Introduction to Nonlinear Dynamics

Santa Fe Institute

Complex Systems Summer School

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Chaos:

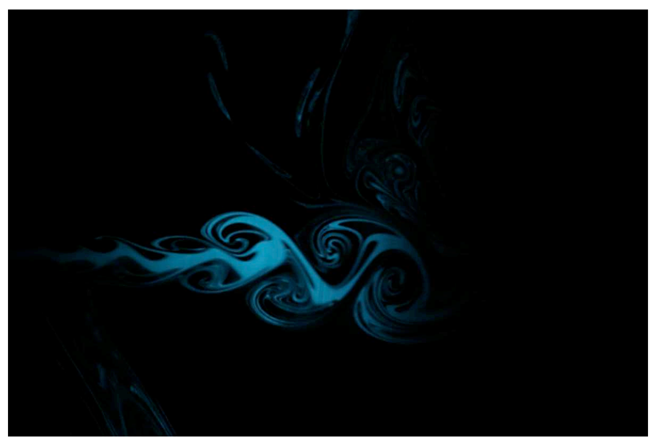
Complex behavior, arising in a deterministic nonlinear dynamic system, which exhibits two special properties:

- sensitive dependence on initial conditions
- characteristic structure…

Systems that exhibit chaos are ubiquitous; many of them are also simple, well-known, and “well-understood”

Where chaos turns up:

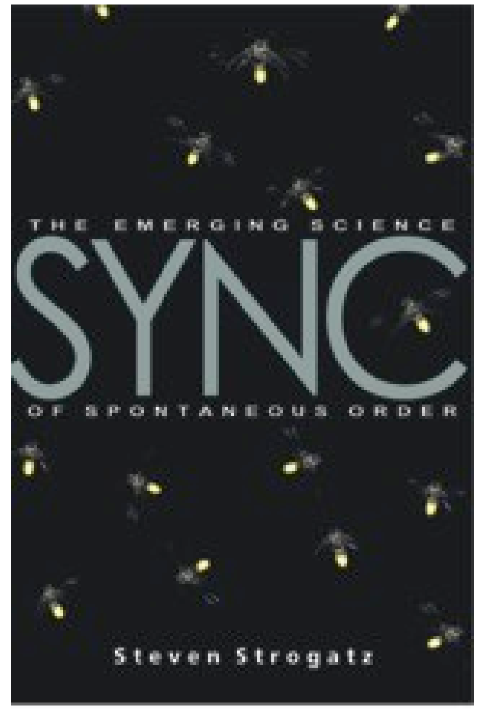
- Flows (of fluids, heat, …)
  - Eddy in creek
  - Weather
  - Vortices around marine invertebrates
  - Air/fuel flow in combustion chambers



Where chaos turns up:

• Driven nonlinear oscillators

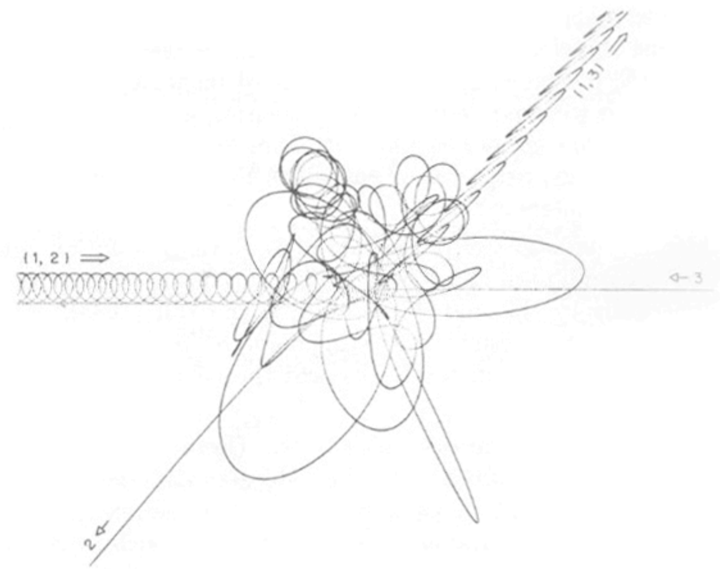
- Pendula
- Hearts
- Fireflies



- and lots of other electronic, chemical, & biological systems

Where chaos turns up:

- Classical mechanics
  - three-body problem
  - paired black holes
  - pulsar emission
  - ….



Hut & Bahcall
Ap.J. ***268***:*319*

- Protein folding
- Population biology
- And many, many other fields (including yours)

- continuous time systems:
    - time proceeds smoothly
    - “flows”
    - modeling tool: differential equations

- discrete time systems:
    - time proceeds in clicks
    - “maps”
    - modeling tool: difference equation

A useful graphical solution technique:

- "cobweb" diagram
- return map
- correlation plot

Bifurcations

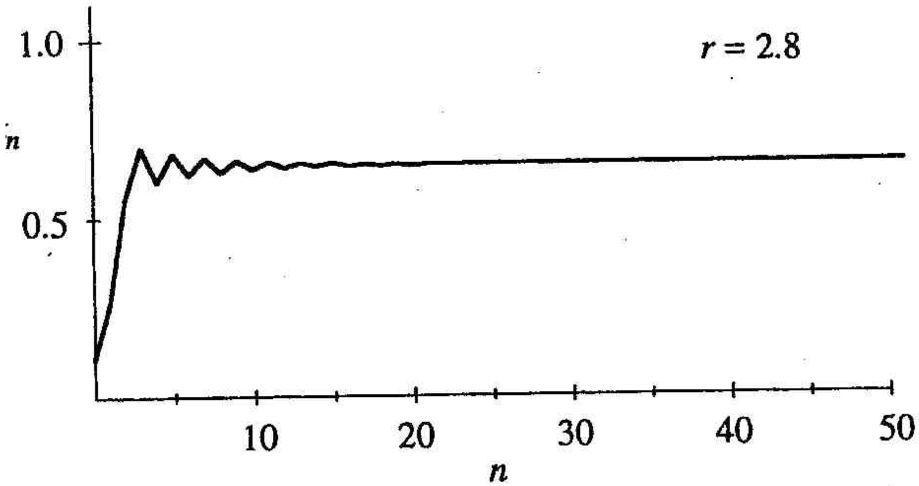
Qualitative changes in the dynamics caused by changes in *parameters*

