Simulation of Exchange Rate Determination with Heterogeneous Strategies

[1]Xiaobing Feng, [2]Lu Tang, [3] Feng Wang,
[1]International Finance, SASS, Shanghai, P.R.China, fxb@sass.org.cn
[2]Academy of Mathematic Science, P.R.China, tanglu.tang@gmail.com
[3]Computer Science, Wuhan University, P.R.China, wangfengwhu@yahoo.com.cn

Abstract:

We develop a nonlinear exchange rate model when agents choose heterogeneous strategies. The simulation results show that when parameters are properly selected, bi-strategy and multi-strategy generate the same exchange rate behaviour even though at individual level they are different strategies. The paper further shows how different strategies interact with each other and formulate the dynamics of exchange rate movement. Additionally we also found the volatility of exchange rates is positively correlated with the transaction costs in the goods market.

JEL classification: F31;F41

Keywords: Exchange rate determination; heterogeneous strategies; fundamentalist and chartist

1. Introduction and literature Review

The breakdown of the Bretton Woods System changed the fixed exchange rate system to a free floating system in the 1970s. As a result an increasing number of puzzles and anomalies have arisen, which the traditional model fails to explain. The first and foremost empirical puzzle has been called the 'disconnect' puzzle. Goodhart (1989), Goodhart and Figlioli (1991) and Faust et al.(2003) found that most of the changes in the exchange rates occur when there is no observable variations in the fundamental economic variables. This finding contradicts the efficient market hypothesis with rational expectations.

The failure of the traditional exchange rate models has led to new attempts to study the exchange rate. There are basically three different modeling approaches to address the exchange rate determination puzzle.(Grauwe and Grimaldi 2006) The first one uses the Obstfeld-Rogoff framework of dynamic utility optimization of a representative agent (Obstfeld-Rogoff 1996); a second approach starts from the analysis of the microstructure of the foreign exchange markets, emphasizing on the aggregated effect of informational order flows; a third approach recognizes agents with heterogeneous beliefs. These different beliefs introduce non-linear features in the dynamic of the exchange rate. This approach was initiated by Frankel and Froot (1987) and further developed mainly in the context of stock prices. (Kirman,1993;Brock and Hommes,1997,1998; Lux,1998;Le Baron et al.,1999;Gaunersdorfer et al.,2003) Later heterogeneous agents has also been introduced in the study of exchange rate behaviour, for instance, Bacchetta and van Wincoop,2003 introduced agents with heterogeneous beliefs .

In Bacchetta and van Wincoop's (2003) model they develop two different strategies for agents to follow, fundamentalist rule and chartist rule, but in the paper it has not been examined how agents with these two rules interact with each other and demonstrates consistent exchange rate behaviour which is disconnected to the fundaments with excess volatility and volatility cluster. In reality agents use various strategies when they make investment decisions. We then enrich agents' strategy space into multi-strategy from two pure strategies. We want to find out under what condition in aggregation the exchange rate will exhibit the same behaviour even though in the micro-individual level, agents with pure bi-strategy and agents with mixed multi-strategies are completely different. The research results for this question will be relevant to practitioners as well as policy makers.

The paper is organized as follows: In section 2, the theoretical model is presented and the dynamics of the exchange rates determination is examined. In section 3 the simulation results are presented and analyzed. Section 4 concludes the paper with policy implication.

2. The Model

In this section, we set up a framework within which exchange rates are determined. We start with defining heterogeneous agents and the fundamentals which is the same as the previous literature. We then set up three processes, going through which exchange rates will be determined. The three steps are agents forecasting process, agents decision making process and market decision making process. The dynamics features are then analyzed based on the constructed exchange rate.

