Key Complexity Terms

Attractor
Autopoiesis
Chaos
Complexity
Developmental Systems Theory
Dissipative System
Distributed Control
Edge of Chaos
Emergence
Fractal
Generation History
Holon
Network/Graph
Nonlinear Dynamics
Order for Free
Phase Space/State Space
Phase Transition
Robustness
Self-Organization
Simulation

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NOTE: These definitions are works in progress

ATTRACTOR

An **attractor** is a point or collection of points in the ***state space*** of a dynamical model towards which a system tends. The region of state space in which the attractor exerts a "pull" on the **trajectory** of the system is called the **basin of attraction**. Like a marble placed somewhere on the inner sides of a bowl that inevitably rolls to the bottom, a trajectory that starts out at any point within the basin of attraction will eventually arrive at the attractor. An attractor may consist of a single state (as in the example of the marble coming to rest), in which case it is called a **point attractor**, or it may consist of a series of states through which the system cycles, in which case it is called a **periodic attractor.** A more complex type of attractor is the **strange attractor**, which is found in ***chaotic*** systems; trajectories in a strange attractor are aperiodic but remain within a bounded region and can often be described through **fractal** geometry. The phase space of complex systems often includes numerous attractors. Such **attractor landscapes** may change over time. For example, in the case of animal behavior, the attractor landscape may correspond to the animal's repertoire of learned or habitual behaviors, and environmental perturbations may modify this landscape as the animal adds or forgets behaviors. The concept of attractors provides a way to qualitatively and quantitatively understand the behavior of complex systems

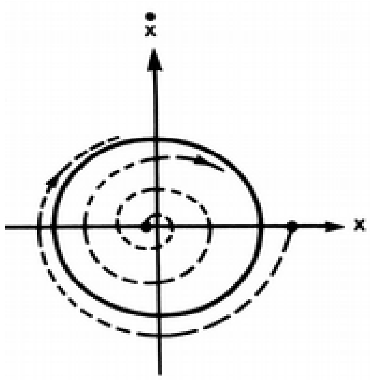

Figure 1: Phase portrait of a system with a periodic attractor. The attractor is represented by the solid line, while the dotted lines represent two different trajectories with different starting points that converge on the attractor. (Saltzman, Kelso 1987)

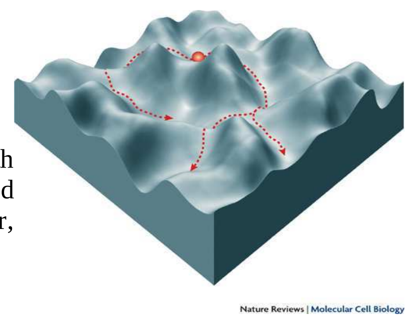

Figure 2: An attractor landscape with numerous attractors and attractor basins, and possible trajectories in red. (MacArthur, Ma'Ayan, Lemischka 2009)

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- MacArthur, B.D., Ma'ayan A., Lemischka, I.R. (2009). Systems biology of stem cell fate and cellular
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AUTOPOIESIS

Autopoiesis (literally “self-creation”) is a concept elaborated by Humberto Maturana and Francisco Varela (1979) to describe what the authors saw as the defining characteristic of life. An “autopoietic machine” displays “*a circular organization which secures the production or maintenance of the components that specify it in such a way that the product of their functioning is the very same organization that produces them*” (Maturana, Varela 1980, p48). For example, this circularity in biological processes is evident in the way that proteins are necessary to synthesize DNA, which is necessary for production of those same proteins. Crucially, the autopoietic entity maintains its identity by continuously producing its boundaries with respect to its surroundings, while continuing to exchange matter and energy with the environment, a characteristic that the authors describe as “**operational closure**.” Rather than sealing it off from its environment, the organism’s operational closure allows it to “couple” closely with its surroundings by only responding to certain aspects of the infinite complexity of its environment (based on what its organization allows it to perceive) and responding in selective ways (again, in ways determined by its organization). The concept of autopoiesis is important to the study of complex systems in that many such systems display a similarly circular logic of organization and act as autonomous or semi-autonomous entities due to their operational closure.

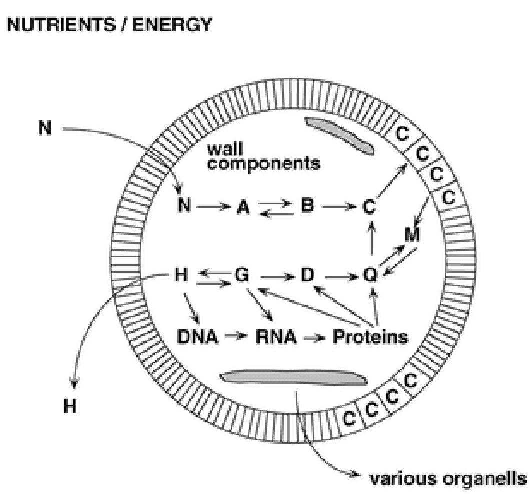

Figure 1: The cell as an autopoietic entity: here a living cell is schematized as an network of reactions that creates as one product a boundary separating the cell from its environment. (Luisi 2003)

References:

- Luisi, Pier Luigi. (2003) Autopoiesis: A Review and a Reappraisal. Naturwissenschaften 90: 49-59.
- Maturana, H., Varela, F. (1980) Autopoiesis and Cognition. London: D. Reidel Publishing Company, 1980.

CHAOS

Chaos is a mathematical term used to describe the behavior of certain kinds of aperiodic dynamical systems with extreme sensitivity to initial conditions. Although the behavior of such systems may appear random and unordered, they are deterministic and may display an underlying structure if analyzed using tools such as phase portraits. The state of chaotic systems at any point in time could theoretically be predicted, given precise values of all the relevant variables at an earlier point in time; however, their behavior is so sensitive to initial conditions that in practice, their trajectories are difficult or impossible to predict over longer time periods. For instance, a system that starts out with one variable initially set to 1.00000 may initially look the same as one that starts out with the same variable at 1.00001, but over time as the nonlinear aspects of the two systems amplify initial differences, their trajectories will diverge radically (figure 1). Despite the appearance of randomness, chaotic systems are not only deterministic but may also display high levels of order when analyzed using dynamical modeling. A phase portrait of a system displaying chaotic dynamics may reveal a **strange attractor** (figure 2), thus demonstrating a nonrandom structure and boundary in state space. Chaos is important in the study of complexity in that the nonlinear nature of complex systems often leads to chaotic dynamics.

