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THE ROLE OF NETWORKS IN COLLECTIVE ACTION WITH COSTLY COMMUNICATION

CHRISTIAN R. JARAMILLO H.*

Abstract

Individuals frequently contribute their resources voluntarily to provide public goods. This paper models the manner in which the linkage between members in a community influences the likelihood of such actions through spontaneous activism in networks. The model I use abstracts from the issue of free-riding behavior by means of small deviations from standard preferences. Instead, it concentrates on the communication aspect of provision through collective action. The solution concept is Nash equilibrium.

I find that the likelihood of efficient provision of a discrete public good in random social networks increases very rapidly for parameter values where the network experiences a phase transition and large-scale decentralized activism becomes feasible. As a result, the model shows that succesful coordination may be more readily achieved the larger the population is, provided its members are sufficiently connected. In contrast with previous results in the literature, this results holds even as the size of the population increases without bound, and it is consistent with the existence of large-scale activism in large populations.

Keywords: Collective Action, Public Goods, Social Networks, Social Capital.

JEL classification: D70, H41, Z13

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* Profesor Asistente. Facultad de Economía. Universidad de los Andes. chjarami@uniandes.edu.co

EL PAPEL DE LAS REDES EN LA ACCION COLECTIVA CON COSTOS DE COMUNICACION

CHRISTIAN R. JARAMILLO H.

Resumen

Es frecuente que las personas contribuyan voluntariamente sus recursos privados para la provisión de bienes públicos. Este trabajo modela la manera en que las conexiones entre los miembros de una comunidad afecta la probabilidad de tales acciones mediante la aparición de grupos activistas. El modelo se concentra en el papel de la comunicación entre individuos y deja de lado el problema del *free-riding*. El concepto de solución es el Equilibrio de Nash.

El trabajo encuentra que la probabilidad de provisión eficiente de un bien público en una red social aleatoria presenta dos regímenes. Si el nivel de conexión de la red es bajo, la probabilidad de activismo es baja y decrece al aumentar la población. En contraste, por encima de una conectividad crítica, la probabilidad de activismo en gran escala es finita y aumenta en relación directa con el tamaño de la población. La transición entre los regímenes es abrupta y corresponde a un cambio de fase. Si la conectividad de la población aumenta al crecer el número de individuos, la probabilidad de provisión del bien público es finita incluso en comunidades grandes. Este resultado es consistente con la existencia de activismo en gran escala.

Palabras clave: Acción Colectiva, Bienes Públicos, Redes Sociales, Capital Social

Clasificación JEL: D70, H41, Z13

1 Introduction

A standard public finance result predicts that, in large populations with incomplete information, efficient private provision of a public good through voluntary contributions is unlikely to obtain due to the free-rider problem. Having many individuals involved in providing a public good may spread the individual cost, but it also makes coordination more difficult to achieve.

In this paper, I argue that the accuracy of this prediction depends strongly on the underlying communication structure of the population itself. Given certain connectivity conditions, successful coordination (and thus provision of the good) may become more likely as the population increases. This is true even if the per-capita cost of provision of the good remains constant.

Most communities are incomplete networks: no coordinating agent, centralized or otherwise, can reasonably expect to reach all other members of it at will. I ask then what degree of coordination among willing members is feasible if communication among them is restricted. At its most fundamental level, my argument relies on the observation that coordination need not be centralized, as two people can cooperate without ever meeting directly, through intermediaries who all agree to act together towards some common goal.[1]

I model the problem of private provision of a discrete pure public good as a game in a community whose members can communicate only by word of mouth. There are two types of individuals with different attitudes towards cooperation. Provision of the good occurs in a decentralized manner, through the actions of private individuals who find it in their own interest to either make a direct monetary contribution, or coordinate with others at a cost (*activism*). Central to this activism is the ability of each community member to talk to others, determined by the linkages among them. This underlying structure of the society, which I call *connectivity,*[2] is loosely related to the concept of social capital in the political science discourse. In its attempt to formalize this idea of social connectivity as applied to a concrete public good problem, this paper makes contributes to the public finance literature. The mathematical results I use, on the other hand, have their roots in the theory of random graphs.

The main prediction of the model is that, in some cases, an increase in population size raises the likelihood of emergence of collective action in a community. Large-scale activism, which may not be incentive compatible–or even feasible–in smaller populations due to communication constraints, becomes optimal as more members join the community. The results holds true even if the average per-capita cost of

[1]Through a friend of a friend of a friend...

[2]I use the term *connectivity* in a manner different from its formal meaning in graph theory. In the context of this model, *connectivity* refers to the general features of the social network and embodies loosely a number of formal properties of random graphs like the degree distribution, clustering and connectivity. For a formal definition of these terms, see [10], [33], [48].

provision is constant. The reason is that the additional members in the community improve the connectivity among existing members. This is consistent with the existence of such activism in large populations, and it stands in contrast with the previous literature on this topic.

Private provision of public goods The literature on private provision of public goods and the emergence of cooperation is very extensive. Its main prediction, available in a wide variety of versions, is that the good will be undersupplied, a problem that becomes worse in the presence of incomplete information. From an isolated individual's perspective, this is the result of benefits that are spread over a group while the costs are not.[42] From the perspective of the potential beneficiaries as a group, the failure to obtain efficient provision stems from a more subtle but pervasive problem, namely the free-rider incentive: each individual within the group has an incentive to delegate to other members the actions (and the costs) necessary to provide the public good. This incentive will exist as long as she knows that her contribution is not indispensable for provision and that someone else may step in in her stead. It is closely related to the incentive to cheat in game theory's prisoner's dilemma and the lack of self-restraint in the tragedy of the commons.[37],[38],[21] Standard approaches to the solution to this problem are coercion, voluntary coordination and the establishment of markets by means of allocation of property rights.[14] It's easy to find examples of each one. Centralized coercion as a mechanism for provision of public goods is represented by government taxation and more generally by regulation, both frequently used in this context. Collections of old clothes of food for charity are a familiar fixture of neighborhood life for many of us. Decentralized market solutions implemented through legislation are not uncommon in pollution issues. Still, while the fundamentals of coercion and property rights allocation are relatively well understood, voluntary coordination–collective action–has proven more troublesome to explain, even though it is patently widespread.[3]

Private provision of public goods is usually modeled in the literature as a game between all potential beneficiaries where each submits a voluntary contribution. Typically, the players have standard egoistic preferences. In the case of continuous public goods, where the level of provision is to be determined, the theory predicts underprovision unless some unwilling individuals can be forced to contribute. (Bergstrom et al. [9]) For discrete public goods, on the other hand, the results are less clear-cut, in part because the choice of game form (mechanism) turns out to be decisive in the feasibility of efficient outcomes. Two types of games are commonly analyzed, namely *contribution* and *subscription* games. In contribution games, the players must pay their contributions as they make them, and no refund is given, regardless of the provision outcome. Subscription games, on the other hand, demand payment if and only if the good is successfully provided, and thus require the availability of contract enforcement to be feasible. Bagnoli and Lipman [5] show

[3]Andreoni (1990) [4] tackles the question of charitable giving, arguably an indirect example of collective action. Ostrom (1990) [38] studies more direct evidence of emergence of collective action.

that, under complete information, one-shot contribution games are inefficient, but the core actually implements the efficient allocations in subscription games, a result that carries over to countable public goods but collapses as the good becomes continous. Admati and Perry [1] derive a similar result for a dynamic setting where the players take turns to make contributions to the good in a game with several rounds, and Marx and Matthews [30] extend this result to games where players can act simultaneously, provided the current aggregate level of collection is public knowledge at each round.

As one should expect, efficiency suffers if information is incomplete. Among one-shot incomplete information games, subscription has a higher probability of achieving efficiency than contribution, but both are less than perfectly efficient (Menezes et al. [31]). Worse yet, Mailath and Postlewaite [29] show that for balanced budget mechanisms the probability of an inefficient outcome converges asymptotically to one in large populations.

All of these results abstract implicitly from the issue of communication. They assume that any relevant contacts between agents take place. However, in large populations, whether a contact can be established between interested parties plays an important role in the feasibility of any outcome. Analyzing the equilibria of repeated games with random matching, Okuno-Fujiwara and Postlewaite [36] prove that, if public signals about a player's previous behavior are available, certain cooperative pooling equilibria exist that amount to social norms. In an influential paper, Kandori [24] shows that decentralized coordination, albeit an equilibrium in infinitely repeated prisoner dilemma games with random matching, is not a robust outcome in large populations when there is no information transmission about past behavior of the players. Nevertheless, allowing for local information transmission succeeds in creating robust equilibria with voluntary local enforcement of good behavior (i.e. punishment of deviators). A degree of communication is hence necessary for decentralized cooperation to be viable. The importance of the communication network's structure is further emphasized by Chwe [13], who analyzes optimal networking for collective action under simple decision rules when each player's decision depends strongly on the perfectly observable actions of a subset of other players.

Summarizing, there are two main issues at hand when considering the private provision of public goods: (1) when are the group size and communication structure such that coordination is feasible and reasonably likely to succeed?; and (2) why does any individual, or group of such, start costly actions towards coordination in spite of the free-rider incentive?

The feasibility question precedes the incentive issue in a logical sense: no costly action would take place if it were known to be doomed to fail. This paper addresses the first question and abstracts from the free-rider problem. Accepted wisdom argues that there is a tension between, on one hand, the need for a group large enough to spread the costs of provision, and on the other hand the rising costs of coordinating such large groups, be they due to informational requirements or otherwise.

However, the theoretical literature on private provision of public goods has failed to include such costs as part of the coordinating mechanisms. The accepted intuition argues that centralized coordination is costly (Kandori [24]), and that as a result increasing group size may deter coordination initiatives from taking place. Moreover, even after such coordination has happened, centralized enforcement of the agreements is more difficult in large groups. I show that this intuition is not necessarily correct: coordination may be easier in larger groups.

The model I use in this paper to model provision of a discrete public good has elements of a contribution game (for activists) and a subscription game (for the rest of the population). The player's payoffs are private information. Preferences are non-standard, as individuals derive a disutility from lying. Centralized coordination is implicitly ruled out by the inability of individuals to directly contact members of the population other than those in a certain small subset, but some limited information transmission occurs within groups of activists. The outcome features a pattern of activism that is local from an individual's perspective (arguably reducing the problem of rising costs of coordination), but global in its reach (thus achieving the necessary large group of contributors).

Random graphs Loosely speaking, a *graph* is a set of elements (called *vertices*) which may be linked to each other (by *edges*). More formally, a graph Γ has is two sets: the set of vertices $V(\Gamma)$ and the set of edges $E(\Gamma)$. A *random graph* adds to this a probability distribution over the existence of each possible edge.[4] The theory of random graphs applies probabilistic methods to analyze a number of macro properties of the graph. The interest in the macro picture rather than individual vertices leads to a treatment that considers permutations of vertices to yield the same graph (i.e. the graph resulting from swapping the names and positions of two vertices is considered to be the same). That is also the case in this paper.

The model used here is a version called the *binomial random graph* model, where each link exists with some probability q. As with almost all results in graph theory, the ones I use are asymptotic in nature. Most arguments presented here are exactly true only for infinite populations. However, a very convincing case with vast evidence has been made that finite graphs converge relatively quickly to their limiting behavior, so that these results are a good approximation of the features of large finite populations.[5] Moreover, the intuition and phenomena I describe in this paper, in particular the formation of a giant component, are robust features of many types of random networks.[6] I argue that they provide insights into some collective phenomena in societies, activism among them.

[4] Throughout the paper, when there's no possibility of confusion, I use the letter Γ to refer both to the random graph and, sometimes, a particular realization of it.

[5] See [2], [10], [33], [45].

[6] See [2], [33], [23], [34], [35], [45].

I have included a summary of the results and properties of binomial random graphs used here in the Appendix A.[7]

In the next section, as a benchmark, I introduce a typical static game that models the decentralized public good provision decision as is usually considered in the public economics literature. Then, in section 3, I add a network structure to the population and build a simple static game where individuals choose between an uncoordinated solution or an approach involving activism to provide the good. In section 4 I discuss the efficiency implications of an incomplete communication network. Section 5 presents some comparative statics results. Section 6 concludes.

2 The problem

Consider a population of n individuals (n a large number), each indexed by $i \in N = \{1, 2, ..., n\}$ and endowed with w_i units of a numeraire good x.

