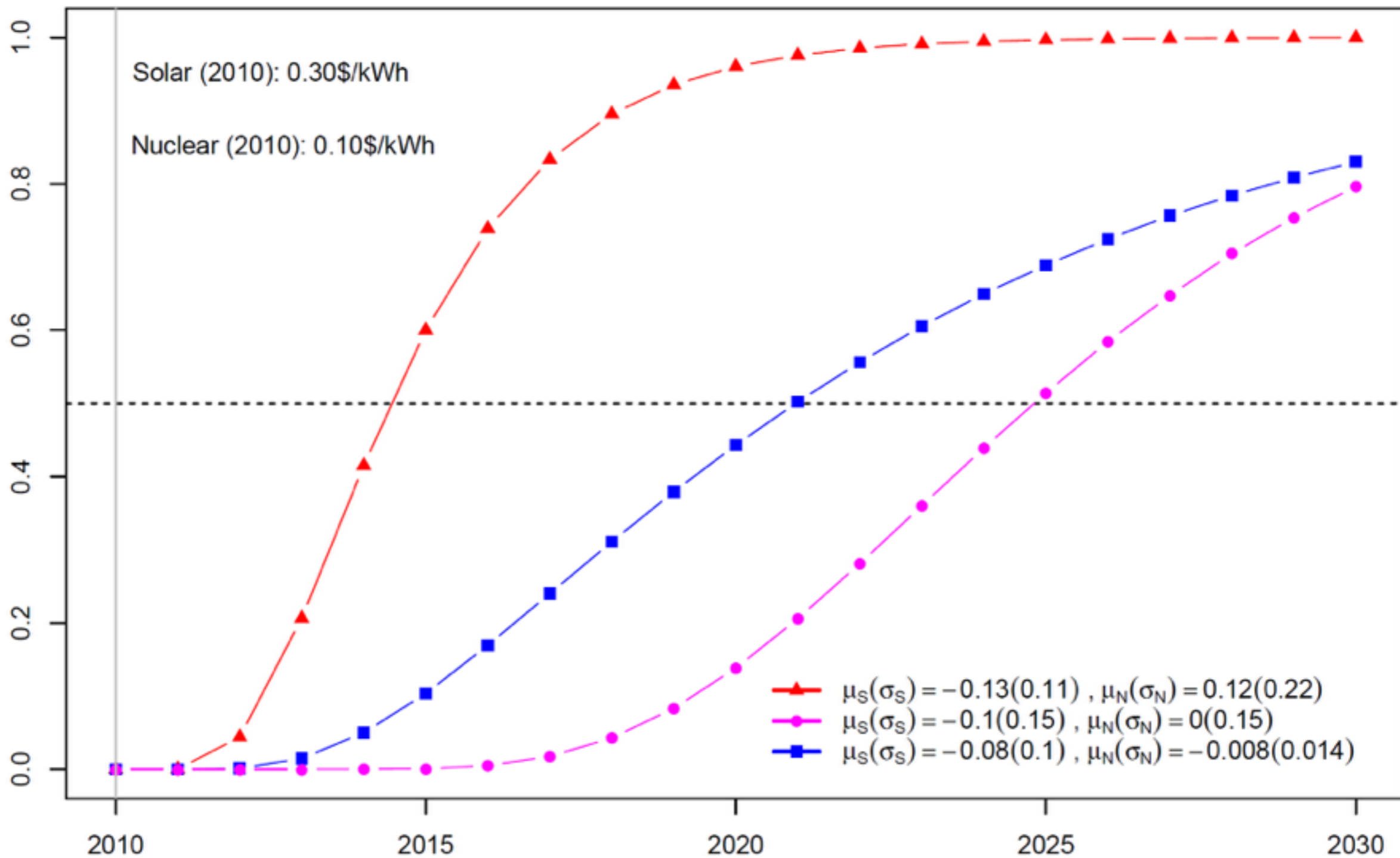




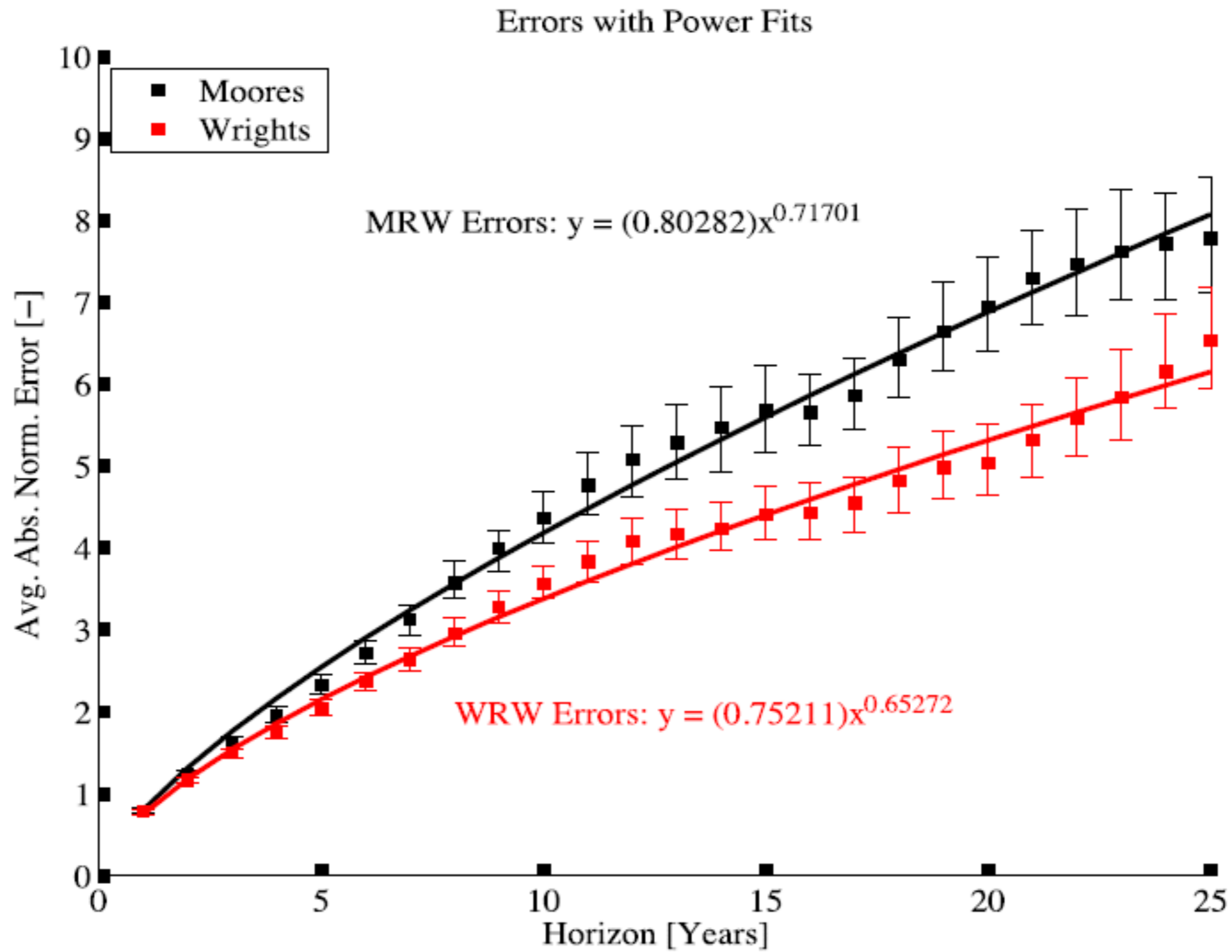




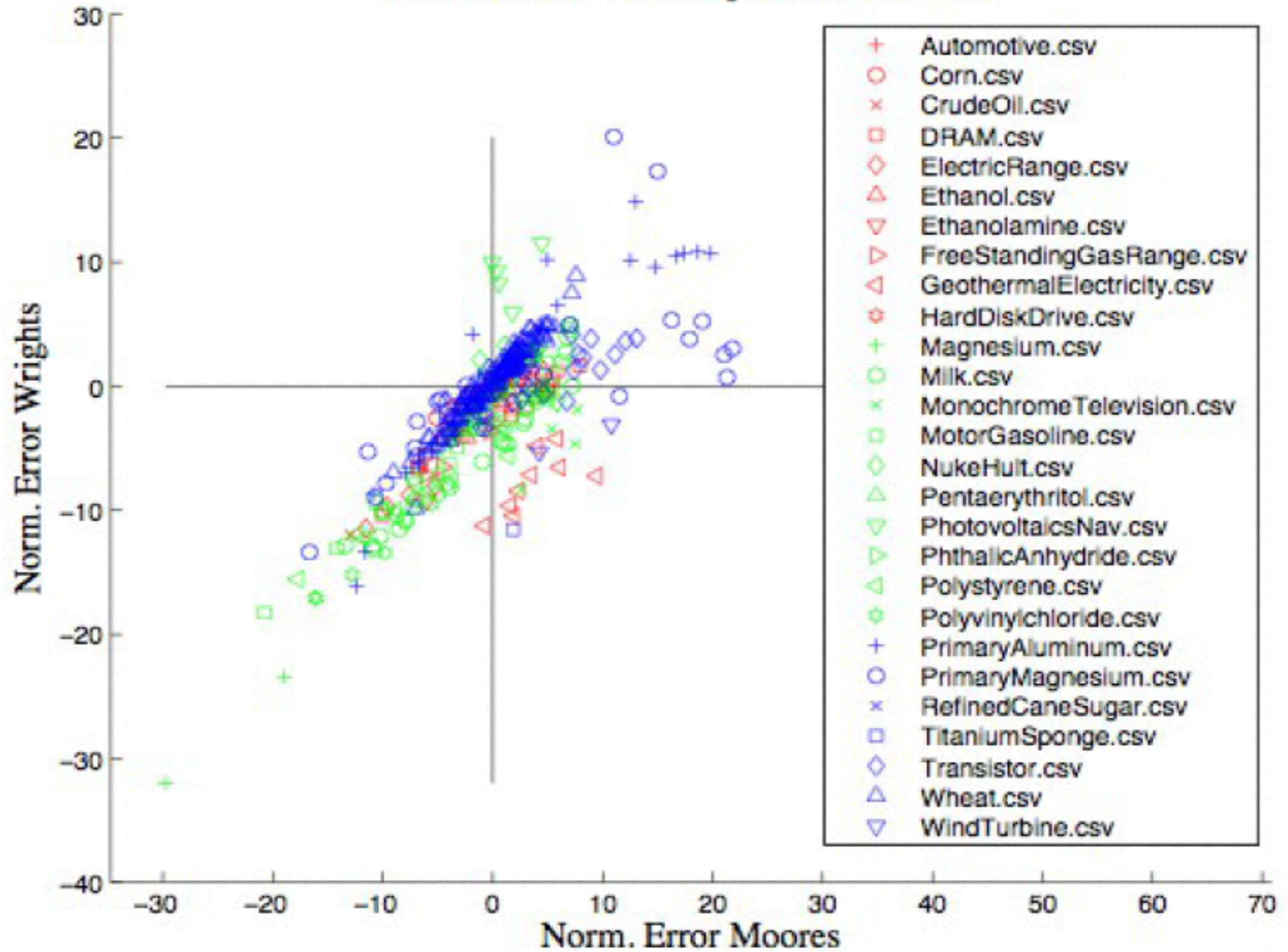
### Probability that solar is cheaper than nuclear