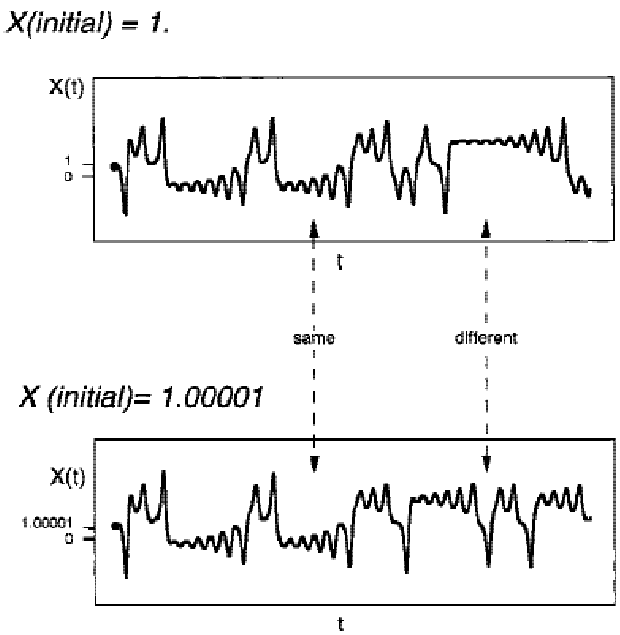

Figure 1: Two different time series for the Lorenz model of atmosphere showing divergence due to slight differences in initial conditions. (Liebovitch, Scheurle 2000)

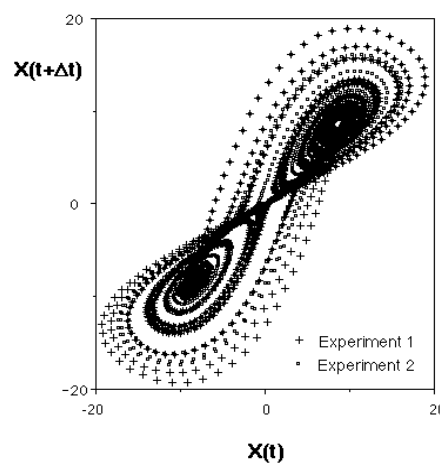

Figure 2: The same two time series plotted in phase space, where they make up parts of the same strange attractor (The Lorenz attractor), thus revealing a level of order not obvious from the time series data. (Liebovitch, Scheurle 2000)

References:

- Liebovitch, LS., Scheurle, D. 2000. Two lessons from fractals and chaos: changes in the way we see the world. Complexity 5(4): 34-43.

STANFORD COMPLEXITY GROUP
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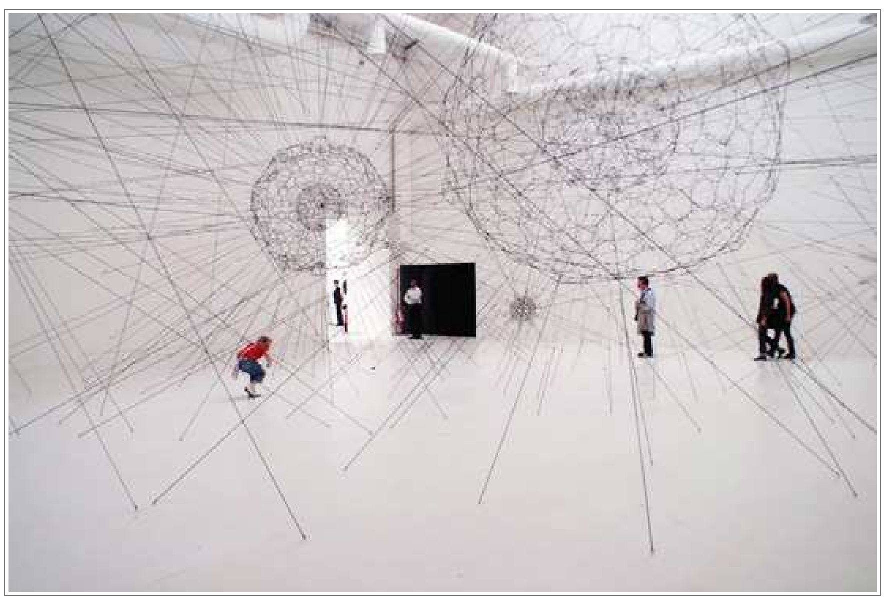
COMPLEXITY

Although there is a growing body of literature on the subject of how to define and even quantify complexity, for the purposes of this class we will refer to **complexity** as an approach to understanding systems rather than a property of systems. Used in this sense, complexity is a systems-based perspective that complements reductionist approaches. At the root of the word "complexity" is the noun "complex," meaning "an integrated whole," and at the root of the complexity perspective is the recognition that certain types of tightly integrated systems cannot be fully understood by studying their parts in isolation. A complex systems approach instead focuses on relationships between parts, elucidating patterns as a function of the dynamic interactions of components as constrained by system-level and environmental feedback. The rich collection of analytical tools that have been developed in the course of the study of such systems, including the concepts of **self-organization, emergence, autopoiesis**, and the mathematics of g**raph theory, fractal geometry** and **nonlinear dynamics**, have the potential to provide a common language and frame of reference for divergent realms of knowledge.



Tomas Saraceno's installation Galaxies Forming along Filaments, Like Droplets along the Strands of a Spider's Web, 2009 (http://e-flux.com/journal/view/217).

DEVELOPMENTAL SYSTEMS THEORY

Developmental systems theory (DST) is a perspective on development, heredity, and evolution (Oyama 2000; Oyama, Griffiths, Gray 2001). DST argues that, contrary to assumptions that arguably shape modern biology, there is no ontological distinction between genes and other factors in development, although biologists may focus on DNA for pragmatic reasons. Thus, instead of seeing development as an "expression" of pre-existing information contained solely within the DNA sequence, DST stresses the co-determination between genes and other developmental factors, as well as between the organism and its environment in evolutionary history. In contrast, the modern synthesis of molecular biology and genetics defines evolution as a change in allele frequency over generations. By definition, genetic inheritance is the extent of the ties that are recognized between parent and offspring; the role of the environment is to "select" the fittest individuals from each generation. DST, on the other hand, argues that "evolutionary systems" consisting of organisms and relevant aspects of their environment are more or less faithfully reconstructed in each generation due to a variety of inherited factors, such as genes, learned behavior patterns or cultural evolution, mutualistic micro-organisms, a built environment, protein conformations, and epigenetic modifications of the DNA. The move from a single locus of control of development (the gene) to distributed control (a network of interactions between genes, internal, and external factors that reshapes itself over time) makes the DST approach congruent with the complexity perspective.

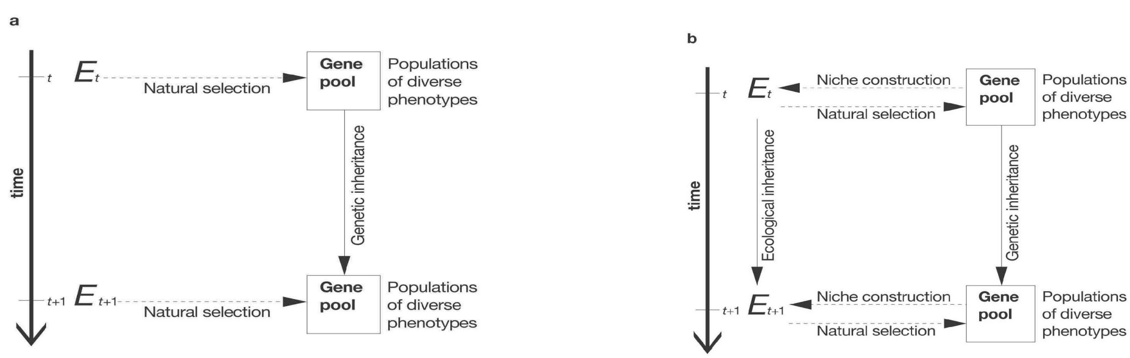


Figure 1: The diagram *1a* represents the standard view of evolution by natural selection acting over two generations, while 1b is updated to include the effects of niche construction and ecological inheritance. (Laland, Odling-Smee, Feldman 2001) Note the *1b* does not depict gene-environment interactions, which are stressed in DST; however, it is a step towards the picture of evolution as "cycles of contingency" painted by DST (note the bi-directional causality between organism and environment as well as the second channel of inheritance that has been added), as opposed to the linear-causal standard model.