Bifurcations

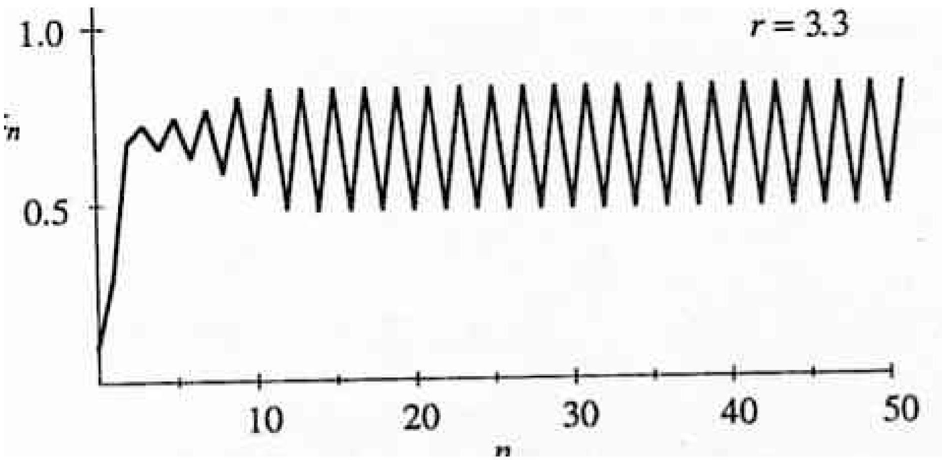
Qualitative changes in the dynamics caused by changes in *parameters:*

- Heart: pathology
- Eddy in creek: water level
- Olfactory bulb: smell
- Brain: blood chemicals
- etc. etc.

Bifurcations in the logistic map:

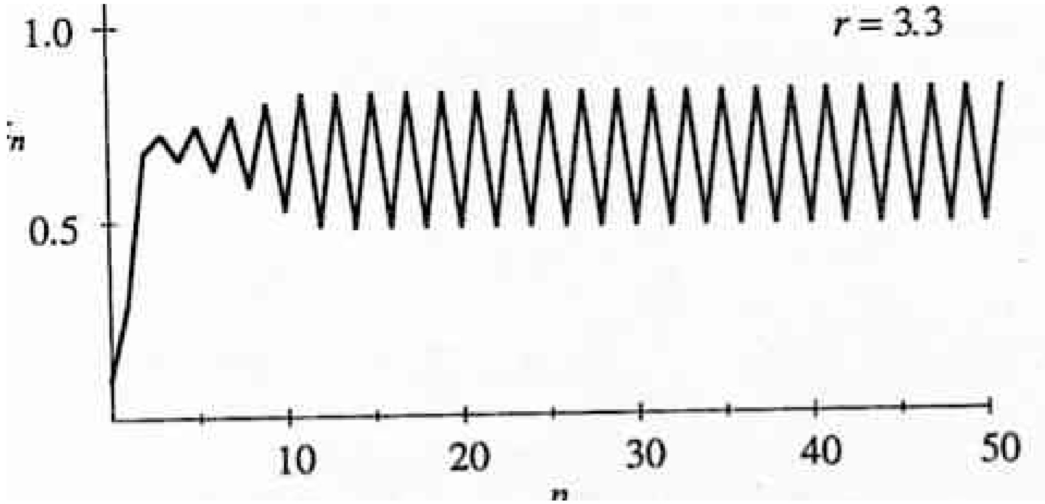


R=2.8

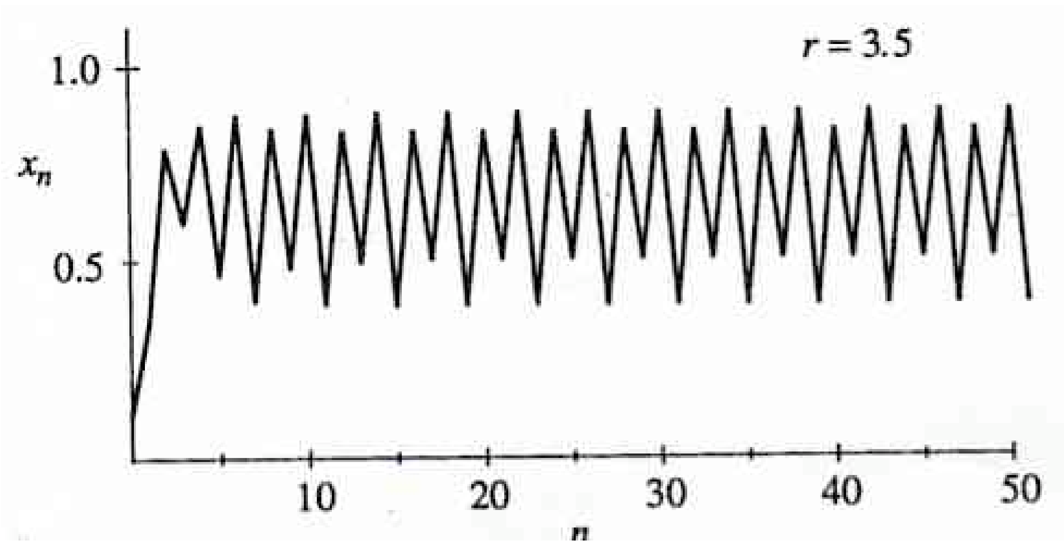


R=3.3

Discrete time: <u>should not connect dots!!</u>



R=3.3



R=3.5

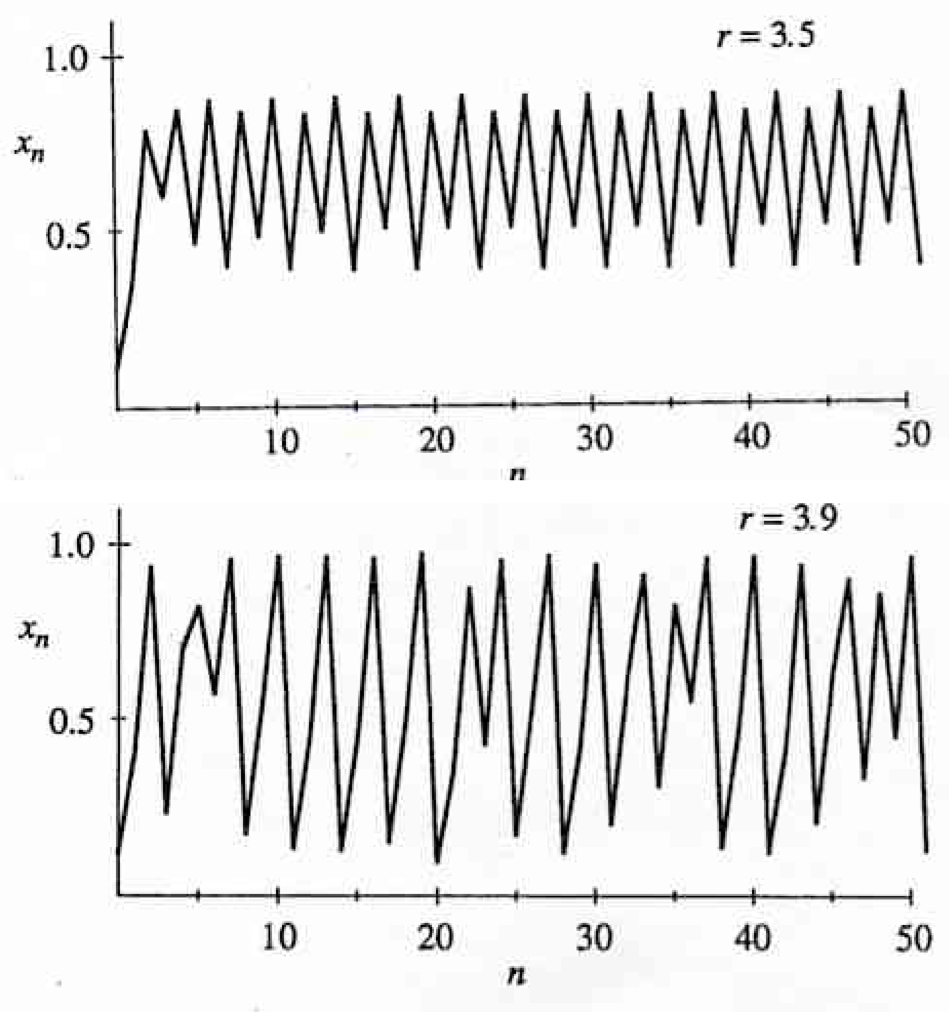
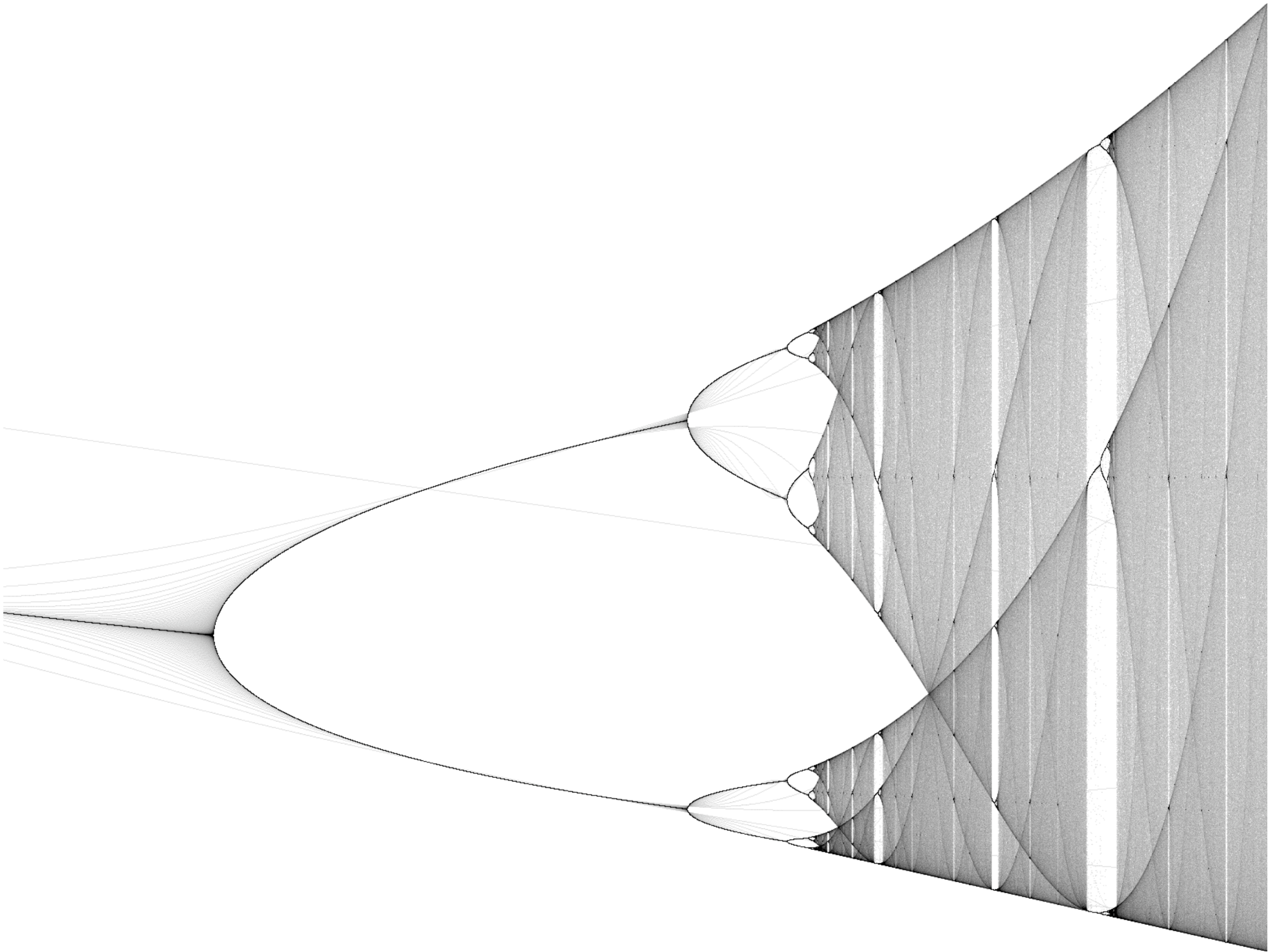