Suppose that the individuals have heterogeneous preferences defined over consumption of a discrete public good $\mathcal{G}$ and the numeraire x:

$$U_i = u_i (x_i, \mathcal{G}) \tag{1}$$
$$\partial u_i / \partial x_i > 0$$

The public good can be either provided ($\mathcal{G} = 1$) at a cost G, or not provided ($\mathcal{G} = 0$). Let person i's willingness to pay for the public good (i.e. her valuation) be given by the scalar $g_i \geq 0$, implicitly defined by

$$u_i (w_i - g_i, 1) = u_i (w_i, 0) \tag{2}$$

and assume that $\sum_{i=1}^{n} g_i > G$ so that it is Pareto-efficient to have $\mathcal{G} = 1$, provided suitable lump-sum transfers of the numeraire can be made among the individuals.[8]

Assume also that for all $j \in N$, $G_{-j} \equiv \sum_{i=1}^{n} g_i - g_j > G$, so that the provision of $\mathcal{G}$ is feasible, even if some of the individuals who have $g_i > 0$ do not contribute to pay the cost G (i.e. even with some free-riders). Further assume that each individual i knows that $G_{-i} > \mathcal{G}$, but she cannot observe other individuals' g_j.

Consider then the static game where each individual, acting strategically, makes a voluntary contribution of z_i units of the numeraire good to be used for provision of the public good. The public good is provided if $\sum_{i=1}^{n} z_i > \mathcal{G}$. All individuals present their contributions simultaneously, so no one knows in advance how much other individuals will contribute. The utility of an individual i will then be

$$\begin{array}{ll} u_i (w_i - z_i, 1) & \text{if } \mathcal{G} = 1 \\ u_i (w_i, 0) & \text{if } \mathcal{G} = 0 \end{array}$$

[7] Further, very detailed references include [2], [10], [22], [33], [45], [46], [47].

[8] Note that the assumption $g_i \geq 0$ implies that $u_i (x, 1) \geq u_i (x, 0)$ for all i, so that $\mathcal{G}$ is indeed a good for everybody.

In general, the Nash equilibria of this static problem need not guarantee $\mathcal{G} = 1$. (Bagnoli and Lipman [5]) Those that imply provision are such that the sum of contributions barely covers the cost of provisions and each contributor is pivotal. The reason for the inefficient outcomes is the free-rider problem: each individual, knowing that her contribution is not necessary to achieve provision of the good, may choose to rely on other people's contributions since, *ceteris paribus*, she's better off consuming all her endowment privately.

In a broader setting, the free-rider problem has been interpreted to mean that some degree of enforceable coordination is necessary to achieve efficiency in this game. If such coordination is costly, the expectation is that it will become more difficult to attain the larger the number of individuals in the population.

One instance of such coordination cost has to do with communication constraints. In a world with incomplete information, it is seldom true that an individual (or a centralized authority, for that matter) can reasonably expect to reach everybody she may want or need to. Indeed, very often she won't even be aware of who it is she should reach. A typical individual knows about the preferences and resources of those she has contact with in daily life (work, social activities) and perhaps of some limited set of public individuals in her community, but not much more. And even if she knew that a particular person could aid her to achieve a certain goal, she might think that the contact cannot be established at a reasonable cost.

As it is, the game presented above has no means to analyze or even suggest these limitations in communications. Therefore, I propose a game that models explicitly the coordination aspect of decentralized public good provision. In section 3 I construct a community where any given member has access to a relatively small random subset of other members. I explore then the feasibility of large scale coordination among individuals to provide a discrete pure public good. The focus of the analysis is not whether some individuals would voluntarily engage in coordinating activities (this is exogenously assumed), but rather whether they should reasonably expect to be successful given their constrained reach and the fact that coordination is costly.

A very interesting result indicates that the traditional intuition against coordination in large groups may be at odds with the realities of a networked community with costly, limited interaction possibilities. In this setting, success may be more likely in larger rather than smaller populations, and a second-best outcome may require that a substantial amount of resources be devoted to increasing the size of the coordinated groups through activism. The driving element behind this result is a sudden transition in the properties of the communication network used by the members of the community.

3 A model with activists

I introduce now some changes to the model seen in the previous section in order to consider a game where activism is an explicit option. I simplify as much as possible the distribution of individual preferences and add a network structure to the population. This setting will enable me to model the formation of activist groups that strive to achieve provision of the public good $\mathcal{G}$.

Two types of individuals First, assume that there are two exogenously given types of individuals, called V (for *volunteer*) and R (for *reluctant*). Each person's type is private information. Volunteers are people who offer their contributions to the community without trying to free-ride. They are a fraction $\delta \in (0,1)$ of the total population n. Reluctant individuals, on the other hand, only give money if asked to, and even then not always, as they may try to free-ride as seen below.

Preferences Individuals' preferences depend on the type. V-types are homogeneous, and their valuation of the public good is given by $g^V > \frac{G}{n}$.

R-type individuals also have homogeneous preferences. They value the public good at $g^R > \frac{G}{n}$, and they also derive utility $h \in \left(\frac{G}{n}, g^R\right)$ from being "truthful" citizens.[9]

Communication network Assume that any two individuals $i \neq j$ in the population are linked ("are neighbors") with a publicly known probability $q \in (0,1]$ independent of i and j. The individuals in this population thus conform a random graph with n vertices, denoted hereafter $\Gamma_{n,q}$. Realizations of such a graph are depicted in figure **??** and figure 5 of Appendix A.

The links allow neighbors to interact with each other in a manner described later. They are exogenously given, and direct interaction between two individuals is feasible if and only if they are neighbors. Given a population size n, the network communication properties are characterized by the connectivity $q \in (0,1]$.

Actions The model is a Stackelberg game with several branches. Each type of individual is assumed to have a different set of available actions.

R-type individuals are followers in all cases, and their set of available options is reduced to accepting or rejecting any proposals made to them at time $t = 2$ by any activist neighbors they have. If an R-type makes a pledge, she is called a *supporter*. Otherwise, she is a *non-supporter*. R-types cannot transmit any information other than this acceptance or rejection of the proposals.

[9]Truthfulness, as will be seen later, has a very precise meaning in this model. In the mean time, a rough approximation should suffice: an individual is truthful if she does not lie about her valuation of the public good when asked.

This type of preference can be obtained as a result of prevailing social norms or, alternatively, of warm-glow utility. (See Appendix B.)

V-type players are the leaders. At time $t = 1$, they have to choose between being passive or active. If some volunteer chooses the passive course, a collection box is placed in the center of town, and each individual can put any voluntary contribution she wants in it.

The active course, on the other hand, involves costly activism. Each V-type who chooses the active course at $t = 1$ (and becomes an *activist*) immediately incurs a cost D_1 and contacts her neighbors, thus starting a coordination drive. This first activist drive has the objective of finding and meeting with other V-type individuals to coordinate future actions with them. Therefore, the cost D_1 is perhaps best viewed as a contribution of time rather than money. The process of contact and coordination occurs by word of mouth, so that contacts can only happen between neighbors. I assume that coordination is achieved between any two activists that establish contact this way. This coordination is also transitive, as i coordinated with j and j coordinated with k necessarily implies that i is coordinated with k.[10]

After the coordination drive is over, each coordinated group of activists is called an *activist cell.* Each activist then knows the size of her own activist cell and whether it is the largest existing activist cell.

Activists who are not in the largest activist cell stop all actions at this point. Activists in that largest cell, denoted C, face the choice between inaction or a fundraising drive at time $t = 2$.[11] If the fundraising drive takes place, each member of C incurs an additional personal cost D_2. The drive involves approaching other members in the population and asking for a monetary contribution in the amount of p (the proposed pledge) to provide the public good. It is assumed that each activist in C must also pledge p. The amount p must be set in advance of the drive, and it cannot be changed after the number of actual contributors is known. Any funds raised in excess of the cost of provision of the public good $\mathcal{G}$ are disposed of and serve no useful purpose.

Communication between neighboring activists is assumed frictionless and costless after the cost D_1 has been incurred. All monetary contributions from players are collected at the end of the game, but only if the cost of the public good is covered.

The influence of the network structure on the extent of coordination is the focus of this paper.

Information At the outset of the game, $n, q, \delta, g^V, g^R, h$ and the type of network are public information. The type of each individual (V or R) is private information.

[10] As a convention, an activist is always coordinated with herself.

[11] If there is more than one group with the largest amount of activists, I assume that all of them go on with the fundraiser decision. While this does not affect the likelihood of activism, it has consequences for efficiency. It is clearly undesirable to have redundant, disjoint efforts to provide the public good. As will be seen later, eliminating such redundancy is one of the sources of efficiency gains of the results in this paper.

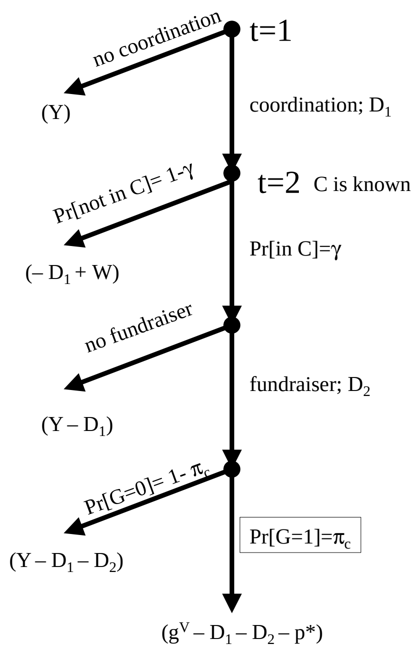

FIGURE 1: Decision sequence for V-types

Notes: All decisions in this tree are made by V-type individuals. W is the expected payoff to activists outside the largest cell from the choices of the activists in the largest cell. p^* is the actual pledge.

As stated above, the cost of activism D_1 is assumed to enable an individual to share information with her neighbors. Consequently, non-activists are not able to transmit any information at time $t = 1$, nor can they take part in coordination in any way. The number of members in any given cell is known to activists within that cell, and each activist knows whether her cell is the largest at time $t = 2$.

Timing The time sequence is presented if Fig.1. The payoff Y represents the best possible outcome without activism.

In the remainder of this section, I derive the optimal choice rules for the agents in this model. In 3.1, I analyze the decision rules for R-type individuals. Then, in 3.2, I present the results for the optimal decisions of V-types in the extreme case where $q = 1$. In this case the community is a complete network and it is possible for any activist to reach any other individual. This is the implicit standard in most of the literature of collective action and coordination, so it is useful to have it as a benchmark. In 3.3 I deal with the the optimal decisions of V-types in an incomplete network. I first consider the case where no activism takes place, which is in some sense the outside option. Then I analyze the alternatives where activism is chosen. In particular, I show that the outcome depends critically on the communication network described here.

3.1 R-type choice: supporter vs. non-supporter

It is easiest to start by analyzing the decision of R-types, since they are followers at every stage. This is the manner of fundraising that may take place: An activist j contacts all her R-type neighbors with a proposed contract. She asks each neighbor whether he would be willing to pledge the amount p in order to get provision of the public good. Payments are due only after all pledges are in, conditional on successful provision.

The R-type neighbor then gives an answer, yes or no, depending on his valuation of the good. Note that there's no immediate cost in becoming a supporter, but there's the issue of whether or not the neighbor is willing to lie about his valuation of the good to save the amount p in the future.[12]

If an R-type accepts a pledge p and at the end of the game she has to contribute, her payoff if $g^R + h - p$. If she does not accept, but there's provision without her contribution, her payoff is $g^R + h$ (if she has been truthful), and g^R (if not). Finally, if there's no provision, she gets h only if she has been truthful, and zero otherwise. As will be seen later, in equilibrium no R-type ever lies.

Clearly, if $g^R < p$, the R-type individual will make no pledge and still be truthful. However, even if i's true valuation is higher than the proposed pledge ($g^R \geq p$), she may still try to free-ride by refusing to pledge any money and answering in the negative, in which case she will lose the warm glow from truthfulness h. In summary, reluctant types only say yes to the proposed monetary pledge if they value the good enough and the warm-glow utility effectively blocks their natural free-riding incentive. However, since $h < g^R$, this means that the decision rule for R-types is simply:[13]

$$\begin{array}{l}\text{Do nothing unless asked directly} \\ \text{Accept pledge} \quad p \quad \text{iff} \quad p \leq h\end{array} \tag{3}$$

[12]In equilibrium, these supporters will end up paying a positive contribution of at most p, so the question has very concrete consequences, which are known by the individuals under rationality assumptions.