2.1 The fundamental and heterogeneous agents

Following Brock and Hommes (1998) and Grauwe and Grimaldi 2006, we define the fundamental exchange rate as a random walk without drift, where the error follows a moving average process, this implies

$$S_{t+1}^{*} = S_t + \varepsilon_t \qquad (1)$$

$$\varepsilon_t = \mu \varepsilon_{t-1} + \xi_{t-1} \qquad (2)$$

We then introduce the assumption that agents will apply two different types of strategies called fundamental strategy F and chartist strategy C and N is the number of population.

$$(1-\Psi_{it})NF + \Psi_{it}NC \qquad (3)$$

$$\sum_{i=1}^{N} [(1-\Psi_{it})F + \Psi_{it}C] \qquad (4)$$

This way of modeling foreign exchange market was first proposed by Frankel and Froot (1986). It was further extended by De Long et al.(1990), De Grauwe et al. (1993) and Kilian and Taylor (2001). We use this framework to build up a basic bi-strategic agent model, i.e. one agent among N will choose one of the two strategies and invest accordingly. (3) denotes the effect in aggregation. We then enrich the strategy space by allowing each agent to randomly choose a proportion $(1-\Psi_{it})$ of F and C to form a convex combination, where Ψ_{it} follows a certain

probability distribution. The rational to construct a multi-strategic agent framework is that it is consistent with the strategy of a statistical hedge fund in practice, which normally allocates capitals according to different strategies.

While the bi-strategic agent model considers the strategies of individual traders, the multi-strategic agent model concerns the same issue from the standard point of the fund. It will then be interesting to find under what condition the two will generate the same exchange rate behaviour in aggregation. Since it is much easier for the regulator to monitor and control when an investor who allocates capital alone its different strategies compared with an investor who does not have such a property in its strategy space. In this case different strategy will lead to different exchange rate behavior which will be discussed in section five.

In order to examine the strategy evolution, we develop a confidence index as follows to specify how agents adjust their capital allocation between fundamental rule and chartist rule from period to period:

$$\lambda_{i,t} = \sum_{t=0}^{T} \rho_{i,t} 1/(1+\Delta\sigma) \qquad (5)$$

Where $\lambda_{i,t}$ is confidence index for trader i at time t, $\Delta\sigma$ is the forecasting error made by agents with different strategies. The smaller $\Delta\sigma$ is in the current round of the transaction, the more confident the agent will be in the next round of transaction. $\rho_{i,t}$ measures the importance of $\Delta\sigma$ at time t in the forecasting and $\sum_{i=0} \rho_{i,t} = 1$. The further away from the current period the less important it will be. This index will control the proportional variation of agents using fundamentalist rule and chartist rule in the population.

$$\Delta\sigma' = \Delta\sigma / (1+ |S_{t-1} - S_{t-1}|^{*2}) \qquad (6)$$

However, (5) needs to be adjusted to address the problem of a systematic error in a standard variation cycle of the exchange rate which will be analyzed in details in section five. In an exchange rate variation cycle, C strategy will win most of the time except for the inflection point whereas F will loose in half of the time. There will be only C left in the market eventually if two strategies and (5) followed closely. In this paper we first define it as over-evaluation error and will make adjustment accordingly. On the contrary for F there is an under-evaluation error. We will discuss this issue in details in section five. We therefore define equation (6) as modified forecasting error.

2.2 Agents forecasting process

The fundamentalists compare the current market exchange rates with the fundamental rate.They believe that the future exchange rate will always revert to the fundamental. They essentially follow a negative feedback rule. We then follow the specification in Dummas (1992)，Obstfeld and logoff (2000) Grauwe,P. and M,Grimaldi (2005) to introduce the transaction costs in goods market which indicates the volatility in the goods market. If the exchange rate variation is larger than the transaction cost in the goods market then the agent will forecast there will exchange rate variation.

If on the other hand, the rate variation is within the range of transaction cost, the agent will forecast the rate will remain the same as in the previous period.

The fundamentalist's forecasting rule therefore can be specified as follows:

$$F: \quad E_{ft}(\Delta S) = -\theta (S_t - S_t^*) \quad \text{if} \quad |(S_t - S_t^*)| > \text{cost} \qquad (7)$$
$$E_{ft}(\Delta S) = 0 \quad \text{if} \quad |(S_t - S_t^*)| < \text{cost}$$

Where $E_{f,t}$ is the forecast of exchange rate variation made in period t by the fundamentalists using information up to time t, S_t is the exchange rate in period t and $\theta > 0$ is the adjustment speed.

