Chile Complex Systems Summer School 2013

Applications of Game Theory

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¿What is a game?

In a game two or more player interact, adopting strategic decisions

The game is characterized by two aspects

1) The set of strategies available to the players

2) The payoff obtained by each strategy when confronting the others

Player 2

Player 1

	Rock	Paper	Scissors
Rock	0, 0	-1,1	1,-1
Paper	1,-1	0,0	-1,1
Scissors	-1,1	1,-1	0,0

Rational choice

The choices made by individuals in a society try to maximize their benefits and minimize their costs and risks.

People make decisions about how they should act **(adopt a strategy)** by comparing the costs and benefits **(payoff)** of different courses of action.

"It is not from the benevolence of the butcher, the brewer, or the baker that we expect our dinner, but from their regard to their own interest." Adam Smith (1776)

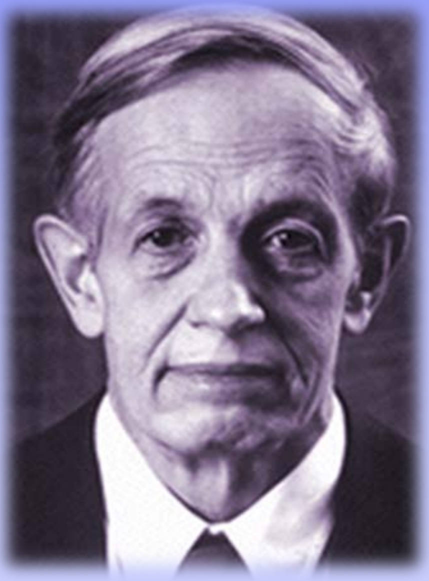
"[Political economy] does not treat the whole of man's nature as modified by the social state, nor of the whole conduct of man in society. It is concerned with him solely as a being who desires to possess wealth, and who is capable of judging the comparative efficacy of means for obtaining that end." John Stuart Mill (1836)

The Homo Economicus acts to obtain the highest possible well-being for him or herself given available information about opportunities and other constraints

Best response – Nash Equilibrium

There are n players

Each player *i* can choose one strategy s_i from a set of strategies S



We say that the strategy $t \in S$ is a best response to s_i if by playing t one gets the highest possible payoff

A Nash Equilibrium is a strategy that is the best response to itself, or a couple of strategies that are mutually best responses to the other

NashEquilibrium

Column

File

	Strategy a	Strategy b
Strategy a	1,2	0,1
Strategy b	2,1	1,0

(b,a) is the Nash equilibrium.

Column best choice is always **a**, File best response is **b**

Fila best choice is always **b**, Column best response is **a**.

NashEquilibrium

Sometimes, the Nash equilibrium is not trivially found

Player B

Player A

	R	P	S
R	0, 0	-1,1	1,-1
P	1,-1	0,0	-1,1
S	-1,1	1,-1	0,0

NashEquilibrium

Sometimes, there are more than one Nash equilibrium

Battle of the sexes

A couple is planning vacations. The woman prefers the beach, the man prefers the mountain. Both prefer spending their time together than separated

Woman

Man

	Mountain	Beach
Mountain	2,1	0,0
Beach	0,0	1,2

Mixed strategy

Consider the game “Matching pennies”

Player B

Player A

	Head	Tail
Head	1,-1	-1,1
Tail	-1,1	1,-1

	p H	(1-p) T
q H	1,-1	-1,1
(1-q) T	-1,1	1,-1

If there is Nash equilibrium, each player would be able to choose an optimum frequency in response to the other player choice

Mixed strategy

As **A** can always change its strategy in response to what **B** does, **B** looks for a choice whose payoff is independent of what **A** does

Player B

Player A

	p H	(1-p) T
q H	1,-1	-1,1
(1-q) T	-1,1	1,-1

If **A** plays **H, B** wins –p+(1-p); If **A** plays **T, B** wins p-(1-p)

$p-(1-p)=-p+(1-p)$ ⇨ $p=1/2$

$q-(1-q)=-q+(1-q)$ ⇨ $q=1/2$

Evolutionary Stable Strategies

Consider a population having adopted a unique shared strategy

Due to mutation or incorporation, suddenly one individual adopts a different strategy.

If due to this different strategy the “mutant” beats the rest of the population, by imitation the individuals will adopt the mutant strategy

In other case, the mutant will be ignored

If the population adopted a strategy such that no mutant can take advantage of the situation, this strategy is called evolutionarily stable

Evolutionary Stable Strategies

Each player has the same set of available pure strategies $R = \{R_1, R_2, \ldots, R_N\}$

The player can choose a pure or mixed strategy **r**

$\mathbf{r}=(p_1, p_2, \ldots, p_N)$, with $p_i \geq 0$ y $\sum p_i = 1$ *(this define a simplex)*

Payoff Matrix

$$A = (a_{ij})$$

a_{ij} is the payoff R_i gets against R_j

The payoff $\mathbf{r}_1$ gets when playing against $\mathbf{r}_2$ is

$$\mathbf{r}_1 A\, \mathbf{r}_2 = \sum_{i,j} p_{1i}\, p_{2j}\, a_{ij}$$

Evolutionary Stable Strategies

A best response to a strategy $\mathbf{r}_1$ is a strategy $\mathbf{r}_2$ such that

($\mathbf{r}_2$ A $\mathbf{r}_1$) is a maximum.

In the case when all the player have the same set of available strategies, a Nash equilibrium is a strategy that is its own best response

$$r_j\, A r_i \leq r_i\, A r_i \quad \forall j$$

If it is the only best response is a Strict Nash Equilibrium

$$r_j\, A r_i < r_i\, A r_i \quad \forall j \neq i$$

A key question in Game Theory is about the existence of a profile of strategies in a population that is stable and resistant to perturbations

Mutants can not take any advantage. If a population with strategy $\mathbf{r}_i$ *is invaded by individuals wit strategy* $\mathbf{r}_j$

$$r_j\, A((1-\varepsilon)r_i + \varepsilon r_j) < r_i\, A((1-\varepsilon)r_i + \varepsilon r_j) \quad \forall j \neq i$$

Evolutionary Stable Strategies

$$r_j\, A((1-\varepsilon)r_i + \varepsilon r_j) < r_i\, A((1-\varepsilon)r_i + \varepsilon r_j) \quad \forall j \neq i$$

$$(1-\varepsilon)(r_i A r_i - r_j A r_i) + \varepsilon (r_i A r_j - r_j A r_j) > 0$$

Strict Nash equilibrium $\quad r_i A r_i > r_j A r_i$

$$r_i A r_i = r_j A r_i \Rightarrow$$

Stability $\quad r_i A r_j > r_j A r_j$

Evolutionary Games

The behavior can be defined by trial and error. Adaptation and learning are key factors

The games are played in a population, where each individual receives a score

Strategies that work better than the average spread while others disappear. The restriction of rational behavior can be relaxed

Each player plays with all the population or only its neighbors (mean field vs. spatial)

The success of each player determines the number of followers or descendants in the next step (Selection)

The descendants or imitators inherit or copy the strategy with some error (mutation)

If you reach the Nash equilibrium (global) no other strategy can invade

Replicator Dynamics

The replicator equation describes the evolution of the frequencies of strategies in a population, with selection proportional to the fitness

Payoff Matrix

$$A = (a_{ij})$$

Population

$$\vec{x} = (x_1, x_2, \ldots, x_i, \ldots, x_N)$$

$$e_i = (0,0,\ldots,0,1,0,\ldots,0)$$

$$\sum_i x_i = 1$$

i Payoff

$$f_i(\vec{x}) = e_i A \vec{x}^T$$

Mean payoff

$$\overline{f}(\vec{x}) = \vec{x} A \vec{x}^T$$

Replicator Dynamics

x_i is the frequency of strategy (phenotype) i, f_i its fitness, the equation is

$$\dot{x}_i = x_i(f_i(\vec{x}) - \bar{f}(\vec{x})), \qquad \bar{f}(\vec{x}) = \sum_i x_i f_i(\vec{x})$$

It can be shown that if a strategy is evolutionary stable then it is a stationary state of the replicator equation

Replicator Dynamics: RPS

	Rock	Paper	Scissors
Rock	0, 0	-1,1	1,-1
Paper	1,-1	0,0	-1,1
Scissors	-1,1	1,-1	0,0

⇓

$$A = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix}$$

Replicator Dynamics: Stability

$$\frac{dx}{dt} = x(-1 + x + 2y)$$
$$\frac{dy}{dt} = y(1 - y - 2x)$$

$$x = x^* + \varepsilon$$
$$y = y^* + \delta$$

$$\frac{d(x^* + \varepsilon)}{dt} = \frac{d\varepsilon}{dt} = (x^* + \varepsilon)(-1 + x^* + \varepsilon + 2(y^* + \delta))$$
$$\frac{d(y^* + \delta)}{dt} = \frac{d\delta}{dt} = (y^* + \delta)(1 - y^* - \delta - 2(x^* + \varepsilon))$$

$$\frac{d\varepsilon}{dt} = x^*(-1 + x^* + 2y^*) + x^*(\varepsilon + 2\delta) + \varepsilon(-1 + x^* + 2y^*)$$
$$\frac{d\delta}{dt} = y^*(1 - y^* - 2x^*) - y^*(2\varepsilon + \delta) + \delta(1 - y^* - 2x^*)$$

$$(\frac{d\varepsilon}{dt}, \frac{d\delta}{dt}) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \varepsilon \\ \delta \end{pmatrix}$$

$$a = 2(x^* + y^*) - 1$$
$$b = 2x^*$$
$$c = -2y^*$$
$$d = 1 - 2(x^* + y^*)$$

Replicator Dynamics: Linear stability

$x^* = 0$

$y^* = 0$

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$\lambda_1 = 1$

$\lambda_2 = -1$

$x^* = 0$

$y^* = 1$

$$\begin{pmatrix} 1 & 0 \\ -2 & -1 \end{pmatrix}$$

$\lambda_1 = 1$

$\lambda_2 = -1$

$x^* = 1$

$y^* = 0$

$$\begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}$$

$\lambda_1 = 1$

$\lambda_2 = -1$

$x^* = 1/3$

$y^* = 1/3$

$$\begin{pmatrix} \frac{1}{3} & \frac{2}{3} \\ -\frac{2}{3} & -\frac{1}{3} \end{pmatrix}$$

$\lambda_1 = i\sqrt{3}$

$\lambda_2 = -i\sqrt{3}$

Replicator Dynamics: Linear stability

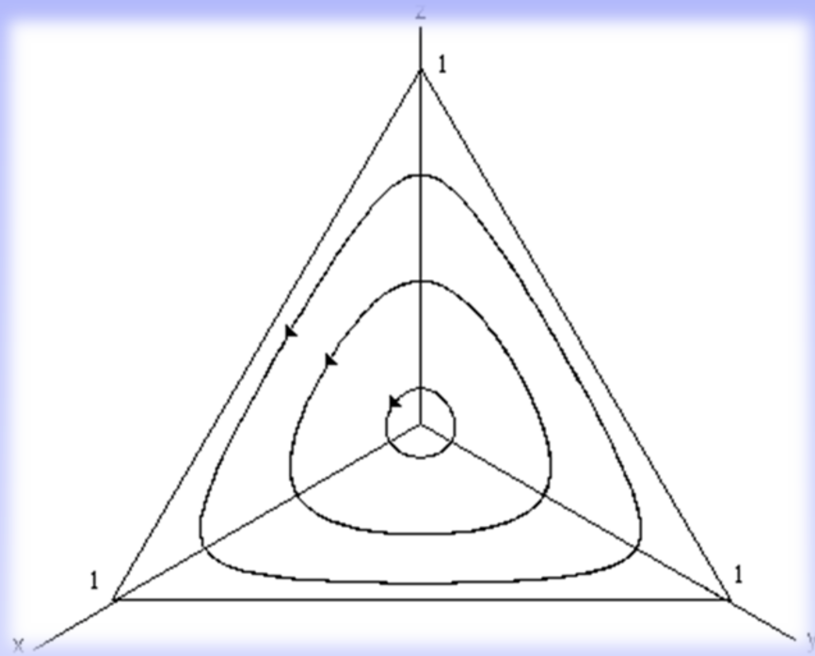
Equilibria

$(1,0,0)$
$(0,1,0)$
$(0,0,1)$ } Saddle

$\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$ Center

$$\frac{dx}{dt} = x(-1 + x + 2y)$$

$$\frac{dy}{dt} = y(1 - y - 2x)$$



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2x2 Symmetric Games

Each player can choose between two available strategies

	E1	E2
E1	a,a	b,c
E2	c,b	d,d

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Consider a population that plays E1 with prob. x and E2 with prob. (1-x)

$$f_1 = xa + (1-x)b$$

$$f_2 = xc + (1-x)d$$

$$\overline{f} = x[xa + (1-x)b] + (1-x)[xc + (1-x)d]$$

$$\frac{dx}{dt} = x(f_1 - \overline{f}) = x(1-x)(x(a-b-c+d) + (b-d))$$

2x2 Symmetric Games

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \longrightarrow \begin{pmatrix} a-c & 0 \\ 0 & d-b \end{pmatrix}$$

$$\overline{f} = x(a-c) + (1-x)(d-b)$$

$$\frac{dx}{dt} = x(f_1 - \overline{f}) = x(x(a-c) - [x^2(a-c) + (1-x)^2(d-b)]$$

$$\frac{dx}{dt} = x(1-x)[x(a-c) - (1-x)(d-b)] = x(1-x)[x(a-b-c+d) + (b-d)]$$

2x2 Symmetric Games

Each player can choose between two available strategies

	E1	E2
E1	a_1, a_1	0,0
E2	0,0	a_2,a_2

$$\longrightarrow \quad A = \begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix}$$