[13]This is actually a stronger condition than necessary. From the perspective of an individual of type R with who is being asked to contribute $p \leq g^R$, the options are:

.	Say yes	Say no
Provision ($\mathcal{G} = 1$)	$g^R + h - p$	g^R
No provision ($\mathcal{G} = 0$)	h	0

Thus, if she has a prior π independent of her own action for the probability of provision, the condition for saying yes to the proposal is $\pi g^R \leq \pi \left(g^R + h - p\right) + (1 - \pi) h$. This can be rewritten as

$$\pi p \leq h$$

Since $\pi < 1$, demanding $p \leq h$ guarantees that i will answer truthfully even if she is sure that she will actually have to pay.

The type of interaction just described happens simultaneously at time $t = 2$ between each activist in the largest cell C and each of her R-type neighbors, provided the fundraiser takes place.

The maximum contribution that a type-R individual i would pledge when asked by an activist is thus h. For proposed contributions higher than this amount, individual i would say she's not interested, even if the contribution were less than g^R.

3.2 V-type choice: complete network $q = 1$

In the case where communication is unconstrained by the network structure of the community, the problem yields results in line with the previous literature.

If no activism is chosen, in practice the individuals in this model set out to play the static one-shot game of private provision of $\mathcal{G}$. By definition of type R, coupled with the condition that no one reluctant individual be pivotal, no individual $i \in R$ will make any voluntary contribution. Thus, $z^R = 0$. The maximum unconditional voluntary contribution that one could possibly get is given by $Z_{\max} = n\delta g^V$. If $Z_{\max} > G$, the static one-shot game will assure efficient provision of the good, paid by the volunteers. On the other hand,

Remark 1 *If the maximum voluntary contribution from V-types in the population is not enough to provide the good*

$$Z_{\max} = n\delta g^V < G$$

the outcome of the one-shot, no-activism game is generally inefficient due to the free-rider problem.

Consequently, if V-types decide not to engage in activism and they are able to provide the good, they alone bear the cost of provision. In principle, they may split that cost in any number of ways, but I focus here on a symmetric solution: each V-type contributes $\frac{G}{n\delta}$ units of the private good, their payoffs are $g^V - \frac{G}{n\delta}$ and the R-types enjoy the public good for free.[14] This outcome, if feasible, is the first-best.

If activism is chosen, the size of the largest activist cell C is simply the total number of V-types in the population, $C = V = n\delta$.[15] In addition, if the funding

[14]If the fund collection is higher than necessary for provision, the extra amount is waste. In the extreme case where the V-types just trow in the collection box their full valuation, the wasted private good is $n\delta g^V - G$, and their utility loss is $u^V\left(w - \frac{G}{n\delta}, 1\right) - u^V\left(w - g^V, 1\right) > 0$. However, I will assume that the collection is barely enough in order to use this option as an efficiency benchmark if it is available.

For a detailed analysis of this matter, see [1],[5],[31].

[15]Throughout the paper, I denote both subsets of the population and their sizes with the same capital letter. Thus, for example, the size of $X \subseteq N$ is denoted simply X. The only exception is the size of the population set N, denoted n.

drive takes place, all R-types are reached and the necessary contribution is $\frac{G}{n}$.[16] The payoffs to the V-types are then $g^V - D_1 - D_2 - \frac{G}{n}$ and to the R-types $g^R - \frac{G}{n}$. The cost of provision is then effectively spread over the population, but the result involves a welfare loss due to activism

$$L = A\left(D_1 + D_2\right) = n\delta\left(D_1 + D_2\right) \tag{4}$$

In general, the V-types may prefer the activist solution, even if it involves a loss. As leaders, they have full choice of actions in this game, so activism will take place if

$$g^V - D_1 - D_2 - \frac{G}{n} > \max\left\{g^V - \frac{G}{n\delta}, 0\right\} \tag{5}$$

where it is natural to assume $g^V - D_1 - D_2 - \frac{G}{n} > 0$.

Thus, two cases exist: for $\delta \leq \frac{G}{ng^V}$, provision without activism (first-best) is not feasible, and the second-best activist choice is the optimal option. It is also guaranteed to work.

For $\delta > \frac{G}{ng^V}$, the outside option is feasible and optimal. Activism only redistributes the cost of provison and generates excess costs. Nevertheless, the inefficient activist solution will then obtain if

$$D_1 + D_2 < \frac{G}{n}\left(\frac{1}{\delta} - 1\right) = \frac{G}{n\delta}\left(1 - \delta\right) \tag{6}$$

and the excess cost to society due to activism is L.

3.3 V-type choice: incomplete network $q \in (0, 1)$

In this case, the outside option Y in Fig.1 is assumed to be the same as in the benchmark case: a collection box is put in the middle of town, and people volunteer their contributions. No communication is necessary, the box is visible to the whole community, and the payoffs to the different types are

$$\left(Y^V, Y^R\right) = \begin{cases} \left(g^V - \frac{G}{n\delta}, g^R\right) & \text{if } g^V - \frac{G}{n\delta} \geq 0 \\ (0, 0) & \text{if } g^V - \frac{G}{n\delta} < 0 \end{cases}$$

If the public good is provided, this outcome is efficient, as the cost is assumed to be exactly covered.[17]

[16] In equilibrium, the cost D_1 is incurred only in if the funding drive takes place, since there's no uncertainty in the game.

[17] As before, I concentrate on the symmetric solution.

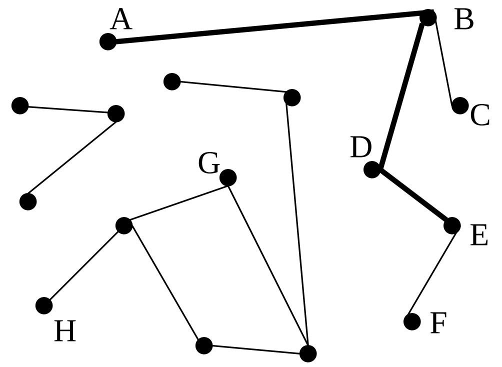

FIGURE 2: A path

Notes: Vertices A and E are connected within any set that contains the path $X = \{B, D\}$. They are not connected within $\{A, B, E, F\}$. Vertices A and G are not connected within the population.

3.3.1 Decentralized activism

I turn now to the optimal choices for V-types in the case where activism is chosen.

Definition 1 *Let A be the set of all activists in N.*

Since the V-types are homogeneous, the set of activists is either empty, or $A = V = n\delta$, where δ is the fraction of individuals of type V in the population.

Definition 2 *For all $m, m' \in N$, define the scalar $a_{mm'}$ to be 1 if m and m' are neighbors, and 0 otherwise. Then $i, j \in N$ are said to be connected by a path within a subset $X \subseteq N$ if there exist $j_1, j_2, ...j_r \in X$ such that $a_{ij_1}a_{j_1j_2}a_{j_2j_3}...a_{j_rj} = 1$. The individuals $j_1, j_2, ...j_r$ then conform a path from i to j within X. If no subset is specified, it is understood that the individuals are connected within the population N.*

Paths are in a sense extended links in the network. They capture the idea of transitivity of connections. Suppose Ann knows Bob, who knows Connie, who in turn knows Dave, who knows–finally!–Elsa. Then Bob, Connie and Dave are a path connecting Ann and Elsa.

To understand the requirement that the path be within a set, consider the following: If Bob, Connie and Dave are members of a social club that encourages introducing their acquaintances to each other, chances are Ann and Elsa have met. On the other hand, if Connie were not in the club, then the path would probably be ineffectual and Ann would not know Elsa. Fig.**??** depicts an example path.

Definition 3 *A set $X \subseteq X'$ is a cluster in X' if any two of its elements are connected by a path within X, but no element of X is connected by a path to an element of $X' \backslash X$ within X'.*

Clusters are easily identified: Start with Bob and find all people in the club who are connected to him by a path within the club, and that's Bob's cluster. The exclusions must be noted, however. Ann does not belong because she's not in the club. Suppose Zack is in the club, but he only knows Ann. He does not belong to Bob's cluster because the path to Zack does not lie within the club.

It's easy to see that communication between members of the population is subject to the following conditions:

Remark 2 *Let A be the set of all activists in the population. (1) For any $i, j \in A$ to be coordinated, they must be linked by a path within A, and each activist in that path must also be coordinated with both i and j. Thus, the coordinated groups are all clusters in A and determine a partition of the set of activists in the population. (2) For any $i \in A$ and $j \notin A$, interactions between i and j are possible if and only if they are neighbors.*

Remark 2 is useful later in determining the scope and reach of feasible coordination in the community. Below, I solve the activists' fundraising choice at time $t = 2$, and then I proceed to find their optimal choice at the first stage, $t = 1$.

Last stage By definition, the largest activist cell at time $t = 2$, called C, is a subset of A. (There may be several such cells.) Note also that all neighbors of members of an activist cell who are not themselves members of the cell are necessarily of R-type.

At the last stage of the game, the activists in C face the choice between carrying out the fundraiser at a cost D_2 or not. Since D_1 is a sunk cost, their choice will favor a fundraiser if

$$\pi_C^* \left[g^V - D_2 - p^*\right] + (1 - \pi_C^*) \left[Y^V - D_2\right] > Y^V$$

where $\pi_C \equiv \pi_C(p)$ is the probability of actually achieving provision ($\mathcal{G} = 1$) for a given proposed pledge p, conditional on C, and $\pi_C^* \equiv \pi_C(p^*)$, with p^* the optimal pledge. The source of this uncertainty π_C is the fact that it is not known how many people will be reached by the fundraising drive at the time p is set because the number of neighbors of activists in C is stochastic.[18] The optimal proposed pledge p^* takes discrete values: if it is optimal to aim to reach s individuals outside C, the optimal pledge would be $p_s = \frac{G}{C+s}$. Consequently, if the number s is increased by one, the optimal pledge decreases discretely to $p_{s+1} = \frac{G}{C+s+1} < p_s$. Proposed

[18] If the number of neighbors were known, the activists would set $p = p_0$ at the exact contribution value necessary to barely cover G. Should $p_0 > h$, the cost of provision could be spread, and the fundraiser would take place if $g^V - D_2 - p_0 > Y = g^V - \frac{G}{n\delta}$ or

$$D_2 + p_0 < \frac{G}{n\delta}$$

pledges between those values would generate excess collection if successful (which is undesirable for the activists, since they pay p^*, too).

The decision rule can then be rewritten as "Carry out fundraiser iff $g^V - p^* - \frac{D_2}{\pi_C^*} > Y^V = \max\left\{g^V - \frac{G}{n\delta}, 0\right\}$". Thus:

Proposition 3 *At time $t = 2$, a fundraising drive takes place if and only if*

$$\begin{array}{ll} p^* + \frac{D_2}{\pi_C^*} < \frac{G}{n\delta} & \text{for } g^V > \frac{G}{n\delta} \\ p^* + \frac{D_2}{\pi_C^*} < g^V & \text{for } g^V \leq \frac{G}{n\delta} \end{array} \tag{7}$$

where p^* is the pledge to be asked.

$p^* = p_{s^*}$ is the solution to the problem

$$\max_s \left\{\pi_C \left[g^V - D_2 - p_s\right] + (1 - \pi_C)\left[Y^V - D_2\right]\right\} \tag{8}$$

Let $\Delta_s \pi_C \equiv \pi_C(p_s) - \pi_C(p_{s-1})$ and $\Delta_s p \equiv p_s - p_{s-1}$. The optimality condition for p^* is

$$\text{Increase from } s-1 \text{ to } s \text{ if } \frac{\Delta_s \pi_C}{\pi_C(p_{s-1})}\left[g^V - Y^V - p_s\right] \geq \Delta_s p \tag{9}$$

where increasing s decreases p_s.

An optimal pledge in the range $p_s > g^V - Y^V$ cannot ever be asked in an equilibrium where activism is present, since it implies that the outside option is better than activism for V-types, even if the costs D_1 and D_2 were zero.[19] Moreover, whenever the solution of (9) is greater than h, asking for pledge p^* would not achieve any redistribution of the provision costs. In consequence, in such a case the activists in C would choose between a fundraiser with proposed pledge h with probability of success $\pi_C(h)$, or the outside option.[20]

[19]Indeed, suppose that at the optimum p^* is in the high range:

$$p_s > g^V - Y^V \tag{10}$$

Then, use $p_s = \frac{G}{C+s}$ to obtain $\frac{G}{g^V - Y^V} > C + s$.