References:

- Laland, K.N., Odling-Smee, F.J. Feldman, M.W. (2001) Niche construction, ecological inheritance, and cycles of contingency in Cycles of Contingency: Developmental Systems and Evolution. Oyama, S., Gray, R., Griffiths, P. (eds) MIT Press. 117–126.
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DISSIPATIVE SYSTEM

Dissipative systems maintain their structural identities through constant inputs of energy or matter while increasing the entropy of their environments, thus constituting potentially complex structures while conforming to the second law of thermodynamics. According to the second law of thermodynamics, entropy (initially defined as a loss of usable energy in the form of molecular motion (i.e., heat); later conceptualized as a measure of disorder) must increase in a closed system. Thus ice melts, mountains erode, and gases spread to fill rooms. However, systems open to exchange of matter or energy with their environments may move far enough from equilibrium that they reach a critical point where non-linear forces balance one another to create dynamic order. Overall entropy increases while a dynamic form or order is maintained within the system. Classic examples of dissipative systems include Bénard cells, which arise as the most efficient means to dissipate heat applied to a thin layer of liquid once the system moves past a critical point, whirlpools and cyclones, and living things, which maintain their internal organization through the metabolism of high energy/order inputs. The concept of dissipative systems is important to the study of complexity in that it provide a means of understanding how dynamic order may arise and be maintained in a universe in which overall entropy is increasing.

Figure 1: A whirlpool funnel. Gravity pulls a pool of water far from equilibrium until centrifugal forces balance one another to create a dynamic structure robust to perturbations (www.imagekind.com/Full-water-funnel-whirlpool-blue_art?IMID=2caeff65-2c41-47af-8926-417619a3d7ab 09/13/2011).

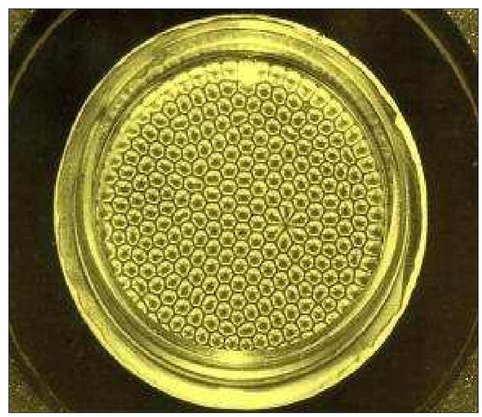

Figure 2: Bénard cells. Uniform heat applied to a thin layer of liquid pushes the system to a critical point where convection cells arise as the most efficient way to dissipate energy (www.metahistory.org/gnostique/telestics/HoneycombLight.php 09/13/2011).

References:

- Nicolis, G.; Prigogine, I. (1989). *Exploring complexity: An introduction*. New York, NY: W. H. Freeman.

DISTRIBUTED CONTROL

Distributed or decentralized control is a phrase used to describe systems whose behavior is not determined at a central location, but rather emerges from the actions of many actors or component parts responding to their local environments. A classic example of distributed control is found in insect colonies, in which no one insect or group of insects commands the rest. Nevertheless, the colony as a whole often displays behavior that seems as though it is being intelligently directed: for example, ants adjust the rate at which they forage according to the current available food supply, and ants may switch tasks (from foraging to colony maintenance, for example) according to the colony's needs (Gordon 2010). The idea that systems can behave adaptively and effectively without any one part encoding the "big picture" has been influential for organizations such as businesses and the military, for example with the concept of "swarm intelligence." The distributed control found in many instantiations of "network organization" is contrasted to more traditional hierarchical or "top-down" organizations. The concept of distributed control is closely related to complexity because it amounts to a consideration of causal powers from numerous parts of the system rather than just one, as well as necessitating a consideration of how those effects interact with each other. Distributed control also makes systems more ***robust*** and more resilient to injury or damage; since control does not reside in one location, there is often no one part of the system that is so crucial that the system could not function without it.

Figure 1: In some species, the rate at which an individual ant interacts with other ants performing specific tasks determines what it will do next (Gordon 2010). (www.superstock.com)

References:

- Gordon, Deborah. Ant Encounters: Interaction networks and colony behavior. Princeton University Press, 2010.

EDGE OF CHAOS

The edge of chaos describes a complex system being in a state of ***phase transition*** between order and ***chaos*** or the tendency of the system to approach such a state. For example, cellular automata can evolve to stable or periodic states (fig 1a) or follow a random (chaotic) evolution (fig 1c). Between these two extremes is the 'edge of chaos' (fig1b) where the system has interesting properties such as memory and information processing (Langton 1990). Though the idea that complex systems will tend to evolve to the "edge of chaos" has not been generally accepted, the metaphor that adaptive systems need to retain some balance of order and randomness in order to remain maximally flexible has been applied broadly to understand systems as diverse as evolving organisms and economies (Kaufman 1993, Mitchell et al. 1993, Pascale et al., 2000).

