Exploring the Delay Embedding of an Electroencephalogram Signal

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1 Introduction

One can hardly argue against the observation that the brain is a nonlinear system. And yet, most traditional methods of analyzing brain activity, whether recorded by electroencephalogram (EEG), magnetic resonance imaging (MRI) or otherwise have tended to stick with linear analysis tools. Much of the reason for this derives from the fact that no one has yet developed a generalized scheme for nonlinear analysis. In this study we adopt a technique for time series analysis known as delay embedding **?**. We apply this to a series of EEG scalp recordings. Delay embedding promises to reconstruct the hidden dynamics of a system by revealing a multi-dimensional space topologically equivalent to the internal phase space. However, this promise depends upon a relatively noise-free signal **?**. Unfortunately, noise frequently dominates EEG signals recorded from the scalp. In this study, we applied delay embedding to several EEG signals. The results looked promising at first, but after further analysis, we found that they appear more ambiguous than we originally hoped necessitating further research.

2 Methods

2.1 Data

In this study we used data from a previous experiment **?** in which the participant was recorded while in three states: 1 "normal" state (non-meditative, eyes-closed, awake), and 2 different "meditative" states (an "one-point focus" meditation and a "deity visualization" meditation). The participant had over 20 years of experience

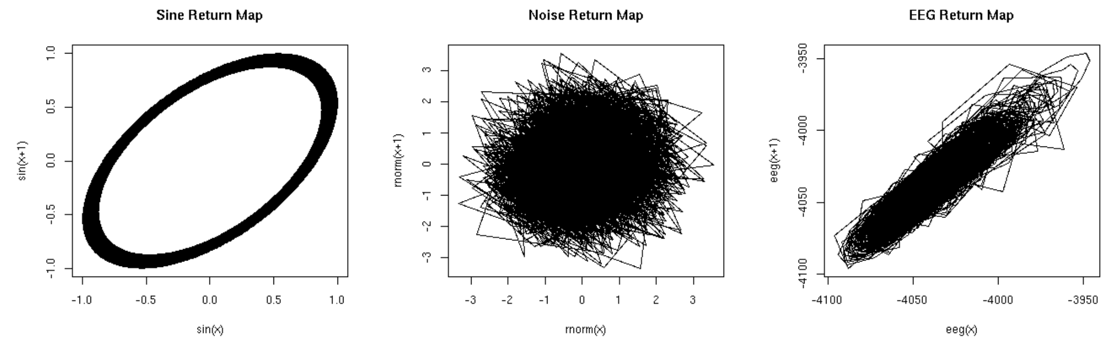

Figure 1: Return map of a sine wave (left), Gaussian noise (center), and an EEG recording (right). (Note that a return map is a delay embedding with $d_E = 2, \tau = 1$.)

practicing meditation. Each state was recorded for approximately five minutes with a 23-electrode array sampling at 512 Hz. We broke the recordings into 8-16 second intevals and embedded them in a multidimensional space using delay coordinates and analyzed them using R **?** with the tseriesChaos and tsDyn packages **??**. We used the scatterplot3d package for many of the figures below **?**.

2.2 Delay Embedding

Delay embedding reconstructs the internal phase space of a dynamic system by rendering a number of dimensions d_E such that each dimension represents the position of the signal at time $t - (d_E - 1)\tau$. The simplest possible delay embedding has a d_E of two and a τ of one yielding a return map (see Fig. **??**).

If a system changes slowly, its return map will stay close to the diagonal. By increasing the delay, τ, you essentially "stretch" the map revealing more of the internal structure. However, there remains the problem of "false nearest neighbors." By projecting a high-dimensional shape onto a low-dimensional space, points may appear close together in the delay embedding than they would be in the phase space. The solution to this is to add dimensions to the embedding such that each added dimension maps a multiple of the delay, τ. For example, a four-dimensional delay coordinate embedding space would have it's dimensions defined by $[f(t), f(t-\tau), f(t-2\tau), f(t-3\tau)]$. We algorithmically add dimensions until we find that adding more does not appear to reduce the number of false nearest neighbors. When we did this over 8-16 second intervals, the number of embedding dimensions necessary ranged from 6 to 11. According to Taken's theorem $d_E > 2d_N$ where d_N is the dimension-