# Comparison of Wright's law and Moore's law



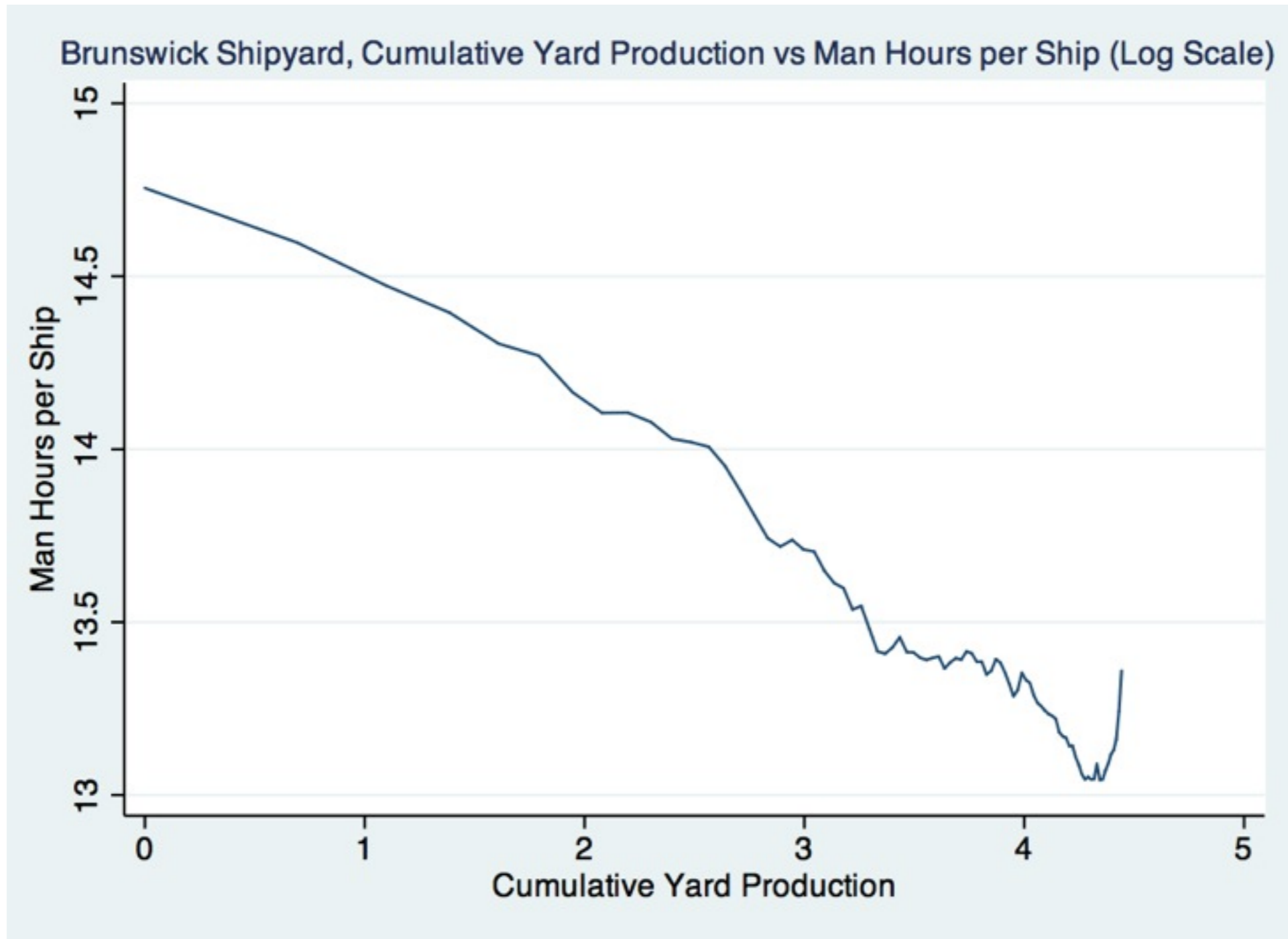
Correlation of Moores and Wrights errors at Horizon12



# SUMMARY OF RESULTS: WRIGHT'S LAW VS. MOORE'S LAW

- Wright's law forecasts based on production better than Moore's law based on time at long horizons
- Production history more useful than time.
- Suggestion: Costs can be driven down by stimulating production (feed-in tariffs).
- Need "artificial experiments", such as WWII, to test properly (correlation v.s causation).
  - Does production drive cost down, or does cost drive production up? Or both?