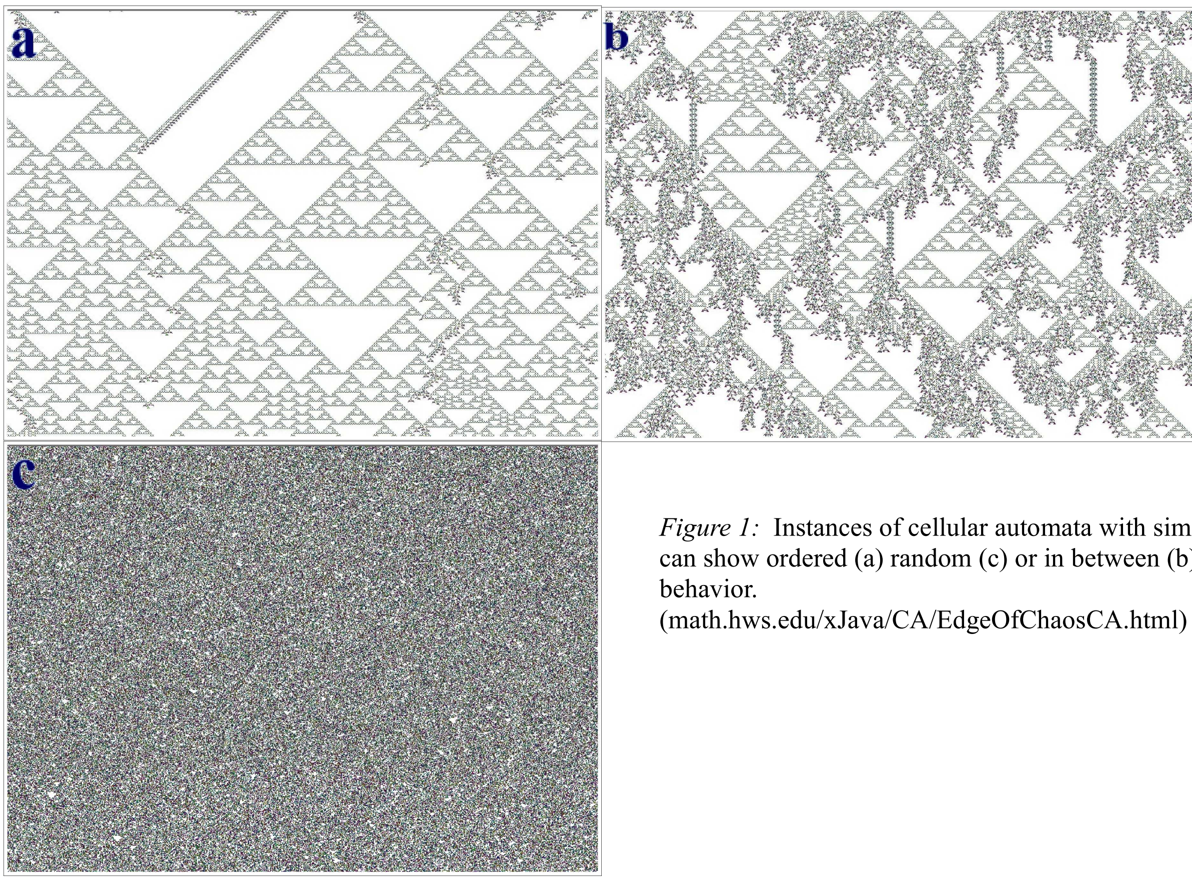

Figure 1: Instances of cellular automata with similar rules can show ordered (a) random (c) or in between (b) behavior.
(math.hws.edu/xJava/CA/EdgeOfChaosCA.html)

References:

- Langton, C. G. (1990). Computation at the edge of chaos: phase transitions and emergent computation. Phys. D, 42(1-3), 12-37. Elsevier Science Publishers
- Kauffman, SA. (1993). The origins of order: self-organization and selection in evolution. New York: Oxford University Press.
- Mitchell, M., Hraber, P. T., Crutchfield, J. P. (1993). Revisiting the edge of chaos: Evolving cellular automata to perform computations. Complex Systems, 7, 89--130.
- Pascale, RT., Millemann, M., Gioja, L.,. 2000. Surfing the edge of chaos the laws of nature and the new laws of business. New York: Crown Business.

EMERGENCE

Emergence refers to the process by which a novel property arises in a system through the interactions of its parts. Such emergent properties are either inapplicable to system constituents or appear to have some autonomy from them. For instance, liquid water arises from the combination of hydrogen and oxygen but the property of liquidity is inapplicable to individual atoms or molecules. Similarly, life is said to "emerge" from the interactions of organic molecules, even though none of these individual molecules themselves are "alive." Important distinctions exist between strong and weak emergence (causally potent properties versus epiphenomena), simple versus complex emergence (aggregate/average properties such as temperature versus the emergence of pattern or organization as in life), and epistemological versus ontological emergence (properties perceived as novel versus truly emergent in an objective sense). At stake are profound philosophical questions regarding the sufficiency of reductionist understanding: If emergent phenomena are "really real" and not just epiphenomena or epistemological constructs, then they may affect the workings of their lower level constituents, thus calling into question the validity of an approach focused solely on one level of analysis. Many, if not all, complex systems display emergent properties by one definition or another.

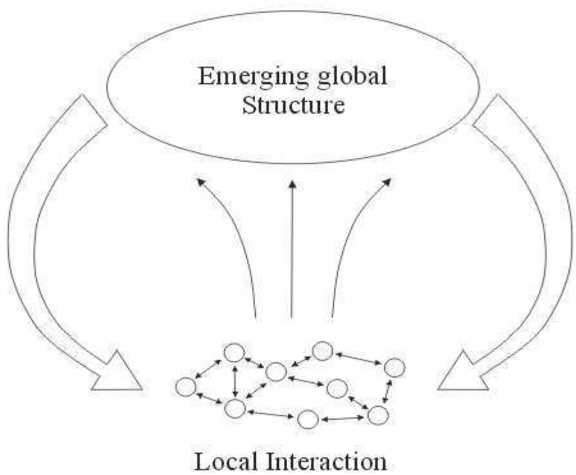

Figure 1: One interpretation of emergence (Lewin, R. 1999)

References:

- Bedau, M A and Humphreys, P (Eds). (2007) Emergence: Contemporary Readings in Philosophy and Science. London: MIT Press
- Bedau, M. (2002) Downward causation and the autonomy of weak emergence. Principia 6.1(2002): 5-50.
- Lewin, R. (1992). *Complexity: life at the edge of chaos*. New York: Macmillan Pub. Co.
- O'Connor, T., Wong, H, "Emergent Properties", *The Stanford Encyclopedia of Philosophy (Spring 2009 Edition)*, E N. Zalta (ed.), http://plato.stanford.edu/archives/spr2009/entries/properties-emergent/
- Haley, J, and Winkler, D. (2008) Classification of emergence and its relations to self-organization. Complexity 13:10-15

FRACTAL

Fractal is a term coined by Benoit Mandelbrot to describe a structure having an irregular or fragmented shape with some degree of self-similarity across all scales of measurement. Fractal geometry recognizes that many surfaces in nature cannot be easily categorized as one-dimensional (e.g., a line), two-dimensional (e.g., a square) or three-dimensional (e.g., a cube). Because of the jaggedness of these irregular objects, a “fractional dimension” more accurately describes their shape than a simple whole number. Consider blood vessels. Arteries are composed of self-similar branching structures across scales from major vessels to capillaries and have evolved to efficiently distribute nutrients across the 3-dimensional tissues of organisms. But they can’t take up all of that 3-dimensional space, or there would be nothing left but blood vessels. Rather, through their fractal, irregular structure they take up almost the full 3-dimensions (2.7 is a good approximation). Fractals relate to Complexity in a variety of ways: 1) Many objects and data sets related to complex systems are better described by fractal geometry and related power laws than traditional geometry and normal distributions; 2) Iterative algorithms used to construct fractal mathematical objects serve as topics of study in their own right as well as models of how natural systems can construct rich structures through the iteration of simple rules.

Figure 1: a) 1-dimensional line, b) 2-dimensional square, c) An irregular object with an effective dimension somewhere in between.