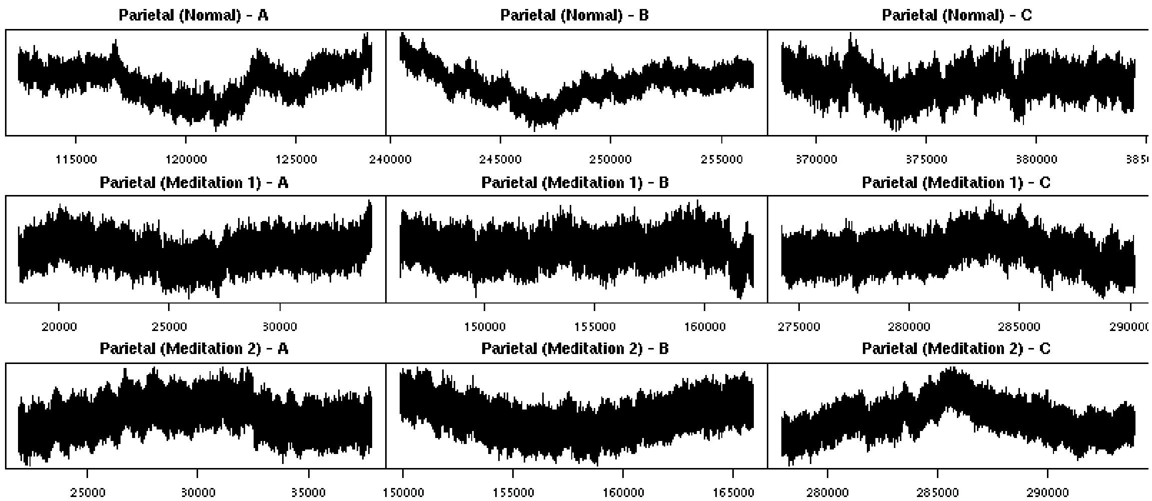

Figure 2: 16 second traces from electrode PZ from the Normal (top) Meditation 1 (middle) and Meditation 2 (bottom) states. We plotted here the residuals from a linear fit to reveal more detail however, we did not use linear regression in the embedded figures below. Time is represented in milliseconds.

ality of the internal phase space **?**. Figure **??** shows the results of recordings from the medial parietal region in the three states at three points in time. Each trace represents 16 seconds. Figure **??** represents the first three principle components of the same signal following delay embedding with the fourth component represented in grayscale.

2.3 Notch Filtering

If we take the images in Figure **??** as representative, we see that delay embedding reveals a certain cylindrical structure. This structure appears more defined in certain states and in certain states and at certain times than others. In Figure **??**, we saw that the return map of a sine wave looks somewhat circular. We find that if we add noise and drift to a sine wave, we can produce similar shapes (see Fig. **??**).

One possible such artifact could come from the 60 Hz line noise known to accompany EEG recordings. To control for this, we employed a notch filter **?** at 60 Hz to see what effect that would have on the embedding (see Figs. **??**, **??**). A problem with using a notch filter on this data comes from the fact that the brain may also show 60 Hz oscillation. Please note that this does change the shape of the signal. However, the underlying structure persists.