The chartists on the other hand follow a positive feedback rule, they extrapolate past movements of the exchange rate into the future, Their forecast is written as

$$C: \quad E_{ct}(\Delta S) = \beta \sum \partial_i \Delta S_{t-I} \qquad (8)$$

Where E_{ct} is the forecast made by the chartists using information up to time t, ΔS_t is the change in the exchange rate, $\sum_i \partial_i = 1$, and $0<\beta<1$ to ensure the dynamic stability. De Long et al., 1990 believe that the chartists compute a moving average of the past exchange rate changes and they extrapolate this into the future exchange rate change. The important point is that chartists do not take into account information concerning the fundamentals. In this sense in the literature they are regarded as pure noise makers.

2.3 Agents decision making rule

Agents are endowed with fixed amount of currencies to begin with and their wealth are measured by net cash flow which will be revalued after each transaction. The net cash flow for individual i at time t period can be calculated as follows:

$$W_{it} = W_{i,0} + \sum_{t=1} Q_{i,t-1}(\Delta S_{i,t-1} - k_{i,t-1}) \qquad (9)$$

$$Q_{i,t} = W_{it} * \lambda / k_{i,t} \qquad (10)$$

Where ΔS_i (t-1) is the unit difference between exchange rate sold and exchange rate bought in the last period. When it is positive the agent makes gross profit. $k_{i,t-1}$ is the unit cost of each currency contract and $Q_{i,t-1}$ is the total amount of currency contracts the agent buys from the market in the last period. W_{it} is the agent's wealth at current period t. After the agent makes its forecasting for the next period, with the updated wealth he will decide how many currency contracts to buy in the currency period. He however will not exhaust his wealth but use proportion λ for the investment. When the agent makes correct forecasting and accumulated profits in the previous period, λ increases and he will raise the proportion of wealth to buy contracts in the next period. The relationship is defined as equation (10).

2.4 Market / Exchange's Decision

When the market receives the orders from agents, it will re-evaluate the value of the currency according to the net demand of supply. When there is an excess demand for a currency in current period, the currency will be appreciated in the next period; when the reverse occurs, the currency will be depreciated. Therefore there are three situations:

$$S_{t+1} = f(NQ_t) \qquad (11)$$

1) $S_{t+1} = S_t$ when $NQ_t = 0$

2) $S_t > S_{t+1}$ when $NQ_t > 0$

3) $S_t < S_{t+1}$ when $NQ_t < 0$

Finally we summarize the forecasting , decision making of agents and decision making of the markets by the following flow chart.