Figure 2: A fractal object in nature showing self similar structure across scale. The rightmost boxes are zoomed in photos of the romanesco on the left. (www.fourmilab.ch/ images /Romanesco 17/08/2011)

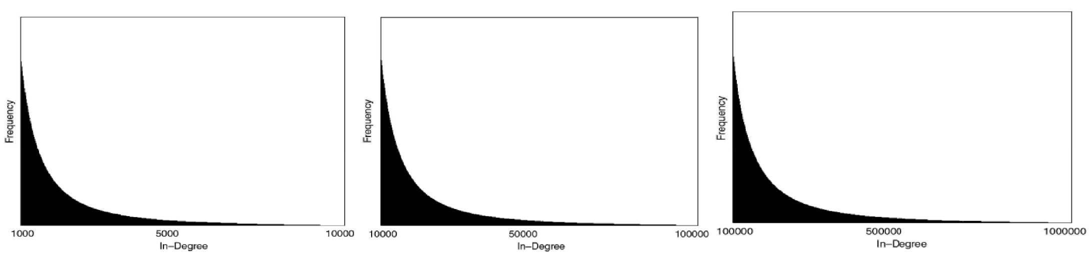

Figure 3: A fractal data set showing a similar relationship between dependent and independent variables across many orders of magnitude. If graphed on a log-log plot, this relationship would trace a straight line, the slope of which would be the characteristic parameter (power) of a “power-law.” (Mitchell, M. Complexity: A Guided Tour. Oxford University Press. 2009)

References:

- Mandelbrot, B. B. The Fractal Geometry of Nature New York: WH Freeman and Co. 1982

GENERATION HISTORY

The **generation history** of a system involves the full set of historical contingencies (including initial conditions) and developmental processes that led to its current state. Natural complex systems do not spring fully formed; they develop. Though such systems may be mathematically modeled, descriptive equations often cannot provide full information regarding how any particular system has arrived at its current configuration or how it will develop in the future. Additional information regarding initial conditions and the specific developmental paths taken by a system may also be needed. For instance, knowledge of the rules (equations) that define a cellular automata (CA) may be insufficient to understand how any particular simulation has arrived at a current configuration since there may be many ways of getting to the same attractor. Similarly, it may be impossible to know how the CA will develop in the future without actually letting the simulation proceed (letting the system trace out its future generation history). Adding simulated contingencies, such as environmental perturbations, to the CA would make knowledge of the particulars of its generation history that much more important. As another example, consider the Mandelbrot Set. Although the Mandelbrot Set is constructed through the iteration of a simple equation, its complexity arises by actually using this equation to generate the mathematical object. Thus, it is the generation of the set– using the equation to develop it by iteration - that leads to its complexity.

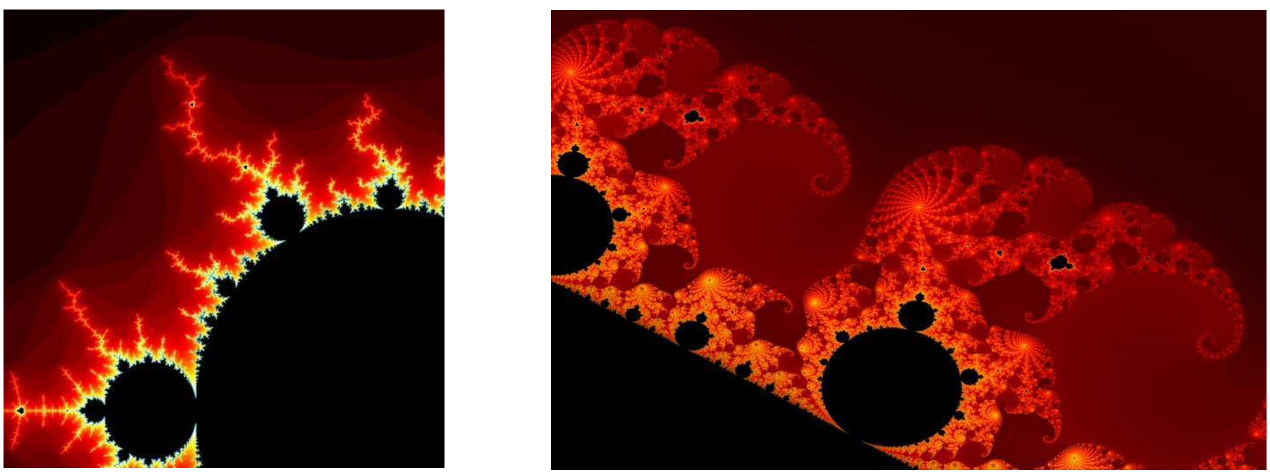

Figure 1: Two examples of the infinitely complex Mandelbrot Set. The equation describing the set is exceedingly simple ($Z = Z^2 + C$); however, its complexity results from generating the set through repeated iterations of the equation. (www.wallpaperhere.com/Abstract/Other/Fractals_the_mandelbrot_set_52447 13/09/2011)

HOLON

The word **holon** was first coined by Arthur Koestler (1967) to describe a unit in a hierarchical organization that is both a part of a larger unit and a whole composed of its own lower-level parts. For example, an organism is composed of organs and tissues but may also be part of a larger social group. Organs and tissues, in turn, are holons in that they are parts of the larger organism while simultaneously being composed of lower-level cells. Cells too are holons in that they are parts of organs and tissues, while simultaneously being composed of lower-level molecules, and so on, up and down the hierarchical organization. More broadly, the concept of holon applies to any type of complex system, such as company that is part of an economy, but in turn consists of local branches, each of which has its own subdivisions.

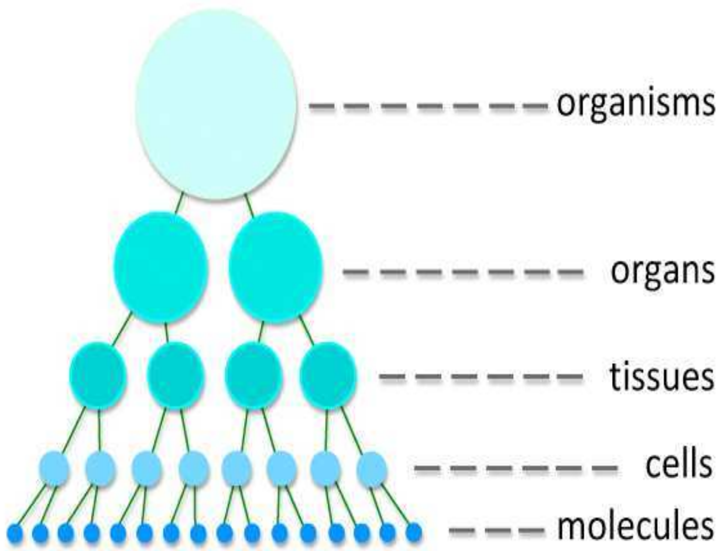

Figure 1: A biological "holoarchy," a hierarchy comprised of holons.
(http://metadesigners.org/tiki/display635 25/08/2011)

References:

Koestler, A. (1967). *The ghost in the machine.* A Gateway ed. Chicago: Henry Regnery.