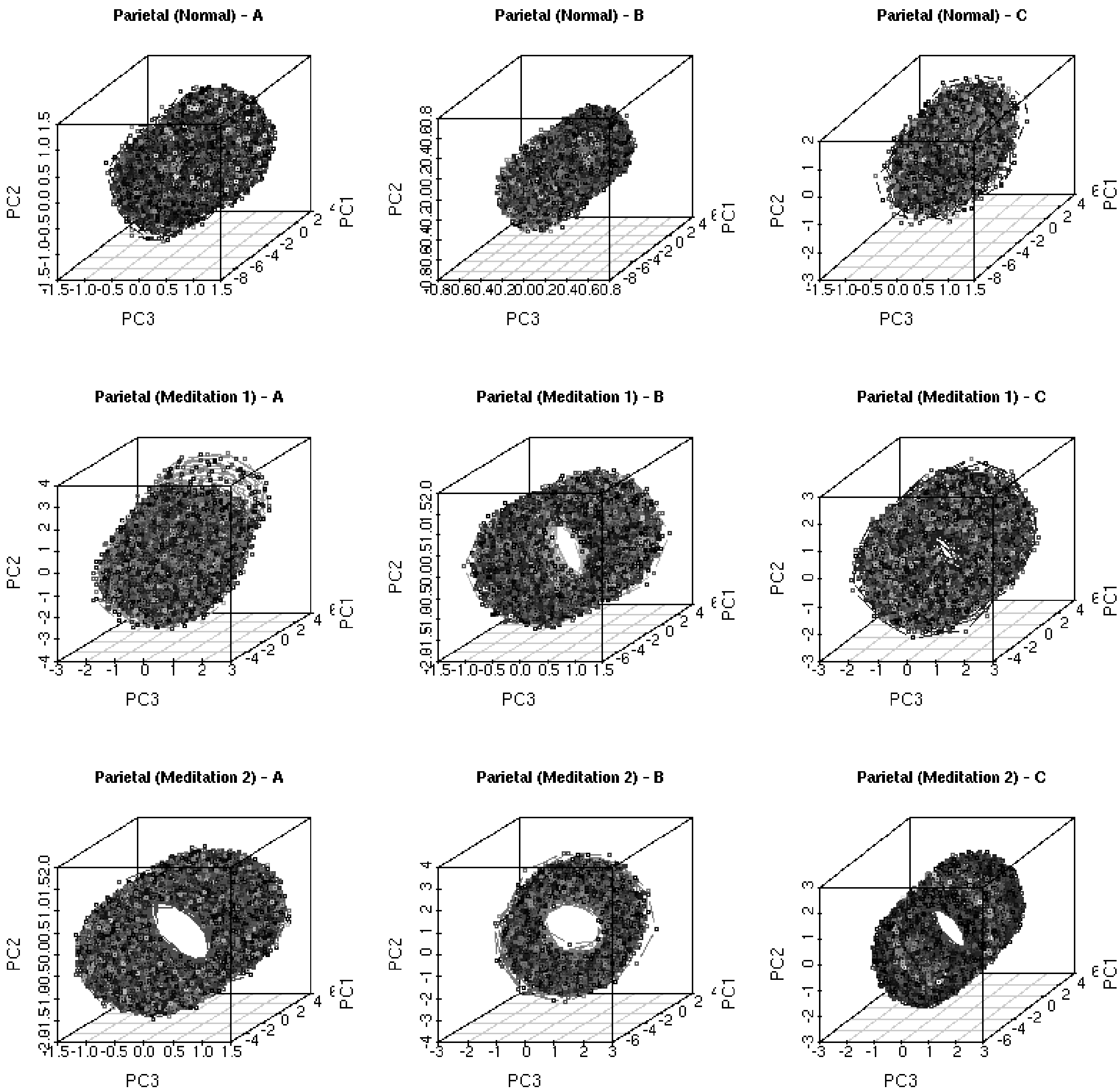

Figure 3: Delay embedding of 16 second intervals at the beginning (A), middle (B), and end (C) of each recording session. Images produced by plotting the first three principle components derived from the hyper-dimensional embedding space with the fourth principle component determining the gray scale value of each point. Embedding parameters determined individually for each 16 second interval: Normal: A ($d_E = 8, \tau = 6$), B ($d_E = 9, \tau = 6$), C ($d_E = 7, \tau = 11$), Meditation 1: A ($d_E = 6, \tau = 3$), B ($d_E = 8, \tau = 5$), C ($d_E = 7, \tau = 11$), Meditation 2: A ($d_E = 9, \tau = 6$), B ($d_E = 7, \tau = 12$), C ($d_E = 8, \tau = 3$)

