A Theory for the Market Impact of Large Trading Orders

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The act of buying or selling something can change its price. This is referred to as *price impact*, or in financial markets, *market impact*.

**Main Question:** Why is market impact a concave function of the size of the order? What is the functional form? NOTE: I focus on large aggressive on-book orders.
(1) Heterogeneous trade sizes (empirical results suggest a power law distribution).
(2) Trades are split into smaller pieces.
(3) This induces predictability in order flow.
(4) Predictability in order flow cannot transfer to predictability in price.
(5) When later pieces of an order are more predictable, they impact the price less (this occurs with power law distributed order size).
The Argument

(1) **Heterogeneous trade sizes** *(empirical results suggest a power law distribution).*
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Cumulative Distribution of Trade Size for AZN from the London Stock Exchange

(Gopikrishnan, Gabaix, Plerou and Stanley, 2000)
(Lillo, Mike, Farmer, 2005)
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Participants split orders for strategic reasons.

- Traders condition the size of their transactions to the available liquidity in the order book.
- For large orders, the available liquidity in the order book is often much less than the full size of the order. Therefore, the trade is split.

For the model below assume the following:

- Trades are split into pieces of standard size.
- The return due to a single transaction is a constant, $\epsilon r_0$. 
One Transaction Impact for AZN

(a) $P(v/v_b)$

(b) $P(+v)$

(c) $E[r|+,v]$  

(d) $R(v,1)$

Empirical Model
The Argument

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Autocorrelation of Trade Sign for VOD

(Bouchaud, Gefen, Potters, and Wyart, 2004)
(Lillo and Farmer, 2004)
Autocorrelation of Trade Sign for AZN (within and across brokerages)

Interpretation: Predictability is due to splitting of orders.
(1) Heterogeneous trade sizes (empirical results suggest a power law distribution).
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(4) **Predictability in order flow cannot transfer to predictability in price.**
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Efficient Market Hypothesis (EMH): (Weak form) Returns cannot be predicted using publicly available historical financial data (Fama, 1970).

\[
E [r_n | \Omega] = E [r_n] \approx 0
\]

- \( r_n \) the return due to the \( n^{th} \) piece of an order.
- \( \Omega \) any set of publicly available historical financial data.
Return Model

\[ r^+_n = \epsilon r_0 - \lambda_n + \eta \]
\[ r^-_n = -\lambda_n + \eta \]

\( r^+_n \) the return given that the \( n + 1^{th} \) piece of an order arrives.
\( r^-_n \) the return given that the order has completed and there is no \( n + 1^{th} \) piece.
\( \epsilon \) whether trade is buyer (+1) or seller (-1) initiated.
\( r_0 \) the return expected due to a single trade.
\( \lambda_n \) a parameter set by the EMH.
\( \eta \) an uncorrelated noise term.
Determination of $\lambda_n$

$\mathcal{P}$ is the probability that the order continues, given the information set $\Omega$.

Applying the EMH:

\[ E[r_n|\Omega] = \mathcal{P}E[r^+_n] + (1-\mathcal{P})E[r^-_n] \]
\[ = \mathcal{P}(\epsilon r_0 - \lambda_n) + (1-\mathcal{P})(-\lambda_n) \]
\[ = 0. \]

Solving for $\lambda_n$:

\[ \lambda_n = \epsilon r_0 \mathcal{P} \]
Final Return Model

\[ r_n^+ = \epsilon r_0 - \epsilon r_0 P + \eta \]
\[ r_n^- = -\epsilon r_0 P + \eta \]

- \( r_n^+ \) the return given that the \( n + 1^{th} \) piece of an order arrives.
- \( r_n^- \) the return given that the order has completed and there is no \( n + 1^{th} \) piece.
The Argument

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When order sizes are power law distributed, the probability of the order continuing is:

\[ P \approx \left( \frac{n}{n+1} \right)^\alpha \]

\[ \alpha \quad \text{the exponent of the cumulative order size distribution.} \]

The return due to a piece of the order:

\[ r_n^+ = \epsilon r_0 \left[ 1 - \left( \frac{n}{n+1} \right)^\alpha \right] + \eta \]
Functional Form of Market Impact

Summing over the pieces of the order:

\[ E[R|N] = \sum_{n=1}^{N} E[r_n^+] \approx \alpha \epsilon r_0 \log(1 + N). \]

- \( R \) is the total market impact (or total return) of the order measured from when the hidden order started to when it completed.
- \( N \) is the total size (or number of pieces) of the order.
Empirical Results: Returns

\[ \frac{R}{\alpha \varepsilon r_0} \]

Graph showing empirical and theoretical data points for \( \frac{R}{\alpha \varepsilon r_0} \) against \( N \).