A Theory for the Market Impact of Large Trading Orders

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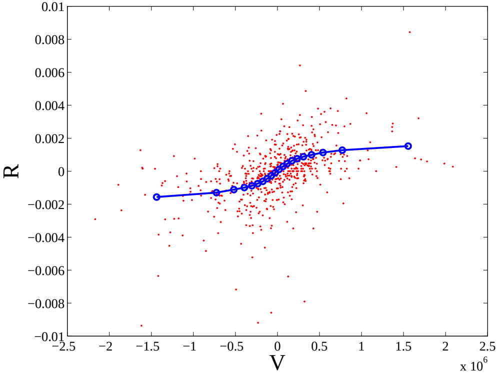
29 July 2008

Main Question

The act of buying or selling something can change its price. This is referred to as *price impact*, or in financial markets, *market impact*.



Main Question: Why is market impact a concave function of the size of the order? What is the functional form? NOTE: I focus on large aggressive on-book orders.

The Argument

(1) Heterogeneous trade sizes (empirical results suggest a power law distribution).

(2) Trades are split into smaller pieces.

(3) This induces predictability in order flow.

(4) Predictability in order flow cannot transfer to predictability in price.

(5) When later pieces of an order are more predictable, they impact the price less (this occurs with power law distributed order size).

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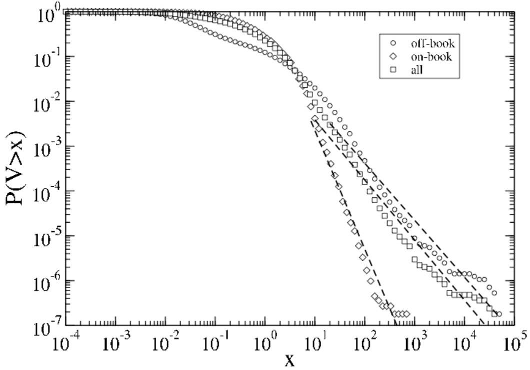
Cumulative Distribution of Trade Size for AZN from the London Stock Exchange



(Gopikrishnan, Gabaix, Plerou and Stanley, 2000)
(Lillo, Mike, Farmer, 2005)

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Splitting of Orders

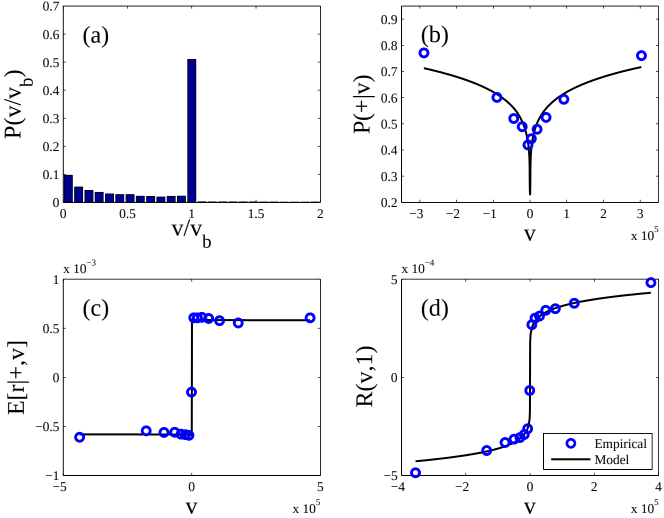
Participants split orders for strategic reasons.

- Traders condition the size of their transactions to the available liquidity in the order book.
- For large orders, the available liquidity in the order book is often much less than the full size of the order. Therefore, the trade is split.

For the model below assume the following:

- Trades are split into pieces of standard size.
- The return due to a single transaction is a constant, ϵr_0.