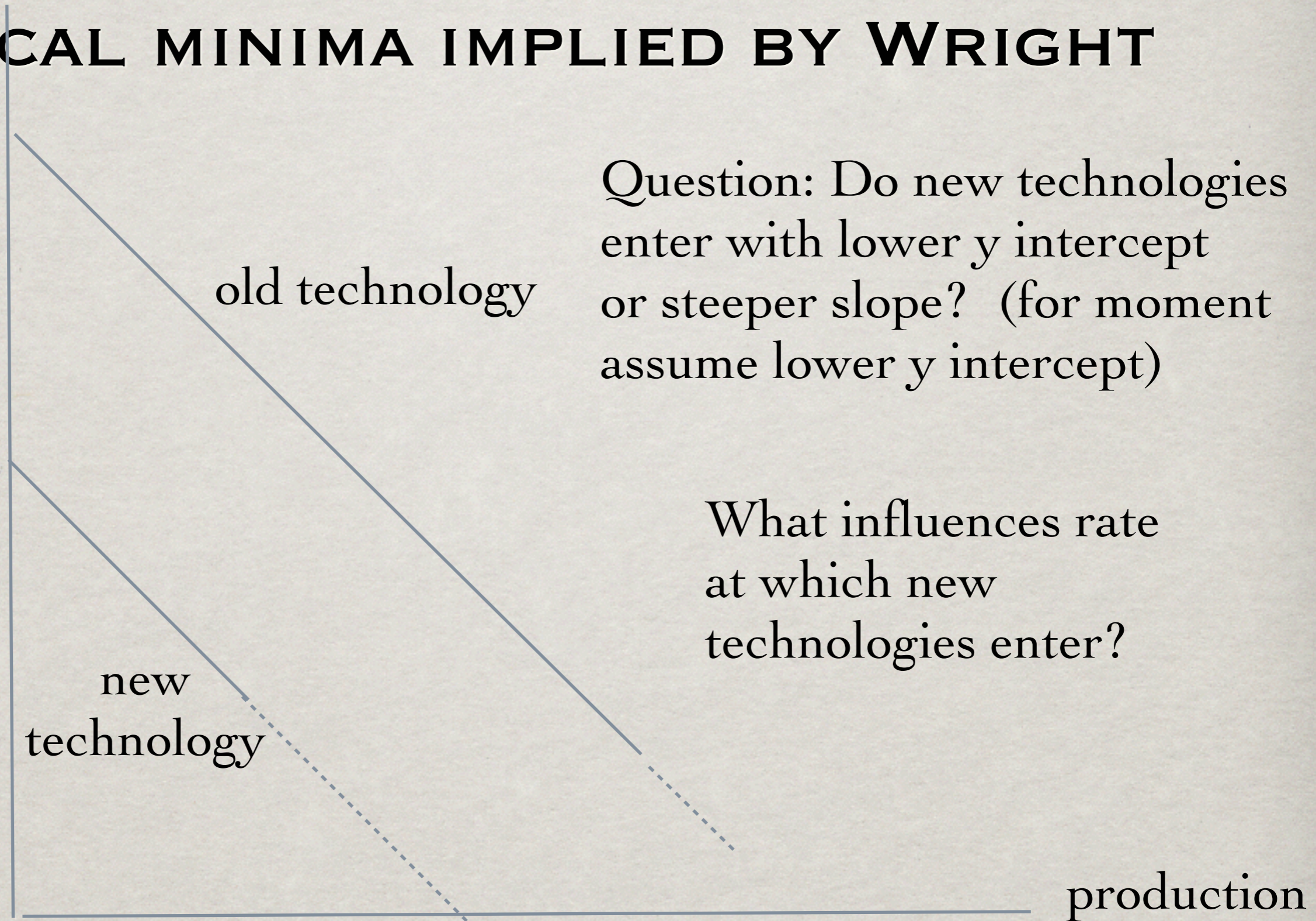
# Liberty Ships





unit cost

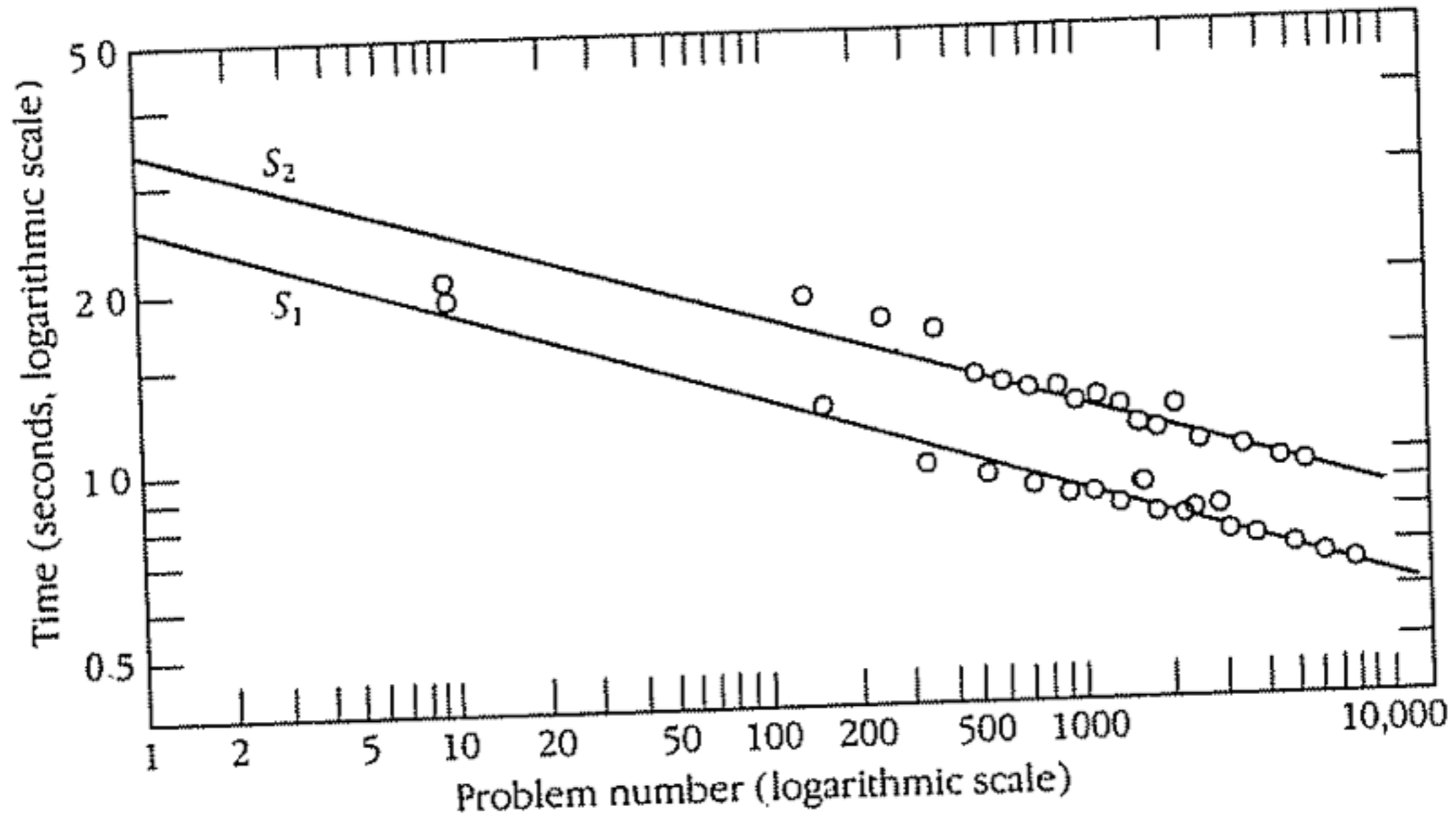
# LOCAL MINIMA IMPLIED BY WRIGHT



# Generality of Wright's law

- Holds at the level of products, firms, industries, or best technology performing a given function.
- Explanation must be correspondingly general.

# POWER LAW OF PRACTICE



Improvement with practice in time to add two numbers  
(Blackburn, 1936)

# RECIPE MODEL OF TECHNOLOGICAL IMPROVEMENT

- Muth (Management Science, 1987)
  - Engineers generate new solutions at random, accept them if they are better. Single component: Implies Wright's law with exponent = -1.
- Auerswald, Kauffman, Lobo and Shell (JEDC, 2000)
  - Multiple components that depend on each other. Accept improvements only if sum score improves.



## RECIPE MODEL (CONTINUED)

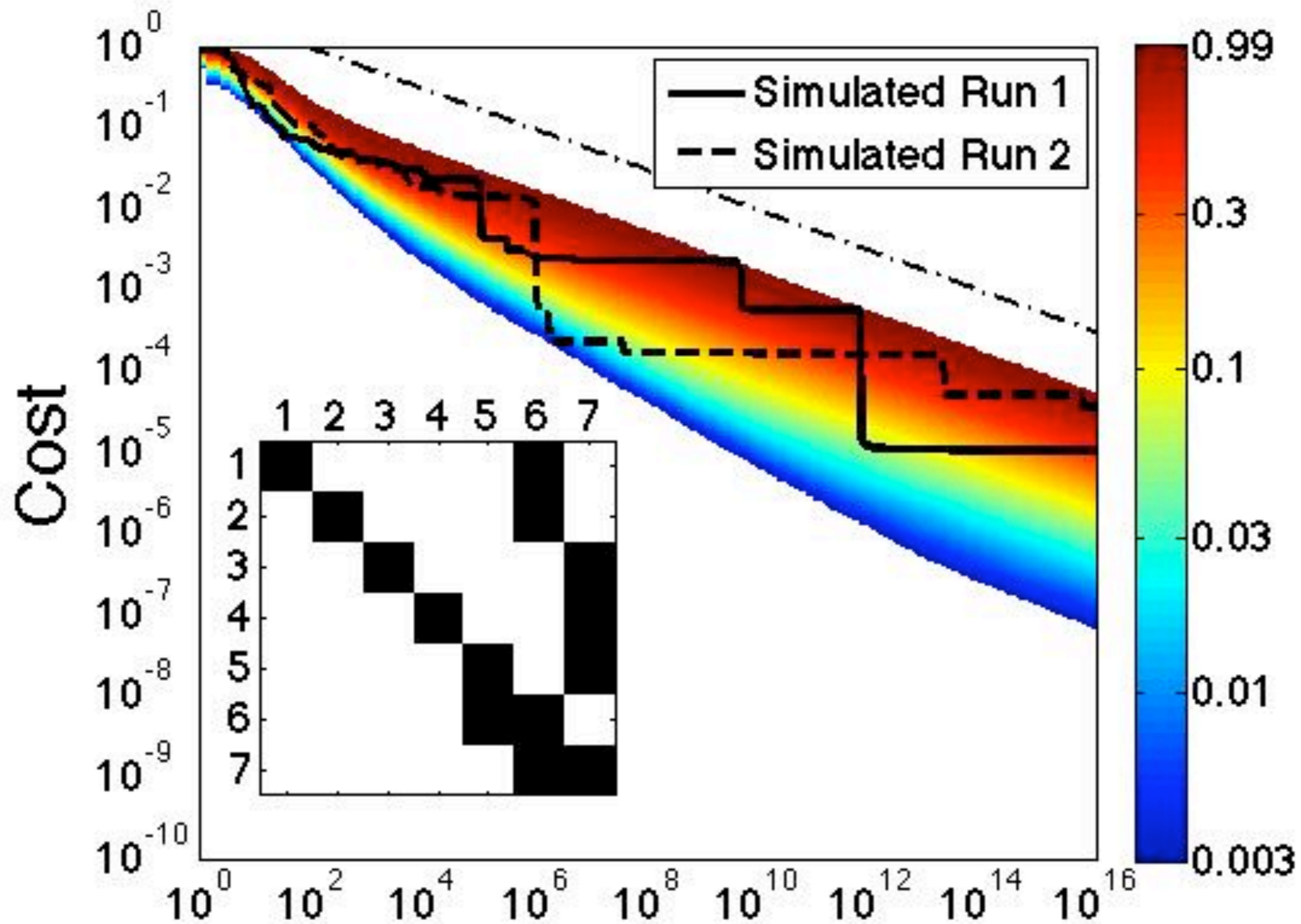
- McNerney, Farmer, Redner, Trancik (PNAS, 2011)

- simplified and solved recipe model

- generates power law with exponent  $-(1/d)$ , where  $d$  = “design complexity”, which depends on DSM. For homogeneous networks  $d$  is in-degree of DSM.