Figure 10.2.5

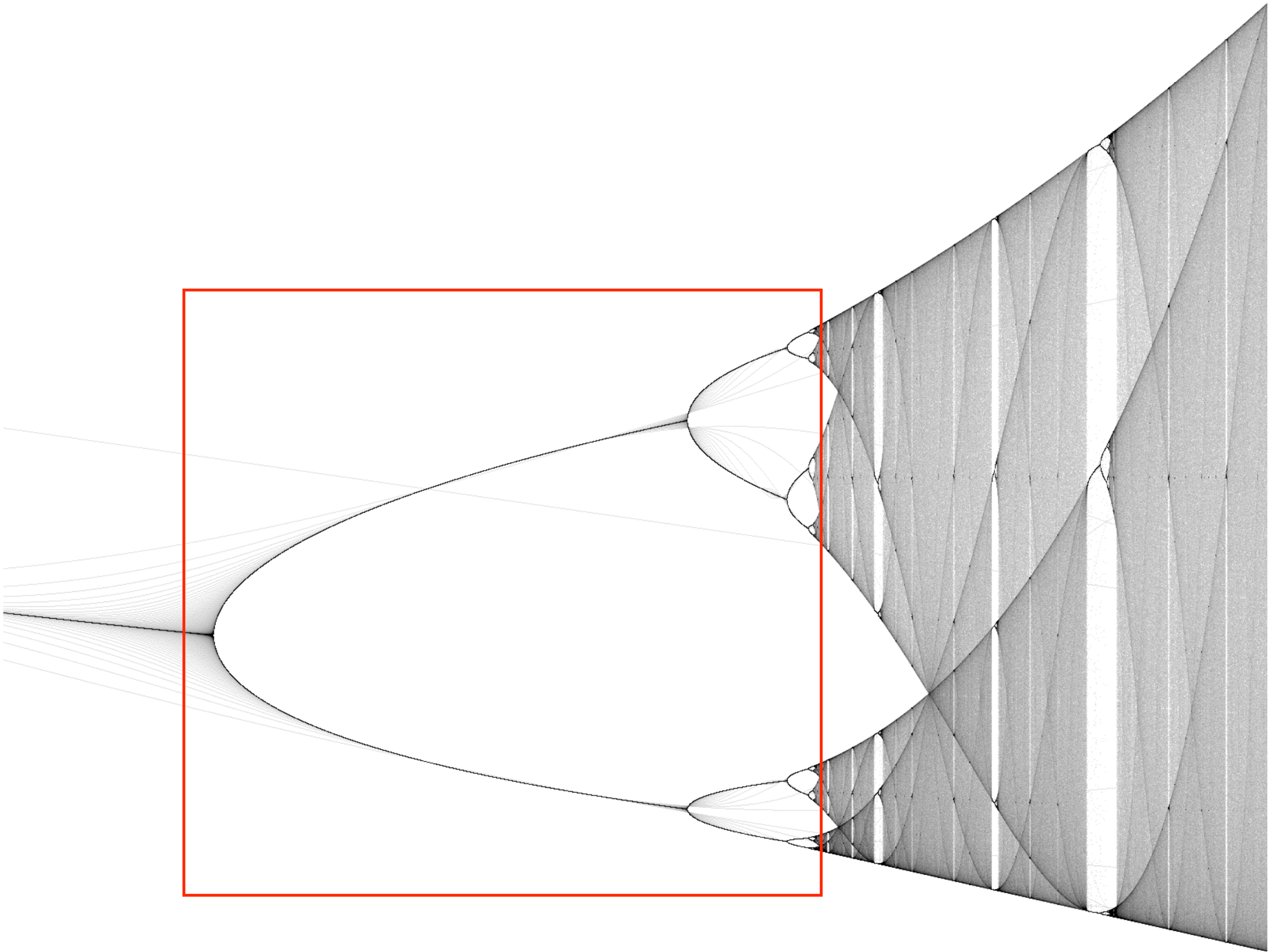
R=3.5

R=3.9

These plots stolen from *Strogatz*

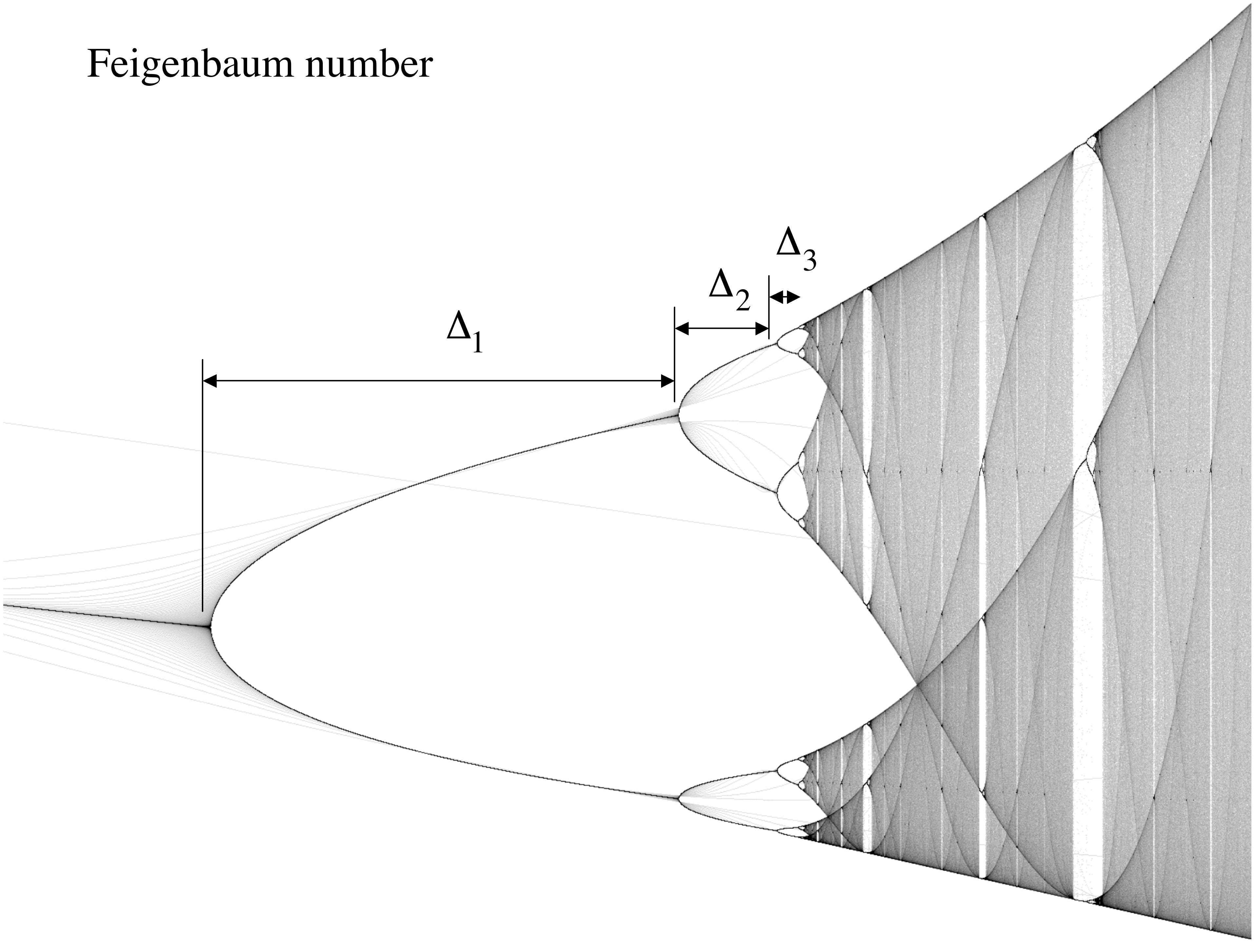


- chaos
- veils/bands: places where chaotic attractor is dense (UPOs)



- chaos
- veils/bands: places where chaotic attractor is dense (UPOs)