NETWORK/GRAPH

A **network** is a way of representing a group of interconnected entities. The connected entities are called **nodes** or **vertices** and their connections are called **edges**. Edges may be **undirected** and simply connect nodes, or they may represent one-way or two-way **directed** flows of information or energy, the strengths of which may be represented by **weights**. For instance, in a neural network, the neurons are nodes, their synaptic connections are directed edges, and the strengths of those connections (e.g., the chances that a receiving neuron will fire if an upstream neuron fires) are captured in the weights of those edges. In a social network, individuals may be represented as nodes, relationships as directed or undirected edges, and the strengths of those relationships (in terms of number of interactions or some other metric) as weights. Networks are often called **graphs**, though the word "graph" is typically reserved for an abstract (mathematical) representation while the word "network" may refer to the physical system. Network analysis lends itself well to the study of complex systems since these systems are often composed of interacting parts.

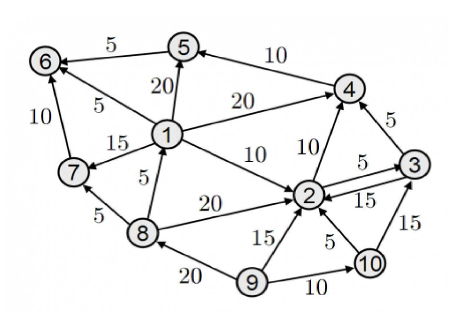

Figure 1: A directed graph. In this model, each node represents an agent who owes another agent a sum of money, represented by the numbers above the edges. (http://stochastix.wordpress.com/2009/07/ 08/25/2011)

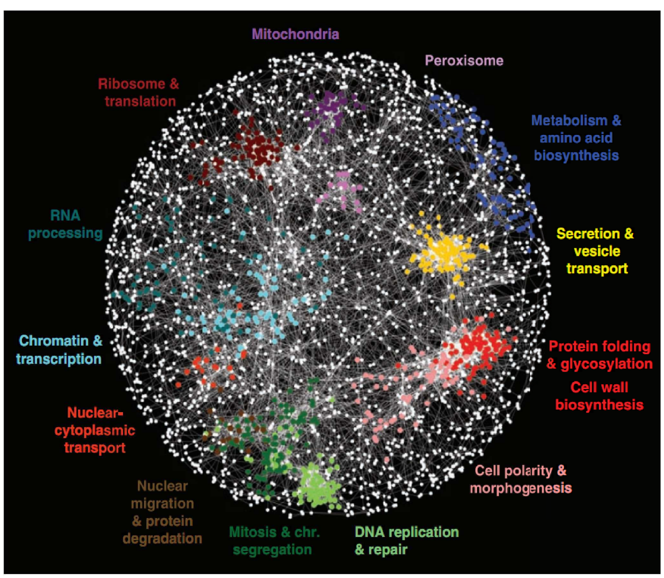

Figure 2: An undirected graph. Each node represents a gene. Undirected edges connect genes known to interactively affect fitness. (Costanzo, M, et al. (2010) The genetic landscape of a cell. Science 327:425-431)

NON-LINEAR DYNAMICS

Non-linear dynamics describe the changes through time (dynamics) of systems modeled by non-linear equations. Linear equations (such as $y = mx + b$) describe the relationship between variables that have a constant slope or rate of change. It doesn't matter where on the graph you are; moving a given number of units on the x-axis always gains you a fixed amount of change on the y-axis. Graphically, the linear relationship is depicted as a straight line. In non-linear equations, in contrast, moving a given amount (say, 2 units) on the x-axis leads to varying amounts of gain in y, depending on where you are on the curve (think of an exponential function such as $y = x^2$). A basic property of linear functions that is not true for non-linear functions is **additivity**: variables can be studied in isolation from each other and their combined effects are the sum of each separate effect. For example, imagine that having a given set of alleles (copies of genes) causes a plant to grow taller, and enriching the soil with phosphorous also causes the plant to grow taller. If these two variables interact linearly, the tallest plants will be those with the "tall" alleles that were grown in the soil with the highest concentration of phosphorous. However, if the variables interfere with each other in some way (perhaps one or more of the "tall" alleles also causes the plant to be extra-sensitive to phosphorous, so that high amounts are harmful to it), then the relationship between genotype and environment is non-linear - and we wouldn't necessarily be aware of the interaction between the two variables without testing each genotype in each environment. Since complex systems are those in which the overall behavior cannot be determined by studying the parts in isolation (they are non-decomposable), complex systems always have non-linear aspects to them. However, they may be approximated by linear models within certain bounds.

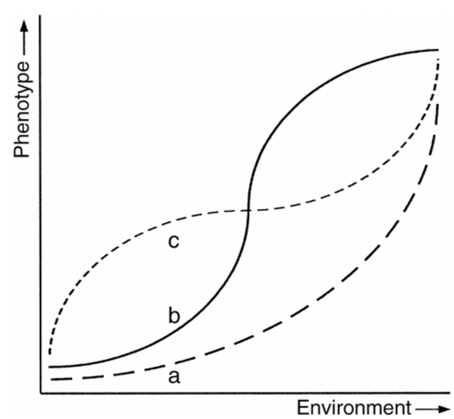

Figure 1: Non-linear "reaction norms" representing possible relationships between environment and phenotype for three different genotypes, a, b, and c (Postma, E., van Noordwijk, A.J. (2005). Ecology, 86, 2344–2357.).

ORDER FOR FREE

“Order for Free” is a term coined by Stuart Kauffman to describe the phenomenon whereby spontaneous order emerges from systems of interacting components without the need for external design or natural selection. Consider a random Boolean network of 100,000 nodes, where each node is randomly connected to two others and the current state of any one node depends on the previous state of the nodes to which it is connected. Depending on initial conditions, this system could wander into any one of 2^{100000} possible trajectories in state space. However, Kauffman showed that these random networks often do not explore such a vast space of possibilities, but rather, tend to settle down to a mere 370 or so stable **attractors**. The lesson is that an architecture based on interactions, even if randomly determined, may constrain the behavior of a system. In other words, interactions serve as a source of “order for free.” Perhaps this finding should not be surprising. We understand plenty of organization in the universe (planets, solar systems, galaxies) as resulting from interactions between particles independent of the guiding hand of natural selection or an active designer. “Order for Free” relates to Complexity in that it offers an alternative and complementary source of the order we see in complex systems. In this view, natural selection further constrains naturally occurring order to create adaptive fit between system and environment, but it is not the sole mechanism for the creation of order.

Figure 1: The universe displays stunning order. “Order for Free” explains this order as an emergent consequence of self-organizing interactions. (http://www.xda-wallpapers.com/ Space/1920x1200/ galaxies-_planets-_stars-_246 08/17/2011)

References:

- Kauffman, S. A. The Origins of Order: Self-Organization and Selection in Evolution. Oxford University Press. 1993.