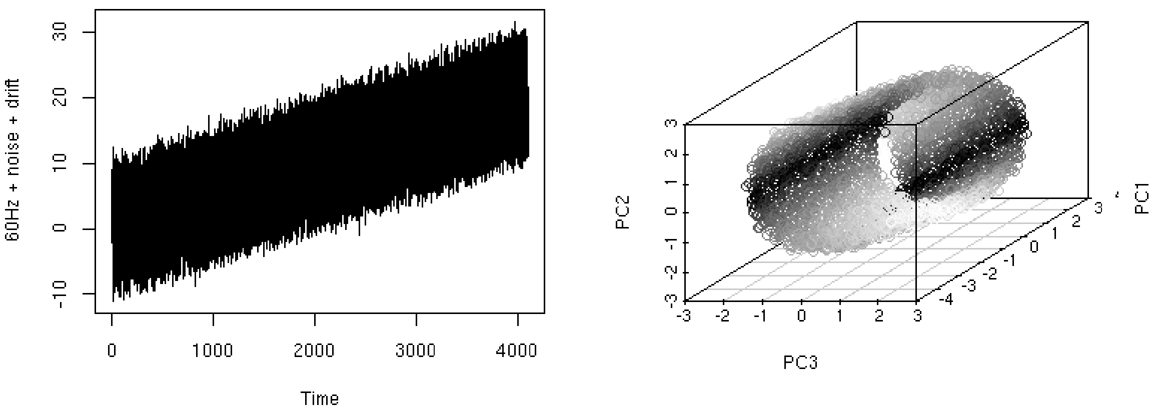

Figure 4: Left: Simulated 60 Hz oscillation with Gaussian noise and linear drift. Right: Same time series embedded with $d_E = 7, \tau = 6$. As before, the three dimensional plot maps the first three principle components against each other with the fourth component determining the gray scale value of each point.

2.4 Phase Randomization

One of the concerns about applying nonlinear time series analysis methods to noisy data raises the question of whether or not this actually tells us anything more than what traditional linear analysis has already told us **?**. Many researchers consider the analysis of the power spectrum as a valuable linear method in the study of EEG. For example, Aftanas and Golosheykin **?** have found Theta rhythms (4-8 Hz) associated with meditation while Alpha waves (8-12 Hz) often appear in correlation with a relaxed, eyes closed state and Beta (12+ Hz) corresponds with active concentration. This leads us to question whether or not delay embedding can tell us any thing more than what the power spectrum tells us. In order to test this, we took the Fourier transform of these data and randomized the phase while preserving the power spectrum **?**. We then converted this back into a surrogate time series to see what structure (if any) would appear upon delay embedding. We have seen that the cylindrical structure does not depend on the 60 Hz line noise (see Fig. **??**). The dependency on the overall power spectrum, however, appears more tentative. In Figure **??**, we randomized the phase of a 16 second segment from the first meditation session using a temporal electrode. In this case, randomizing the phase appears to have no effect on the structure suggesting that the power spectrum suffices in determining the structure.

However, when we embedded a 16 second segment from a frontal recording during the second meditation session, we see a different cylindrical structure that appears

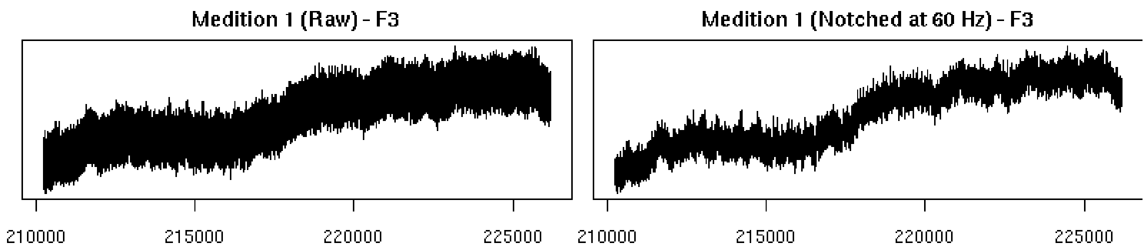

Figure 5: Left: 16 seconds from the right frontal region during the first meditation session. Right: Same as left with 60 Hz notch filter applied to remove line noise.