- *period-doubling cascade @ low R*

Feigenbaum number

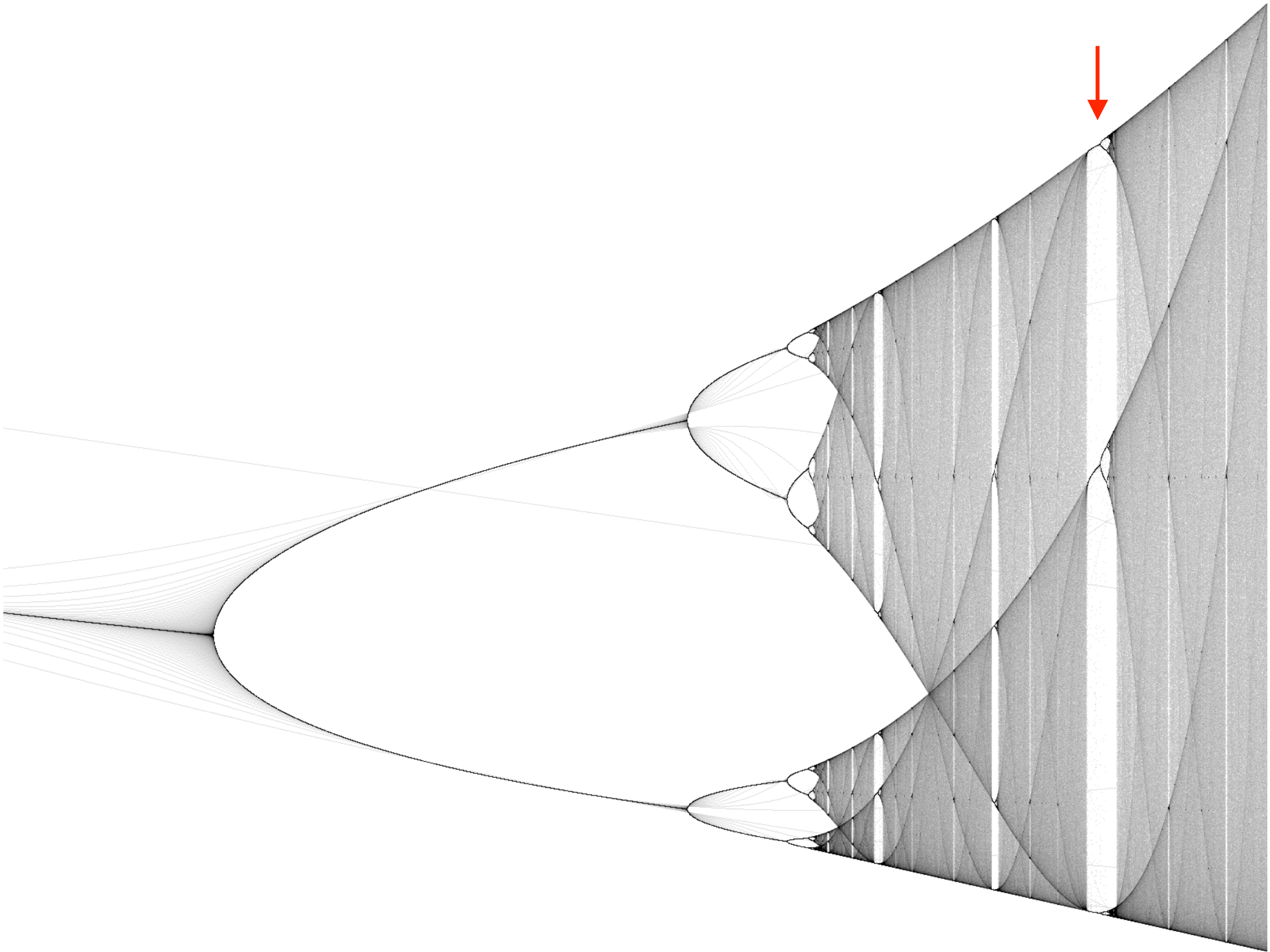


Universality!

Feigenbaum number and many other interesting chaotic/dynamical properties hold for any 1D map with a quadratic maximum.