In the case where the outside option is available, this case reduces to $n\delta = A > C + s$. Surely, it does not make sense to incur the cost of activism to reach fewer expected contributors $(C + s)$ than those that can be reached costlessly through the box in the middle of town (A)! So, in the presence of an outside option with provision, a necessary (but not sufficient) condition for activism to obtain is $p^* < g^V - Y^V$.

If the outside option is not available, (10) implies $p^* > g^V$, and provision with this pledge would just result too expensive for the V-types. In the absence of a certain alternative for provision, however, it may be worth trying to ask for a pledge $\min\left\{g^V, h\right\}$ (corner solution) and hope that enough R-types are reached to achieve provision, but that is again in the low-p range.

[20]See Appendix C

First stage At the first stage, the V-types incur the cost D_1 if

$$E\left[\begin{array}{c}(1-\lambda)\left(Y^V-D_1\right)+\lambda(1-\gamma)\left[W-D_1\right]+\\ \lambda\gamma\Big[\pi_C^*\left[g^V-D_1-D_2-p^*\right]+(1-\pi_C^*)\left[Y^V-D_1-D_2\right]\Big]\end{array}\right]>Y^V$$

where $W=\pi_C^* g^V+(1-\pi_C^*)Y^V$, $\lambda=\Pr\left[p^*+\frac{D_2}{\pi_C^*}<g^V-Y^V\right]$ and the expectations are taken over all possible outcomes of the size of the largest activist cell C, conditional on a fundraiser occurring. γ is the probability that a given activist ends up being a member of one of the largest activist cells–each has size C. Rearranging this expression one obtains the much more accesible condition:

Proposition 4 *At time $t=1$, a coordination drive takes place if and only if*

$$D_1<\lambda\Big[E\left[\pi_C^*\right]\left(g^V-Y^V\right)-E\left[\gamma\right]D_2-E\left[\gamma\pi_C^* p^*\right]\Big] \tag{11}$$

where $\lambda=\Pr\left[p^*+\frac{D_2}{\pi_C^*}<g^V-Y^V\right]$, the ex-ante probability that a fundraiser takes place. The expectations are also conditional on the fundraiser.

(11) says that activism is more appealing for a V-type if the fundraiser is likely to take place and succeed (high λ and $E\left[\pi_C^*\right]$), or if it is unlikely that she will be part of it (low $E\left[\gamma\right]$). Of course, it is always desirable for her that the necessary contributions be low. How do $E\left[\gamma\pi_C^* p^*\right]$, $E\left[\pi_C^*\right]$, $E\left[\gamma\right]$ and λ relate to each other?

The probability distribution for the size of the largest activist cell is critical to answer this question. While the exact solution of this problem is available only for infinite n, an extensive body of literature on random graphs has shown that in large populations the realized size of C is very likely to be close to its expected value.[21] I make use of this feature to approximate the behavior of (11). Moreover, many of the properties of the distribution of C as n and q grow are known, so that meaningful comparative statics can be examined in this model. The interesting feature ultimately is that the expected value of C in an infinite population is not continuous in the parameters that describe the network. This discontinuity affects the right hand side of (11).

(11) is analogous to (5) in the benchmark case. Together with (7) and (3), it fully characterizes the optimal decisions for all players of the game.

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[21] Janson et al. ([22] Ch.2) give bounds for the deviations from the mean of realizations of random variables in graphs. However, the specific evidence for the case of the existence and size of the giant component stems mainly from a large simulation literatute on percolation and networks [2], [33], [23], [34], [35], [45].

P. Erdös and A. Rényi (1959) [18] prove a more general result relating the size of the set X and the probability of existence of link between any two of its elements.

This and some other features of random graphs are discussed in the Appendix A. For a more detailed analysis of random graphs and their properties, see Bollobás (1985) [10], Janson et al. (2000) [22].

4 Efficiency in the incomplete-network population

In this game, as in the complete-network benchmark, any possible outcome that implies provision of $\mathcal{G}$ is more efficient that an outcome with $\mathcal{G} = 0$. However, if the outside option is able to provide the public good, activism is simply a costly way to shift the cost of the public good away from the V-type people. It is thus undesirable from the perspective of a social planner. This case, while possible, is not the focus of my analysis. I concentrate here on the case where the public good cannot be provided by the V-types alone. This seems the more interesting scenario for practical applications, and it is cumbersome but not difficult to derive the corresponding results when the first-best is feasible.

When activism takes place in an incomplete network, three possibilities arise that may have an impact on the expected efficiency of the outcome. First, ex-post efficiency may decrease because the redistribution mechanism is less effective. There is a chance that not enough V-types are reached in the coordination drive, or that too few R-type people are reached by the largest activist cell in the fundraiser. In this case the V-types would have to take the outside option and the costs of activism would have been wasted. If the outside option has $\mathcal{G} = 0$, even worse.

Second, since activism now does not guarantee success but does imply costs, the V-types may actually be deterred from becoming activists at all. This would be good for efficiency if the outside option provided the good, but it's bad in the case analyzed here.

Third, depending on the shape of $\pi_C(p)$, there's the possibility that one can reach enough R-types to achieve redistribution of the cost G without every activist having to pay D_2. Indeed, if just a relatively small number of activists are in the largest cells, but they reach many R-types, the total cost of fundraising in the community will be lower and efficiency will be higher.

No outside option $\left(g^V < \frac{G}{n\delta}\right)$ In the case where the first-best with voluntary contributions and no activism is not attainable because $g^V < \frac{G}{n\delta}$, the existence of activism may be a desirable second-best outcome. A proposed pledge p^* that stands a chance of achieving provision (i.e. $p^* > \frac{G}{n}$) may enhance the expected efficiency of the outcome. Whether such activism actually obtains depends on condition (11). As stated above, however, provision may still fail.

If there is no activism, the welfare loss is certain: $n\bar{g}$, where $\bar{g} \equiv \delta g^V + (1-\delta) g^R$ is the average valuation of the good in the population. Conditional on activism taking place, the expected welfare loss is $A(D_1 + \lambda E[\gamma] D_2) + \lambda(1 - E[\pi_C^*]) n\bar{g} + (1-\lambda) n\bar{g}$. Rearrange to obtain:

$$E[L] = \begin{cases} n\bar{g} & \text{if no activism} \\ n\{\delta D_1 + \lambda E[\gamma]\delta D_2 + (1 - \lambda E[\pi_C^*])\bar{g}\} & \text{if activism} \end{cases} \quad (12)$$

Proposition 5 *Suppose that the public good cannot be provided without activism. The population with $q \in (0,1)$ can achieve higher efficiency than the complete-network population whenever (i) the latter does not feature activism, but the former does and*

$$\delta D_1 < \lambda \Big[E\left[\pi_C^*\right] \bar{g} - E\left[\gamma\right] D_2 \Big] \tag{13}$$

or (ii) both feature activism and

$$\Big[1 - \lambda E\left[\gamma\right] \Big] \delta D_2 > \Big[1 - \lambda E\left[\pi_C^*\right] \Big] \bar{g} \tag{14}$$

Proof. Immediate from (4) and (12). ■

As stated previously, $E[\gamma]$ should be low, while $E[\pi_C^*]$ and λ should be high for activism to take place in the incomplete network. The first two requirements also lower the expected welfare loss from activism in case (i), so that it is not only more likely but also more desirable. It is interesting to note that in the limit of infinite population, the condition for existence of activism (11) is sufficient to guarantee that (13) holds.

For case (ii), high $E[\pi_C^*]$ and low $E[\gamma]$ also enhance efficiency. However, the partial effect of λ is ambiguous. If $\frac{E[\pi_C^*]}{E[\gamma]} > \frac{\delta D_2}{\bar{g}}$, an increase in λ makes the incomplete network more efficient relative to the benchmark.

This makes sense, since the gains from the network should come precisely from having good chances of success (high π_C^*) with relatively few people incurring the cost of fundraising (low γ). The average valuation of the good reflects the stakes: the higher it is, the worse the chance of failure seems. The role of λ is somewhat more complex. On the one hand, it increases the likelihood of incurring cost D_2, but on the other it also decreases the chances of failure of the activism.

As in (11), the relative efficiency of the different network structures depends strongly on the probability distribution of the size C of the largest activist groups and their number, among others. In the next section I show that, in some parameter ranges, any changes in (11), (13) and (14) are driven almost exclusively by C.

5 Comparative statics in the network

In this section I carry out comparative statics analyses to determine the effects of the network parameters n and q on the existence of activism and the efficiency of the activist outcome. I concentrate on the case where the outside option is not available. This is done for simplicity, and the results are easily generalized.

Assumption 4 *In all cases below, the average per-capita cost of the public good remains constant. Thus, $G(n) = n\hat{g}$ with $\hat{g}$ constant.*

Assumption 4 insures that the increases in population size do not directly decrease the average individual cost of provision of $\mathcal{G}$. This rules out the this possibility as the driving force behind the results in this section.

Several types of comparative statics analyses are of interest in this model. First, changes in the extensive parameter n give rise to what I call scaling of the graph. They correspond to increases in the size of the target population. Second, changes in the intensive connectivity parameter q are akin to increasing the level of involvement of individuals with other members within the community. Third, I analyze the possibility of scaling the population while keeping the expected number of neighbors of each individual constant. This implies decreases in q and is thus a mix of the two previous cases. A last case involves changes in the fraction δ of type V individuals. Its interpretation provides a loose link to standard results in the literature.

These analyses require four assumptions. First, n is assumed to be large, so that the the asymptotic results are good descriptions of the phenomena. In an infinite population, the variables of interest converge in probability to their expected values.

Second, several changes in this model are inherently discrete, as C and n must be integers. Nevertheless, for large C and n, I approximate them by continuous function analysis when considering their effects on (9) and (11).

Third, I need to specify the composition of the population N as it is grows. I assume that the population is scaled up in a manner that does not alter the distribution of reservation values or types among its members. In other words, δ is unaffected by the scaling.[22] The number of V-types and the aggregate willingness of the population to contribute will then increase. The cost of provision G will increase as well. I argue that these changes are not driving the results.

Finally, I assume that the binomial random graph structure remains a good approximation of the social network. Even with the number of activists in the population a constant fraction of the total, it is reasonable to expect that the underlying linkage structure changes. This is certainly true in a real community. However, if the number of people added to the community is relatively small, so will be the structure changes. In contrast, I'll show that the properties of that structure may change dramatically.

I show below that the most salient aspect of these comparative statics is related to the scope and reach of the activist groups that are formed in equilibrium. In infinitely large populations, activism undergoes a phase transition when, on average, a typical activist knows at least one other activist. Below this threshold, there are many groups of size C, but $C/A = 0$, i.e. C is small compared to A. Above the

[22]Suppose for example that the initial set N is composed of all individuals within certain five blocks of a city landmark. A scaling (roughly by a factor of four) is then obtained by increasing the activists reach to people within ten streets of the same landmark, provided the population has similar characteristics in the whole neighborhood.

threshold, there is one single such group, and C is a finite fraction of the whole population. The transition between these regimes is discrete for the limit of infinite n. For finite populations, although not discontinuous, it is very fast.[23] As a result, (11), (13) and (14) experience large variations with small parameter changes, and so does the outcome of the game.

I comment next on the comparative statics of the optimal pledge condition and analyze the effect of the network structure parameters on λ, $E[\gamma]$, $E[\pi_C^*]$ and $E[\gamma \pi_C^* p^*]$. Then, I determine the relationship between them and the size of the largest activist cell C. Finally, I derive results for the effect of the population size and its connectivity on the likelihood of the cooperative outcome.

5.1 Changes in the network structure

5.1.1 Behavior of λ, $E[\gamma]$, $E[\pi_C^*]$ and $E[\gamma \pi_C^* p^*]$

As a first step, consider the optimal pledge condition (21). Write

$$s^* = s^*(n, C, \theta) \quad p^* = p^*(C, s^*) \quad \pi_C^* = \pi_C^*(n, \theta, s^*)$$

with $\theta = \theta(q, C)$ and $C = C(n, q, \delta)$.

It is difficult to obtain unambiguous results for the comparative statics of s^* in all ranges. For instance, it can be shown that, if $s^* \gg C$ (if one needs many contributors per activist), $\frac{\partial s^*}{\partial C} > 0$ and thus $\frac{\partial p^*}{\partial C} = \frac{-G}{(C+s^*)^2}\left[1 + \frac{\partial s^*}{\partial C}\right] < 0$.[24] Also, $\frac{\partial \pi_C^*}{\partial C} > 0$. These conditions are likely to hold if there are relatively few people in the fundraiser but the cost G needs to be distributed among many supporters, arguably the most common rationale for activism. Fig.3 illustrates the response of s^* to increases in C or q in this case.