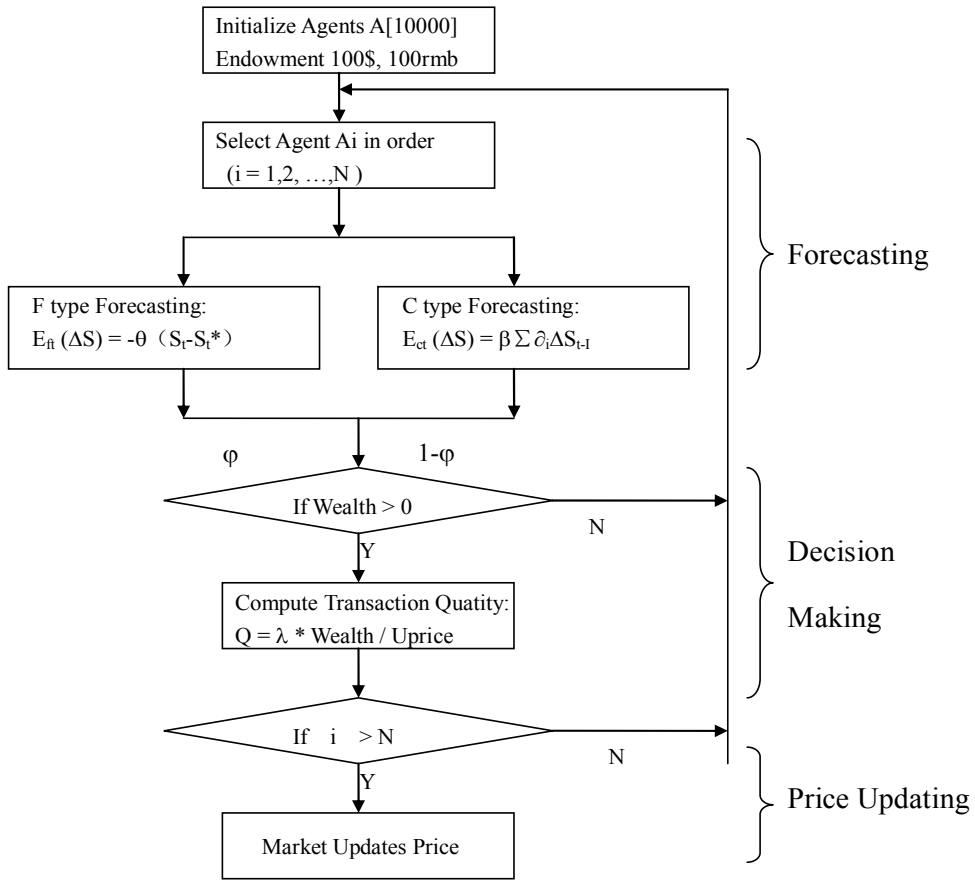


Fig.1. Determination of Exchange Rate

2.5 The dynamics of exchange rate determination

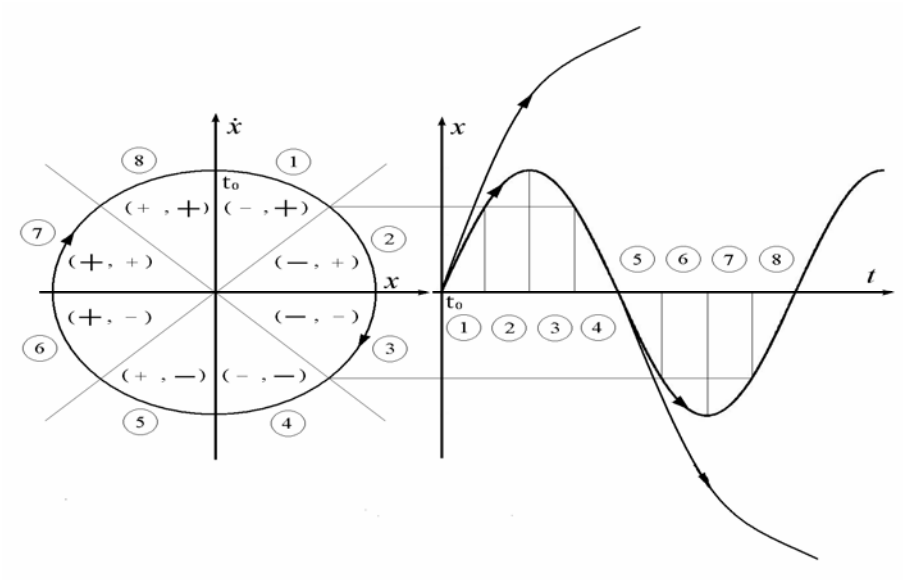


Fig.2. The Forecasting and Decision Making of Agents

It is well recognized in finance literature there exists two strategies for forecasting the fundamentalist rule and chartist rule, but the question how traders with different strategies in the market interact with each other and formulate the exchange rate dynamics has not yet been answered. We attempt to do this in the following section.

In figure two, $X_t = S_t - FV$, where FV is the fundamental value of the exchange rate, X_t is the deviation of the rate to the fundamental rate. When the fundamental is set to zero as the horizontal axis, X_t is the exchange rate too.