Proof: renormalizations. See Strogatz §10.7

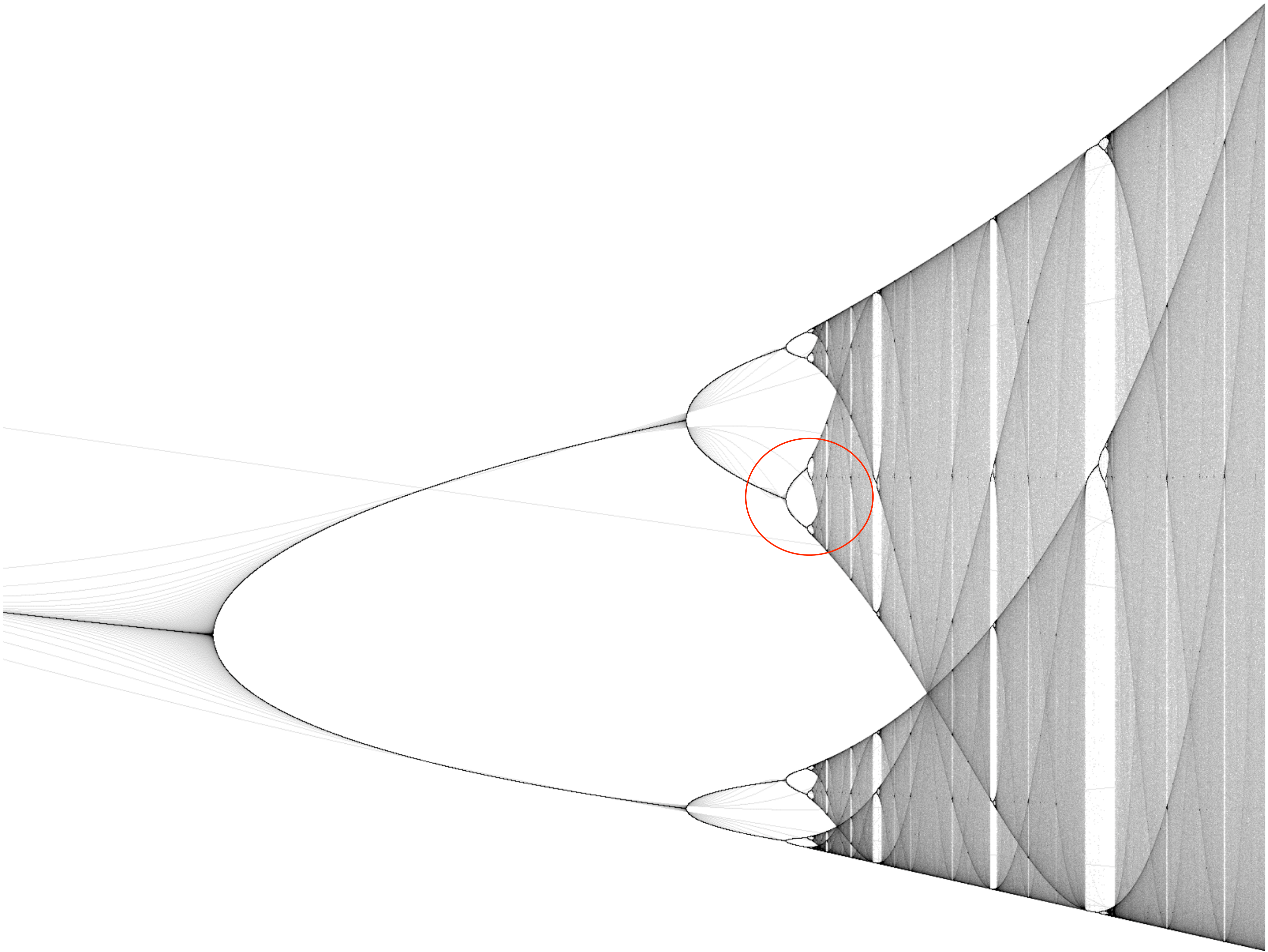
Don’t take this too far, though…



- chaos
- veils/bands: places where chaotic attractor is dense (UPOs)
- period-doubling cascade @ low R
- *windows of order within the chaos, complete with their own period-doubling cascades (e.g., 3 to 6 to 12)*

A bit more lore on periods and chaos:

- Sarkovskii (1964)
- Yorke (1975)
- Metropolis *et al.* (1973)



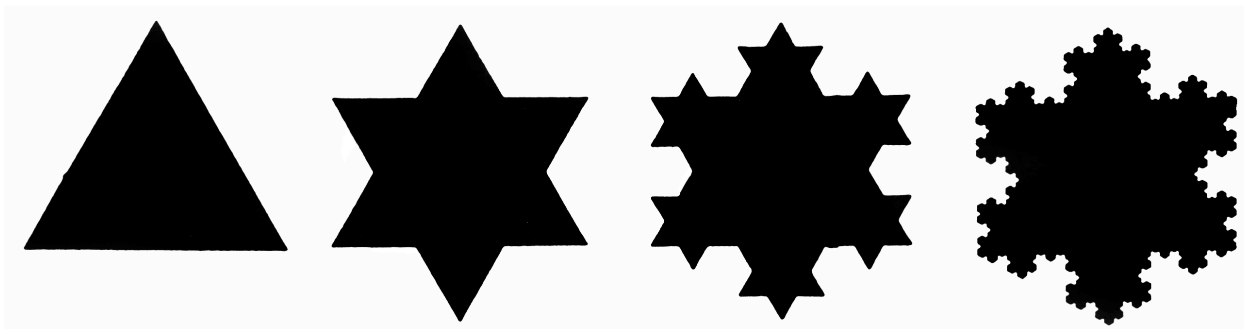
- chaos
- veils/bands: places where chaotic attractor is dense (UPOs)
- period-doubling cascade @ low R
- windows of order within the chaos, complete with their own period-doubling cascades (e.g., 3 to 6 to 12)
- *small copies of object embedded in it (fractal)*

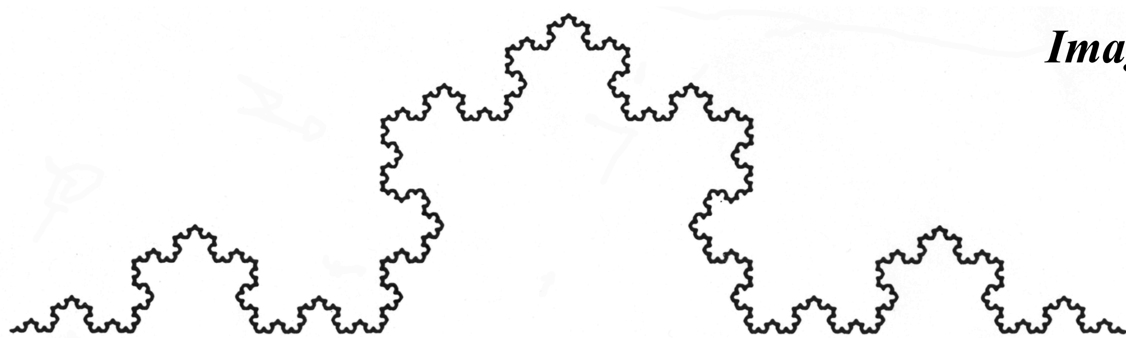
Fractals and Chaos…

The connection: *many (most)* chaotic systems have fractal state-space structure.