One Transaction Impact for AZN



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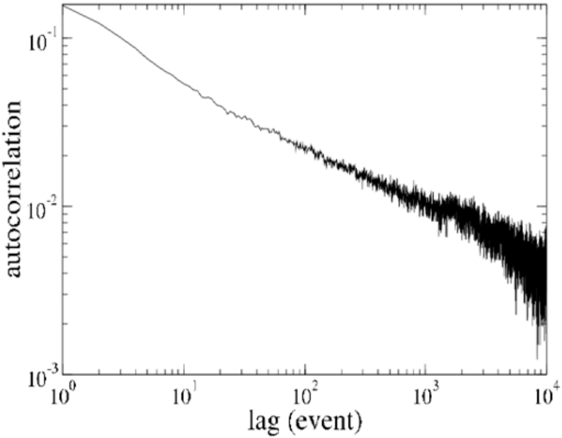
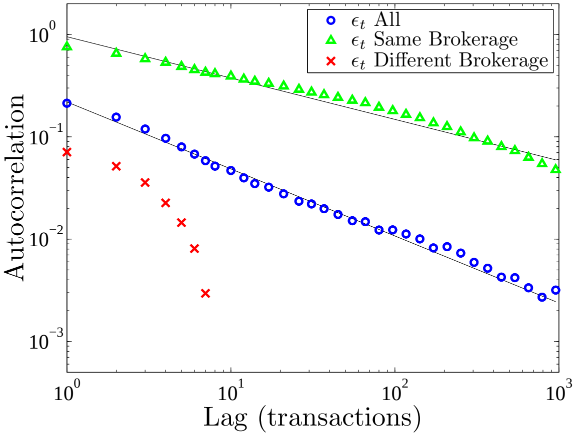
Autocorrelation of Trade Sign for VOD



(Bouchaud, Gefen, Potters, and Wyart, 2004)
(Lillo and Farmer, 2004)

Autocorrelation of Trade Sign for AZN (within and across brokerages)



Interpretation: Predictability is due to splitting of orders.

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Efficient Market Hypothesis (EMH)

Efficient Market Hypothesis (EMH): (Weak form) Returns cannot be predicted using publicly available historical financial data (Fama, 1970).

$$E\left[r_n|\Omega\right] = E\left[r_n\right] \approx 0$$

r_n	the return due to the n^{th} piece of an order.
Ω	any set of publicly available historical financial data.

Return Model

$$\begin{aligned} r_n^+ &= \epsilon r_0 - \lambda_n + \eta \\ r_n^- &= -\lambda_n + \eta \end{aligned}$$

r_n^+	the return given that the $n+1^{th}$ piece of an order arrives.
r_n^-	the return given that the order has completed and there is no $n+1^{th}$ piece.
ϵ	whether trade is buyer (+1) or seller (-1) initiated.
r_0	the return expected due to a single trade.
λ_n	a parameter set by the EMH.
η	an uncorrelated noise term.

Determination of λ_n

$\mathcal{P}$ is the probability that the order continues, given the information set Ω.

Applying the EMH:

$$\begin{aligned} E[r_n|\Omega] &= \mathcal{P}E[r_n^+] + (1-\mathcal{P})E[r_n^-] \\ &= \mathcal{P}(\epsilon r_0 - \lambda_n) + (1-\mathcal{P})(-\lambda_n) \\ &= 0. \end{aligned}$$

Solving for λ_n:

$$\lambda_n = \epsilon r_0 \mathcal{P}$$

Final Return Model

$$
\begin{aligned}
r_n^+ &= \epsilon r_0 - \epsilon r_0 \mathcal{P} + \eta \\
r_n^- &= -\epsilon r_0 \mathcal{P} + \eta
\end{aligned}
$$

r_n^+ the return given that the $n + 1^{th}$ piece of an order arrives.

r_n^- the return given that the order has completed and there is no $n + 1^{th}$ piece.

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Return due to a Piece of the Order

When order sizes are power law distributed, the probability of the order continuing is:

$$\mathcal{P} \approx \left(\frac{n}{n+1}\right)^{\alpha}$$

α the exponent of the cumulative order size distribution.

The return due to a piece of the order:

$$r_n^+ = \epsilon r_0 \left[1 - \left(\frac{n}{n+1}\right)^{\alpha}\right] + \eta$$

Functional Form of Market Impact

Summing over the pieces of the order:

$$
\begin{aligned}
E[R|N] &= \sum_{n=1}^{N} E[r_n^+] \\
&\approx \alpha \epsilon r_0 \log(1+N).
\end{aligned}
$$

- R is the total market impact (or total return) of the order measured from when the hidden order started to when it completed.
- N is the total size (or number of pieces) of the order.

Empirical Results: Returns