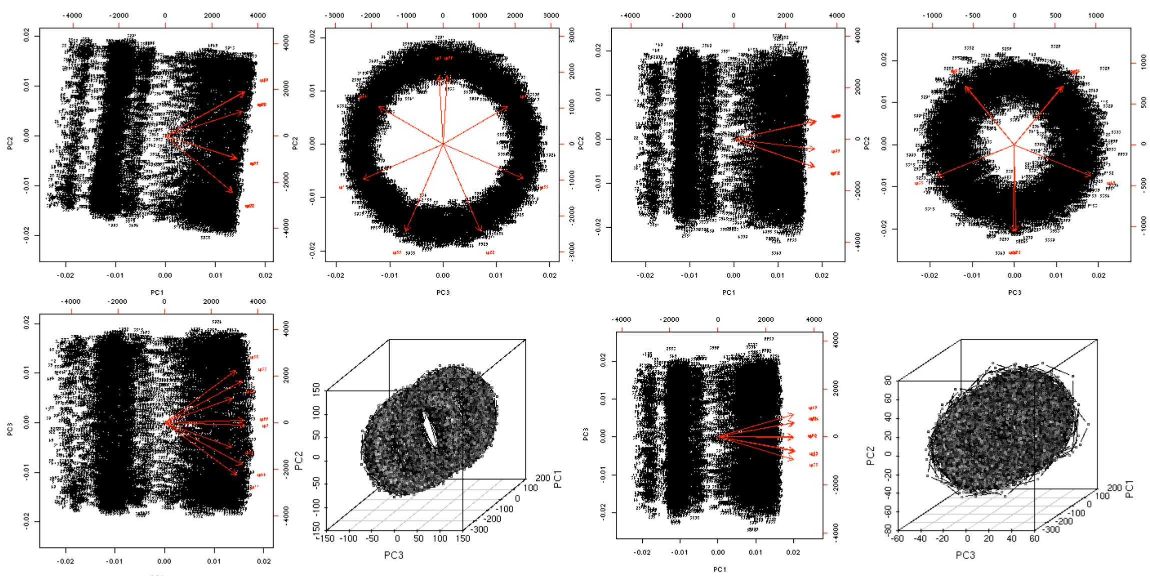

Figure 6: Delay embedding of F3 electrode: left — raw data, right — notched at 60 Hz. Here also we have rendered the images by virtue of PCA. The two dimensional plots show, pairwise, the first three components mapped onto one another for better clarity.

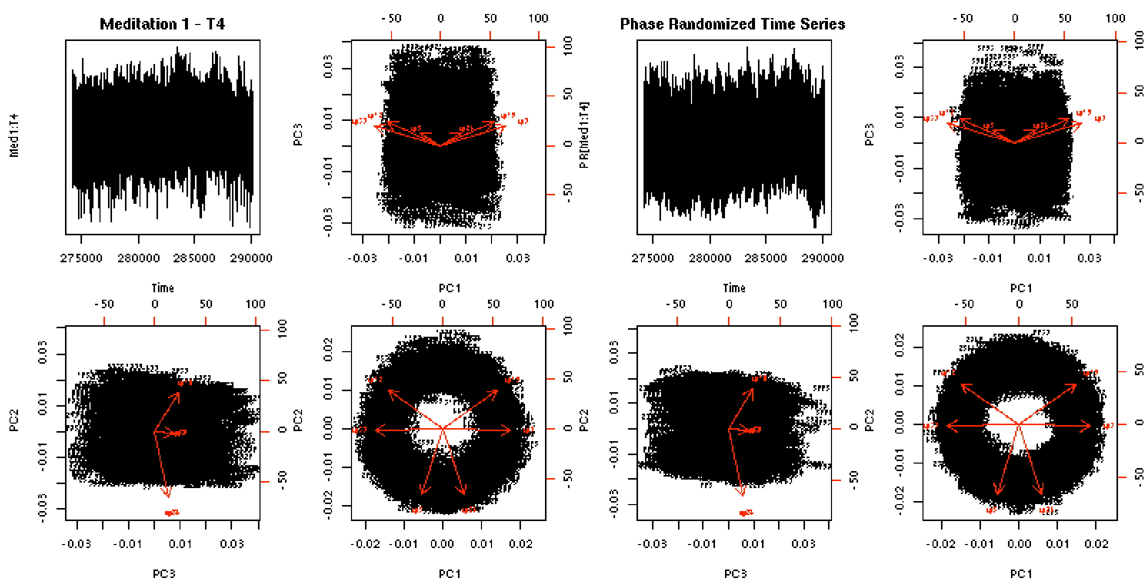

Figure 7: Left: 16 seconds from a temporal electrode recording during the first meditation session. Upper left corner represents a trace of this signal. Other subfigures show first three principle components mapped onto one another. Right: same as left following phase randomization. Note that the principle components from the delay embedding look virtually identical.