Fractals

• non-integer Hausdorff dimension
• self-similar

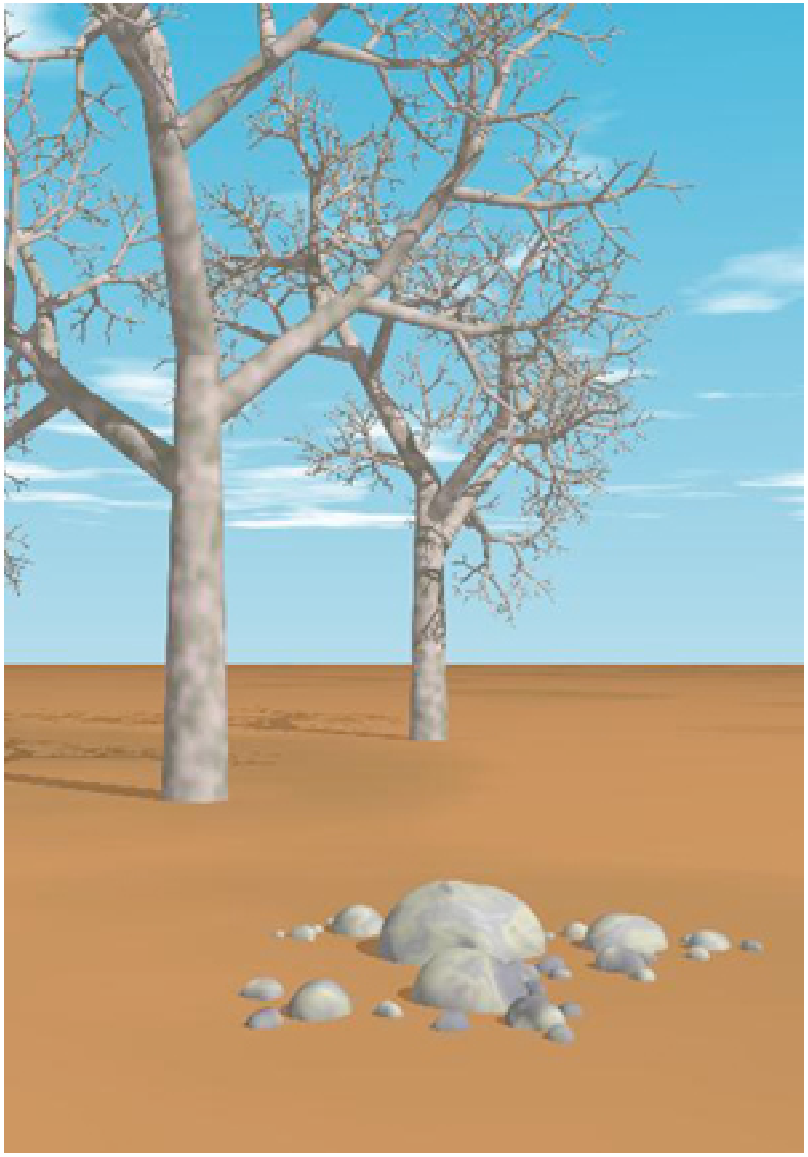




Images from Gleick.