- for heterogeneous networks there are typically bottleneck components,  $d$  is more complicated to compute, and progress typically occurs via a sequence of punctuated equilibria

# COST VS. TIME FOR RECIPE MODEL



# Need to go beyond recipe model

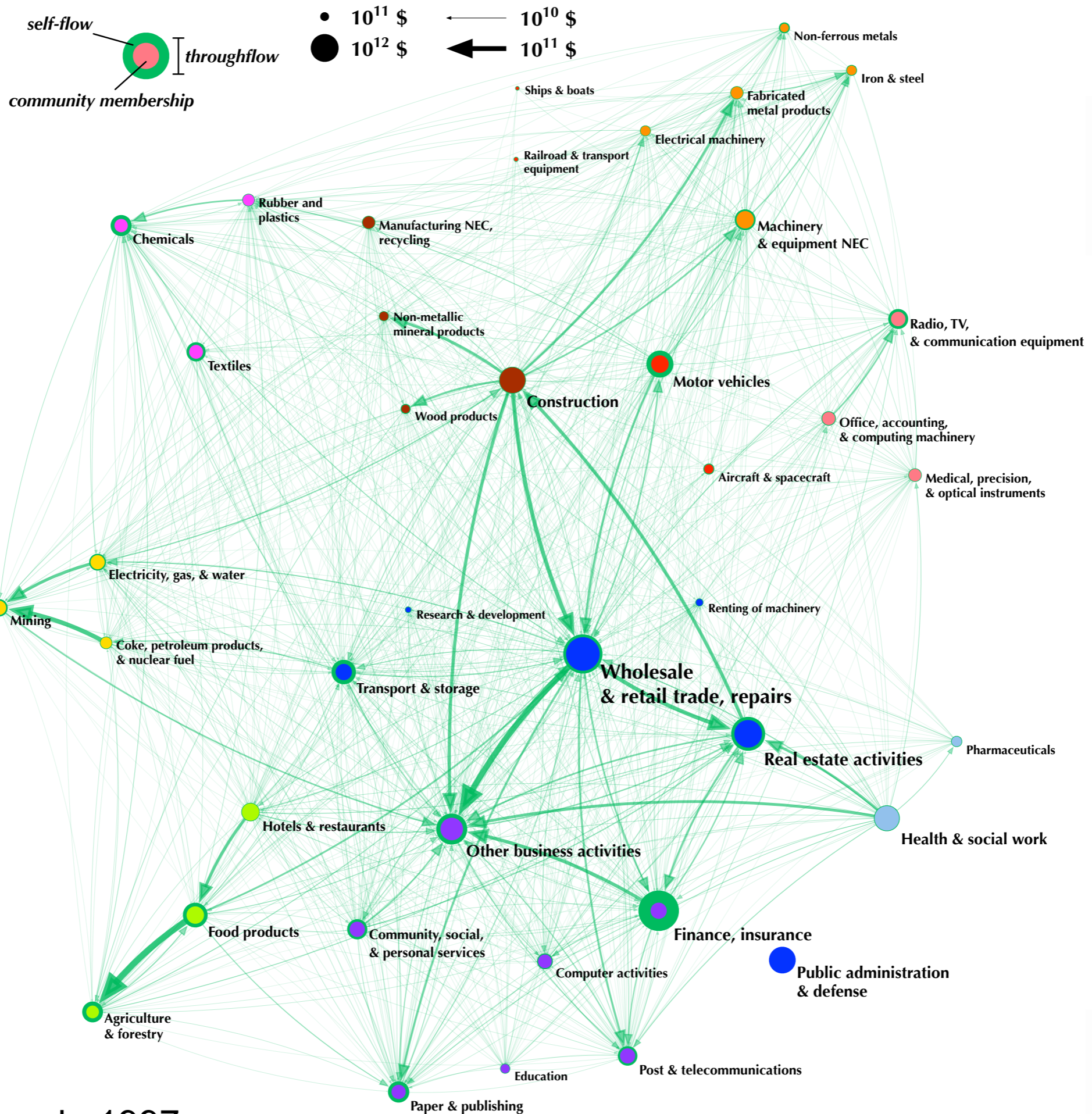
- Nice start, but only part of story
- Anecdotally: Innovations in one industry often drive innovations in others
  - solar PV, laser printers, digital cameras, ...
- Interactions between technologies are key
  - must model evolution of entire technological ecology to understand a single technology





## LEONTIEF: INPUT-OUTPUT MODEL OF AN ECONOMY

- Nodes are industries, (weighted) directed links are inputs to each industry.
- Can be based on physical flows or on monetary flows.
- Precise analogy to equilibrium chemical kinetics (allowing non-integer stoichiometric parameters)
- Conservation laws lead to linear system of equations
- Used in national accounting, central planning.



U.S. industry network, 1997

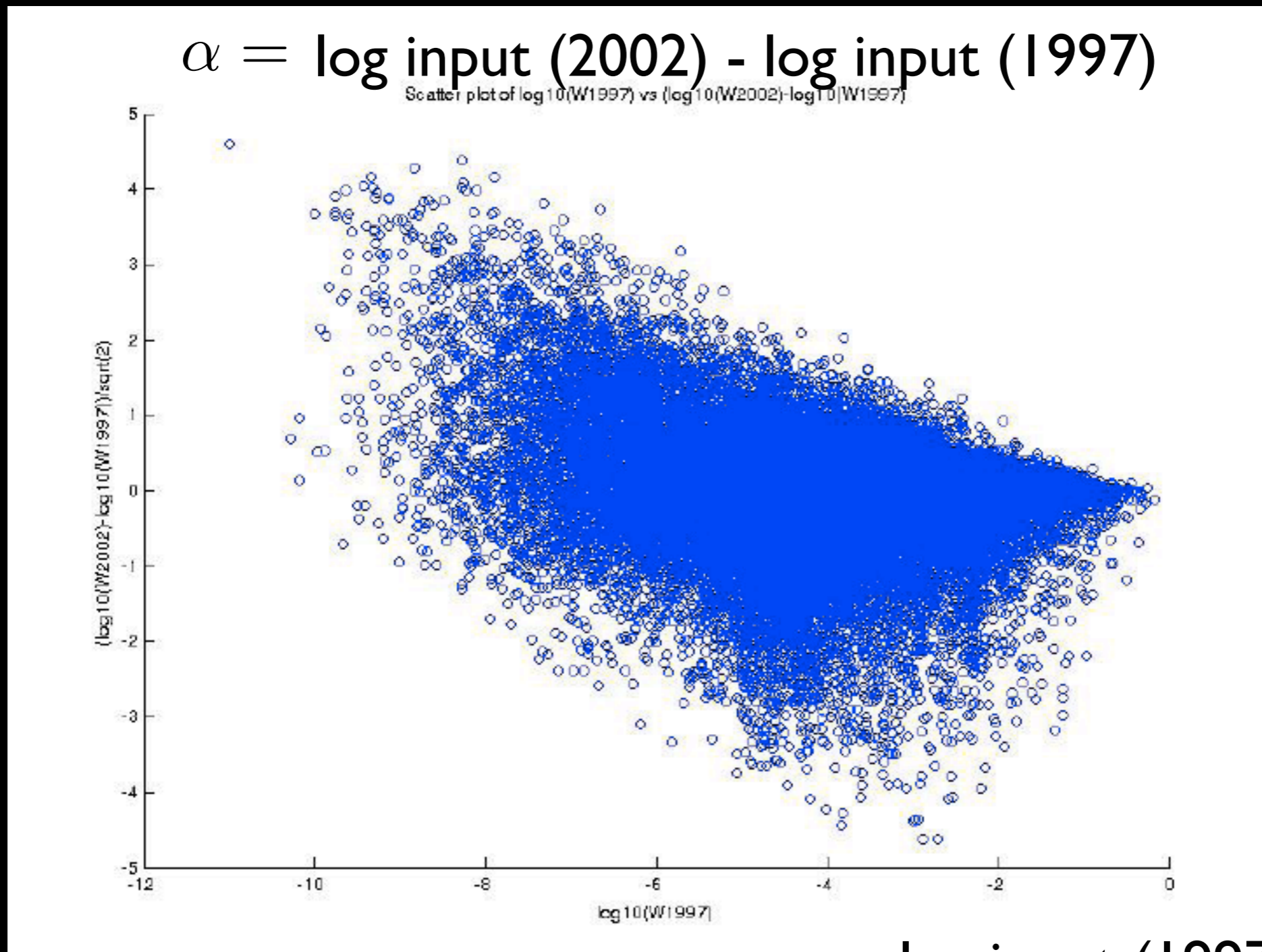
# OUR APPROACH

EVOLUTIONARY EXTENSION  
OF LEONTIEF FORMALISM

with James McNerney,  
Francesco Caravelli

- Design improvement happens through
  - Input tuning
  - Substitution of cheaper or better inputs
  - Creation of new goods: Network growth
  - Improved social technologies of production, distribution, ... invisible to Leontief network
- Increase in combinatorial possibilities --  
palette gets larger and more powerful

# Input fluctuations of I/O matrix for USA



“Deductive tinkering”

# Simplest model: Efficiency improvements

- The net result of a design improvement is an overall decrease in material inputs to perform same function

$$\phi_{ij} \rightarrow \alpha \phi_{ij}$$

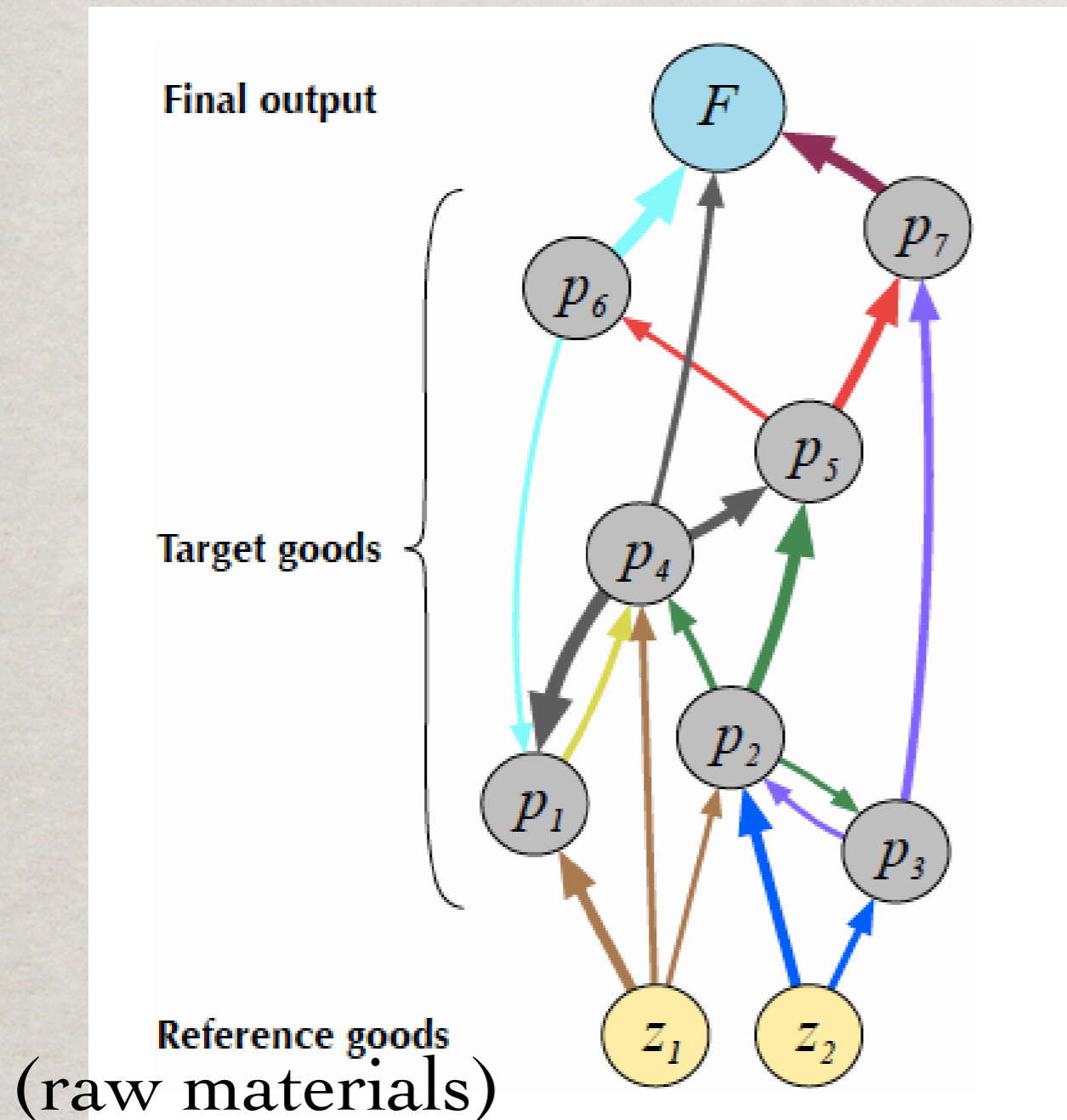
- Leads to conclusion that technologies have a trophic structure, like food webs