PHASE SPACE/ STATE SPACE

Phase or **state space** is a mathematical construct that represents all of the possible states of a dynamic system. A **phase portrait** is a geometrical representation within phase space that illustrates the trajectory or trajectories of system through its state space. The "state" of the system (represented by a single point in phase space) is defined as the values of all of its variables at that moment. By graphing sequential system states on to an n-dimensional space, where n is equal to the number of variables taken into consideration, one is able to represent the global behavior of a system. Phase space and phase portraits are important in the study of complex systems for several reasons. First, the patterns traced by systems in their state space may provide information regarding regularities that are not obvious from time-series data (for example, well-defined **attractors** in a seemingly random data set is an indicator of **chaos**). Second, phase portraits may be particularly useful for **non-linear** systems because they provide a way of understanding the behavior of analytically intractable equations. Third, phase or state space represent a way of visualizing attractor landscapes, which provide a prominent means of conceptualizing the behavior of complex systems.

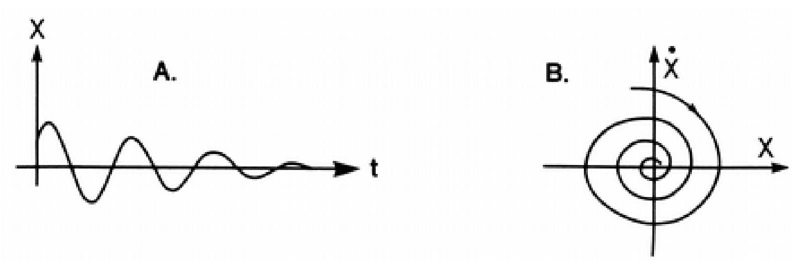

Figure 1: A swinging pendulum whose amplitude decreases over time until it comes to rest is a dynamical system with a **point attractor**. 1A plots the amplitude as a function of time, while 1B represents a 2-d phase portrait of the system, with amplitude and velocity on the axes and the point attractor at the origin. (Saltzman & Kelso 1987)

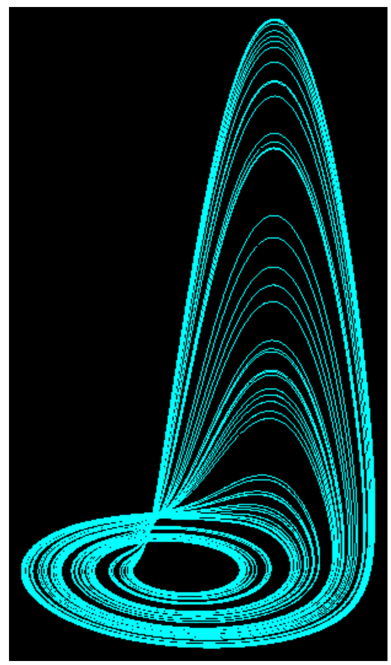

Figure 2: Phase portrait depicting the trajectory of a system of non-linear equations (the Rössler attractor) (http://chaos4.phy.ohiou.edu/~thomas/chaos/ode.html)

References:

- Saltzman, E., Kelso, J. A. (1987). Skilled actions: A task-dynamic approach. *Psychological Review, 94*(1), 84-106.

PHASE TRANSITION

The term **phase change** or **phase transition** comes from thermodynamics, where it is used to describe the transition between different states of matter. For example, when boiling water a change in quantity (temperature) leads, at some threshold or critical point, to a change in quality as water changes from liquid to gas. In an analogous sense, one also sees discrete changes in the behavior of complex systems as certain parameters (such as the interconnectivity of the parts of the system) are smoothly varied. Cellular automata and neural networks both display phase transitions from ordered to chaotic behavior as certain parameters are changed. In mathematics, these transitions are described as "bifurcations." In the terms of dynamical systems theory, they might be described as a transition from one **attractor** to another. The concept of phase transitions has been influential in the study of diverse complex systems such as the mental development in children (Jansen, Van der Maas 2001) and changes in society (Engels 1883).

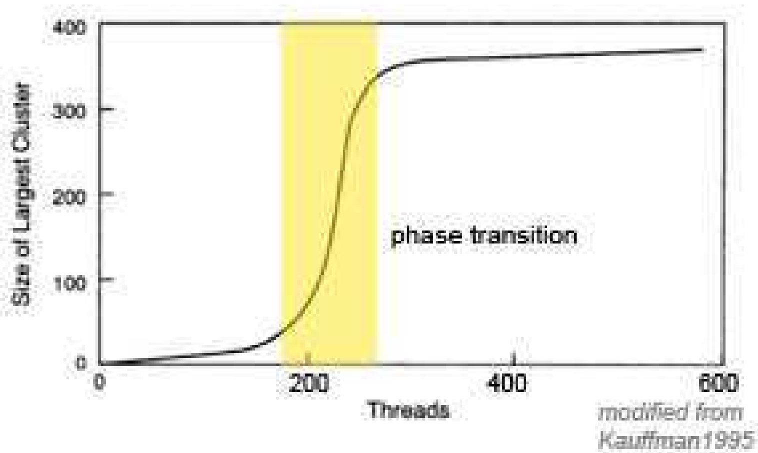

***Figure 1**:* Kauffman (1995) imagines spreading 400 buttons on a table, and tying one button to another with a piece of thread, one connection at a time, to create a simple **network**. At first, the clusters of buttons remain small; however, around 200 threads (edge:node ratio of 1:2), the size of the largest cluster of buttons dramatically increases, before leveling off again. The region connecting the two areas of steady increase is the phase transition. (www.dataspora.com/category/analytics/page/2/)

References:

- Jansen R.J.J, Van der Maas, HL J (2001) "Evidence for the Phase Transition from Rule I to Rule II on the Balance Scale Task." *Developmental Review* 21, 450–494.
- Kauffman, S. (1995) At Home in the Universe. New York: Oxford University Press, 1995.
  Engels, F, (1883) (Dutt C. P (transl 1973) *Dialectics of nature*. New York: International Publishers.

ROBUSTNESS

Generally a system is considered **robust** if it can function or sustain its structure and characteristics despite perturbations. This may be because it actively resists such perturbations (homeostasis, canalization, or buffering), because many configurations of the system have the same function or structure (redundancy, resilience), or because the system is able to alternate between varying steady states. An example of the first case is the temperature of a mammal which, through altered behavior, sweating, or shivering, will remain relatively constant despite changes in the environment. An example of the second type of robustness is the redundancy of genetic code with which a gene can produce the same protein even if a third of its nucleotides have changed. An example of the third type is neural dynamics. Such dynamics may be envisioned as moving from one chaotic attractor to another depending on internal and external perturbations. Although neural network states change moment by moment, the brain as a whole continues to function. In a similar way, development can often shift to varying basins of attraction depending on environmental stimuli – for instance, some amphibians can develop to become carnivorous or herbivorous depending on environmental conditions. Their development does not break down despite being pushed by circumstances into drastically different dynamic trajectories. Most, if not all, complex systems display one or multiple types of robustness, or they would cease to function and thus soon cease to be.