Examples: Cantor set, coastlines, trees, lungs, clouds, drainage basins, …

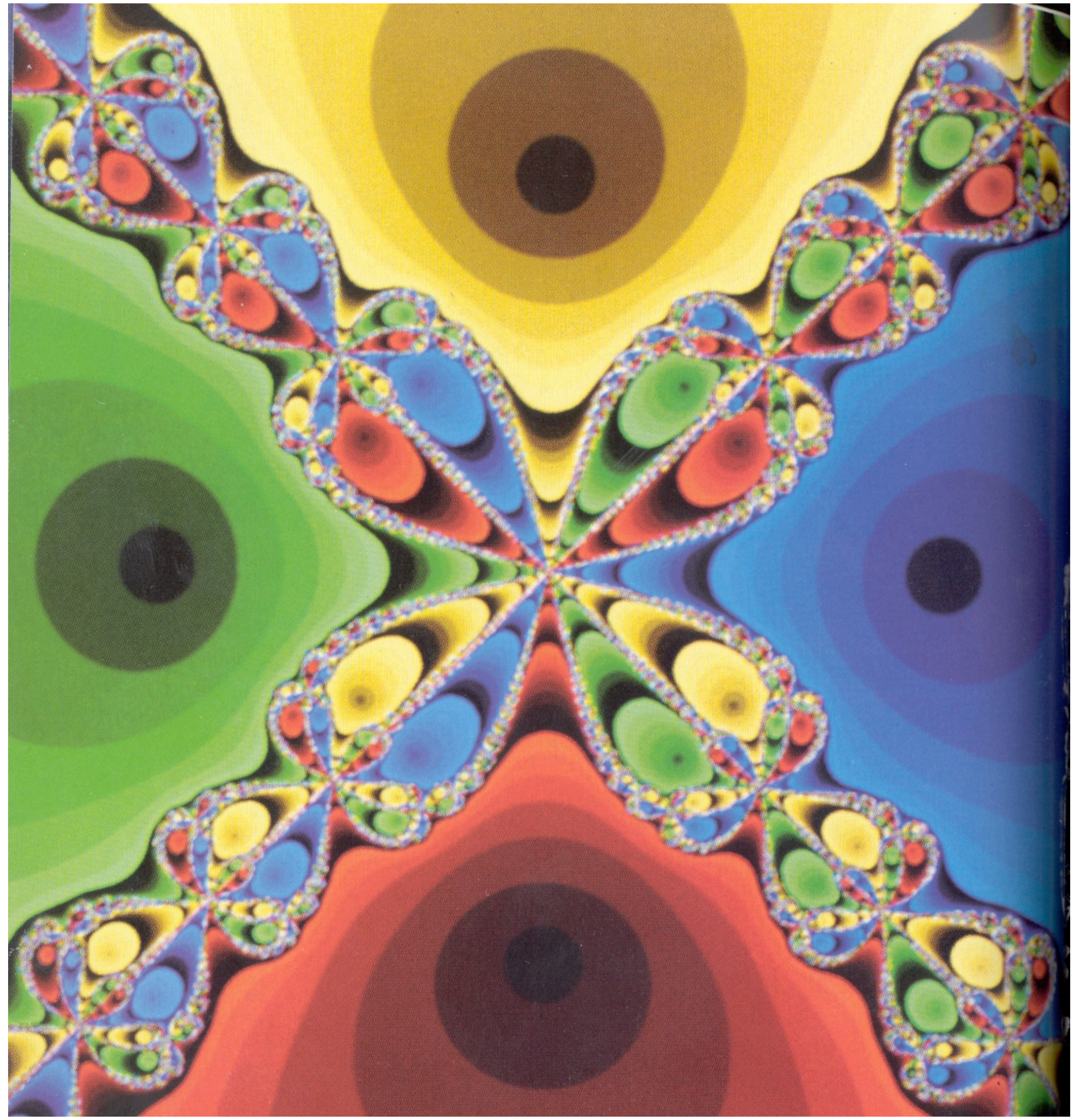
In computer graphics…



Matthew Ward, WPI
http://davis.wpi.edu/~matt/courses/fractals/trees.html

In maps:

Newton's method
on $x^4 - 1 = 0$



From Strogatz

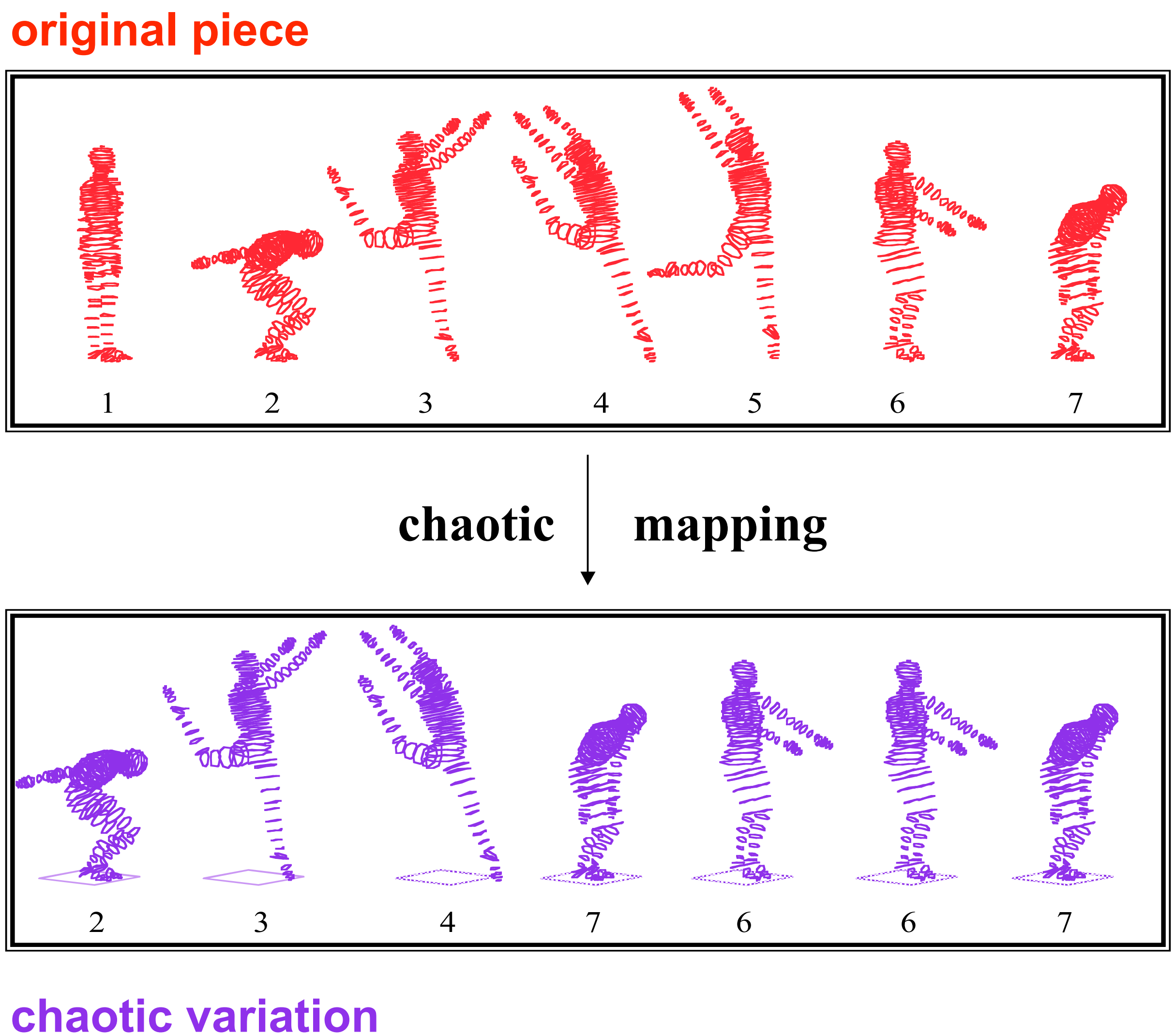
That was all about *maps*.

- discrete time systems:
  - time proceeds in clicks
  - “maps”
  - modeling tool: difference equation

Next: *flows.*

- continuous time systems:
    - time proceeds smoothly
    - “flows”
    - modeling tool: differential equations

But first…


original piece
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chaotic
mapping
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chaotic variation