# TECHNOLOGIES CAN BE ARRANGED IN TROPHIC LEVELS

with James McNerney and Francesco Caravelli

Hypothesis:  
All else equal, technologies with high trophic level improve faster than technologies with low trophic level.

Reason: Improvements are amplified multiplicatively.

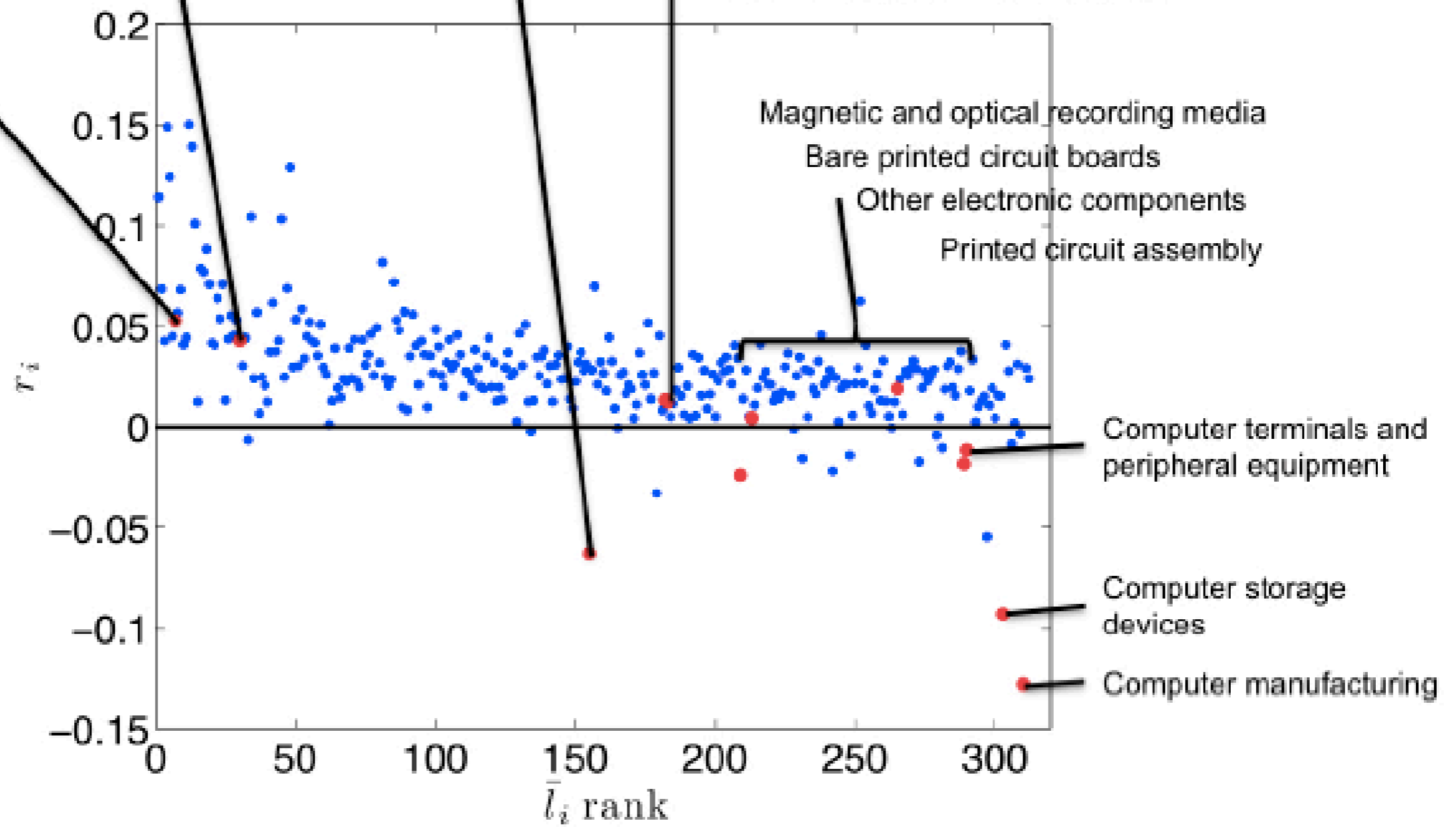


**b**Sand, gravel, clay,  
and ceramic miningGround mineral and  
earth manufacturing

Semiconductors

Electronic connectors

Electronic capacitors, resistors, etc.



# Provides alternate explanation for super-exponential population growth

- Paul Romer's theory: population and technology co-evolved.
- Our theory: Trophic structure accelerates growth, graphic structure has grown.



# Physics matters to economics

- Evolutionary search finds physical processes capable of rapid improvement
- Interaction between physics, which determines what is possible, and economics, which determines what is wanted
- Physics is key determinant of technological improvement (Funk and Magee)
- Migration toward “good physics” can result in dramatic improvements

# Analogy: Evolution of autocatalytic networks

- Autocatalytic metabolism: Set of chemical species that jointly produce each other via catalyzed chemical reactions involving only other members of the set.

# METADYNAMICS

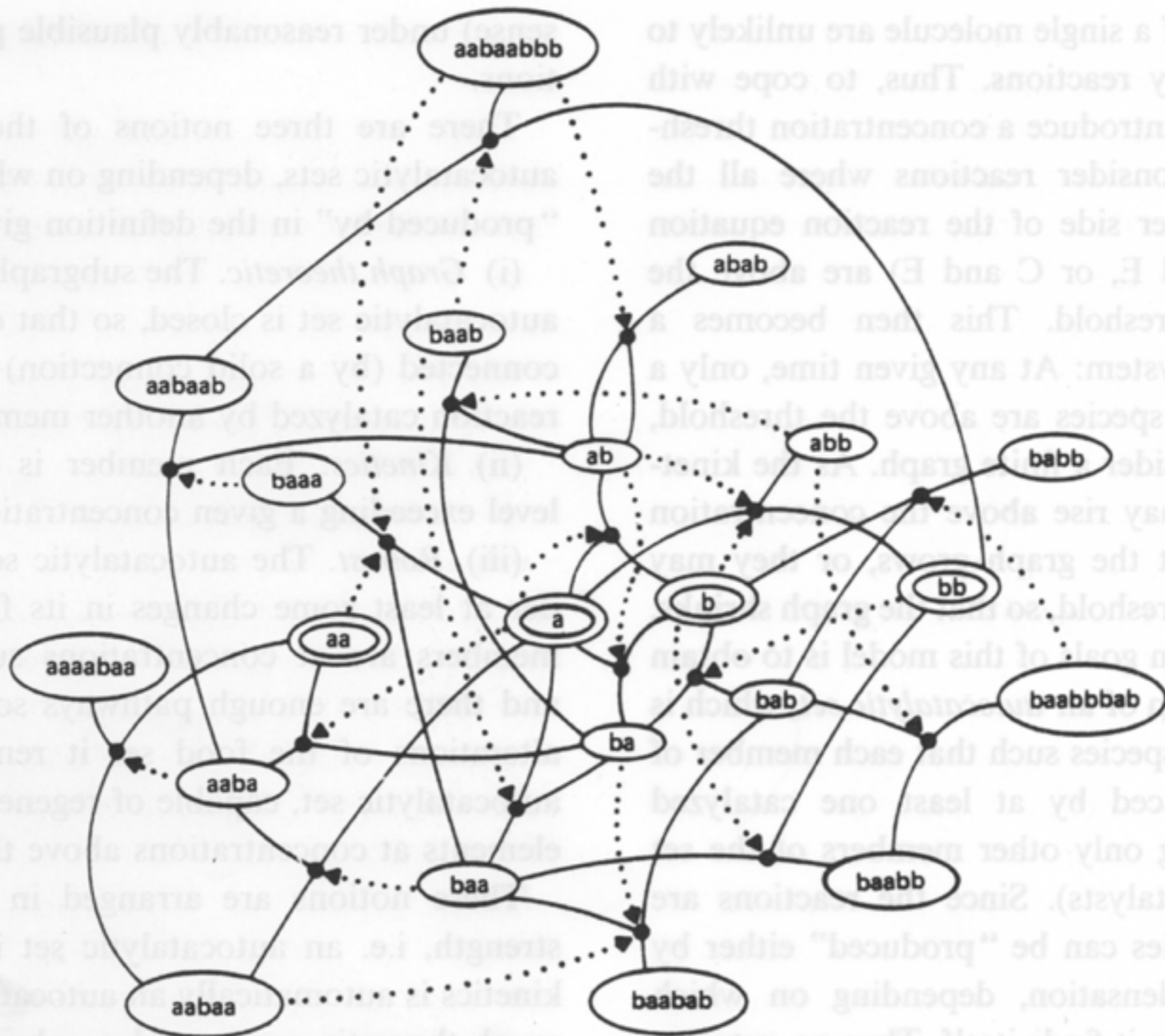
(WITH NORMAN PACKARD)





- A *metadynamics* model is a dynamical systems model on a dynamic network. The dynamics induce changes in the network, which in turn induces changes in the dynamical system.
- For example, consider modeling a potentially infinite set of possible chemical reactions.
  - ~ Chemical kinetics are solved on a network of dominant reactions. This network is defined by the set of existing chemical species, which can themselves change through time. As they change, they change the network.
- Key idea: Evolution toward the adjacent possible (Kauffman).