Consider a population that plays E1 with prob. x and E2 with prob. (1-x)

$$f_1 = xa + (1-x)b$$

$$f_2 = xc + (1-x)d$$

$$\frac{dx}{dt} = x(f_1(\vec{x}) - \overline{f}(\vec{x})) = x(1-x)(xa_1 - (1-x)a_2)$$

Steady states $\frac{dx}{dt} = 0 \quad \Rightarrow x = 1, \quad x = 0, \quad x = \frac{a_2}{a_1 + a_2}$

2x2 Symmetric Games

$$\dot{x}_1 = x_1(1 - x_1)(x_1 a_1 - (1 - x_1)a_2)$$

$$x_1^* = \frac{a_2}{a_1 + a_2}$$

If $a_1 > 0$ and $a_2 < 0$, the flux is always

If $a_1 < 0$ and $a_2 > 0$, the flux is always

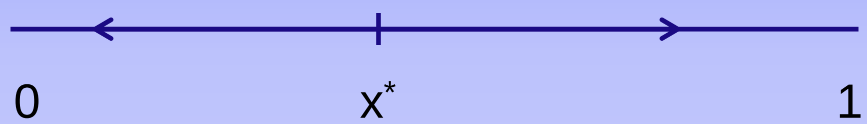
$$|x_1^*| > 1$$

2x2 Symmetric Games

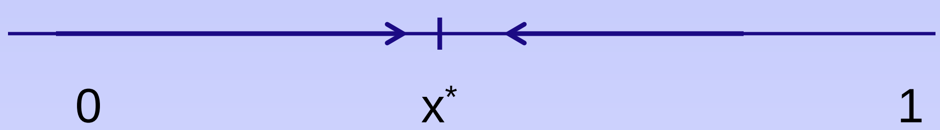
$$\dot{x}_1 = x_1(1-x_1)(x_1a_1-(1-x_1)a_2)$$

$$x_1^* = \frac{a_2}{a_1+a_2}$$

If $a_1 > 0$ and $a_2 > 0$, the flux close to x^*, where the derivative is null



If $a_1 < 0$ and $a_2 < 0$, the flux close to x^*, where the derivative is null



2x2 Symmetric Games

There are three (four) types of 2x2 Symmetric Games

$$A = \begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix}$$

Games type I (IV) : $a_1 < 0$ y $a_2 > 0$ ($a_1 > 0$ y $a_2 < 0$) — Prisoner's dilemma

Games type II: $a_1 > 0$ y $a_2 > 0$ — Coordination Games

Games type III: $a_1 < 0$ y $a_2 < 0$ — Hawks and doves

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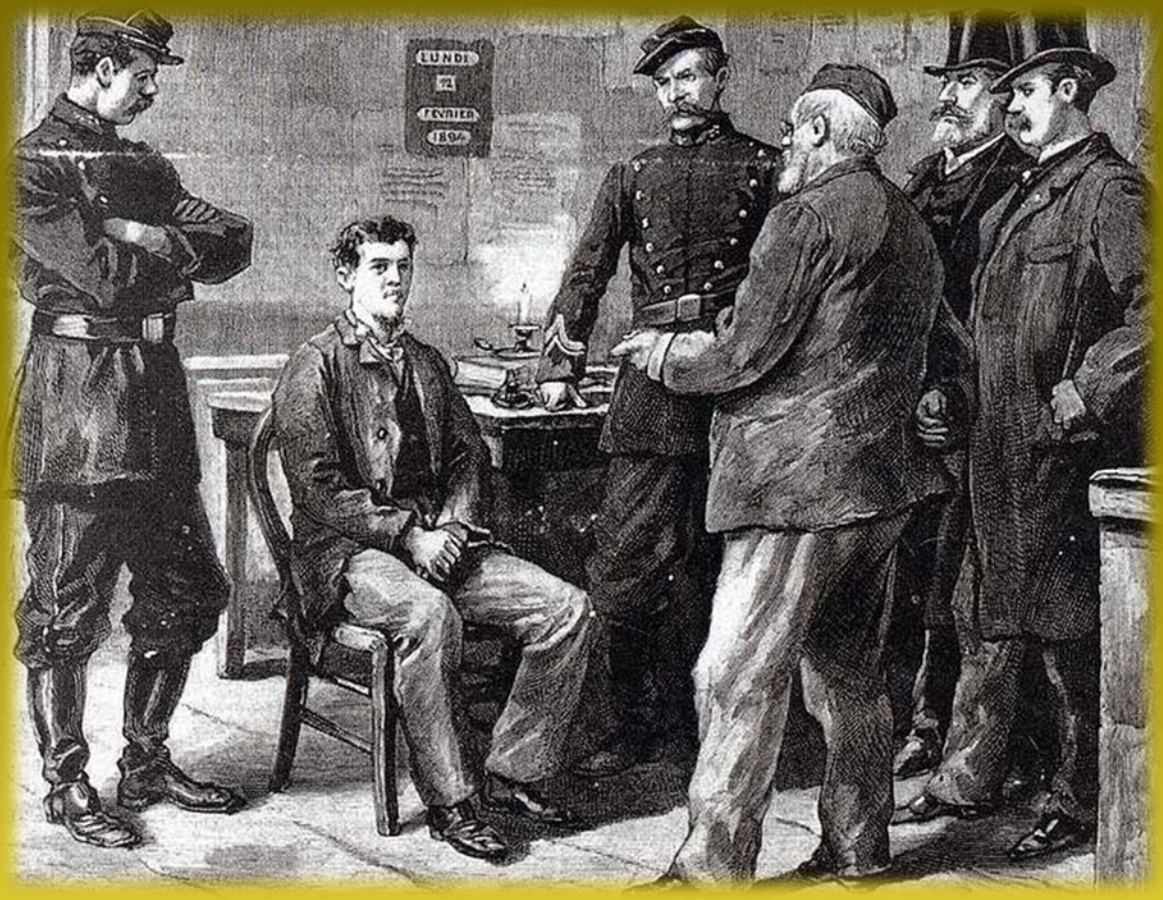
Prisoner’s Dilemma



Cooperation is more frequent than suggested by models based on rational behaviour

Prisoner's Dilemma

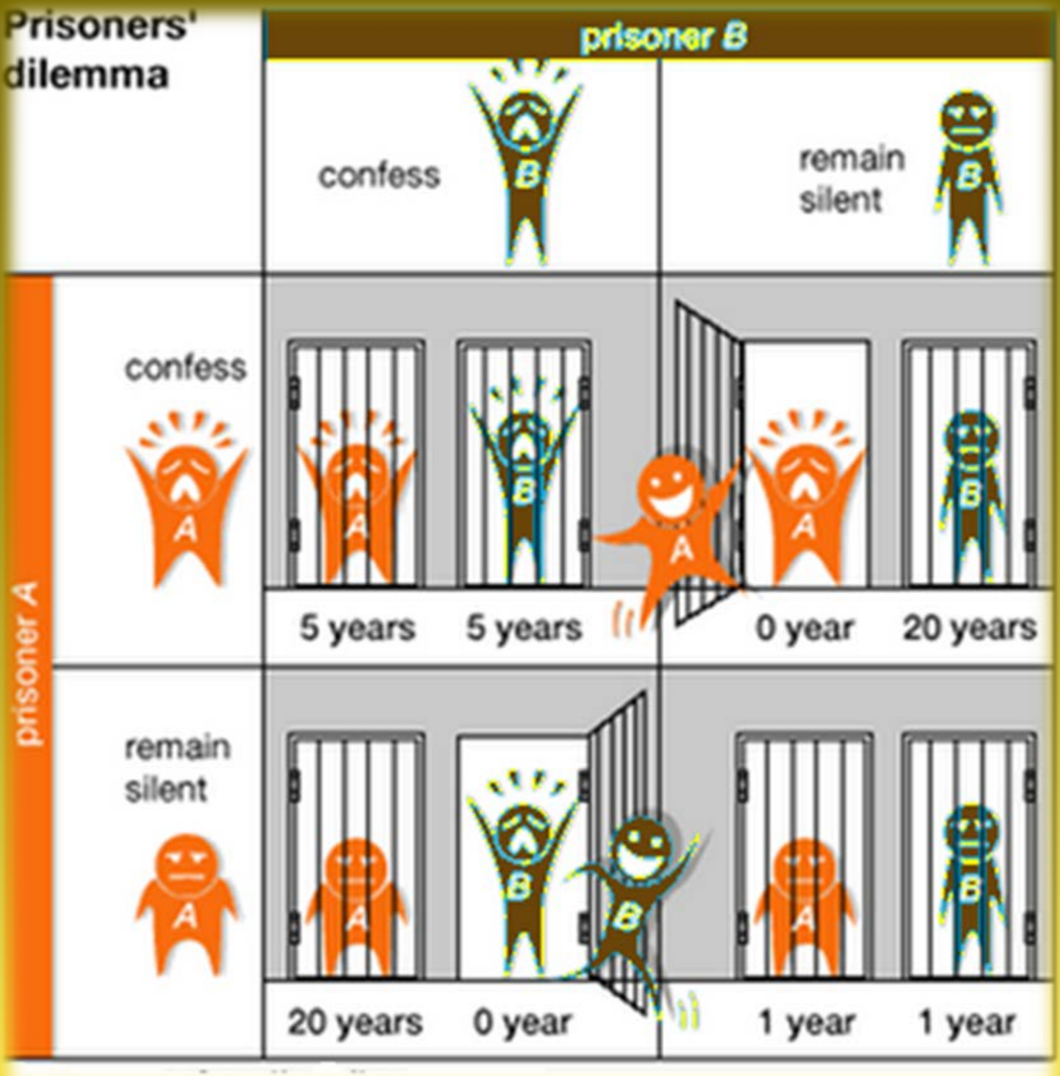
Two partners in crime are separated into separate rooms at the police station and given a similar deal.

If one implicates the other, he may go free while the other receives a 20 years in prison.

If neither implicates the other, both are given moderate sentences (1 year)

If both implicate the other, the sentences for both are severe (5 years).

Prisoner's Dilemma



Prisoner's Dilemma

	Cooperate	Defect
Cooperate	R, R	S, T
Defect	T, S	P, P

R	REWARD	for mutual cooperation
S	SUCKER's	payoff
T	TEMPTATION	to defect
P	PENALTY	for mutual defection

With $T>R>P>S$ and $R > (T+S)/2$

Prisoner's Dilemma

(D) is dominant for player 1

	Cooperate	Defect
Cooperat e	R	S
Defect	T	P

and player 2

	Cooperate	Defect
Cooperate	R	T
Defect	S	P

T>R
P>S

Both are better off if the other cooperates T>P

Prisoner's Dilemma

Each player has a dominant strategy to implicate the other. Thus in equilibrium each receives a harsh punishment, but both would be better off if each remained silent.

In simple instances, rational decision prevails. Always defect.

However, in the iterative defect is not always optimal and that mutual cooperation can cause a net gain in the two agents

In a repeated or iterated prisoner's dilemma, cooperation may be sustained through trigger strategies.

From the individual point of view desertion is the rational choice, while cooperation is the collective rational behavior.

The lack of cooperation is the tragedy of the commons. A situation in which multiple individuals, motivated only by self-interest and acting independently but rationally, end up destroying a limited shared good, even when it is clear that it is in their interest, either as individuals or together that such destruction do not happen.

Prisoner's Dilemma

	Cooperate	Defect
Cooperate	R	S
Defect	T	P

T>R

P>S

$P(D, D) > P(C,D) \Rightarrow$ ESS? $P(D, D) > P(C,D) \Rightarrow P > 0$

$P(C, C) > P(D,C) \Rightarrow$ ESS? $P(C, C) > P(D,C) \Rightarrow R > T$ **False**