Around the transition threshold, the relevant fact for my analysis is that in general $\frac{\partial s^*}{\partial C}$, $\frac{\partial p^*}{\partial C}$ and $\frac{\partial \pi_C^*}{\partial C}$ are continuous and different from zero. As a result, so is $\frac{\partial \lambda}{\partial C}$. Moreover, the same is true for the partial changes of s^*, p^*, π_C^* and λ with respect to n or q.

λ has a very straightforward behavior. In the limit $n \to \infty$ it must be either one or zero, as there is no uncertainty regarding whether activism takes place. In finite populations, a larger expected C raises λ. It does not follow that it is one only if C is very large; it may be that even with relatively small expected C, activism takes place. However, if the type of activism required is large-scale, the necessary C will likely be large compared to N.

γ is somewhat more difficult to analyze. It is equal to the ratio

$$\gamma = \frac{(\text{number of cells of size } C) * C}{A}$$

[23] Janson et al. [22, Ch.5, section 2].

[24] See Appendix D.

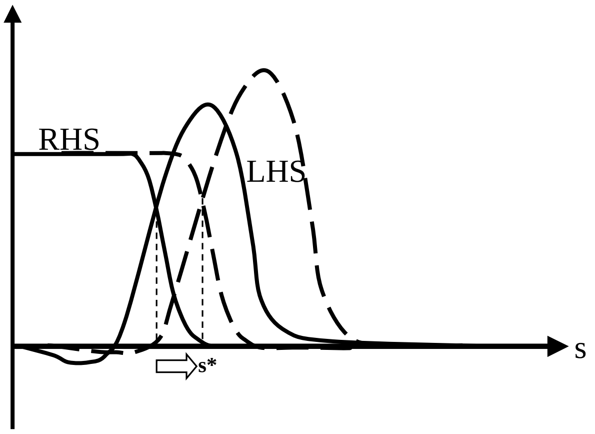

FIGURE 3: Optimal pledge

Notes: The arrows indicate the direction of change of s^* as C or q increase, given $C+s$ large enough. The associated proposed pledge is $p^* = p_{s^*} = G/(C+s^*)$.

I show below that there may be multiple cells C in some parameter ranges. It is not clear in general how the sum of its members compares to A. However, at some point all those cells merge into a unique largest cell C. Then γ is very likely C/A and rises with C. At the transition, γ may first fall and then rise, or it may rise all the time.

Consider then (11) with $Y^V = 0$:

$$D_1 < \lambda\Big[E\left[\pi_C^*\right] g^V - E\left[\gamma\right] D_2 - E\left[\gamma \pi_C^* p^*\right]\Big]$$

In the limit of $n \to \infty$, and $E\left[\gamma \pi_C^* p^*\right]$ converges in probability to $E\left[\gamma\right] E\left[\pi_C^*\right] E\left[p^*\right]$. Then, for very large n, the latter is very likely a good approximation for the former. Drop the expectation operators for simplicity of notation and write

$$D_1 < \lambda\Big[\pi_C^* g^V - \gamma\left[D_2 + \pi_C^* p^*\right]\Big] \tag{15}$$

In a similar manner, approximate (13) and (14) respectively by

$$\delta D_1 < \lambda\left[\pi_C^* \bar{g} - \gamma D_2\right] \tag{16}$$

$$0 < \left[1 - \lambda\gamma\right] \delta D_2 - \left[1 - \lambda \pi_C^*\right] \bar{g} \tag{17}$$

Let $\Phi \equiv \lambda\left[\pi_C^* g^V - \gamma D_2 - \gamma \pi_C^* p^*\right]$, $B_1 \equiv \lambda\left[\pi_C^* \bar{g} - \gamma D_2\right]$ and $B_2 \equiv \left[1 - \lambda\gamma\right] \delta D_2 - \left[1 - \lambda \pi_C^*\right] \bar{g}$. It follows that

$$\frac{\partial \Phi}{\partial \lambda} < 0 \quad \frac{\partial \Phi}{\partial \pi_C^*} > 0 \quad \frac{\partial \Phi}{\partial \gamma} < 0 \quad \frac{\partial \Phi}{\partial p^*} < 0$$

$$\frac{\partial B_1}{\partial \lambda} \gtrless 0 \quad \frac{\partial B_1}{\partial \pi_C^*} > 0 \quad \frac{\partial B_1}{\partial \gamma} < 0$$

$$\frac{\partial B_2}{\partial \lambda} \gtrless 0 \quad \frac{\partial B_2}{\partial \pi_C^*} = \frac{\partial B_1}{\partial \pi_C^*} > 0 \quad \frac{\partial B_2}{\partial \gamma} = \delta \frac{\partial B_1}{\partial \gamma} < 0$$

and $\frac{\partial B_1}{\partial \lambda} > 0$ implies $\frac{\partial B_2}{\partial \lambda} > 0$.[25]

Other things equal, decreases in γ and increases in π_C^* raise the likelihood of activism while making it more efficient as well. A decrease in λ also facilitates activism, but it does not always make it more desirable. Finally, a decrease in the equilibrium pledge p^* favors the activist outcome, but it has no direct effect on its efficiency.

5.1.2 Size of the largest activist cell

First, note that the set $A = V$ of activists in the population is itself a binomial random graph $\Gamma_{V,q}$. By Remark (2), C must be precisely the size of the largest component of the graph $\Gamma_{V,q}$.

Second, the possibility of multiple groups of size C means that $\gamma \geq \frac{C}{A}$, with equality holding only if there is one largest group. On the other hand, p^* and π_C^* are not affected by the number of groups, only by C.

Finally, λ, γ and π_C^* are strictly increasing in C, while p^* is decreasing in C.

I define here a property of the structure of linkages that will be central to the outcome:

Definition 5 *Let $i \in N$, $X \subseteq N$. Then, the degree $k_X[i]$ of the individual i within the set X is the number of neighbors of i who belong to X.*

$$k_X[i] \equiv \sum_{j \in X, j \neq i} a_{ij}$$

Notation 6 *Denote the expected degree of $i \in N$ within $X \subseteq N$ by $\langle k_X[i] \rangle$. Then, $\langle k_X[i] \rangle = q(X-1)$ if i is a member of X and $\langle k_X[i] \rangle = qX$ if not. The expected average degree of the members of X within X is called the "average degree of the set X" and denoted $\langle k_X \rangle$.*

$$\langle k_X \rangle = q(X-1) \tag{18}$$

For instance, $\langle k_N \rangle = q(n-1)$ and since the set of activists $A = V$ has size $n\delta$, $\langle k_A \rangle = q(n\delta - 1)$.

The following theorem from the random graphs literature, which I reproduce without proof, helps relate the average degree of the activist set A to the size of C in an infinite population.[26]

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[25] See Appendix D.

[26] See Appendix A and Janson et al. [22, Ch.5. p.109, Theorem 5.4].

Theorem 6 *Let $nq = k$, where k is a constant. (i) If $k < 1$, then a.a.s. the largest component of $\Gamma_{n,q}$ has at most $\frac{3}{(1-k)^2} \ln n$ vertices. (ii) Let $k > 1$ and let $\beta = \beta(k) \in (0,1)$ be defined by the equation $\beta = 1 - e^{-\beta k}$. Then $\Gamma_{n,q}$ contains a giant component of $(1 + o(1)) \beta n$ vertices. Furthermore, a.a.s. the size of the second largest component of $\Gamma_{n,q}$ is at most $\frac{16k}{(1-k)^2} \ln n$.*

Part (ii) of Thm.6 holds for $nq = 1 + \eta$, for $\eta(n) > 0$ decreasing to zero slowly enough as n increases, in which case the size of the giant component is at least $(1 + \eta) \beta n$ vertices.[27] Hence

Proposition 7 *Let $A \subseteq N$ be the set of activists. Suppose also $n > \frac{1}{\delta q}$. Then the probability that the largest activist cell in the steady state of the activism game contains at least $\beta_A A$ members of A converges uniformly to one as $A \longrightarrow \infty$, where β_A is an exponentially increasing function of $\delta[\langle k_N \rangle + 1]$ only, $\beta_A(1) = 0$, $\lim_{\xi \longrightarrow \infty} \beta_A(\xi) = 1$. If such a large cell is present, it is called the giant component.*

Proof. Given that N has a random graph's structure with a constant uniform probability of link q between any two individuals, any subset of N along with the links among its members also does. In particular, the subset A of activists with all links among themselves is a random graph $\Gamma_{A,q}$, with the probability that any two activists $i, j \in A$ be linked given by q. Since $A = n\delta$, the probability of appearance of a giant component in A converges to 1 as $A \longrightarrow \infty$ if $n\delta q > 1$. By (18) the (expected) average degree of A is $\langle k_A \rangle = q(A-1)$ and $\langle k_N \rangle = q(n-1)$. Combine these to obtain $n\delta q = \delta[\langle k_N \rangle + 1]$. By Thm.6 the result follows. ■

This result implies that the communication characteristics of the community of V-types in the infinite population are not a smooth function of the network parameters. In some range, any activism will involve relatively small activist groups, and there may be several of size C–the community of V-types is subcritical. In a higher connectivity range, the outcome changes discretely to one single largest group whose size is a finite fraction of the whole population. A is then supercritical. The critical property of A is whether its average degree is above or below one, with $\langle k_A \rangle = \delta \langle k_N \rangle - (1 - \delta) q$.[28]

Fig.4(a) depicts a phase diagram (n, qn) where each point Γ^i is a possible population with n vertices and probability of link q. As one moves right, letting $n \to \infty$, the expected fraction of people in the largest activist cell, given by C/n, converges monotonically in probability to some value. If the limit graph is above the dashed

[27] Janson et al. [22, Ch.5, section 2].

[28] The expected size of C behaves approximately as:

$$E[C] \sim \begin{cases} \frac{3}{(1-\langle k_A \rangle)^2} \ln A & \text{if} \quad \langle k_A \rangle + q < \langle k \rangle_c = 1 \text{ (subcritical)} \\ \beta_A A & \text{if} \quad \langle k_A \rangle + q > \langle k \rangle_c = 1 \text{ (supercritical)} \end{cases} \tag{19}$$

See Thm.6 and Appendix A. These are actually bounds: for subcritical graphs, the largest graph is of size at most $\frac{3}{(1-\langle k_A \rangle)^2} \ln A$, and for supercritical graphs it is at least $\beta_A A$.

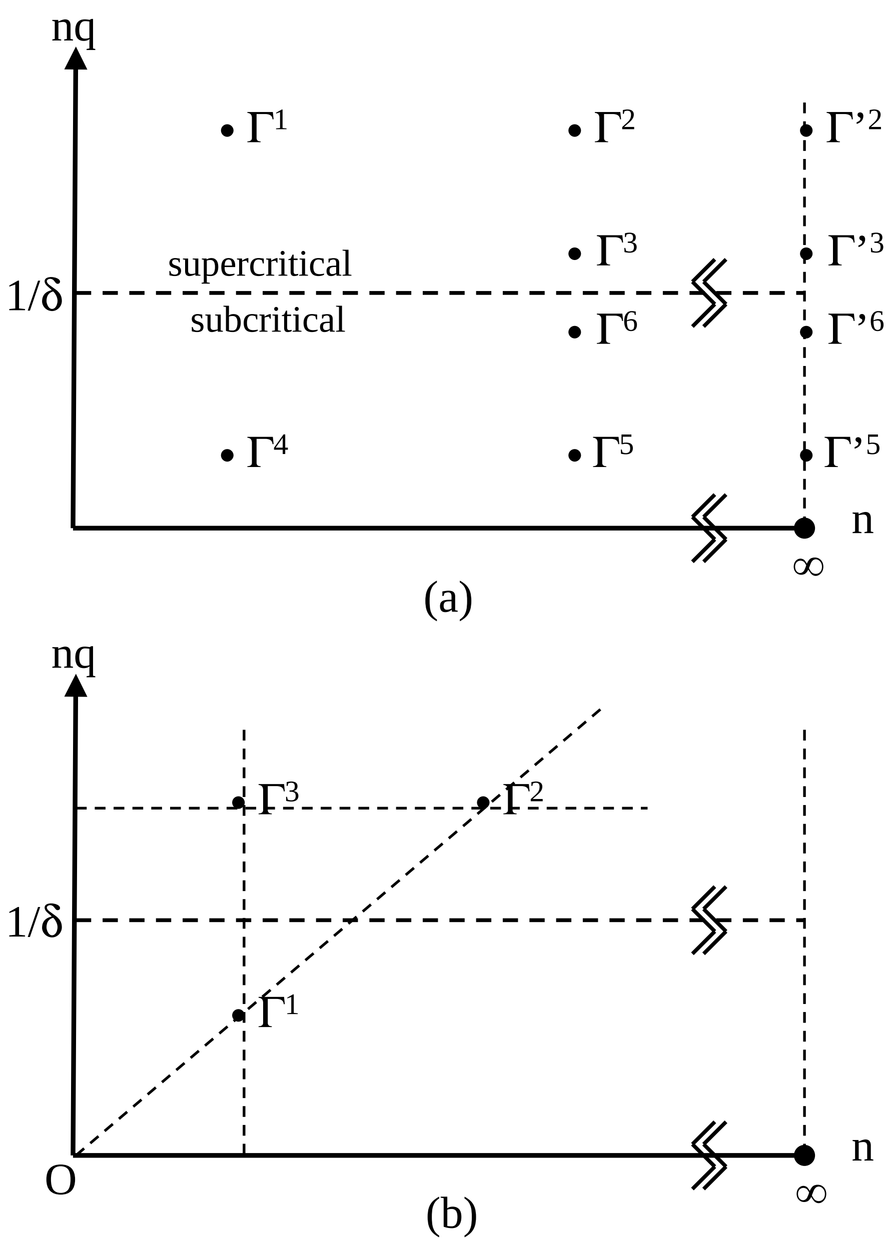

FIGURE 4: Phase diagram

Notes: Each point represents a possible population size and probability of link.