# METADYNAMICS PAPERS

- Farmer, J.D., S. Kauffman, N. Packard. “Autocatalytic Replication of Polymers.” *Physica D* (1986).
- Farmer, J. D., N. H. Packard, A. Perelson. “The Immune System, Adaptation, and Machine Learning.” *Physica D* (1986)
- Bagley, R. J., and J. D. Farmer. “Spontaneous Emergence of a Metabolism.” In *Artificial Life II* (1991).
- Bagley, R. J., J. D. Farmer, and W. Fontana. “Evolution of a Metabolism.” In *Artificial Life II* (1991).





-  = food set
-  = other chemicals
-  = reactions
-  = action of catalysts

(19)

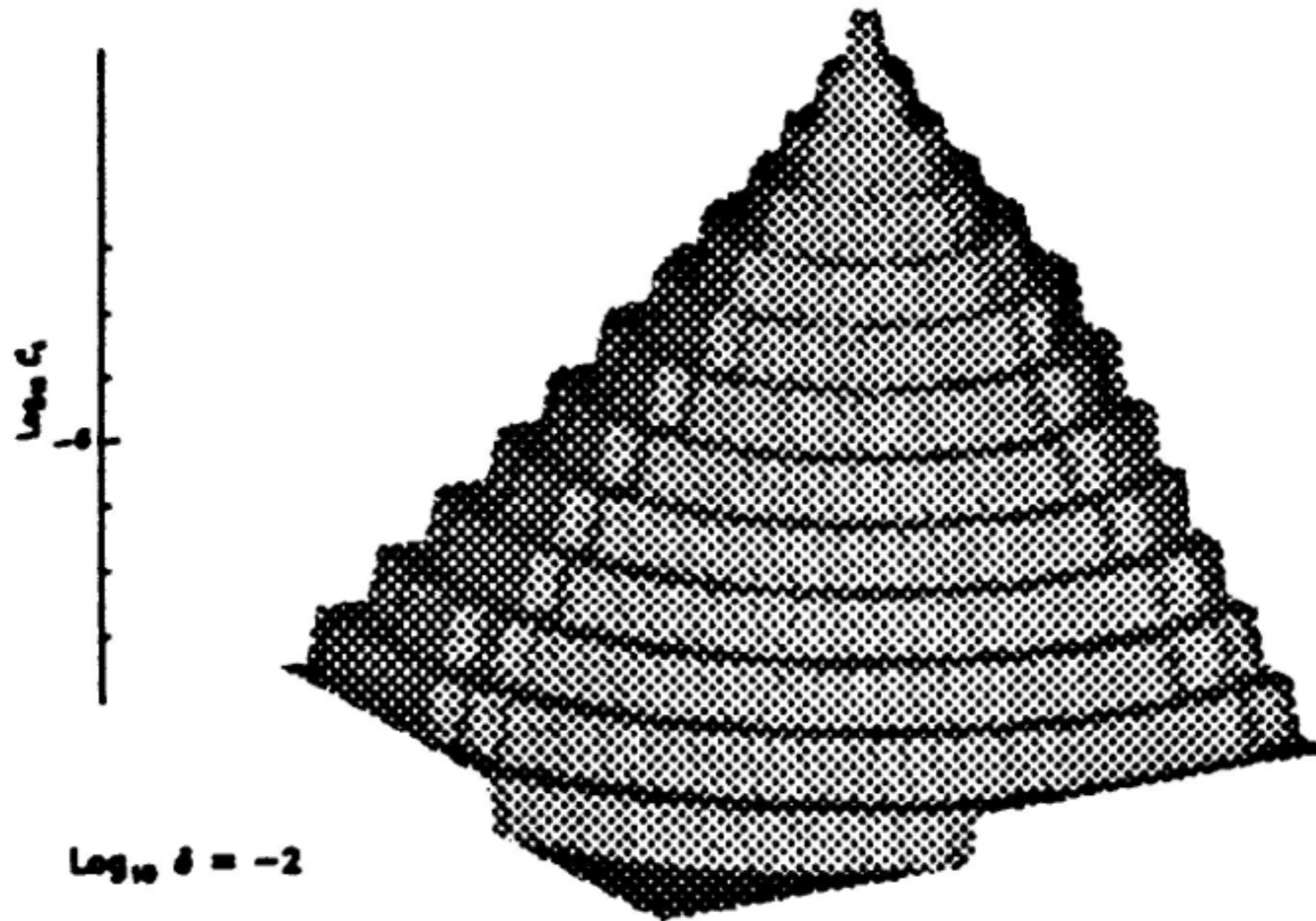


is an autocatalytic set, and so is

# SIMULATION OF AN AUTOCATALYTIC METABOLISM

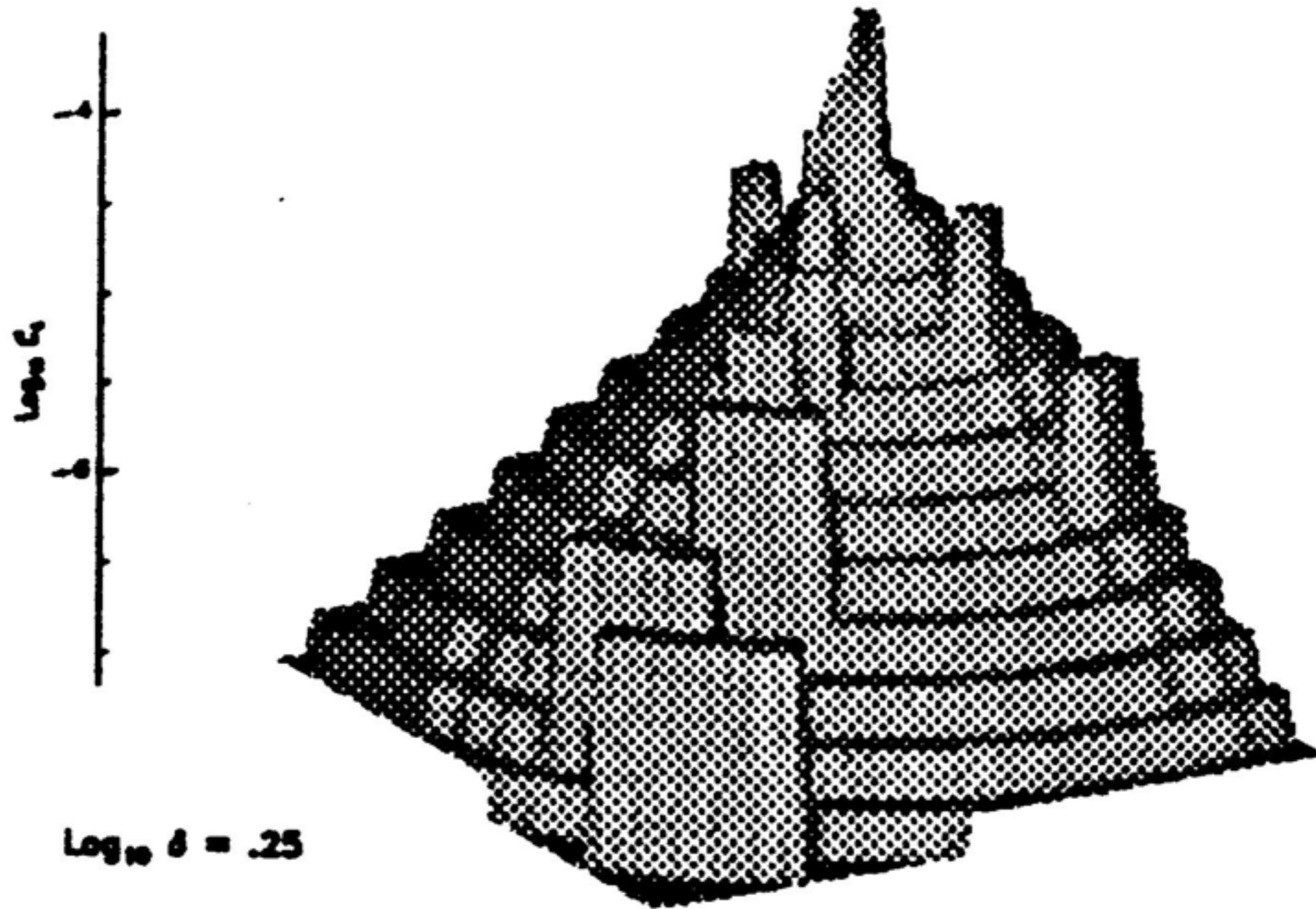
- Start with simple food set (e.g. 5 species)
- Implement kinetics for catalyzed reactions among food set (which defines initial network).
- Define shadow set as species that can be reached by uncatalyzed reactions within network
- Create a new species from shadow set with probability depending on reaction rates.
- If this adds new catalyzed reaction, alter network of catalyzed reactions accordingly.
- Repeat.