Prisoner's Dilemma

Payoff Matrix

$$A = \begin{pmatrix} R & S \\ T & P \end{pmatrix}$$

$$f_C = (1,0)A\begin{pmatrix} x \\ 1-x \end{pmatrix}$$

$$f_D = (0,1)A\begin{pmatrix} x \\ 1-x \end{pmatrix}$$

$$\overline{f} = (x,1-x)A\begin{pmatrix} x \\ 1-x \end{pmatrix}$$

Replicator Equation

$$\frac{dx}{dt} = x\left(f_C - \overline{f} \right)$$

Prisoner's Dilemma

$$\frac{dx}{dt} = x(f_C - \overline{f})$$

$$\overline{f} = x f_C + (1-x) f_D$$

$$f_C = xR + (1-x)S$$

$$f_D = xT + (1-x)P$$

$$\frac{dx}{dt} = x(1-x)(f_C - f_D)$$

$$\frac{dx}{dt} = x(1-x)(x(R-T) - (1-x)(P-S))$$

With T>R>P>S and R > (T+S)/2

$$\frac{dx}{dt} = 0 \Rightarrow x = 0$$

$$x = 1 \text{ is unstable, } \left| \frac{P-S}{(R-T)+(P-S)} \right| > 1$$

Axelrod Tournament

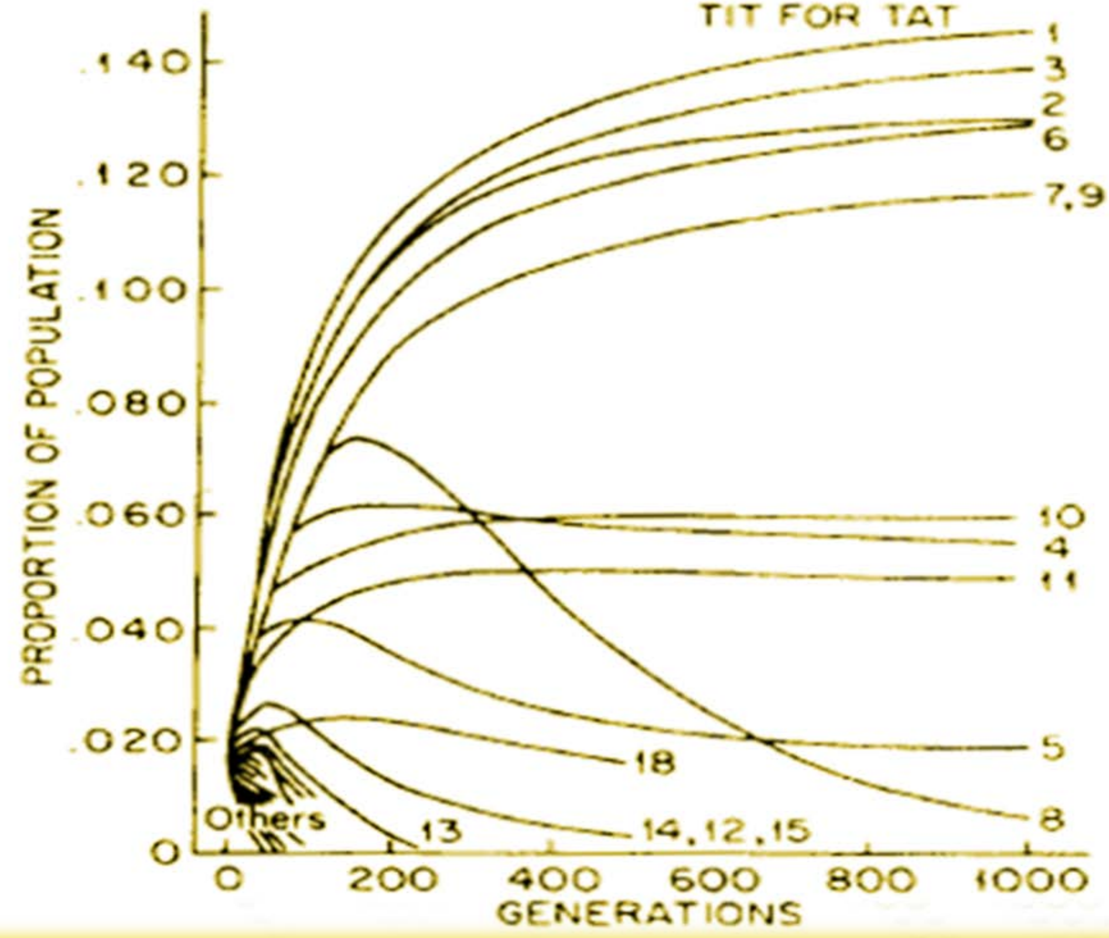
R. Axelrod invited a group of Game Theory researchers to propose different strategies for an iterated P.D.

Each strategy played against all the others including itself and a randomly alternating strategy

Prisoner's Dilemma

Tit for tat (TFT)	• Always cooperate on the first round; defect only after the other player has defected
Tit for 2 tats(TF2T)	• Always cooperate on the first round; defect only after the other player has defected two consecutive times
Suspicious Tit for tat (STFT)	• Similar a *TFT*, but starts defecting
Naive Probe(SI)	• Starts cooperating , defects after a defection and also sporadically
Remorse probe(SR)	• Similar a *SI*, but never avenges a defection responding his own defection
Explorer (E)	• Starts defecting and if the opponent responds defecting plays TFT,. If the opponent does not avenge a defection alternates D and C
Vindictive (V)	• Starts cooperating, but once the opponent defects, always defects
Free Rider (AD)	• Always defects
Cooperator (AC)	• Always cooperates

Iterated Prisoner's Dilemma



Axelrod Tournament

TFT was the winner, exploiting the following attributes

(i) being affable

(ii) Being sensitive to provocation

(iii) Not being rancorous

Don’t be envious

- Don’t play as if it were a zero sum game
- You don’t have to beat your opponent for you to do well

Be nice (don’t be the first to defect)

- Start by cooperating, and reciprocate cooperation

Retaliate appropriately

- Always punish defection immediately,
- But use “measured” force — don’t overdo it

Don’t hold grudges

- Always reciprocate cooperation immediately

Prisoner’s Dilemma

A spatial variant of the iterated prisoner's dilemma

A model for cooperation vs. conflict in groups

It shows spread of

altruism

exploitation for personal gain

in an interacting population of agents learning from each other

Initially population consists of cooperators and a certain amount of defectors

Advantage of defection is determined by the 'payoff matrix'

A player can change strategy, by selecting the most favourable strategy from itself and its direct neighbours

Prisoner's Dilemma

In spatial games, players interact with their neighbors and adopt the state that is more convenient.

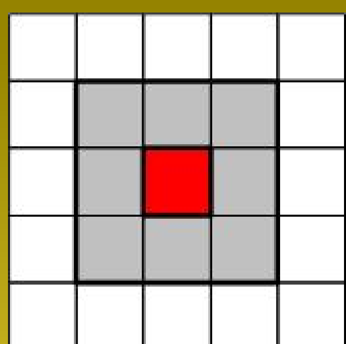
Reveals the importance of social topology (complex networks) and bounded rationality (expectation based on local conjecture) to describe how cooperative behavior spreads in the population.

In models where complex networks are considered, the cooperators can invade defectors.

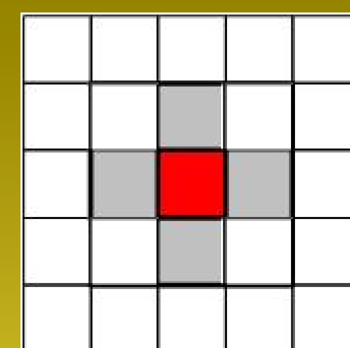
Prisoner's Dilemma

The interaction topology can be described by a network or graph

First models: Square lattices



Moore

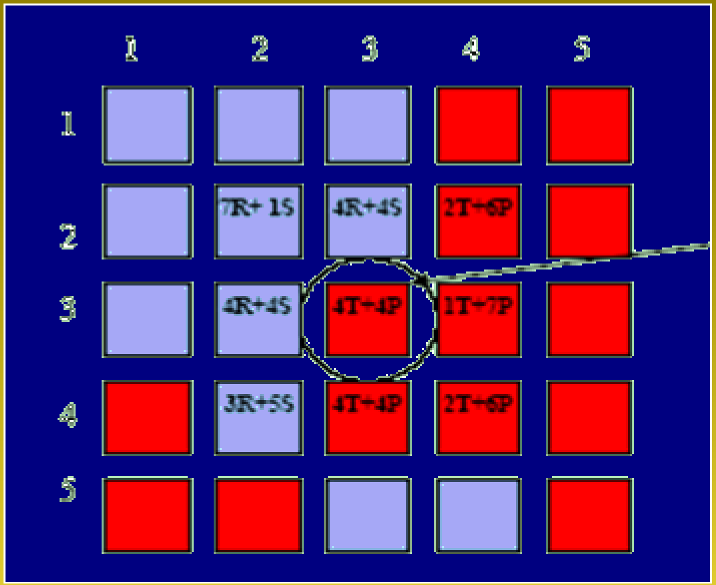


Von Neumann

Prisoner's Dilemma

Strategies : C (blue) D(red)

Consider a Moore neighborhood (8 neighbors)
The central site imitates the strategy of the neighbor who accumulates the greatest benefit



Site(2,2) is the selected one ⇒ (3, 3) changes from red to blue

Prisoner's Dilemma

	Cooperate	Defect
Cooperate	1	0
Defect	b	ε

Prisoner's Dilemma

Depending on the value of b there can be four scenarios. When a single defector is inserted into a sea of cooperators, always expands to a 3 x 3 block of cooperators and then

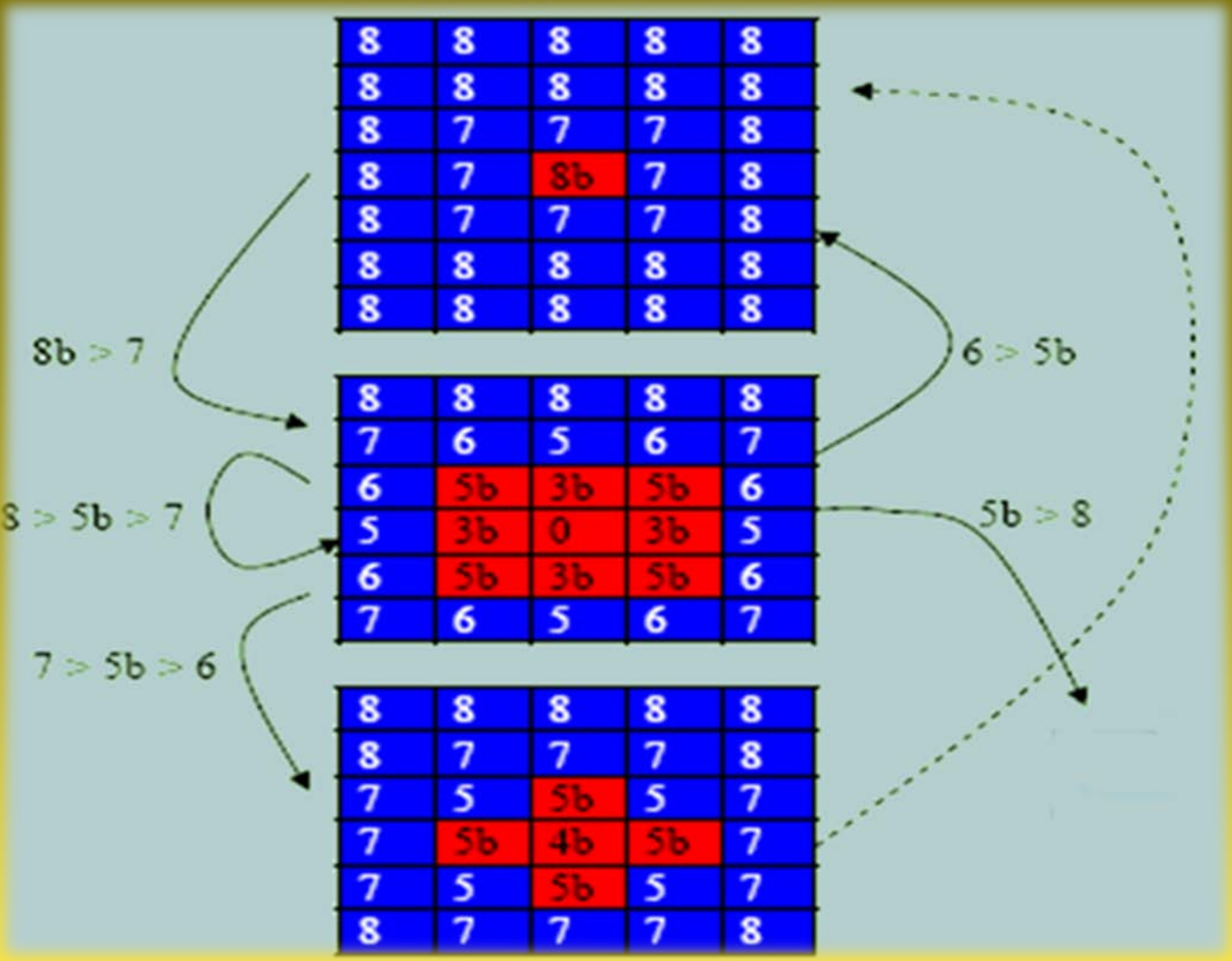
(i) returns the state of the starting block

(ii) remains there indefinitely

(iii) creates a cross-shaped cluster and returns to the initial state

(iv) spread defection

Prisoner’s Dilemma



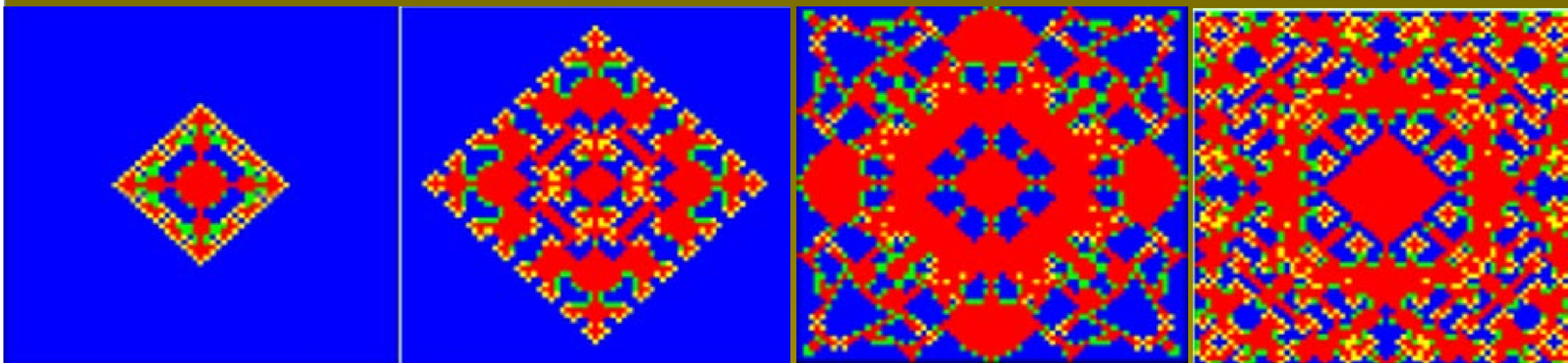
Prisoner's Dilemma

A cooperator can not expand in a sea of defectors

Cooperation can propagate only if inserted in a cluster, e.g. for b <3/2, It can start with a cluster 2 x 2, which in the next period evolves to 4 x 4, and then to 6 x 6.