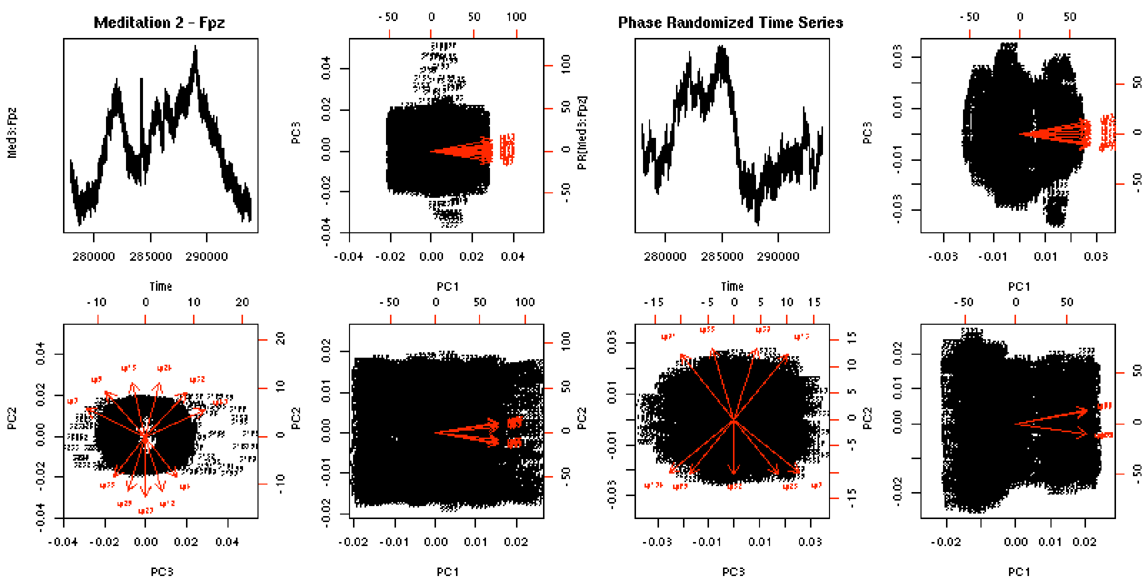

Figure 8: Left: 16 seconds from a frontal electrode recording towards the end of the second meditation session. Upper left corner represents a trace of this signal as in Figure ??. Right: same as left following phase randomization. Note how, unlike in Figure ??, the embedded structure has completely changed.

quite sensitive to the phase. The cylindrical structure appears despite considerable variance in the signal (over three times as much as in the previous example). When we randomize the phase of this signal, the cylindrical structure completely disappears.

3 Results

Delay embedding promises to produce a shape that is topologically equivalent to the internal phase space of a dynamical system. However, just as a donut is topologically equivalent to a coffee mug, mere topological equivalence may not suffice in explaining the subtleties of the data. Under certain conditions, EEG recordings, when delay embedded, take on a cylindrical form. The source of this form remains ambiguous and may arise from a variety of factors. One cause may hide in the power spectrum of the signal. However, while this may be sufficient (Figure **??**), it appears not to be the only way of producing this structure (Figure **??**). We remain confident that delay embedding may hold some potential in the interpretation of EEG signals. However, pulling the revealed structure apart into its various components remains an open question.

4 Discussion

In this study, we focused on 16 second intervals. We did this as we wanted to represent as large a section of each state as possible while keeping our computations feasible. If one were to consider event-related potentials, one might consider working on the level of a second or less. The advantage to this would be greater visibility with regard to subtleties in the structure. On the other hand, one could easily obtain spurious results in this fashion. As with traditional linear analysis, one would want to compare across samples.

In this study we have focused entirely on the qualitative shape of the embedded signal. The next step is to quantify these results into a model and compare that model across states.

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