equilibrium concentration with only uncatalyzed reactions

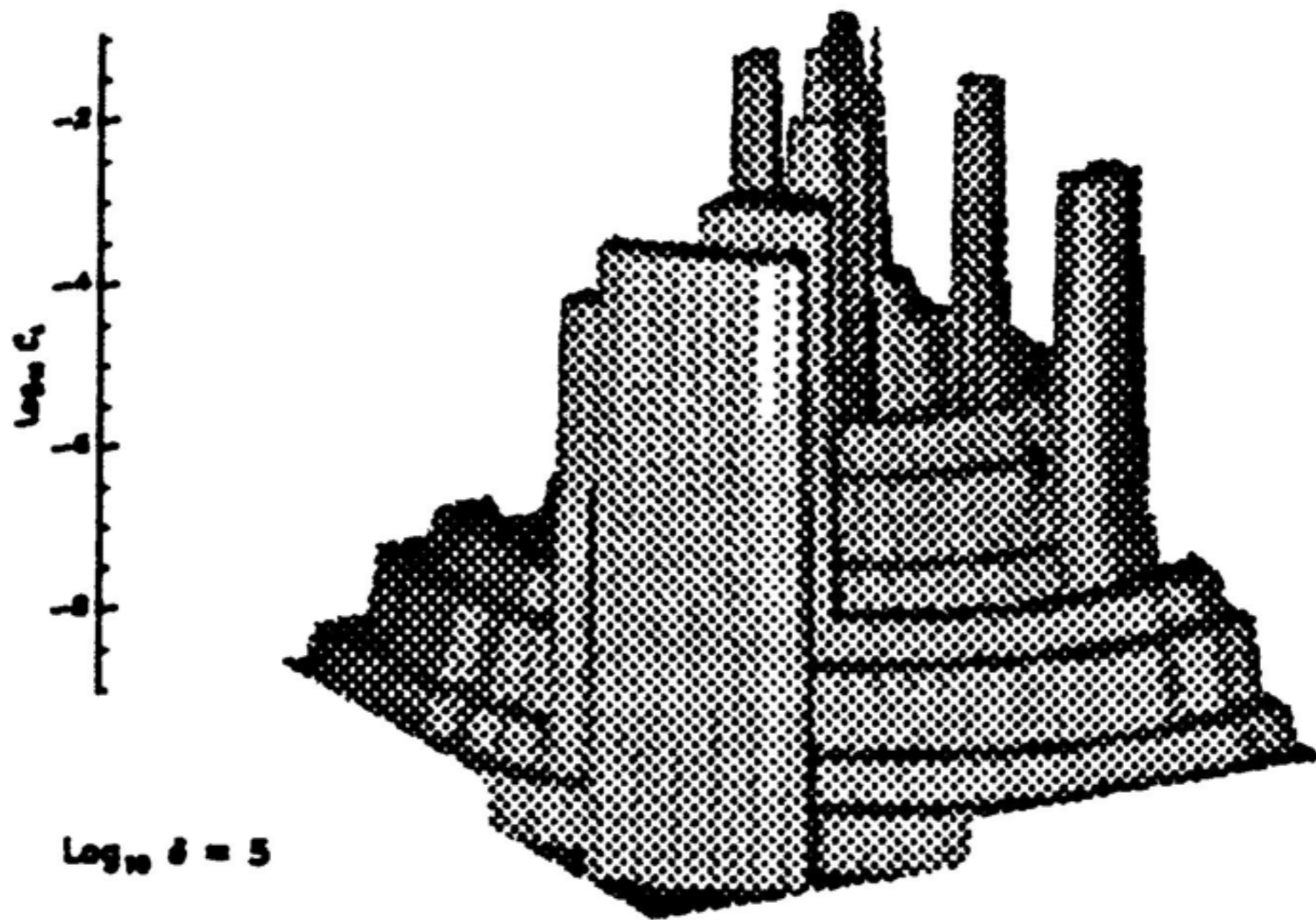


from: The Functional Self-Organization of Autocatalytic Networks  
In a Model of the Evolution of Biogenesis, Richard James Bagley, Ph.D. thesis (1991)





from: The Functional Self-Organization of Autocatalytic Networks  
In a Model of the Evolution of Biogenesis, Richard James Bagley, Ph.D. thesis (1991)



from: [The Functional Self-Organization of Autocatalytic Networks](#)  
In a Model of the Evolution of Biogenesis, Richard James Bagley, Ph.D. thesis (1991)

# SPONTANEOUS EMERGENCE OF AN EVOLVING METABOLISM (WITH R. BAGLEY, W. FONTANA)

- Set of specific chemical species
- Capable of “digesting” many possible food sets
- Composition of of species evolves through time under random variation and selection.
- Metadynamic model generates network through dynamics of components of network.

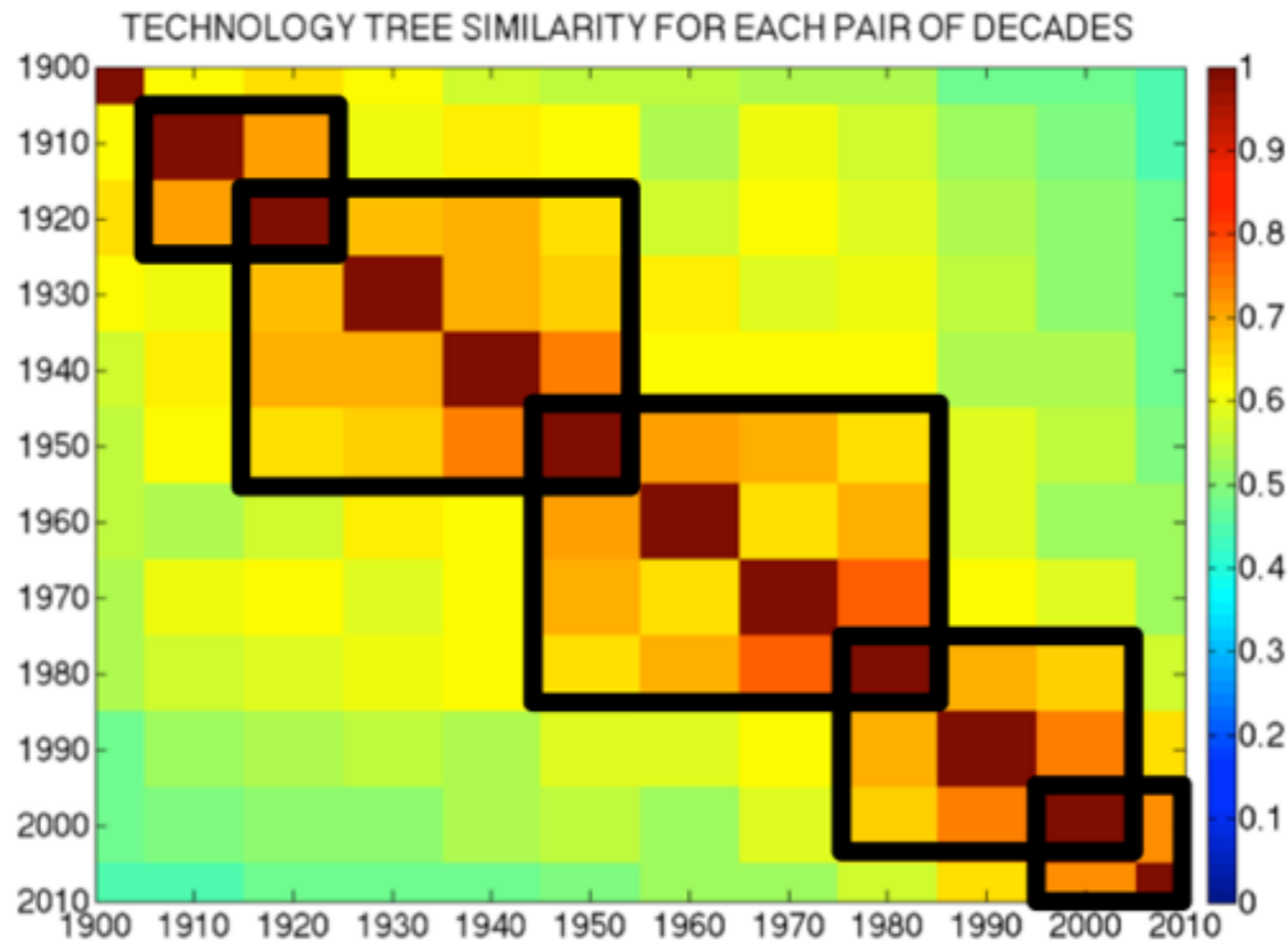
# Investigating technological evolution via US patents

- 9M patents
- 1790 to present:
  - 10,000 tech. codes -> 150,000 tech. codes
- Closest thing to a fossil record of tech. change

# Co-occurrence network

- Can define co-occurrence network as the frequency with which two technology codes appear together.
- Defines network with technology codes as nodes and co-occurrence frequency as weighted links.
- Provides a way to understand how technologies interact with each other and how this evolves through time.