The right hand side of the graph depicts the relationship between the level and time. In region 1 and 2, when fundamentalists see the exchange rate appreciates, they sell because they know it will revert to fundamentals. The difference between region 1 and 2 is that as the deviation gets larger, fundamentalists will sell more as they are more certain about its future reversion. Meanwhile the chartists will buy as soon as the exchange rate starts to appreciate, and forecast that it will rise more in the future. They therefore start to buy in region 1 then increase the speed in region 2 depending on the slope of the curve. The combination of opposite buy-sell forces of fundamentalists and chartists reaches zero at the inflection point, the deviation reaches its maximum and from then on the exchange rate starts to depreciate.

In region 3 and 4, fundamentalists will still sell since the deviations remains, but sell less in region 4 than in region 3 as in region 4 the exchange rate gets closer to the fundamental. The chartists will sell too when they see price falls, and will sell more in region 4 than in region 3 depending again on the slope of the curve providing short-sell is allowed. Hence in region 1 and 2, fundamentalists make wrong forecasting and chartists make correct forecasting; in region 3 and 4, both fundamentalists and chartists make correct forecasting, so two selling forces generates some strong force and inertia, the exchange rate continues falling in region 5 and 6.

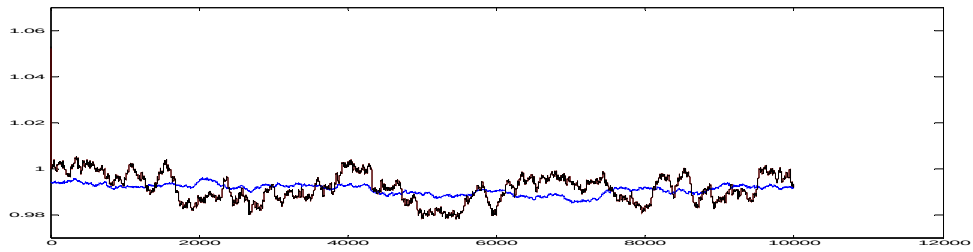
The behavior of the exchange rate in region 5 and 6 is the mirror image of case 1 and 2 but the opposite direction. The same is true if we compare between region 7 and 8 with region 3 and 4. In region 1 and 2, 5 and 6, if there are many chartists in the market and they dominate we will see bubbles as the exchange rate continues rising and collapse as the exchange rate continues falling, which will be discussed further in the next section.

If we take first derivative of x and plot it with respect to level x, we get the phase diagram. The plus sign denotes buy and the minus sign denotes sell. The bigger sign denotes buy/sell in a larger quantity, the smaller sign denotes the opposite. In parenthesis, the left sign is the behavior of a fundamentalist and the right sign is the behaviour of a chartist. Region 1 to 8 of left diagram is corresponding to region 1 to 8 on the right which shows the speed of the adjustment and it is consistent with the diagram in level.

To summarize the performance of fundamentalists and chartists, it is obvious that fundamentalists make the right forecasting half of the time, but the chartists make it right most of the time except for the inflection point. The result is caused by the definition of x, the deviation between the fundamentalists and the realized exchange rate behave in a cyclic fashion. Additionally the definition leads to certain bias when fundamentalists' rule is used for forecasting. When the deviation gets larger, the forecasting error for fundamentalists gets larger too. But the fact is as the deviation gets larger, the chances for the exchange rate reverts to fundamental gets larger too, so the more likely the fundamentalists get the forecast correct. To take into account these two factors we have modified $\Delta\sigma$ in equation (5) into $\Delta\sigma / (1+ |St\text{-}1 - St\text{-}1|^{*2})$.

3. The Solution of the Model and Analysis of the Results

In this section we investigate the properties of the solution of the model. The top graph of figure three presents the exchange rate behaviour when half of the agents take the fundamentalist rule and the other half take the chartist rule and the transaction costs are not considered. The bottom one represents the exchange rate when all the agents are hybrid of fundamentalist and chartist again the transaction costs are ignored. We found even though at the micro-individual level, these two are very different strategies, we get the same simulation results in the aggregation. This is true when Ψ in equation (3) takes the value 0.5 and Ψ in equation (4) follows uniform distribution at an interval [0,1] .