Space games allow you to show the possibility of co-existence of cooperation and defection

Prisoner's Dilemma



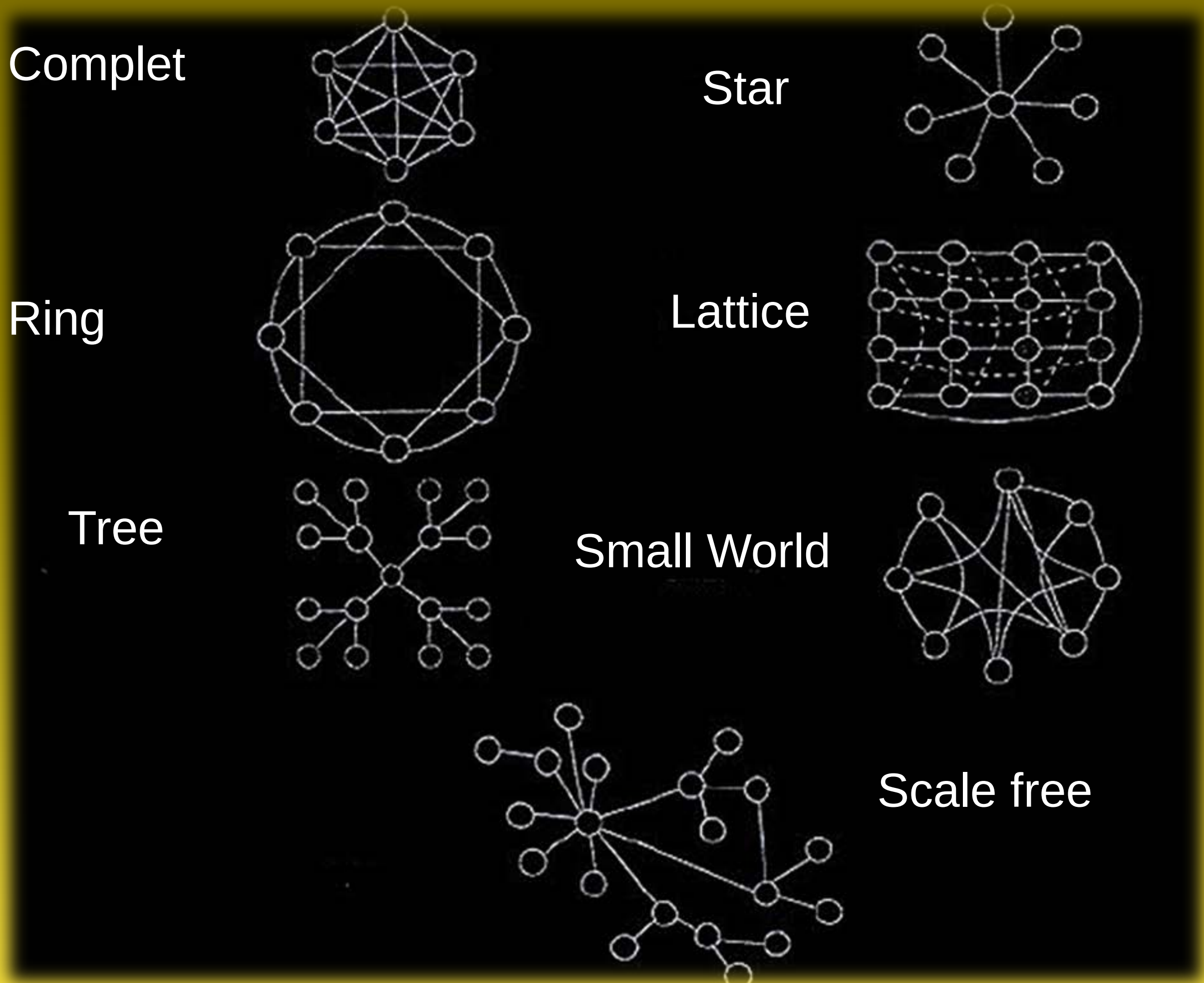
Cooperators

No Cooperators

NC → C

C → NC

Prisoner's Dilemma



Prisoner's Dilemma

	Synchronous updating		Asynchronous updating	
	Proportion cooperating	Rounds to steady-state	Proportion cooperating	Rounds to steady-state
Complete	no cooperation	1	no cooperation	27.2 (4.19)
Star	P(all Cs) = ½ P(all Ds) = ½	1	P(all Cs) = ½ P(all Ds) = ½	27.3 (4.10)
Ring	0.967 (.0075)	189.2* (57.45)	0.998 (0.0010)	very large number of rounds
Grid	0.358 (0.071)	no steady state	0.538 (0.071)	no steady state
Tree	P(all Cs) = 0.6 P(all Ds) = 0.4	14.4 (1.12)	0.894 (0.003)	no steady state
Small-world	0.713 (0.021)	150.2* (96.79)	0.700 (0.014)	no steady state
Power	0.947 (0.011)	20.5 (5.22)	0.944 (0.012)	58.4 (13.0)

Ultimatum Game

Two players interact to decide how to divide a sum of money **M** that is given to them.

The first player, the offerent **O**, proposes how to divide the sum between the two players **(M-x, x)**

The second player, the acceptor **A**, can either accept or reject this proposal.

If the **A** player rejects the offer, both receive nothing.

If the **A** player accepts, the money is split according to the proposal.

Rational players: Offer x very small, Accept x>0

Ultimatum Game

One solution, Nash equilibrium

$$D = (M - \varepsilon, \varepsilon)$$

It is the "rational" solution based on the axioms

1 Each player prefers a payoff α to β if $\alpha > \beta$

2 Both players know 1

3 **O** can calculate the optimal offer

Ultimatum Game

University students

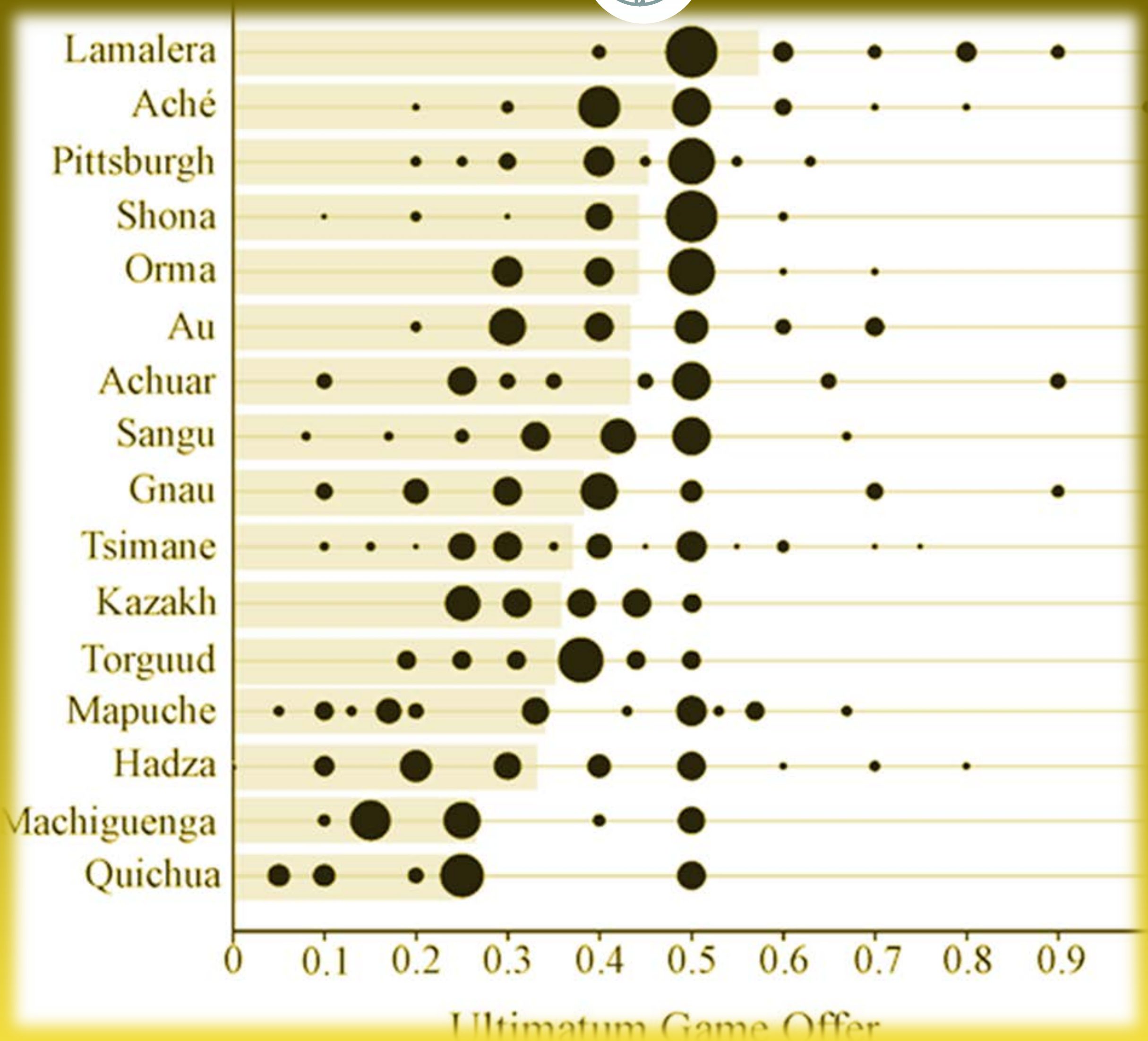
Modal offer = 50%.

Mean offer = 40%–50%.

Offers < 20% rejected

Ultimatum Game

Ultimatum Game



Ultimatum Game - Replicator

The trade amount is 1

The players have equal chance of being **A** or **O**

When *i* is **O** offers p_i

When *i* is **A** rejects any offer below q_i

The strategy of a player is defined by (p,q)

$$1 - p_i \geq q_i$$

Ultimatum Game - Replicator

Payoff of a player (p_1 , q_1)

$1- p_1 + p_2$	$p_1 \geq q2 \wedge p_2 \geq q_1$
$1 - p_1$	$p_1 \geq q2 \wedge p_2 < q_1$
$p2$	$p_1 < q2 \wedge p_2 \geq q_1$
0	$p_1 < q2 \wedge p_2 < q_1$

Ultimatum Mini Game

Offers l (low), h (high): $0 < l < h < 1/2$

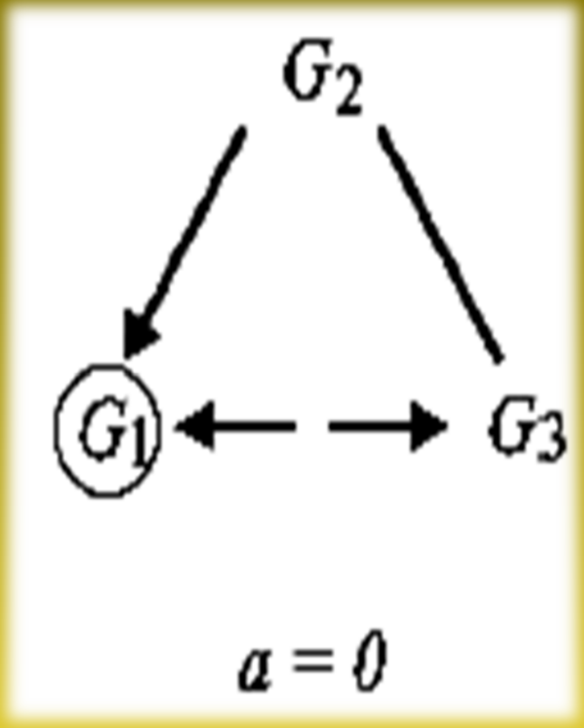
- 4 strategies: G1 - G4
- G1= (l,l) : reasonable
- G2 =(h,l) : altruist
- G3 = (h,h) : fair
- G4 = (l,h) : ambicious

Table 1. Payoff matrix for the mini-ultimatum game.

	G_1	G_2	G_3	G_4
G_1	1	$1-l+h$	h	l
G_2	$1-h+l$	1	1	$1-h+l$
G_3	$1-h$	1	1	$1-h$
G_4	$1-l$	$1-l+h$	h	0

Ultimatum Mini Game

- G1 is a fix point
- A mixed population G1 and G3 converge to G1 or G3
- A mixed population G1 and G2 tends to G1
- A mixed population G2 and G3 is neutrally stable



Ultimatum Mini Game

Consider that accepting a low offer affects the reputation

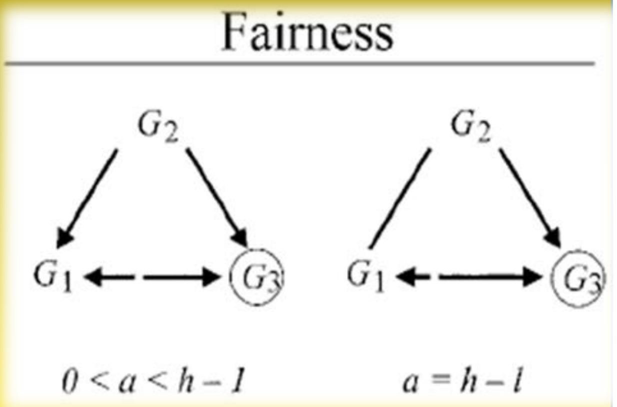
The mean offer of a O-h to a A-l is reduced by a

In an extreme case, when h-l=a, O has all the information about A, G3 is stable, G1 y G2 are neutrally stable.