# Evidence for technological epochs

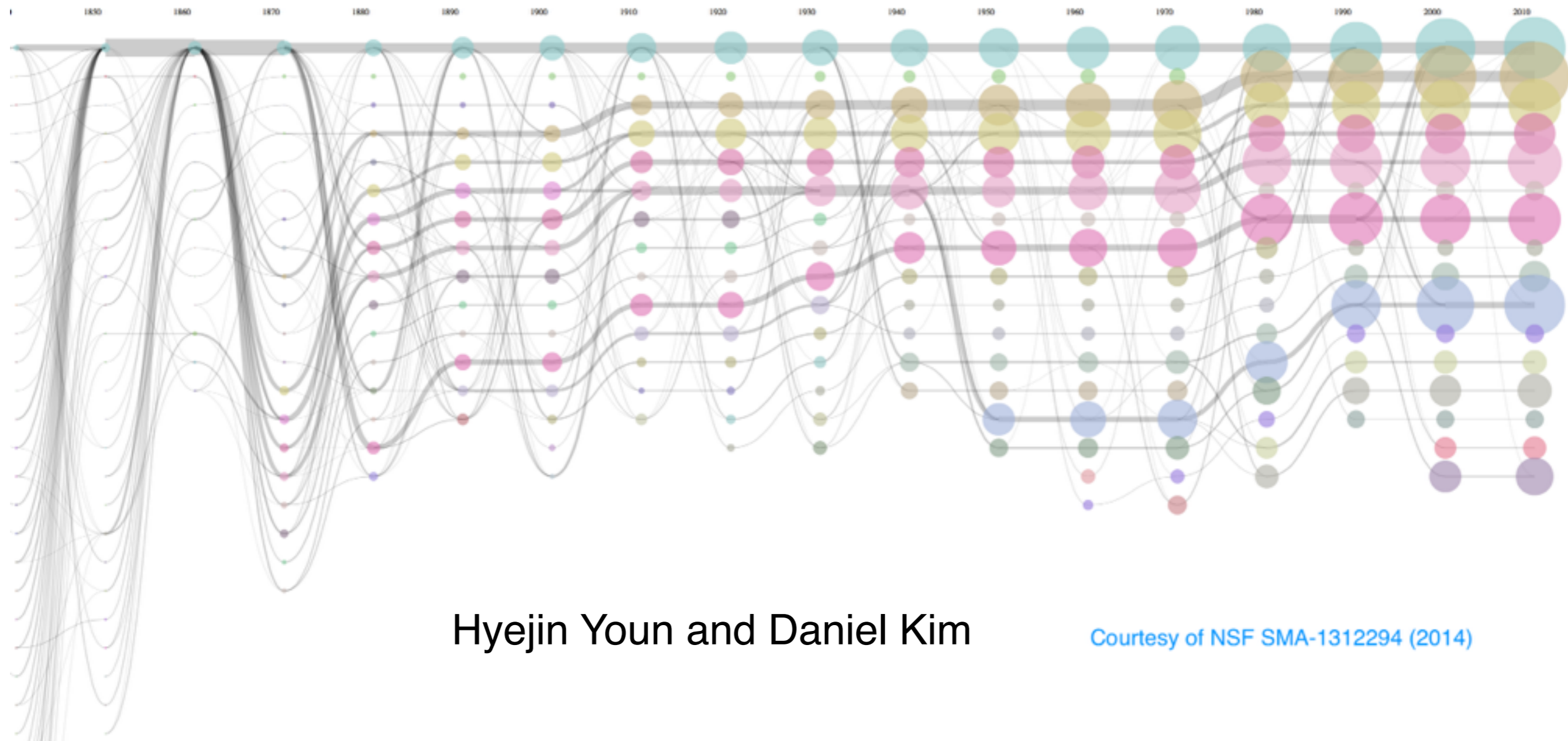


- Inventing activity changes, based on technological, economic, social, even geopolitical trends.
- We investigate the self-organization patterns of technologies across time, given our combination-based communities at each decade.
- Our analysis reveals clusters of self-similarity across time, corresponding to various historical eras (WWI, WWII, Cold War, modern era).
- Cluster boundaries correspond to *technological shifts*, that allow us to “sandbox” different models at different time periods.

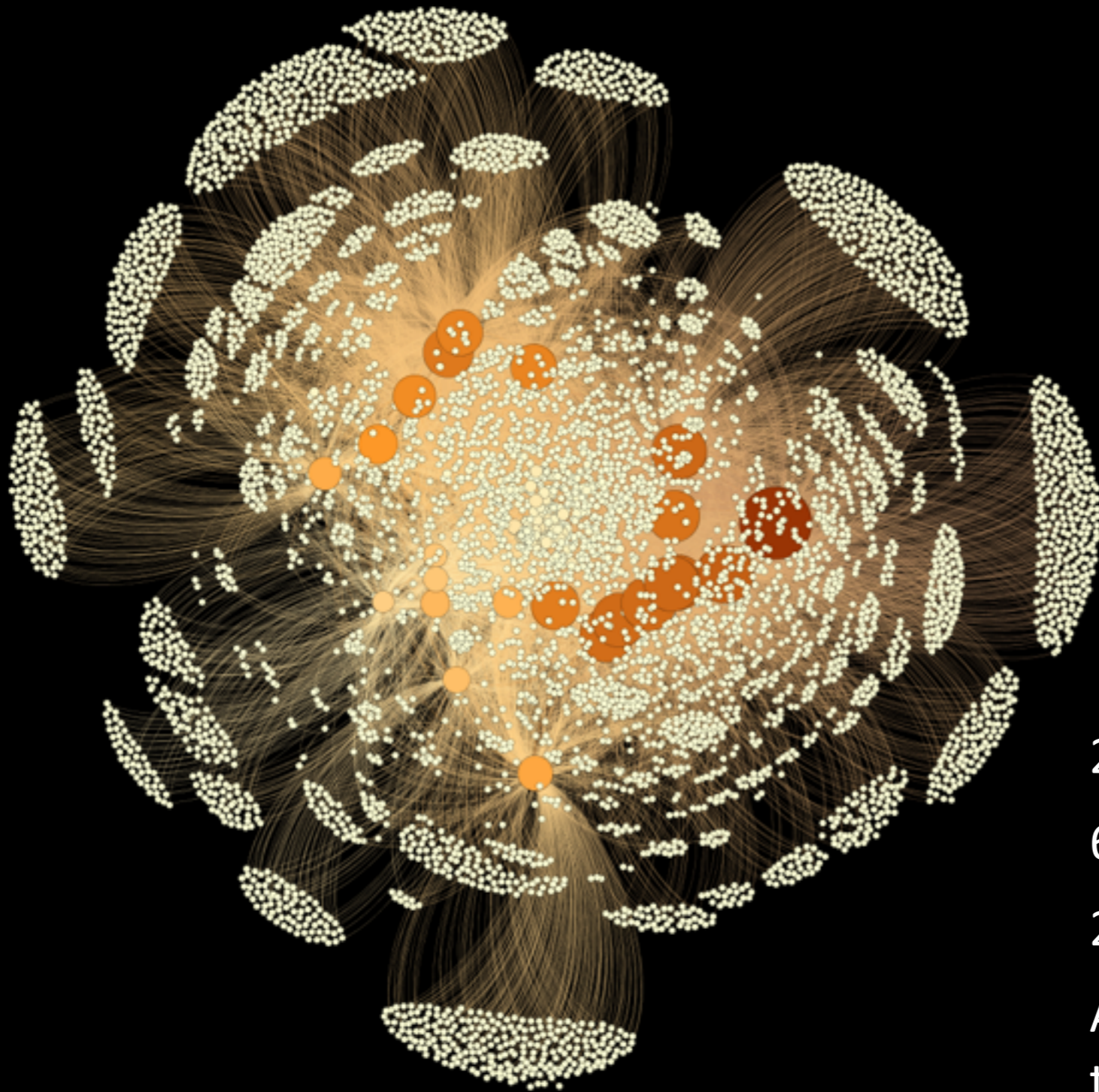
with Ioannis Psorakis

# Community dynamics

Fuzzy early communities, taxonomy stabilizes over time



# The PV technology ecosystem



22 PV-specific technologies

6198 PV-related technologies

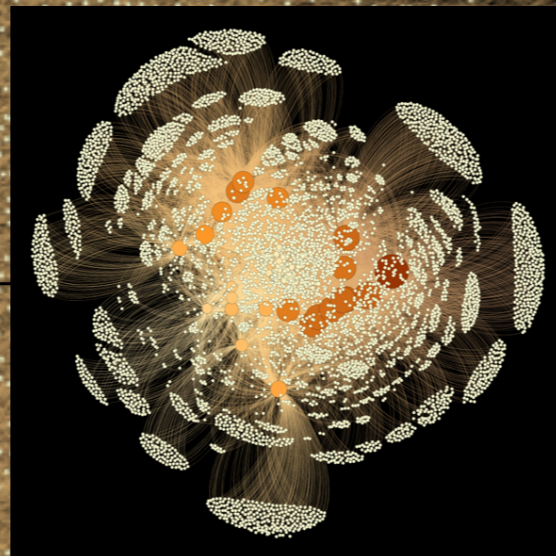
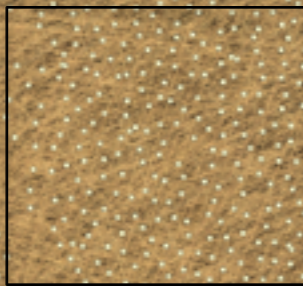
20,697 combinations

Aggregated across entire  
time history



# The technological ecosystem is vast

- ~150,000 technologies
- Connected via ~10m patents
- Dynamic and changing through time – 1790 to now.
- Temporal changes (through patenting activity ) reflect:
  - Creation of new technological capabilities
  - Refinement of existing technologies



# Rebooting the economy?

- Suppose all technology were destroyed
  - Library with all explicit knowledge remains
  - All tacit knowledge remains (100M technicians?)
  - Century supply of freeze dried food
- Could we reboot the economy?
- How would we do it?

*Economy is strongly autopoietic*

# Last Sander question

Did being a scientist change your view of the world in general, and in what sense?

# OPTIMAL TECHNOLOGY INVESTMENT PORTFOLIOS

- How should a decision maker invest in substitutable technologies (e.g. green energy)?
  - Depends: Is Wright or Moore correct?
- If Moore: Don't bother -- investments don't matter
- If Wright: Investment can play key role
  - cost decrease depends on investment
  - tradeoff between concentration and diversification
  - critical to have error estimates for forecasts.
  - compromise between risk and performance
- Expert forecasts vs. time series forecasts?
- R&D vs. production stimulus? Patenting activity?