line $nq = 1/\delta$, the limit of C/n is positive: a finite fraction of the total population is coordinated. This is the case of Γ'^2 and Γ'^3. (Graphs higher on the diagram have higher expected C/n.)

On the other hand, if the limit graph is below the line $nq = 1/\delta$, like Γ'^5 or Γ'^6, C/n converges to zero. Still, C is larger in Γ'^6 than in Γ'^5, for example.

The main feature of Thm.6 is that, even if one lets Γ'^6 and Γ'^3 be arbitrarily close to the threshold line, the difference in their respective largest cells remains discrete in the following sense. At $n = \infty$, graphs above the line have supercritical activist communities. As Γ'^3 approaches the threshold, C/n converges to zero, but the structure of C always spans the whole population in the sense that it is of the same magnitude as n. Also, C is unique. In constrast, as Γ'^6 gets closer to the threshold, it still has a multitude of cells of size C, and each one is local – its size is not of the same order of magnitude as n. Thus, the transition is not quantitative, but rather qualitative.

For finite size graphs, being higher in the diagram for a given n also means having a larger expected C. For instance, Γ^2, Γ^3, Γ^6 and Γ^5 are ordered from largest to smallest expected C. However, for all of them the expected C/n is positive. Moreover, the convergence to the limit graph as n grows is smooth through every path, even though the limit has a discontinuity at the threshold $nq = 1/\delta$. As a result, for finite graphs far to the right, the difference in expected C/n just across the threshold must be very large.

Fig.4(b) illustrates three comparisons among finite populations. Γ^1 and Γ^3 have the same size n but different q. Γ^2 and Γ^3 have the same average degree but different n and q. Finally, Γ^1 and Γ^2 have the same q, given by the slope of the straight line $O\Gamma^1\Gamma^2$, but different size n.

Clearly, increases in n with any fixed $q > 0$ will always result in a supercritical A for infinite n. The same is true for any q that does not decrease quickly enough as n grows.[29]

Let me summarize what these results mean in our game. If the average degree of A is below the critical threshold, most activists are not able to meet each other. They are most likely isolated or in relatively small groups. Their ability to reach R-types is in consequence very limited. Activism, if it takes place, is local. Moreover, there are probably several groups of activists of similar size.

On the other hand, if the average degree of A is above the critical threshold, the local communication between neighboring activists described here suffices to bring together a large fraction of all existing activists. A unique largest activist cell appears with a size of the same order of magnitude of the population size n. In this

[29]See Note 21.

case activism has a global reach. In a sense, it permeates the whole population, regardless of size.

This largest supercritical cell of activists scales its size in linear proportion with increases in the number of activists A in the population, as long as the average degree $\langle k_A \rangle$ is kept constant. Even as $n \to \infty$, the fraction of members of the population involved in activism remains positive. However, $\langle k_A \rangle$ increases with n for given q. Thus, if one fixes q and allows $\langle k_A \rangle$ to increase with the population, the size of the largest cell of activists scales up at a faster rate than the population itself.

In the rest of this section I identify what parameter changes may tip the value of $\langle k_A \rangle$ across the critical threshold and analyze the comparative statics of C that do not involve a transition.

5.1.3 Phase transitions

Scaling n with constant $\langle k_N \rangle$ and δ This is perhaps the most interesting type of scaling. It implies adding members to the population while making sure that the average number of neighbors remains constant. In Fig.4(b), this corresponds to a movement to the right along a path that becomes roughly parallel to $nq = 1/\delta$ for large n. It is a very strong condition, as one might expect that any given individual have more acquaintances in a larger population–even if the increase is less than proportional.

In this case, since $\langle k_A \rangle = \delta \langle k_N \rangle - (1 - \delta) q$ and $q = \frac{\langle k_N \rangle}{n-1}$

$$\langle k_A \rangle = \langle k_N \rangle \left(\frac{n}{n-1} \delta - \frac{1}{n-1} \right)$$

and thus $\langle k_A \rangle$ converges to $\delta \langle k_N \rangle$ from below as n grows. For large n, an increase in population size is in general unlikely to tip A through the threshold, but it may happen. If so, the graph goes from the subcritical to the supercritical region.

If the random network of activists $\Gamma_{A,q}$ is close to its critical threshold, the likelihood that an increase in n be enough to change the outcome of the game will be very high. Equivalently, the expected return of an increase in the population is very large for a critical network.

The model yields stronger results for two other types of scaling, presented below.

Scaling n with constant q and δ If n grows but q remains constant, the resulting change is a movement outward along a ray from the origin in Fig.4(b). The expected number of neighbors of each individual grows proportionally with n. This effect is added to the first-order effect of the increase in C. Most importantly, this type of change guarantees that any subcritical set A of activists will eventually reach the critical average degree, as $\langle k_A \rangle$ grows with $A = n\delta$.

Increases in q for given population size n and δ An increase in q is a vertical displacement upwards in Fig.4(b). It can clearly cause the transition to large scale activism. Increasing q may reflect a direct effort to improve the connectivity of the community, so that individuals have more "neighbors". If so, $\left.\frac{d\Phi}{dq}\right|_{n,\delta}$ is arguably a measure of the expected return to investment in the social network.

Changes in the composition of the population δ for given n, q Although the type of an individual is not an endogenous decision in this model, this analysis in particular provides some ties to the standard literature of public goods provision by means of an analogy to a pivotal player's incentives. It is not a scaling of the population, nor does it reflect connectivity improvements. Rather, it measures the return to magically switching the type of some individuals in the population from R to V, thus gaining some extra activists. In Fig.4(b), it corresponds to a downward shift of the threshold curve $1/\delta$.

(15) does not depend directly on δ. However, the shift in the transition threshold affects Φ through C. If the initial population is just below the threshold, it may become supercritical as δ increases.

In (16) the direct effect of a larger δ is to make the incomplete network less desirable by increasing the number of activists who incur D_1. At the same time, however, $\bar{g}$ grows. The net direct effect is positive if $\lambda\pi_C^*\left[g^V - g^R\right] > D_1$. In any case, if C experiences the phase transition, this effect is dominated by the changes in the right hand side of the condition.

Finally, rises in δ have a positive direct effect on the efficiency of the game in (17) if $\pi_C^*\left[g^V - g^R\right] > \gamma D_2$. Again, any transition-related changes dominate the outcome.

5.1.4 Comparative statics within each regime

Even if a change in the network's parameters does not tip A across the critical threshold, it is interesting to ask what its impact on the number of supporters reached may be. The following proposition addresses this question.

Proposition 8 *Let A_R denote a set consisting of the activists in C and all their R-type neighbors. That is, A_R is the set of supporters of the public good. (i) If $\langle k_A\rangle > 1$, then A_R/n converges in probability to a positive number as n increases. (ii) If $\langle k_N\rangle < 1$, then $A_R/n < \frac{3}{(1-\langle k_N\rangle)^2}\frac{\ln n}{n}$ as n increases.*

Proof. To prove (i), simply notice that $N > A_R > C$. Since C/n converges to a positive number, the same is true for A_R/n. For (ii), N itself is a subcritical graph, and so is A as well. In this case, there may be several cells C, each with its corresponding A_R. However, each of those sets A_R is at most as large as the largest clusters in N, since it is necessarily a subset of some cluster: a cluster in N that contains C must also contain its neighbors. By Thm.6, the largest clusters in N are of size $\frac{3}{(1-\langle k_N \rangle)^2} \ln n$. The result follows. ■

The comparative statics analyses in the previous section dealt with the effects of the transition on the game's outcome. However, the question remained open whether the communities at either side of the threshold had the same behavior otherwise. This theorem shows that is not the case. The supercritical and subcritical regimes are qualitatively diffferent in that increases in the population size have different effects in the potential reach of collective action. If the community of activists is supercritical, increases in the population likely improve the chances of emergence of cooperation. If the population as a whole–and therefore the community of activists as well–is subcritical, increases in the population size lead to decreases in the fraction of people that can be reached by means of activism. The likelihood of activism taking place then decreases.

Proposition 8 does not describe all possible cases. It provides no results for the range where $\langle k_A \rangle < 1$ but $\langle k_N \rangle > 1$. However, it succeeds in establishing that even small changes in the parameters of the game have different effects on its outcome depending on whether the activist community is supercritical or subcritical.

6 Conclusion

Let me first state clearly what this model is not meant to argue.

It is by no means implied that the structure of the linkages among the people in a real society (or within a particular group of it) has a binomial random graph structure. Research in social networks within professions and economic agents suggests that the actual structure of such networks is different in at least two ways: it has a certain lumpiness to it, and some individuals have a very large degree, i.e. many neighbors.[30] The binomial random graph model does not have these features, which seem essential to a realistic model of social networks.[31]

Neither am I suggesting that activism, or its motivation, originate in some intrinsic quality of each individual, although that is a matter of controversy. I am not so much concerned with the initiation of activism as I am with its reach.

[30] The formal name for this lumpiness is *clustering*. Measured in the random graphs literature by the *clustering coefficient*, it reflects the transitivity of the linkage structure. Say Elsa knows John, and John knows Laura; what is the probability that Elsa know Laura? If it is high, the expected clustering coefficient is high, and the result is likely to be a network structure with lumps: there will be groups of people where everyone knows almost everyone else, but the links across such groups are relatively few.

See [2], [23], [35], [47], [12], [27].

[31] Several alternative models exist which lay a claim to be more accurate in this sense. Perhaps the best known is the *Small World model*, proposed by Watts and Strogatz [47] (see also Newman et al. [23]), roughly the superimposition of a binomial random graph on a dense network of spatially local links.

The underlying premises in this paper are twofold. First, any implementable mechanism to coordinate individuals in a society is bound to have limitations in the amount of interested parties that can actually be reached. Second, coordination can be achieved through indirect interaction between individuals, without the need for strongly centralized institutions, as long as the costs of such an endeavor are not prohibitive for any one activist. If that is the case, a question of interest for a given potential activist is what her chance of success in reaching enough supporters is. This model shows that a critical feature of the society in this context is its connectivity.

It is surely no surprise that in a society whose members are well interconnected, the existing activists stand a high chance of being themselves well interconnected. It is not surprising either that this has a positive effect on the ability to gather enough supporters for any given cause. What is new in this paper is the insight that the transition between "not sufficiently interconnected" (subcritical network structure) and "well enough interconnected" (supercritical) is not a gradual one, but rather a sharp jump as one changes the network's connectivity. If the network is close to this transition threshold, small variations in its linkages may yield large changes in the occurrence and success rates of collective action.

A second result has to do with the increases in the size of the population under certain scaling conditions. In this model, the connectivity threshold for the transition falls as the population grows. It may then be easier to obtain successful decentralized activism in larger communities. In contrast with the previous literature, this result holds as the the number of agents becomes infinitely large. This is consistent with the existence of large-scale activism in large populations, a phenomenon not explained by previous models.