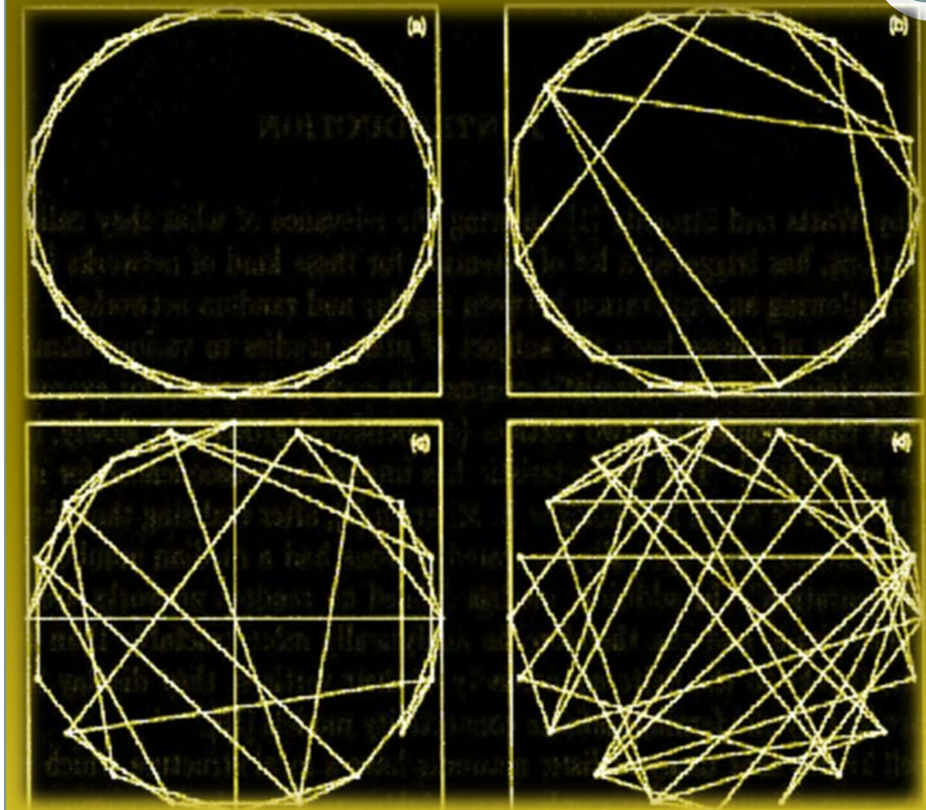
With information, fairness dominates

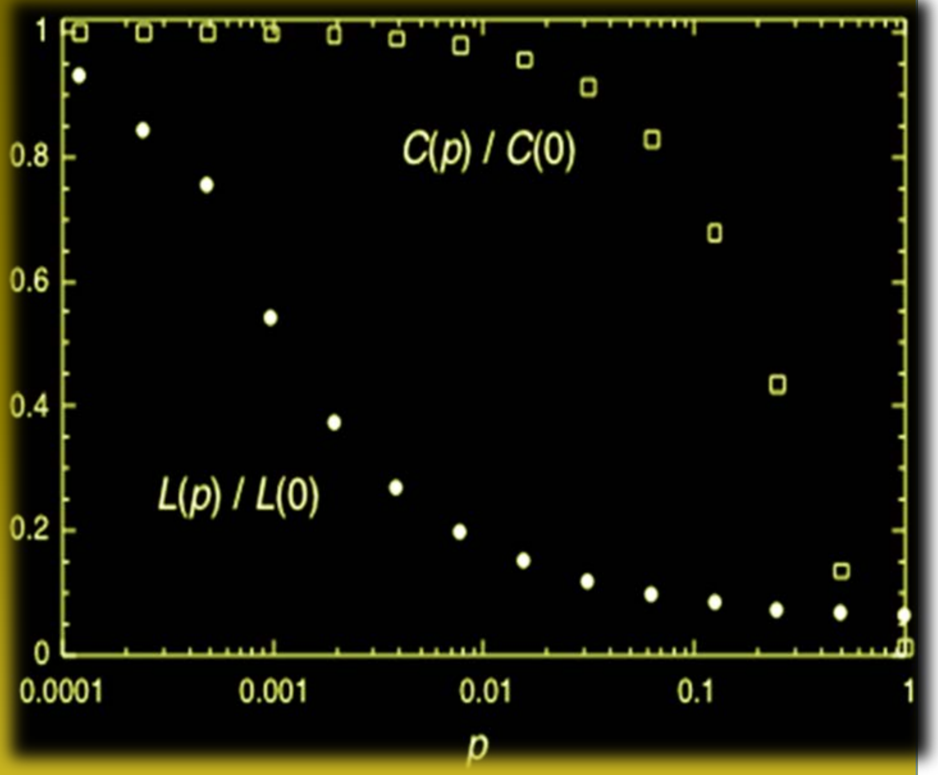
Table 2. Payoff matrix for the mini-ultimatum game with information.

	G_1	G_2	G_3	G_4
G_1	1	$1 - l + h - a$	$h - a$	l
G_2	$1 - h + l + a$	1	$1 - a$	$1 - h + l$
G_3	$1 - h + a$	$1 + a$	1	$1 - h$
G_4	$1 - l$	$1 - l + h$	h	0



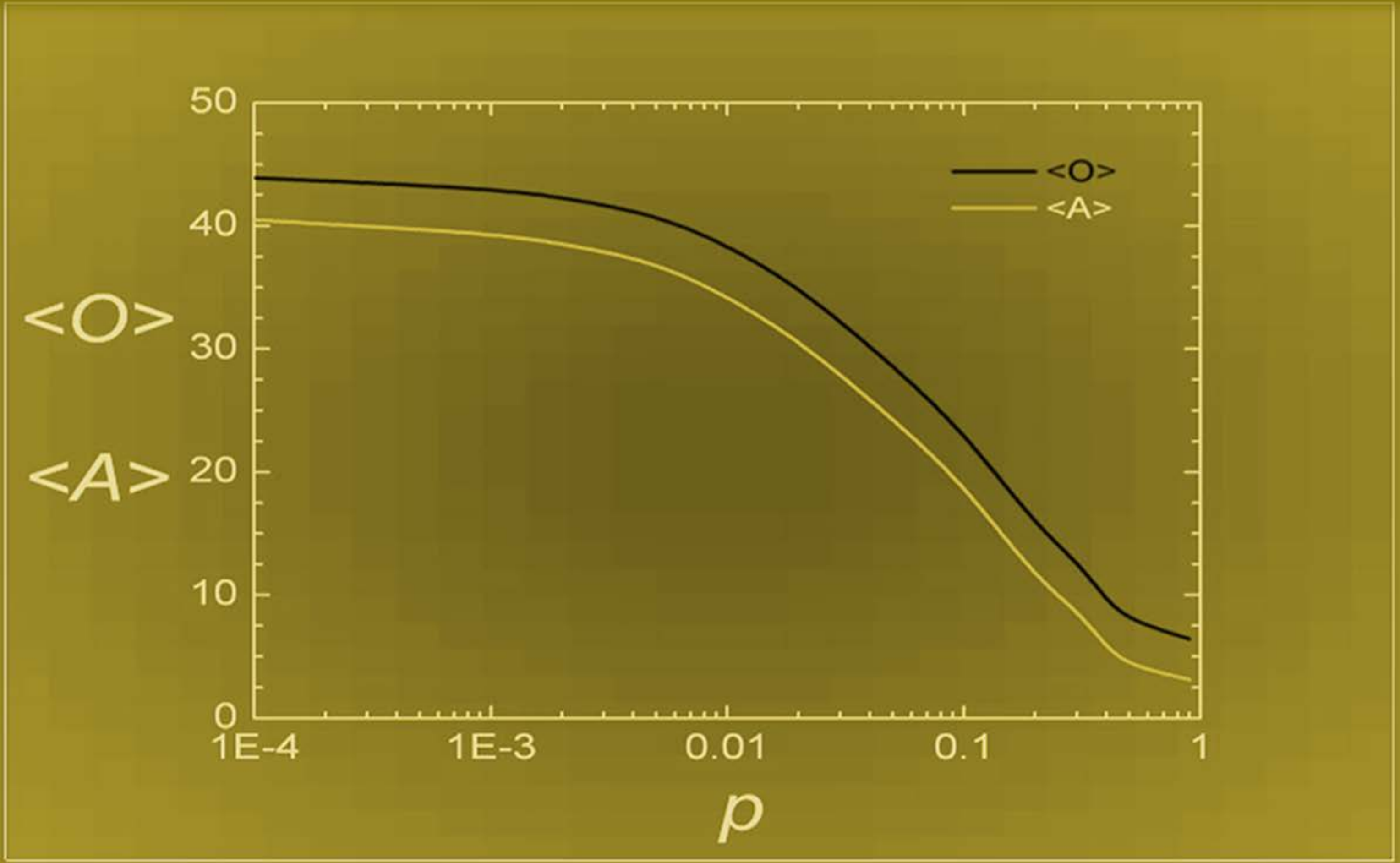
Evolutionary Ultimatum Game





p: disorder parameter

Spatial Ultimatum Game



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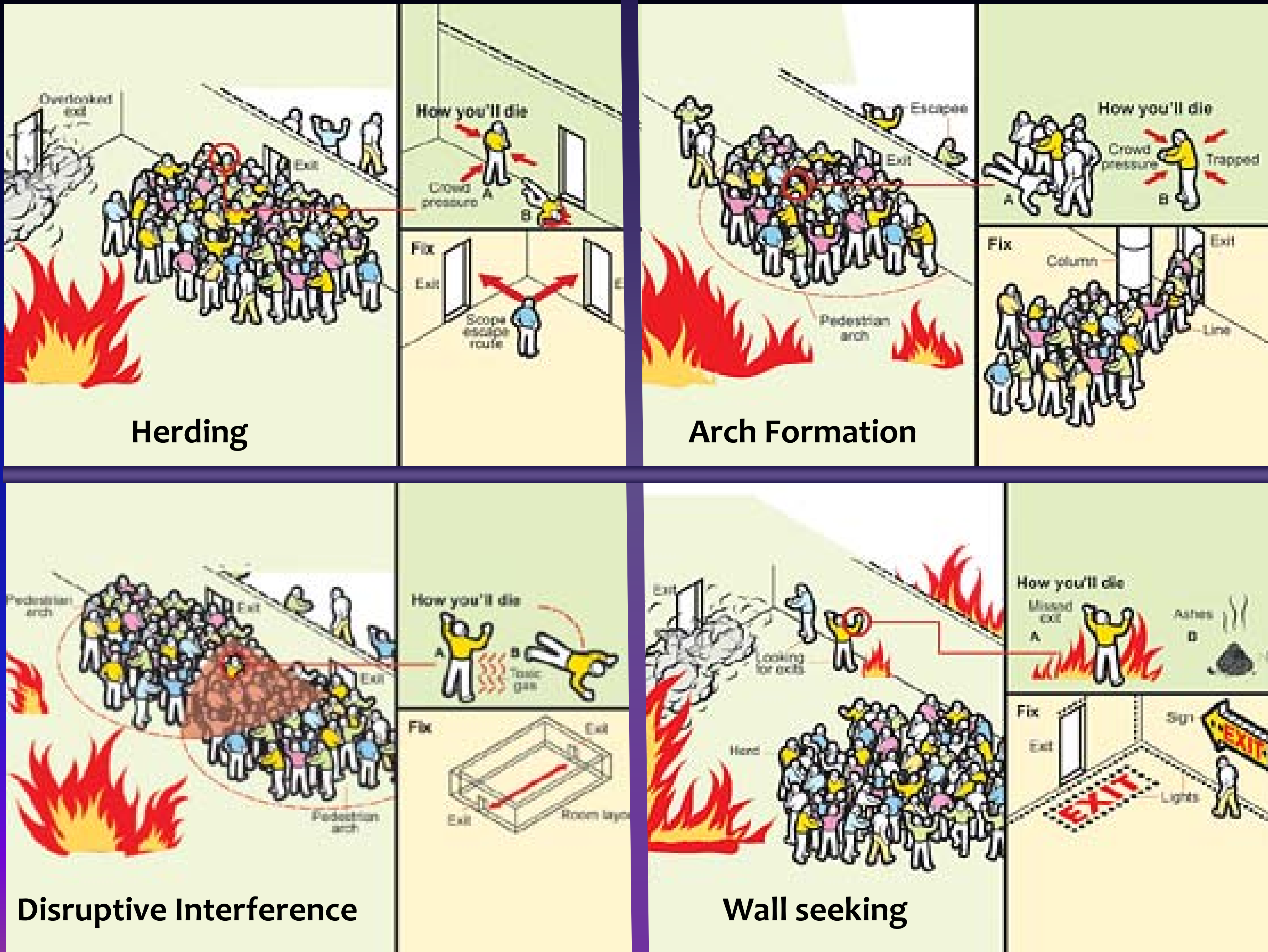
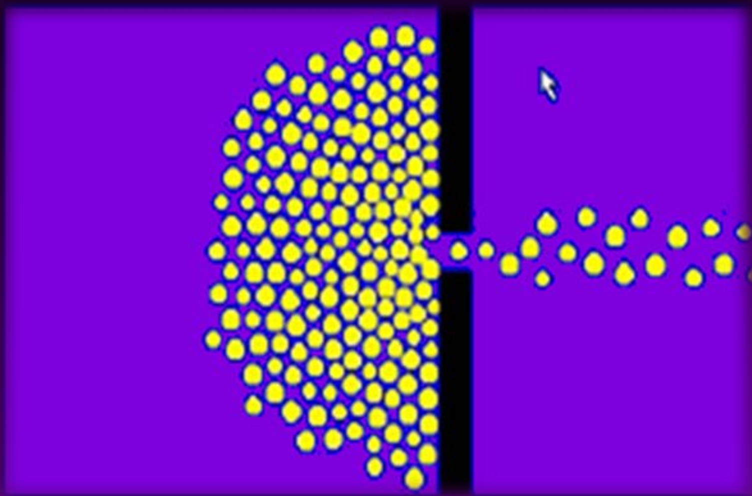

Overlooked exit
Exit
How you'll die
Crowd pressure
A
B
Fix
Exit
Scope escape route
Herding
Escapee
Exit
Pedestrian arch
How you'll die
Crowd pressure
Trapped
A
B
Fix
Column
Exit
Line
Arch Formation
Pedestrian arch
Exit
Exit
Pedestrian arch
How you'll die
A
B
Toxic gas
Fix
Exit
Exit
Disruptive Interference
Exit
Looking for exits
Herd
How you'll die
Missed exit
A
Ashes
B
Fix
Exit
Sign
EXIT
EXIT
Lights
Wall seeking