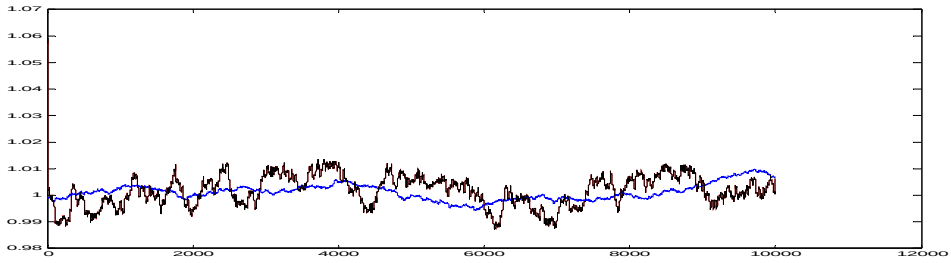


Fig.3. The exchange rate with bi-strategy and multi-strategy agents when transaction cost is zero

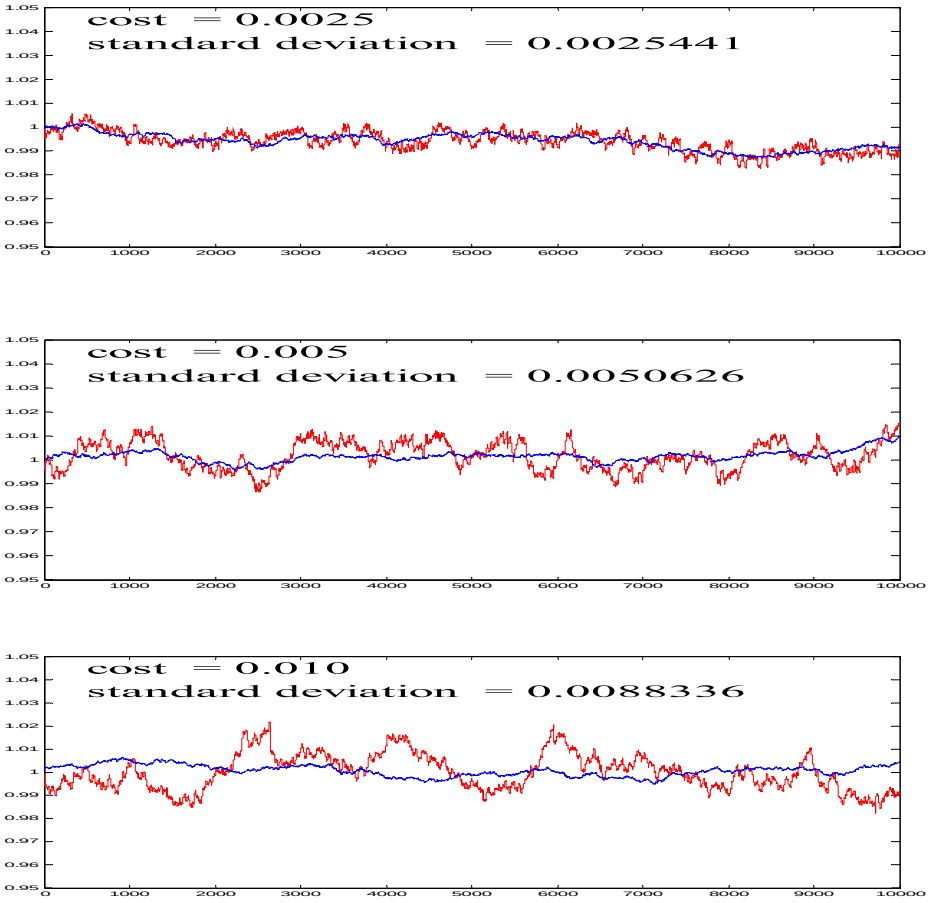


Fig.4. The exchange rate with bi-strategy agents when transaction cost is non-zero

Grauwe and Grimaldi's (2006) found when transaction costs are taken into consideration, the behaviour of exchange rate will show the property of clustering, excess volatility etc. In our study we further discover the volatility of the exchange rates is positively correlated with the size of the transaction costs. This is true for both bi-strategy agent case and multi-strategy agent case as shown in figure four and five. In figure four the costs increase from 0.0025, to 0.005 to 0.010. We have also calculated the standard deviation to show the increase in volatility when transaction costs rise, they are 0.0025441，0.0050626 and 0.0088336 respectively. In figure five the costs increase from 0.0025, to 0.005 and 0.010. The volatility measured by standard deviation are 0.0026981，0.0048495 and 0.010737. Figure six shows two extreme cases with bubbles and

collapses when the markets are dominated by chartist rule.