Even without a transition from one regime to another, the model shows that small increases in population have very different effects on the likelihood of collective action in each regime. In supercritical populations, having more people increases the chances of activism taking place. In at least some range of subcritical populations, the result is the opposite.

In the specific case of my model, the results indicate that a supercritically networked activist community within a population will not face increasing per capita marginal costs of activism when expanding its reach to gather support for its cause (e.g. the provision of a public good). In practice, if the population interested in the cause is large enough, one may expect an institutionalized coordinating agent like a government to take action. Thus, the main bearing of these results are (1) medium-sized populations, not large enough to grant direct intervention by a coordinating agent; or (2) public goods that preclude institutional action.

These results suggest that investment in the construction of social networks may have very large returns for societies that are close to the transition region. In consequence, one may not require large changes in social networks to explain great variance in outcomes of collective endeavors. A further, more controversial suggestion stems from the comparative statics on δ. Under imperfect information, proximity to the transition threshold may provide the rationale for an individual to think that she has a high probability of being pivotal in the provision of the good.

While the particular network structure model in this paper was chosen for analytical tractability, these regime transitions in networks are relatively well-studied and robust phenomena in the mathematics and physics literature dealing with random graphs and percolation. Features like clustering and large degree members are the focus of an extensive body of research and can yield valuable insights into the dynamic aspects of collective action and other collective economic phenomena that take place in networks. Further research avenues in this direction should include extensions of graph models to allow for costly endogenous construction of new links (investment in social networks), more heterogeneous linkage structures, the superimposition of competing activist groups and perhaps the possibility of dynamic learning by individuals.

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A random network

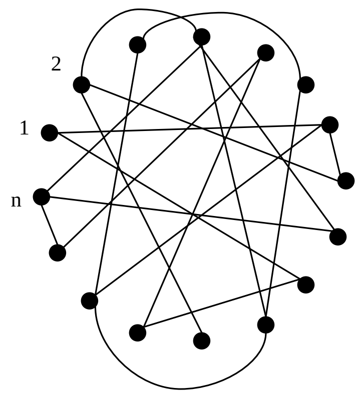

FIGURE 5:

Notes: A typical random network. The lines indicate links between individuals. This is a graph with 16 vertices and $p = 0.15$ approximately.

A Random graphs

Random graphs were first studied using probabilistic theory by Paul Erdős and Alfred Rényi in a series of papers starting in 1959.(Erdős and Rényi [18]) The model they used, called the *uniform random graph* model, was different from the one in this paper in that they specified a graph by the number of nodes and links rather than using a probability of link and the number of nodes. However, the results are esentially the same in both models (they are equivalent in the limit of large graphs), and the intuition is interchangeable. The version of the results I present in this appendix follows closely Barabási [2], and it pertains to the *binomial random graph* model. For an exhaustive analysis of random graphs, the reader should refer to Bollobás [10], Janson et al. [22] and Tutte [46].

One can think of a graph G as consisting of a set $V(G)$ of elements called *vertices* and a set $E(G)$ of elements called *edges*. Each edge is a pair $\{x, y\}$ of elements, where $x, y \in V(G)$. It is useful to visualize this as points (the vertices) joined by lines (the edges), as in figure 5. Thus, the edge $\{x, y\} \in E(G)$ joins the vertices x and y, both of which are in $V(G)$.

A random graph $\mathbf{\Gamma}$ is one where the edges in $E(G)$ are chosen randomly with some probability from the universe of all possible pairs $\{x, y\}$ where $x, y \in V(G)$. Exactly how they are chosen is determined by the particular model used. In this paper, I use the binomial random graph model, where, for any two vertices x, y, the probability that the edge $\{x, y\}$ be in $E(G)$ is given by the constant scalar $q \in [0, 1]$. To avoid confusion, one may want to distinguish between a particular realization of the set of edges, which I denote $E(G)$ (and the graph $G = (V(G), E(G))$), and the random graph $\Gamma_{V(G),q}$. Also, because the elements of $V(G)$ are interchangeable, we can speak of $\Gamma_{n,q}$ where n is the number of vertices.

It should be noted before going any further that most results in the theory of random graphs are by nature asymptotical. They hold with certainty in the limit of $n \longrightarrow \infty$ only. What makes them interesting even in finite (albeit large) n is that they hold for the overwhelming majority of possible realizations.

In the binomial formulation of the random graph model, the total number of edges m is a random variable. Its expected value is $E[m] = qn(n-1)/2$. The probability of obtaining a given graph G_0 with m edges is $\Pr[G_0] = q^m (1-q)^{\frac{n(n-1)}{2}-m}$. Although a higher probability of link q does not guarantee a higher number m of links, the mathematical literature on random graphs does associate these two parameters.[32] Thus, $q = 1$ corresponds to $m = \frac{n(n-1)}{2}$ and $q = 0$ to $m = 0$.

The main issue in this model is to find out the probability of link at which certain properties become common in the graphs. Perhaps surprisingly, many important properties of the graphs appear quite suddenly, and they become almost universal very quickly.

For many properties, one can find a probability threshold below which almost no graph has them, and above which almost every graph does. This probability threshold depends on the number of vertices in the graph, and is thus a function $q_c(n)$.[33] For random graphs with constant probability of link q, increasing the number of vertices increases the number of existing links very quickly. This has the particular effect that $q_c(n)$ is decreasing (indeed, it may converge to zero) for many of the more interesting properties. One particular property, however, defines a critical threshold independent of n: the average degree of the graph[34]

$$\langle k \rangle = q(n-1) \simeq qn$$

In other words, there is a critical average degree of the graph, $\langle k \rangle_c = 1$ for all n, which defines the critical probability threshold for a number of properties:

$$q_c(n)(n-1) = \langle k \rangle_c = 1$$
$$q_c(n) = \frac{1}{n-1}$$

Among the properties of interest is the size of the largest cluster present.[35] If $q(n) < q_c(n)$ [$\langle k \rangle < \langle k \rangle_c$], the largest cluster has a simple structure and its size is proportional to $\ln(n)$ or $\ln(\langle k \rangle / q)$, i.e. grows only slowly with the size of the graph. However, for $q(n) = q_c(n)$ [$\langle k \rangle = \langle k \rangle_c = 1$], the size of this cluster is approximately $n^{2/3}$ and its structure becomes very complex. Finally, for $q(n) > q_c(n)$ [$\langle k \rangle > \langle k \rangle_c$], this complex cluster, called the *giant component*, contains a finite fraction $1 - f(\langle k \rangle)$ of the n vertices available, where $f(\langle k \rangle)$ is an exponentially decreasing function with $f(1) = 1$ and $f(\infty) = 0$. I reproduce here without proof a version of this result from Janson et al. [22, Ch.5. p.109, Theorem 5.4], Thm.6:

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[32]Indeed, one can prove that they are equivalent in the limit of large n (Janson [22, Ch.1]).

[33]Let q be a function of the number of vertices, $q = q(n)$. Then, $q_c(n)$ the critical probability threshold for the property Q satisfies

$$\lim_{n \longrightarrow \infty} \Pr[\Gamma_{n,q} \text{ has property } Q] = \left\{ \begin{array}{ccc} 0 & \text{if} & \frac{q(n)}{q_c(n)} \longrightarrow 0 \\ 1 & \text{if} & \frac{q(n)}{q_c(n)} \longrightarrow \infty \end{array} \right\}$$

If this limit is 1, then , for $q = q(n)$, almost every (or almost all) graphs have property Q. Alternatively, $\Gamma_{n,q}$ has property Q almost surely.

[34]The degree of a vertex is the number of edges that are incident on it. The average degree of a graph is the average degree over all vertices. Since there $n(n-1)/2$ possible edges, each ocurring with probability q, the expected number of edges is $qn(n-1)/2$. Dividing by the number n of vertices and multiplying by two (each edge touches two vertices, after all), one obtains the *expected* average degree of the graph. Or, according to common usage, the *average degree of the graph.*

[35]A cluster is a subset of vertices, each of whom can be reached from any other through a path of interconnected vertices in the graph, all of which are also part of the cluster.

Theorem 9 (6) *Let $nq = k$, where k is a constant. (i) If $k < 1$, then a.a.s. the largest component of $\Gamma_{n,q}$ has at most $\frac{3}{(1-k)^2} \ln n$ vertices. (ii) Let $k > 1$ and let $\beta = \beta(k) \in (0,1)$ be defined by the equation $\beta = 1 - e^{-\beta k}$. Then $\Gamma_{n,q}$ contains a giant component of $(1 + o(1))\, \beta n$ vertices. Furthermore, a.a.s. the size of the second largest component of $\Gamma_{n,q}$ is at most $\frac{16k}{(1-k)^2} \ln n$.*

The transition described in Thm.6 is abrupt in the value of k for infinite n. However, for a given n, as q increases, it appears that the size of the largest cluster grows very quickly, but smoothly, during the transition (Janson et al. [22, Ch.5, Sec 2]). This is also the case for changes in n given q.

Other properties of $\Gamma_{n,q}$ help obtain some insight about the implications of the appearance of the giant component. In the supercritical regime, all other large (non-giant) clusters tend to dissappear. Only small clusters survive unattached to the giant one, and relatively few vertices belong to them. This means that the graph is spanned by a single cluster of size of the same order as her own, and not by a collection of small isolated clusters.

Perhaps an alternative thought experiment is more useful to understand the intuition behind these results. Consider for a moment a construction of a given subset $X \subseteq N$ by adding members one by one according to the random sequence $i_1, i_2, i_3, ...i_X$. The first member, i_1, cannot possibly have neighbors, as she is alone. The second member i_2 is linked to i_1 with probability q. i_3 is linked to each one of these with probability q, and she is isolated with probability $(1-q)^2$. i_4 is more interesting. For the sake of argument, suppose that i_1 and i_2 turned out to be linked, but i_3 was isolated as seen in Fig.6(b). Then, i_4 is linked to i_3 with probability q, but to *either* i_1 *or* i_2 (a multi-activist group) with probability $1-(1-q)^2 > q$. Further consideration of this scenarios yields two insights: First, the likelihood of $i_{m<X}$ being linked to at least another member increases the more activists there are already, since she has a probability q of being a neighbor of any particular one. Second, and due to the same reason, the likelihood of i_m being linked to an already existing group of interconnected members increases the more members that group has.

If m is a small number, i_m stands a fair chance to be isolated. Even if she turns out to be linked to some previous member $i_{m'<m}$, chances are that member had no neighbors, so that they are now a group of two. At most, i_m will be in a small group. If m is instead a "medium" number (loosely speaking), there will likely exist several groups of various sizes. In that case, i_m has a disproportionately higher probability of being linked to them precisely because they have more members (Fig.6(c)). Moreover, among those groups, i_m is most likely to link with the larger ones. Even further, i_m may even act as a bridge, linking two or more existing groups, an event that is more likely the larger those groups are. This last possibility will in time (i.e. for large m) determine the appearance of an unusually large group, more than twice the size of any other, that will quickly annex all other groups, except for very small ones. Any remotely big groups still unlinked to the large group are very likely to be annexed thanks to the next member that is added.

The literature on random graphs provides some interesting results concerning the size and properties of such a large group, called the *giant component*. Perhaps counter to intuition, its expected size scales up linearly with the number of members in X, for fixed average degree $\langle k_X \rangle$, and even more than linearly for fixed q. Moreover, while it is not certain that it will exist in small subsets (even if $X > \frac{1}{q} + 1$), the probability of its

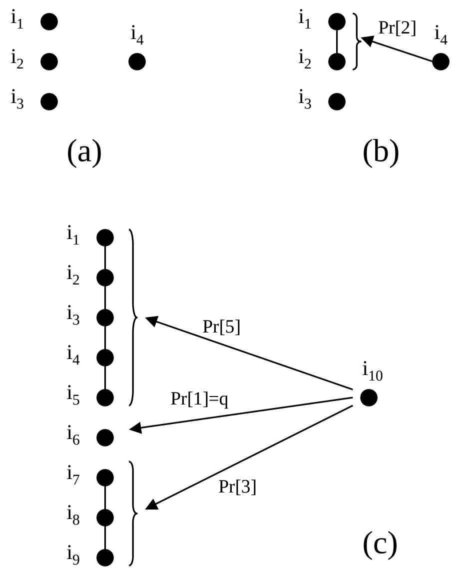

FIGURE 6: Thought experiment

Notes: In (a), all links are equally probable. In (b) i_4 is linked to i_3 with probability q, but to *either* i_1 *or* i_2 (a multi-activist group) with probability $1-(1-q)^2 > q$. The larger a pre-existing group is, the more likely it is that the next member is linked to it. In general, the probability of being linked to a group with k members is $Pr[k] = 1-(1-q)^k$

appearance converges very quickly to 1 as $X \longrightarrow \infty$.[36]

The dramatic consequences of the existence of the giant component become most evident if one considers a change the scale of the graph. Suppose that the graph duplicates its size n, but keeps k constant. If the graph is subcritical, the size of the largest cluster will increase by $\ln 2$ (not much). Most of the effects will be felt in the number of clusters present. If the graph is supercritical, however, the number of clusters (in particular large ones) will decrease, but the size of the largest cluster will double.