Agent based models



Each individual has personal attributes

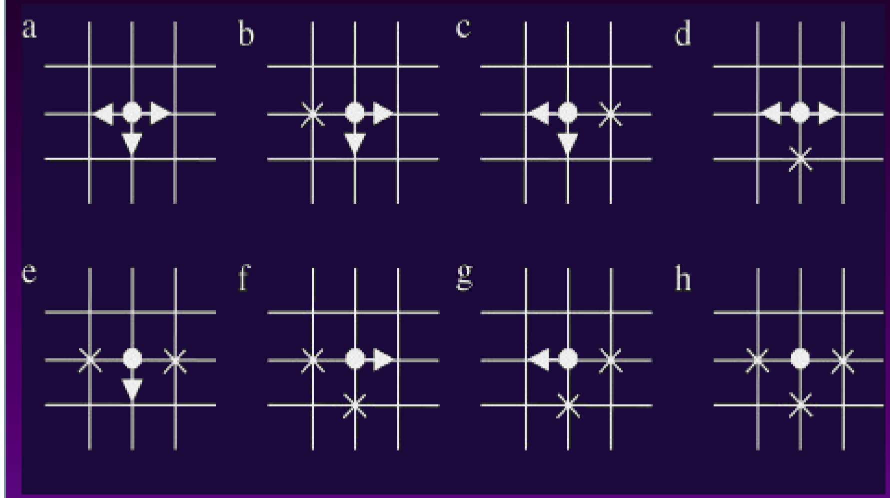
Rational agents get optimal escape route

Simulation based on Social Forces models or collision avoiding

Newton's Second Law
Repulsive interaction force
Interactions with the walls

$$m_i \frac{d\vec{v}_i}{dt} = m_i \frac{\vec{v}_i^o(t)\vec{e}_i^o(t) - \vec{v}_i(t)}{\tau_i} + \sum_{j \neq i} \vec{f}_{ij} + \sum \vec{f}_{iw}$$

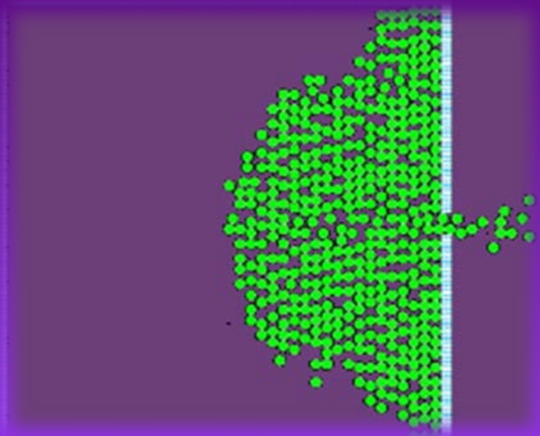
Gas Lattice models



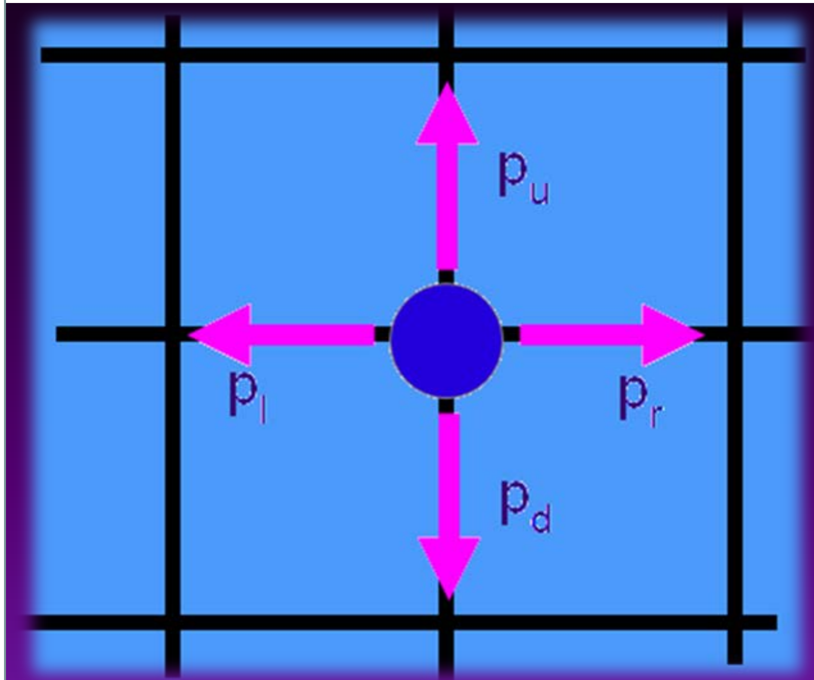
Discretize space into cells

Individuals on a grid

Lack of social behaviour



The movement of pedestrians



At each time step, pedestrians prioritize the target direction (up, down, left, right) according to rational choice (reach the exit)

There is also a probability *R<1* of a non rational choice (random selection).
The movement will be decided according to a combination of both strategies

$$p_u = \frac{1}{4}R + (1-R)\frac{(-\cos(\alpha))(1-\mathrm{sgn}(\cos(\alpha))}{2Z} \qquad p_r = \frac{1}{4}R(1-R)\frac{(-\sin(\alpha))(1-\mathrm{sgn}(\sin(\alpha))}{2Z}$$

$$p_d = \frac{1}{4}R + (1-R)\frac{\cos(\alpha)(1+\mathrm{sgn}(\cos(\alpha))}{2Z} \qquad p_l = \frac{1}{4}R + (1-R)\frac{\sin(\alpha)(1+\mathrm{sgn}(\sin(\alpha))}{2Z}.$$

The interaction among pedestrians

Once they have made a choice, the pedestrians try to move to the selected site. But there are some restrictions.....

1) The site must be empty
2) The site might have been chosen by more than one pedestrian

When the site is empty and was chosen by more that one pedestrian, there is a competition among the interested individuals to decide who will make the move

There is a sort of game between the involved competitors, where individuals can adopt either a ***cooperative*** or a ***defective*** (non cooperative) behaviour

The "payoff" matrix

	C	D
C	1/2	0
D	1/P	1/(2P)

If $1<P<2$ ⟶ Prisoner's Dilemma

If $2<P$ ⟶ Stag Hunt

	(n-1)C	(n-m-1)D
C	$1/n$	0
D	$1/P$	$1/((n-m)^2P)$

The temptation to defect

	$(n-1)C$	$(n-m-1)D$
C	$1/n$	0
D	$1/P$	$1/((n-m)^2P)$

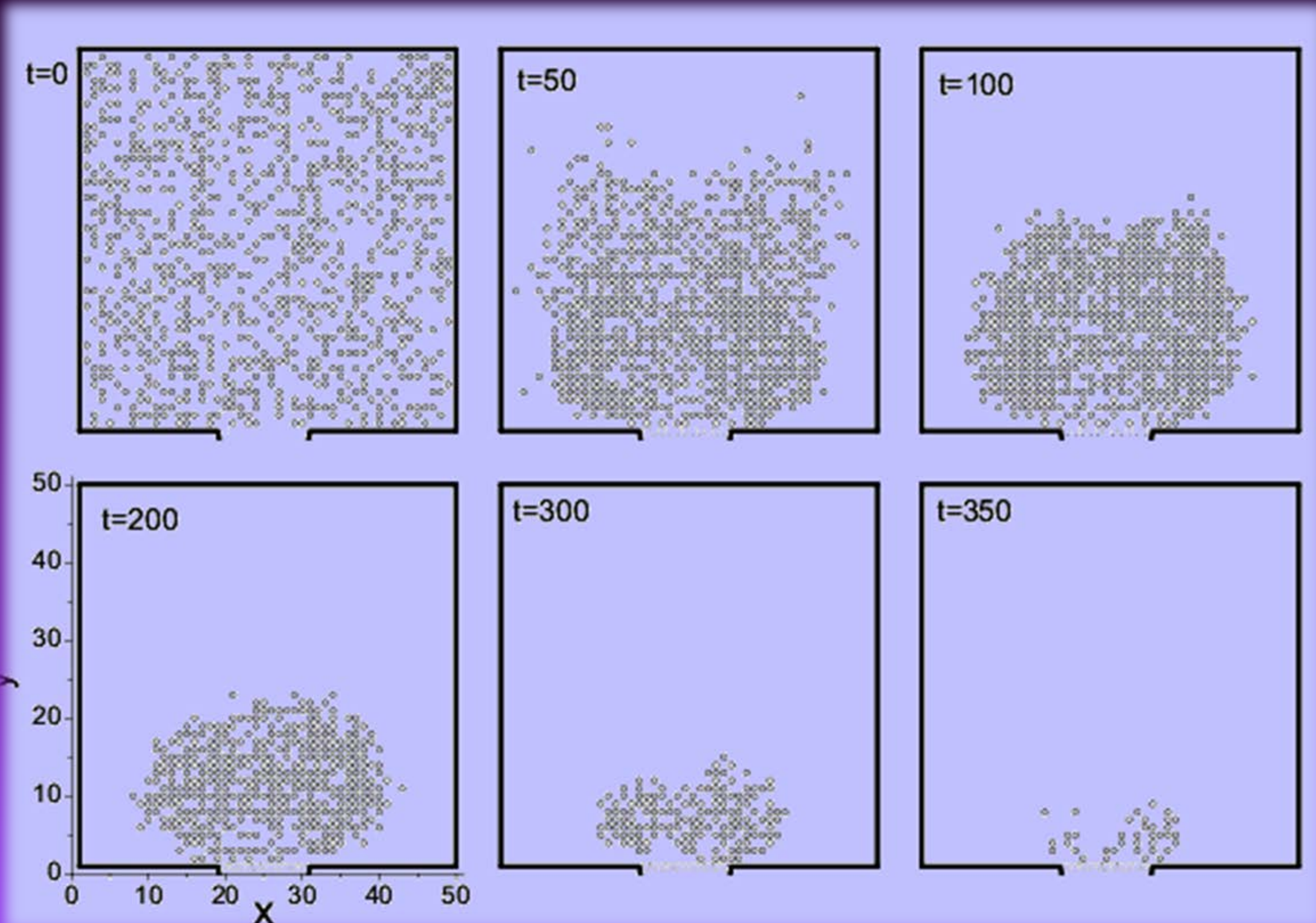
From the individual point of view defecting is always better than cooperate ($P<2$)

From the global point of view, at each encounter of two cooperators the chance of at least one of them moving to the desired site is higher than when defectors are involved

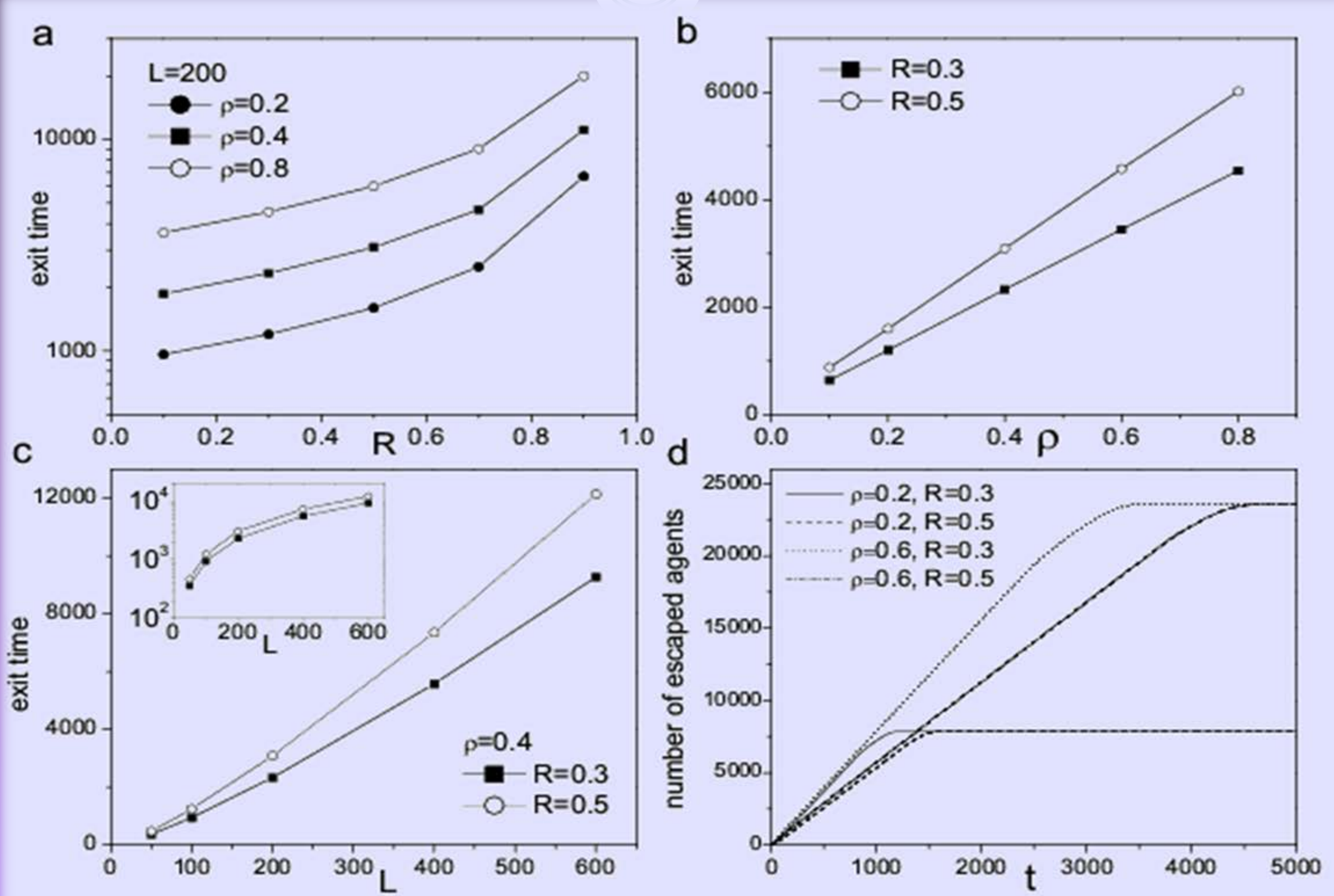
This is the analogous conditions on P.D. and S.H. iterative games:

$P(D,C)+P(C,D)<2\ P(C,C) \longrightarrow 1/P+0<2\times 1/2$

Snapshots

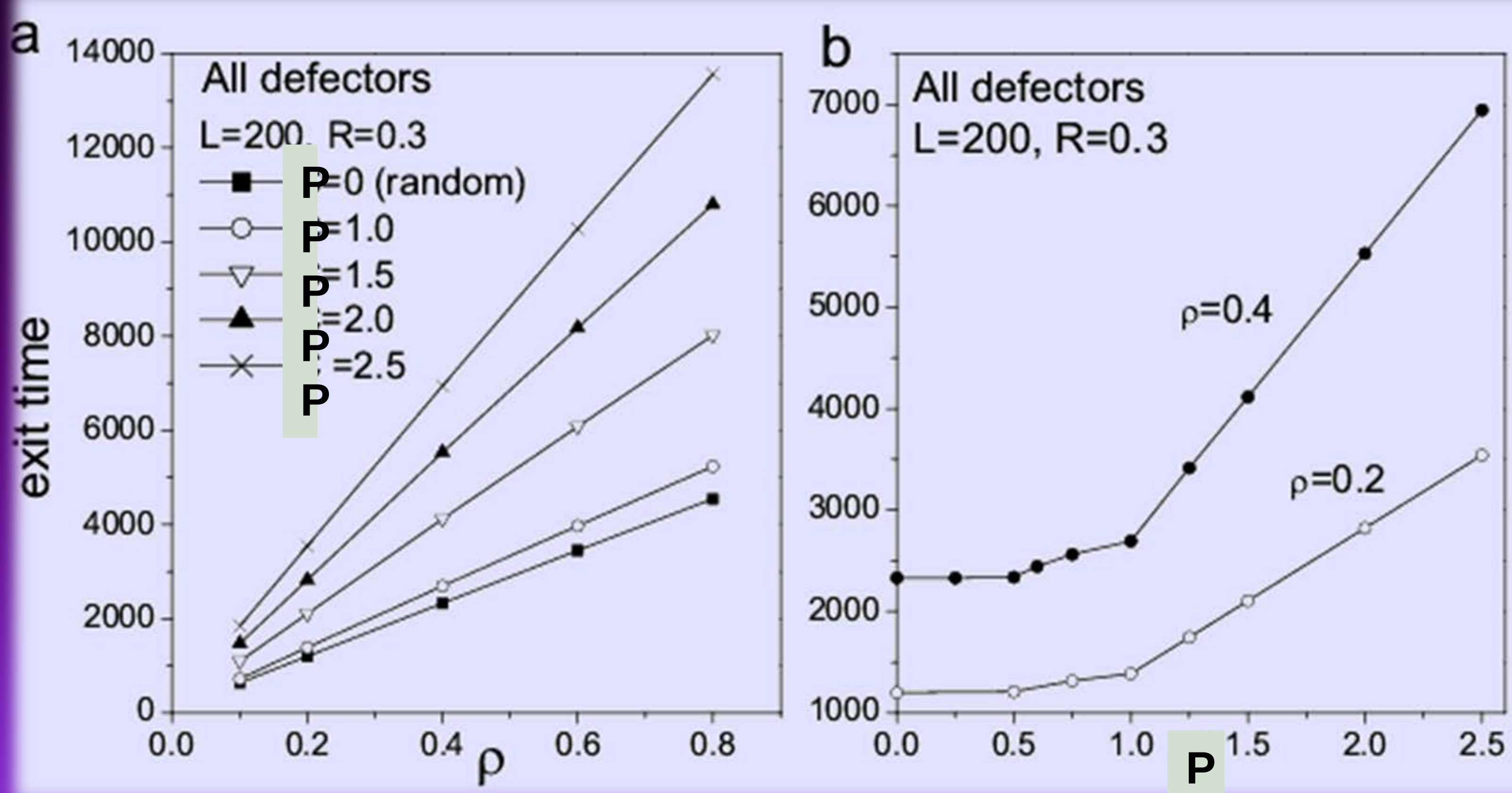


Only cooperators



R: probability of random choice ρ: Initial density L: Size of the room

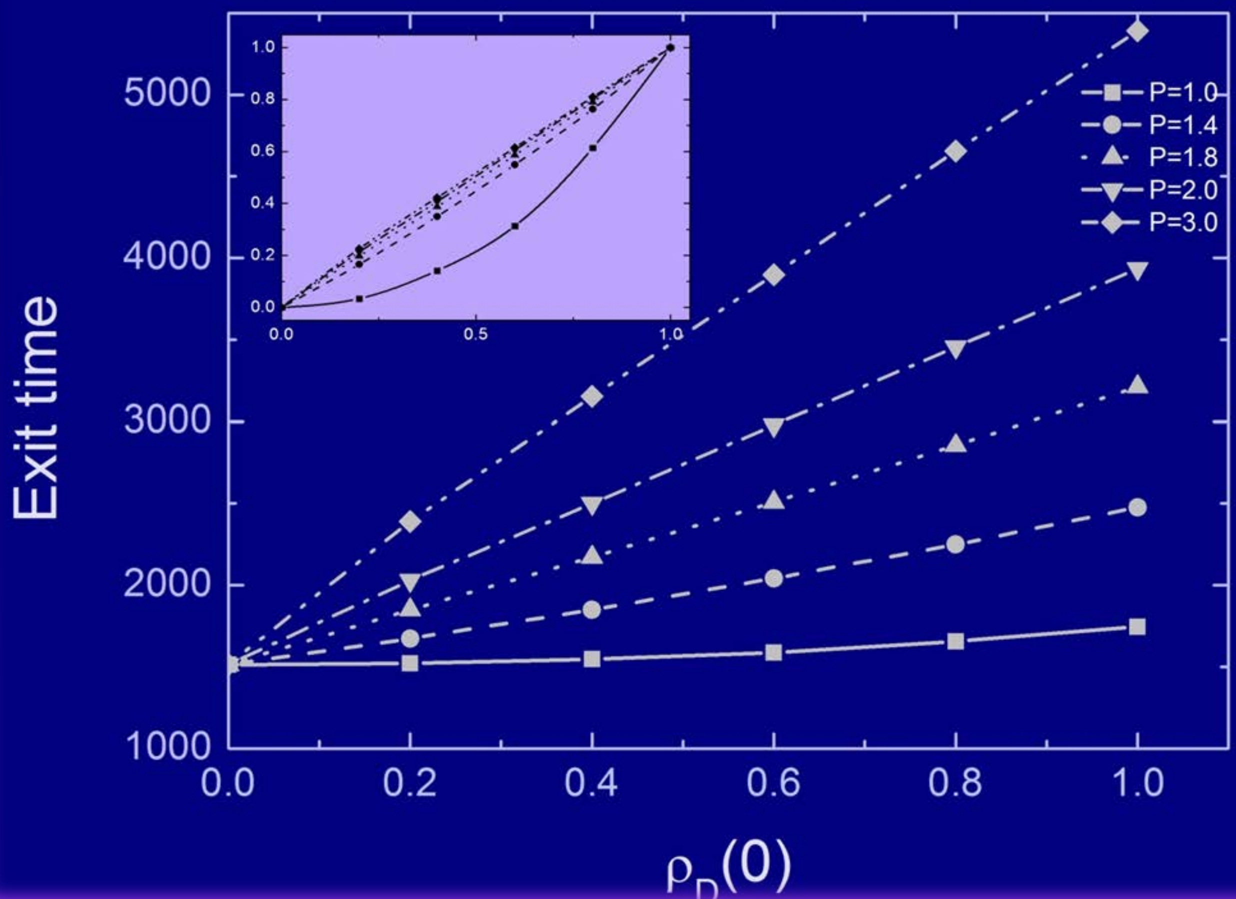
Only defectors



ρ: Initial density P: Defector penalization

Mixed populations

$t_n(\rho_D) = (t(\rho_D) - t(0))/t(1)$



$\rho_D(0)$: Initial D proportion

P: Defector penalization

Rationale

Cooperators are always overcome when competing with defectors

Nevertheless, the emerge and prevalence of cooperation has been observed in several examples as an effect of the advantage of mutual cooperation

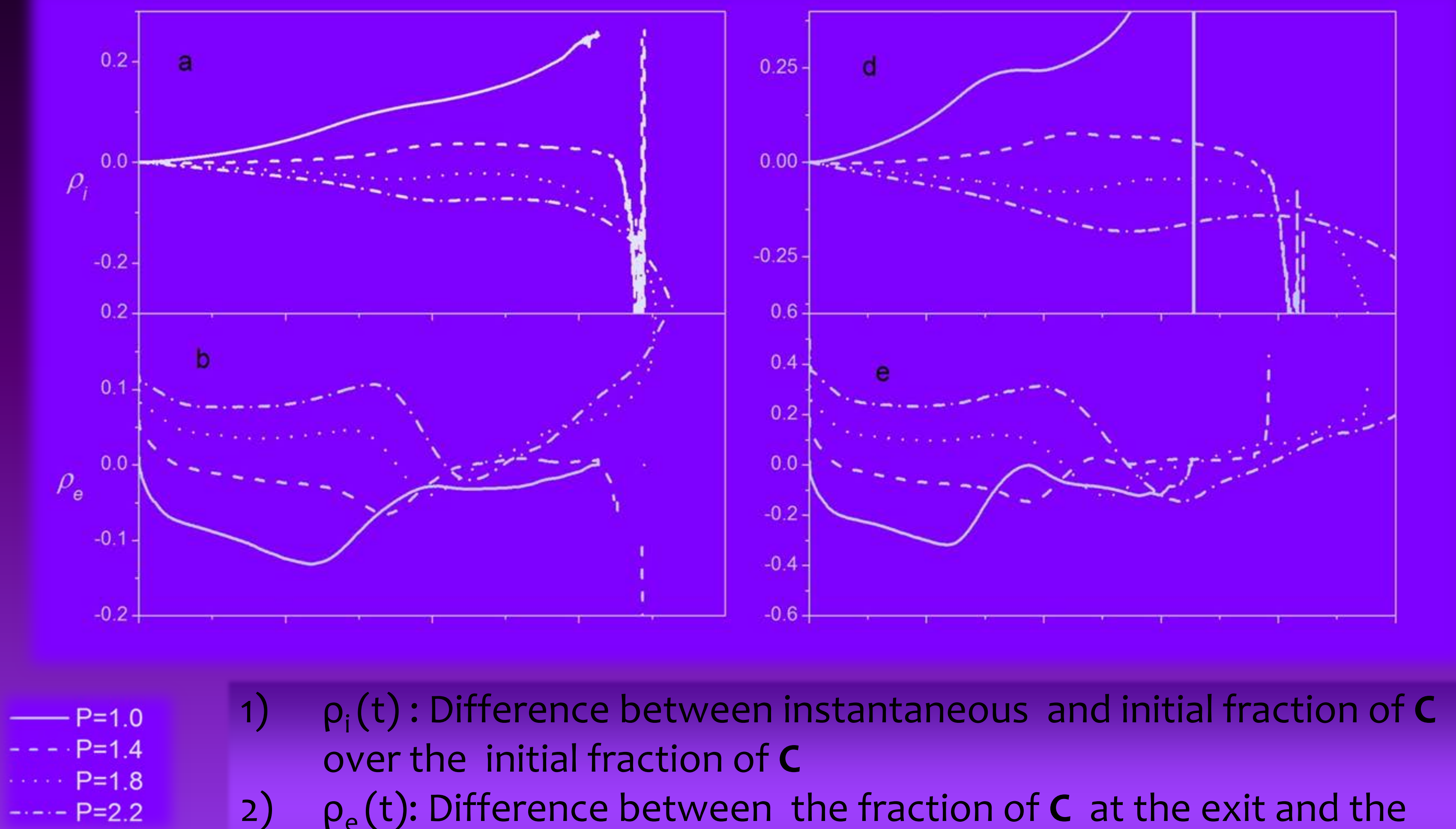
To take profit from mutual cooperation, cooperation must conform clusters, resisting the invasion by defectors

We will measure

1) $\rho_i(t)$: Difference between instantaneous and initial fraction of **C** over the initial fraction of **C**
2) $\rho_e(t)$: Difference between the fraction of **C** at the exit and the fraction of **C** within the room over the fraction of **C** within the room
3) C_C: Ratio between the fraction of **C** neighbours of a **C** and the instantaneous fraction of **C** in the room

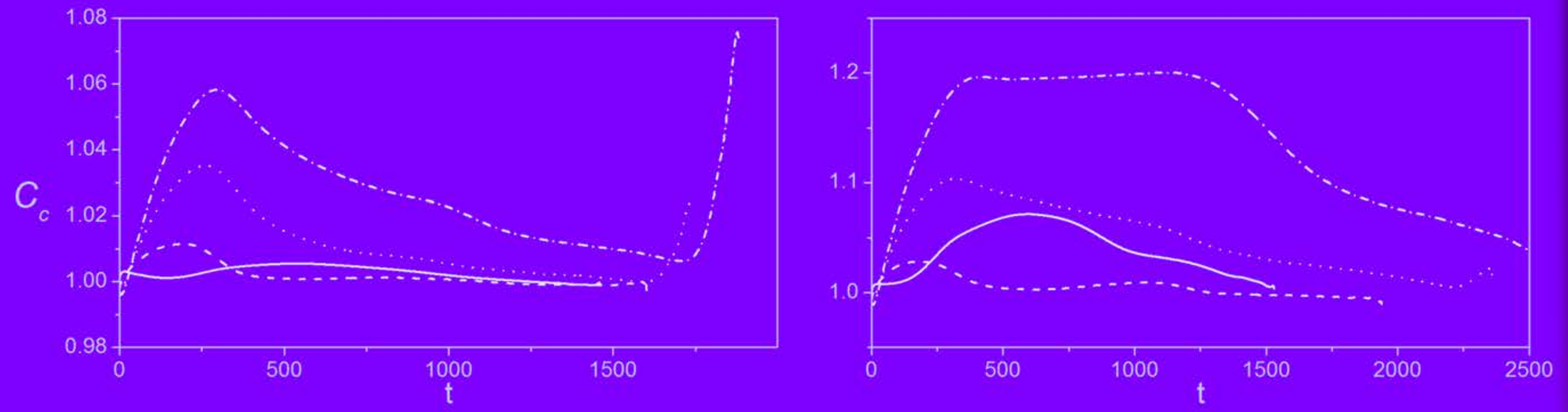
We look for effects of mutual cooperation and the formation of clusters of cooperators