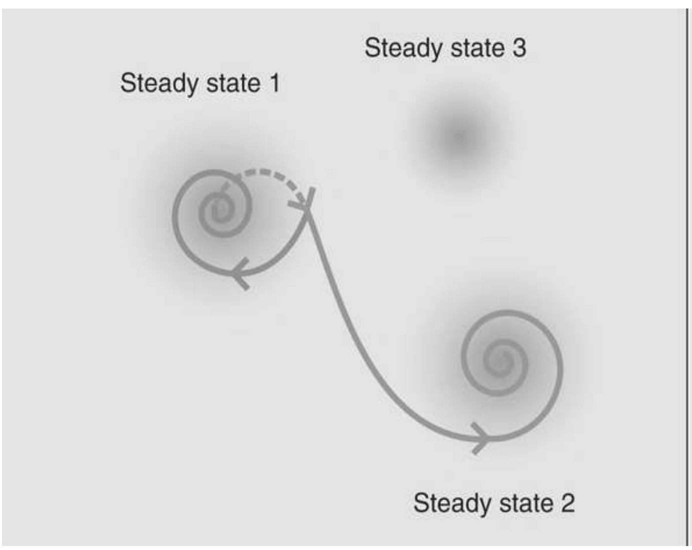

Figure 1. Robustness can involve movement of a system from one dynamic steady state to another according to internal or environmental perturbations, as long as overall function of the system is not compromised (Kitano 2007)

References:

- Kitano, H. (2007). Towards a theory of biological robustness. Molecular Systems Biology, 3(1)
- Wagner, A. (2005). Robustness and evolvability in living systems. Princeton, N.J.: Princeton University Press.
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SELF-ORGANIZATION

Self-organization refers to the production of pattern or order in a system through the interactions of the system's parts. For instance, insect colony nests can display incredible organization, yet these structures are built without blueprints or leaders; they are the result of the self-organizing dynamics of interactions between individual insects, each following simple rules based on local information. Similarly, flocking birds or schooling fish display striking coherence of movements at the population level despite the fact that no individual animals are in charge; each time a school of fish turns to evade a predator, new fish take their places at the front of the group. Self-organization is an important concept in the study of complex systems in that many complex systems lack centralized control, and hence their order arises through self-organizing dynamics.

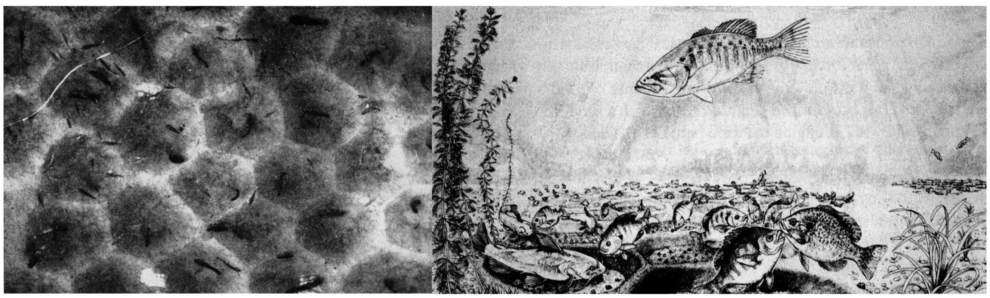

Figure 1: Left – Polygonal pattern of male *Tilapia mossambica* territories. Right – Similarly shaped territories of bluegills. These patterns are the result of local interactions between fish. Each territory is a pit dug in the sandy bottom of the water and results from a combination of positive and negative feedback from neighbors. (Camazanine et al., 2001)

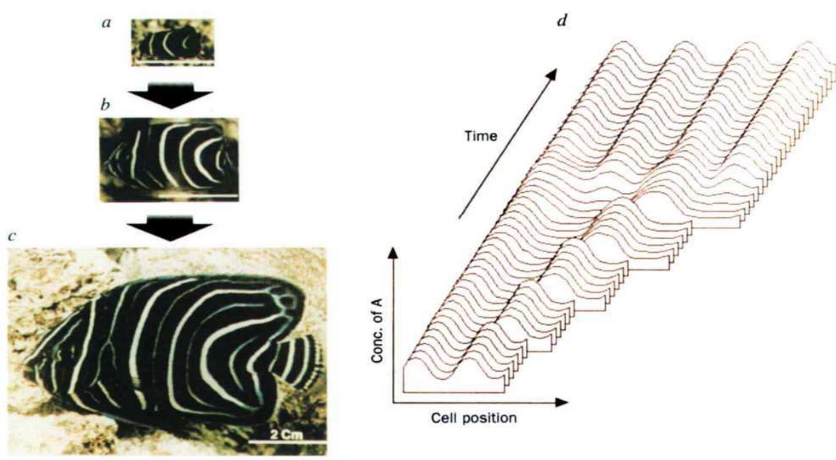

Figure 2: Many patterns in nature arise from self-organizing dynamics. For instance, during development of *Pomacanthus* patterns arise from the interactions of cells, each operating according to local information.(Kondo, Asai., 1995)

References:

- Camazine, S., et al. (2001). *Self-organization in biological systems.* Princeton [NJ]: Princeton University Press.
- Kondo, S., Asai, R. A (1995) Reaction-diffusion wave on the skin of the marine angelfish *Pomacanthus*. *Nature* **376**, 765-768.

SIMULATION

A **simulation** is a dynamic model of a real world system at some level of abstraction. In the contemporary sense of the term, “simulation” usually refers to a model created on a computer by which it is thought that we can gain information about the properties of the real object. For example, a simulation of the earth's climate can be carried out by combining measured data, such as the physical dimensions of the planet or details of the solar radiation that reaches it, with physical laws, such as the transfer of heat in liquids. By incorporating time as a parameter, such a model can simulate a system as it changes through time. In the earth climate simulation, we could thus extrapolate to past and future states of the climate. Simulations are often undertaken to understand systems when analytical solutions are difficult or impossible to achieve. This is often the case with complex systems given their highly nonlinear nature. Two of the most popular ways to model complex systems are agent-based simulations and genetic algorithms. In **agent-based simulations**, agents are modeled as autonomous entities which interact according to set rules. From these interactions, system-level behaviors and patterns may emerge. In **genetic algorithms**, agents undergo some analog of selection and the system as a whole learns to solve some arbitrary problem.

Figure 1: A snapshot at day 130 from a dynamic simulation showing how oil released at the location of the Deepwater Horizon disaster in the Gulf of Mexico would spread. (http://www2.ucar.edu/news/2154/ocean-currentts-likely-carry-oil-along-atlantic-coast 08/23/2011)

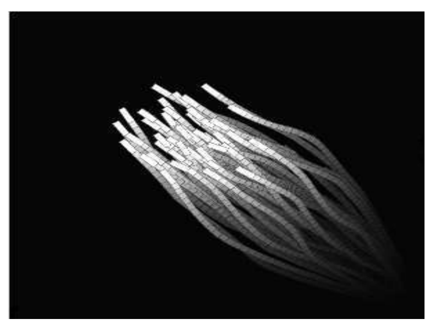

Figure 2: An implementation of the boids simulation, showing how flocking behavior can emerge from agents following simple rules using local information. (http://www.local-guru.net/blog/2009/03/15/boids-demo 08/23/2011)