If one considers increases in n keeping q rather than k constant, as in this paper, even graphs that are originally subcritical become eventually supercritical. The size of the largest cluster grows then at a rate faster than n itself in the neighborhood of the transition.

B Individual preferences

The R-type preferences can be argued as a result of community enforcement of norms [24]. Alternatively, they are similar to the "warm glow" utility in the impure altruism literature, as shown next. Let R-type individuals have homogeneous preferences defined over consumption of a discrete public good $\mathcal{G}$, the numeraire x, and a parameter T, intended to capture each invidual's private benefit from being a truthful citizen:

$$U^R = u\left(x, \mathcal{G}\right) + \varepsilon T$$

[36]See Note 21.

where

$$T = \left\{ \begin{array}{ll} 1 & \text{if the individual is truthful} \\ 0 & \text{otherwise} \end{array} \right\}$$

and $\varepsilon > 0$ is a measure of the importance of this "warm glow" for the reluctant individuals. ε is assumed to be the identical across the V-type population for simplicity. Moreover, assume that each individual is endowed with w units of the private good.

Let g^R, the valuation of the public good for R-types, be defined as before in (2)

$$u\left(w - g^R, 1\right) = u\left(w, 0\right)$$

One can further define implicitly for type R individuals the "personal valuation of truthfulness" $h > 0$ by

$$u\left(w - h, 1\right) + \varepsilon = u\left(w, 1\right)$$

Andreoni [3],[4] shows that this warm glow, in practice a private benefit derived from the act of giving to the public good, may increase the level of provision and make it a function of the income distribution. It reduces but does not eliminate the free-rider problem. More important for the purpose of the present paper, it does not address the issue of the coordination failure in the game.

C Optimal pledge proposal

I now characterize the interior solution for p^* in the required low range $p_s \leq g^V - Y^V$. First, I need the probability $\pi_C = \pi_C(p)$ of a successful fundraiser given coordination. Let $\theta \equiv (1-q)^C$, the probability that any given R-type not be a neighbor of C. If the number of R-type supporters is s, and $p \leq h$, the probability of a successful fundraiser for given C is

$$\pi_C(p) = \Pr\left[p(s+C) \geq G\right] = \Pr\left[s \geq \frac{G}{p} - C\right]$$

$$\pi_C(p) = \sum_{j=\left|\frac{G}{p}-C\right|+1}^{n-A} \binom{n-A}{j} (1-\theta)^j \theta^{n-A-j} \text{ for } p \in (p_{s-1}, p_s]$$

$$\pi_C(p) = 1 - \sum_{j=0}^{\left|\frac{G}{p}-C-1\right|} \binom{n-A}{j} (1-\theta)^j \theta^{n-A-j} \text{ for } p \in (p_{s-1}, p_s]$$

Substituting $s = \frac{G}{p_s} - C = \left|\frac{G}{p_s} - C\right|$

$$\pi_C(p) = \pi_C(p_s) = 1 - \sum_{j=0}^{s-1} \binom{n-A}{j} (1-\theta)^j \theta^{n-A-j} \text{ for } p \in (p_{s-1}, p_s] \qquad (20)$$

Since $\Delta_s p = \frac{-G}{(C+s)(C+s-1)} < 0$ and $\Delta_s \pi_C = -\binom{n-A}{s-1}(1-\theta)^{s-1}\theta^{n-A-s+1} < 0$, (9) always holds for $p_{s^*} \geq g^V - Y^V$, regardless of C, which confirms that any interior solution must

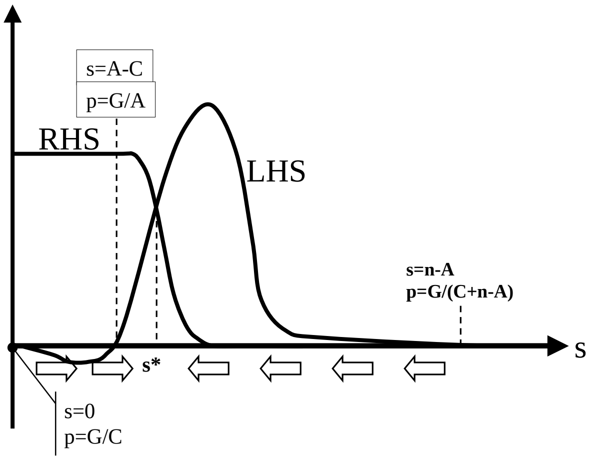

FIGURE 7: Optimal pledge

Notes: s is the minimum number of supporters that must be reached for successful provision of the good. The arrows indicate the direction of increasing payoff to the V-types, so s^* is the optimum. The associated proposed pledge is $p^* = p_{s^*} = G/(C + s^*)$.

involve $p^* = p_{s^*} < g^V - Y^V$. Replace to obtain

$$\begin{gathered}\text{Increase } s \text{ if} \\ (C+s-1)\left[\frac{g^V - Y^V}{G}(C+s)-1\right]\binom{n-A}{s-1}(1-\theta)^{s-1}\theta^{n-A-s+1} \\ \leq 1-\sum_{j=0}^{s-2}\binom{n-A}{j}(1-\theta)^j\theta^{n-A-j}\end{gathered} \tag{21}$$

Both sides of this condition are depicted in Fig.7, along with the optimal solution.

D Some calculations

At the optimum,

$$\begin{aligned} s^* &= s^*(n, C, \theta) \\ p^* &= p^*(C, s^*) \\ \pi_C^* &= \pi_C^*(n, \theta, s^*) \end{aligned}$$

with $C = C(n, q, \delta)$, $\theta = \theta(q, C)$. Totally differentiating s^* and p^* yields, for the optimal solution

$$
\begin{aligned}
ds^* &= \left[\frac{\partial s^*}{\partial C} + \frac{\partial s^*}{\partial \theta}\frac{\partial \theta}{\partial C}\right] dC + \frac{\partial s^*}{\partial n} dn + \frac{\partial s^*}{\partial \theta}\frac{\partial \theta}{\partial q} dq \\
dp^* &= \frac{\partial p^*}{\partial C} dC + \frac{\partial p^*}{\partial s^*} ds^* = \frac{-G}{(C+s^*)^2}\left[dC + ds^*\right] = -\frac{p^{*2}}{G}\left[dC + ds^*\right] \\
&= -\frac{p^{*2}}{G}\left\{\left[1 + \frac{\partial s^*}{\partial C} + \frac{\partial s^*}{\partial \theta}\frac{\partial \theta}{\partial C}\right] dC + \frac{\partial s^*}{\partial n} dn + \frac{\partial s^*}{\partial \theta}\frac{\partial \theta}{\partial q} dq\right\} \\
d\pi_C^* &= \left\{ \begin{array}{l} \left[\frac{\partial \pi_C^*}{\partial n} + \frac{\partial \pi_C^*}{\partial s^*}\frac{\partial s^*}{\partial n}\right] dn + \left[\frac{\partial \pi_C^*}{\partial \theta} + \frac{\partial \pi_C^*}{\partial s^*}\frac{\partial s^*}{\partial \theta}\right]\frac{\partial \theta}{\partial q} dq \\ \quad + \left[\left(\frac{\partial \pi_C^*}{\partial \theta} + \frac{\partial \pi_C^*}{\partial s^*}\frac{\partial s^*}{\partial \theta}\right)\frac{\partial \theta}{\partial C} + \frac{\partial \pi_C^*}{\partial s^*}\frac{\partial s^*}{\partial C}\right] dC \end{array} \right.
\end{aligned}
$$

where $\frac{\partial \theta}{\partial C} = \theta \ln(1-q) < 0$ and $\frac{\partial \theta}{\partial q} = \frac{-C}{1-q}\theta < 0$.

Consider the expressions $\Phi \equiv \lambda\left[\pi_C^* g^V - \gamma D_2 - \gamma \pi_C^* p^*\right]$, $B_1 \equiv \lambda\left[\pi_C^* \bar{g} - \gamma D_2\right]$ and $B_2 \equiv [1 - \lambda\gamma]\,\delta D_2 - [1 - \lambda \pi_C^*]\,\bar{g}$. Differentiate to obtain

$$
\begin{aligned}
dB_1 &= \left[\pi_C^* \bar{g} - \gamma D_2\right] d\lambda + [\lambda \bar{g}]\, d\pi_C^* - [\lambda D_2]\, d\gamma \\
dB_2 &= \left[\pi_C^* \bar{g} - \gamma \delta D_2\right] d\lambda + [\lambda \bar{g}]\, d\pi_C^* - [\lambda \delta D_2]\, d\gamma \\
d\Phi &= -\left[\pi_C^* g^V + \gamma D_2 + \gamma \pi_C^* p^*\right] d\lambda + \lambda\left[g^V - \gamma p^*\right] d\pi_C^* - \lambda\left[D_2 + \pi_C^* p^*\right] d\gamma - \lambda\gamma\pi_C^* dp^*
\end{aligned}
$$

It follows that

$$
\begin{array}{llll}
\frac{\partial \Phi}{\partial \lambda} < 0 & \frac{\partial \Phi}{\partial \pi_C^*} > 0 & \frac{\partial \Phi}{\partial \gamma} < 0 & \frac{\partial \Phi}{\partial p^*} < 0 \\
\frac{\partial B_1}{\partial \lambda} \gtreqless 0 & \frac{\partial B_1}{\partial \pi_C^*} > 0 & \frac{\partial B_1}{\partial \gamma} < 0 & \\
\frac{\partial B_2}{\partial \lambda} \gtreqless 0 & \frac{\partial B_2}{\partial \pi_C^*} = \frac{\partial B_1}{\partial \pi_C^*} > 0 & \frac{\partial B_2}{\partial \gamma} = \delta \frac{\partial B_1}{\partial \gamma} < 0 &
\end{array}
$$

and $\frac{\partial B_1}{\partial \lambda} > 0$ implies $\frac{\partial B_2}{\partial \lambda} > 0$.

E Notation

I provide here a summary of notation:

N The population set.

n Number of elements (size) of N.

$\mathcal{G}$ The public good.

G Cost of provision of $\mathcal{G}$.

q Connectivity of the network. Denotes the probability of link between any two individuals, so $q \in (0, 1]$.

V, R Types of individuals: Volunteer and Reluctant.

δ Fraction of individuals of type V.

g^V, g^R Valuation of the good to each type of individual.

$\bar{g}$ Average social value of $\mathcal{G}$. $\bar{g} = \delta g^V + (1 - \delta) g^R$.

h Warm glow from telling the truth for an R-type.

$\Gamma_{v,e}$ A random graph with v vertices and e edges.

A (In normal font) it denotes both the set of activists and its size.

C Both the largest activist cell and its size.

A_R (In normal font) a set consisting of C and all its R-type neighbors. Also, the size of this set.

γ Probability that an activist be a member of C.

λ $= \Pr\left[p^* + \frac{D_2}{\pi_C^*} < g^V - Y^V\right]$, the ex-ante (time $t = 0$) probability that a fundraiser takes place.

D_1 Cost of becoming an activist.

D_2 Cost of taking part in a fundraising drive.

p Proposed pledge in the fundraising drive.

$Z_{\max}$ Maximum attainable voluntary contribution from V-types.

L Welfare loss due to the costs of activism.

$Y = \left(Y^V, Y^R\right)$ Payoffs if no activism takes place (outside option).

W Expected payoff to activists not in C.

θ Probability that any given R-type not be a neighbor of C.

π_C Probability of success of the fundraising drive, conditional on C.

s Number of R-types that C plans to reach.

p_s Optimal pledge as a function of s.

$\langle k_X \rangle$ Average degree of X. It is the expected number of neighbors, in X, of any member of X.

Optimal values of the choice parameters are denoted everywhere by an asterisk (*).