Cooperators dynamics: Fractions of C



1) $\rho_i(t)$: Difference between instantaneous and initial fraction of **C** over the initial fraction of **C**
2) $\rho_e(t)$: Difference between the fraction of **C** at the exit and the fraction of **C** within the room over the fraction of **C** within the room

Cooperators dynamics: Clustering



P=1.0
P=1.4
P=1.8
P=2.2

C_C: Ratio between the fraction of **C** neighbors of a **C** and the instantaneous fraction of **C** in the room

Contents

Naming Games

Interactions of N agents who try to communicate and need to conform a lexicon, i.e. a system of Name-Object associations

The agents can keep in memory different words, can speak or hear

At each time step 2 agents, a transmitter and a receiver, are randomly selected

The transmitter communicates a name to the receiver
(if the transmitter has nothing in memory –at the beginning- it invents a name)

-if the receiver already has the name in its memory ⟶ success
else ⟶ failure

Naming Games

Success : the speaker and hearer retain the uttered word as the correct one and cancel all other words from their memory

Speaker	Hearer		Speaker	Hearer
JUYFE	PUFC	→	PUFC	PUFC
PUFC	GIUT			
RETS	BOPI			

Failure : the hearer adds to its memory the word given by the speaker

Speaker	Hearer		Speaker	Hearer
JUYFE	KREC	→	JUYFE	KREC
PUFC	GIUT		PUFC	GIUT
RETS	BOPI		RETS	BOPI
				PUFC

Bimatrix Naming Game

In the model, the individuals can communicate through a simple system of sounds or signals .

The use and interpretation of each one of the signals is defined by a couple of matrices, the transmitter matrix *T* and the receiver matrix *R.*

There are *s* sounds or signal and *o* objects or concepts,
The element t_{ij} of the $o \times s$ transmitter matrix contains information about the probability that the individual refers to concept i^{th}, using the j^{th} signal.

The $s \times o$ receiver matrix contains the reciprocal information, i.e. r_{ij} is the probability that the individual associates the signal i^{th}, to the j^{th} concept.

R is not necessarily the transpose of T

Bimatrix Naming Games

Signal

Object

$$\begin{pmatrix} t_{11} & t_{12} & t_{13} & t_{14} \\ t_{21} & t_{22} & t_{23} & t_{24} \\ t_{31} & t_{32} & t_{33} & t_{34} \end{pmatrix}$$

Object

Signal

$$\begin{pmatrix} o_{11} & o_{12} & o_{13} \\ o_{21} & o_{22} & o_{23} \\ o_{31} & o_{32} & o_{33} \\ o_{41} & o_{42} & o_{43} \end{pmatrix}$$

$$\sum_{j=1}^{s} t_{ij} = \sum_{j=1}^{o} r_{ij} = 1$$

Bimatrix Naming Games

Communication Matrix: $C(a,b) = T^a R^b$

Perceived object

Signaled Object

$$\begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{31} \end{pmatrix}$$

$$c_{ij} = \sum_{k=1}^{s} t_{ik} r_{kj}$$

$$CP(a,b,k) = \sum_{i=1}^{s} t_{ik}^{a} r_{ki}^{b} = c_{kk}(ab)$$

Bimatrix Naming Games

Communicative Power: individuals *a* and *b*, object *k*

$$CP(a,b,k) = \sum_{i=1}^{s} t_{ik}^{a} r_{ki}^{b} = c_{kk}(ab)$$

Communicative Power between *a* and *b*

$$CP(a,b) = \frac{\sum_{k=1}^{o} CP(a,b,k)}{o} = \frac{Tr\, C(a,b)}{o}$$

Bimatrix Naming Games

Imitator

- The individual samples the transmission and reception behavior of the environment to build up its own behavior, by imitating the others.
- The imitators adopts the average transmission and
- reception behaviours of the system.

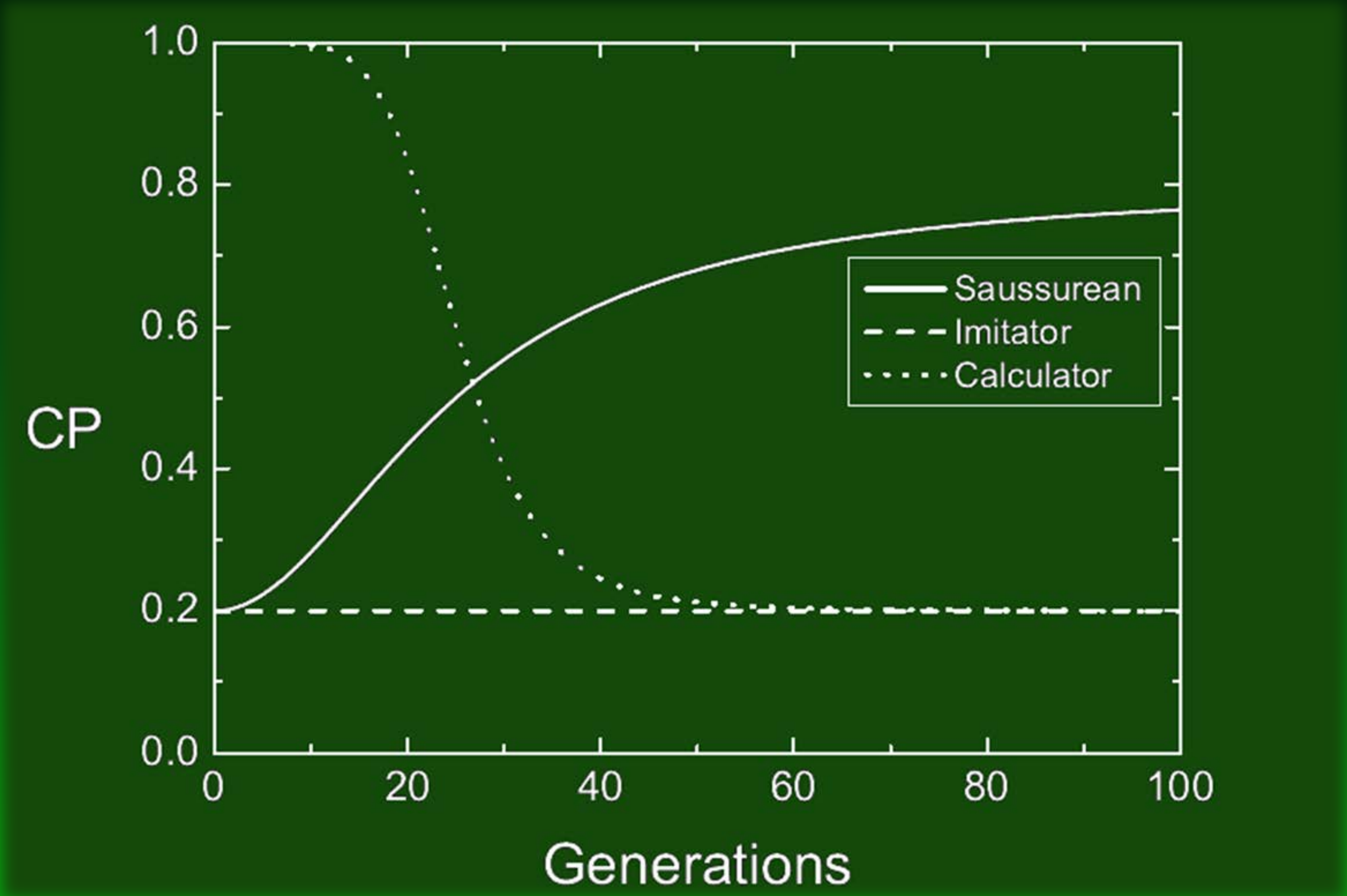
Calculator

- This individuals seeks to optimize its role as transmitter and receiver.
- With this rationale, the individuals adopts the mean transmission behavior to build up its receiver matrix and vice versa

Saussurean

- This individuals only samples the transmission behavior of the population and coordinates its reception behavior to be affine to its own transmission.

Bimatrix Naming Games



Bimatrix Naming Games

Each lexicon is a strategy

Individuals can be transmitters and receiver

The payoff is the communicative power

$$CP(a,b) = \frac{1}{2o} \sum_{i=1}^{o} \sum_{j=1}^{s} (t_{ij}^{a} r_{ji}^{b} + t_{ij}^{b} r_{ji}^{a})$$

Consider evolutionary dynamics

Bimatrix Naming Games

$$CP(a,a) = \frac{1}{o}\sum_{i=1}^{o}\sum_{j=1}^{s}(t_{ij}^{a} r_{ji}^{a})$$

The optimum *R matrix has* $r_{ji} = 1$ *when* t_{ij} *is the largest value*

The maximum communicative power will be obtained when *T has at least one 1 in every column* (if $o > s$) *or in every row (if* $o < s$*).*

When $o = s$ *then* $T = R^{\dagger}$*.*

Matrices containing either 0 or 1 in their elements are called binary matrices

Bimatrix Naming Games

$$P(a,b)=\sum_{i=1}^{o}\sum_{j=1}^{s}(t_{ij}^{a}r_{ji}^{b}+t_{ij}^{b}r_{ji}^{a})=\frac{1}{2}(Tr(T^{a}\cdot R^{b})+Tr(T^{b}\cdot R^{a}))$$

A lexicon a *is a Strict* Nash equilibrium if and only if *o = s with T being a permutation matrix* and R *its corresponding transpose one*.

A permutation matrix is a binary matrix with the additional constraint of having only one element equal to 1 in each row and column.

This strong condition implies that in such a lexicon there are bijective relations between the set of signals and objects, one word to each object and vice versa.

Bimatrix Naming Games

The cases *o≠ s are more interesting.*
No Strict Nash Equilibria or Evolutionary stable lexicons ⟶ Simple Equilibria

(1) The elements of *T* and *R* must be numbers in the interval *[0,1]*.
(2) all the non zero elements of a column of *T* and *R* are identical.
(3) $R^{\dagger}$ is in the support of T , that means that if the element t_{ij} is non null, r_{ji} must be non null.
Homonymy is possible, but with restrictions. If some objects are associated to the same set of signals, none of them can have associations to signals not belonging to the set.

A reciprocal condition exist for synonymy. If some signals are associated to a group of objects, none of them can have associations to objects outside this set.

Bimatrix Naming Games

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\ \begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}\ \begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}\ \begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}\ \left\{\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}\ \begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}\right\}$$

Strict Nash Equilibrium

$$\left\{\mathcal{T}=\begin{bmatrix}0&1&0\\0&1&0\\1-x&0&x\end{bmatrix}\quad \mathcal{R}=\begin{bmatrix}0&0&1\\1-y&y&0\\0&0&1\end{bmatrix}\right\}$$

Nash Equilibrium

$$\left\{\mathcal{T}=\begin{bmatrix}1-x&x&0\\1-x&x&0\end{bmatrix}\quad \mathcal{R}=\begin{bmatrix}1-y&y\\y&1-y\\0&0\end{bmatrix}\right\}.$$

Nash Equilibrium

$$\begin{bmatrix}1-x&x&0\\&x&1-x\end{bmatrix}\ \begin{bmatrix}y&0\\y&1-y\\0&1-y\end{bmatrix}.$$

Not Equilibrium

Evolutionary Naming Games

N individuals

Lexicon dynamics

Underlying complex network

Network dynamics

Evolutionary Naming Games

A given lexicon will be defined by the strength of the association between a given word s_k *and an object* o_l. *These strengths will* be upload to a *o × s matrix,* ***M*** *adopting values within the interval [0, 1].,*

The success of the interaction occurs when both individuals share the same object-word association.

During the interaction, the speaker chooses a given meaning and uses a word to express it according to the lexical matrix, using the stronger object-word association.
The hearer will then compare whether his lexicon also associates the chosen word with the meaning denoted by the speaker. If this happens the interaction is considered a success

Evolutionary Naming Games

The defined lexical matrices *M are not normalized.*

The normalization is not unique and depends on the role of the agent and the chosen normalization is in correspondence with previously discussed ideas.

When the individual *i is a speaker, the matrix will be normalized according to the* rows, such that the sum of the value in each row equals one.

When the individual plays the role of hearer, the normalization will be performed according to the columns.

$$t_{ij}^{a} = \frac{m_{ij}^{a}}{\sum_{j=1}^{s} m_{ij}^{a}}$$

$$r_{ij}^{a} = \frac{m_{ij}^{a}}{\sum_{i=1}^{s} m_{ij}^{a}}$$

Network Dynamics

A node i is randomly chosen

One of its neighbors, j, is selected.

A third node k, not connected with i is randomly chosen

The lexical distance between i-j, i-k is compared

$$d_{ab}^{l} = \frac{1}{o \cdot s} \sqrt{\sum_{i=1}^{o} \sum_{j=1}^{s} (t_{ij}^{a} - t_{ij}^{b})^2}$$

The link between i-j is broken and a new link between i-k is created according to certain probability, depending on the lexical distance.

Both Dynamics

The simulation performs N^*t_r cultural steps followed by $N^*\ t_n$ network steps

Repertoire dynamics favor convergence

Network dynamics favors fragmentation and freezes the lexical dynamics