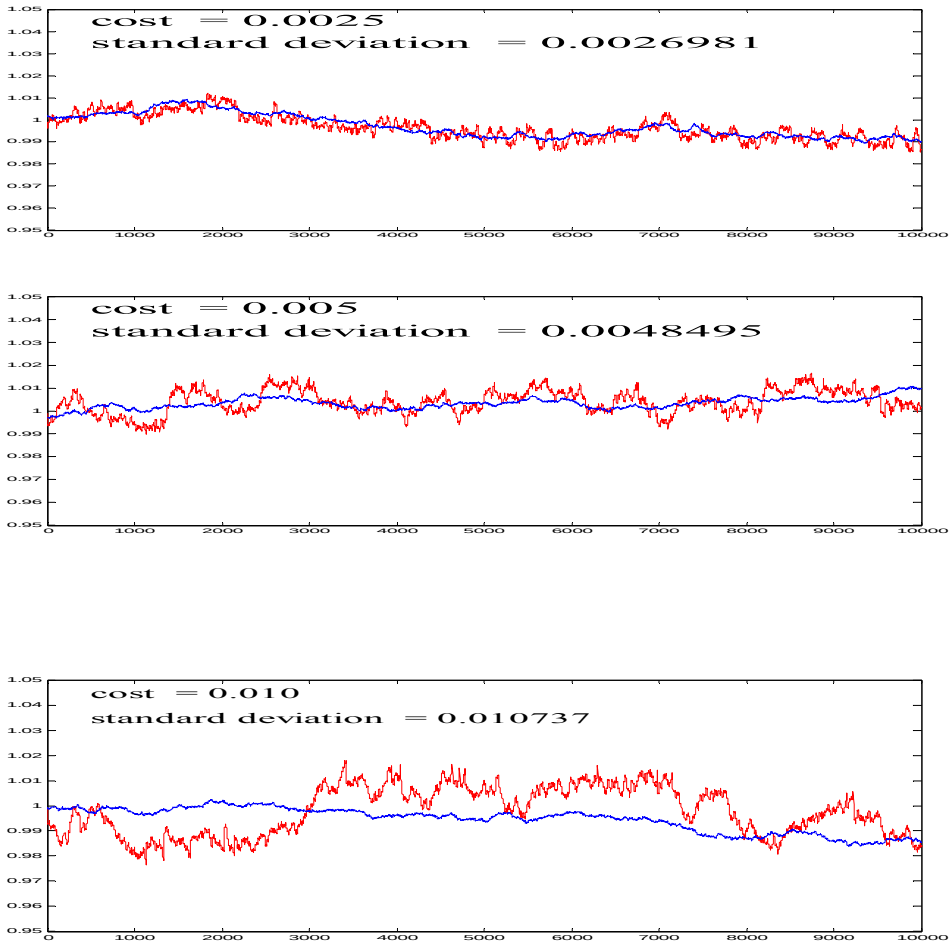
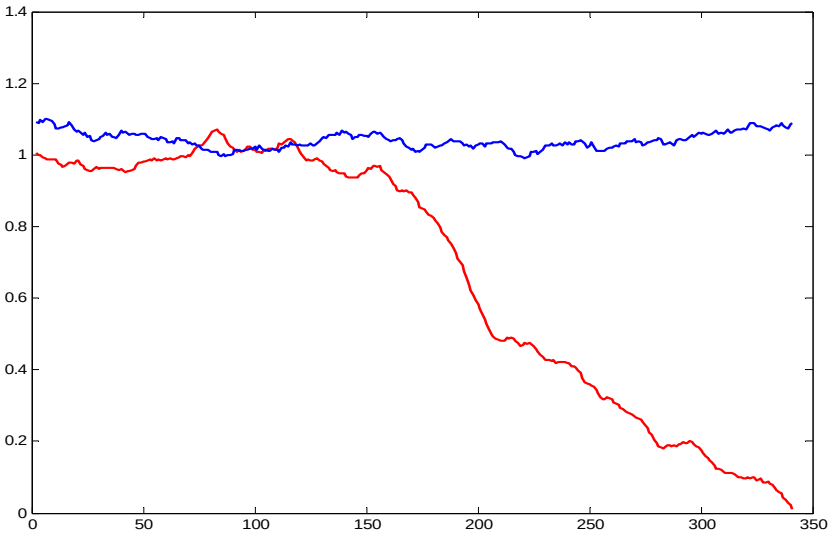


Fig.5. The exchange rate behaviour with multi-strategy agents when transaction cost is non-zero



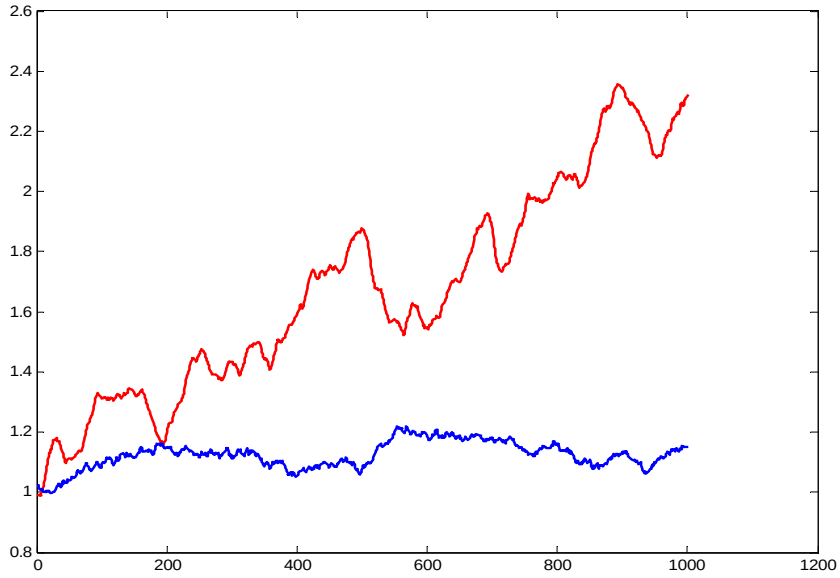


Fig.6. Two extreme cases with bubbles and collapses when the markets are dominated by chartist rule

4. Conclusion and Policy implication

In this paper we have developed an exchange rate model with agents that take hybrid strategy of fundamentalist and chartist. The model produces the following features: first it allows for heterogeneity of agents' forecasting strategies. When the distribution for the weights are properly chosen between fundamentalist strategy and chartist strategy, in aggregation the behaviour of the exchange rate remains the same as when there are only two pure strategies in the entire population. This finding explains the fact that the statistic hedge funds allocate their capital according to different strategies and evaluate their performance accordingly. On the contrary when individual investors are taken into the consideration, it is hard to tell which strategy they take for the investments.

The policy implication is that the statistic hedge fund is a recommended approach for investment. Compared with the individual investment the statistic hedge fund approach is able to generate the same macro-behaviour of the asset pricing, but they are much easier to impose controls from the perspective of fund allocation along the line of different strategies. When observing the strategy preference of the fund, regulator will be able to expect how exchange rate will behave. They will then in advance advice certain strategy profile to funds in order to achieve certain macro-effect in the behaviour of exchange rate. In that way, the bubble and collapse is much easier to prevent from.

The paper also analyses the path of exchange rate dynamics when fundamentalist and chartist strategies are taken into the consideration. The simulation results also show the positive relationship between the transaction costs and the volatility of the exchange rate.

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