

# Evolution: I-3

- **Foundations** - *evolutionary dynamics*
- **Phylogeny and Ontogeny** - *evolution of development*
- **Frontiers** - *Demons, coevolution, niche construction and cultural change*

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# Replicator-Mutator (Quasispecies) Equation

$$\dot{g}_i = \sum_j^{2^n} g_j r_j(\mathbf{g}) m_{ij} - g_i \bar{f}$$

Kimura-Maruyama Equation - population genetics

$$\dot{g}_i = \sum_{j=0}^N \frac{r_{N-j} g_{N-j}}{\bar{f}} \frac{\mu^j}{j!} e^{-\mu}$$



# *Eigen Error Threshold*

*Delta function:*

$$\mu < \frac{s}{L} = \frac{1}{L}$$



# Kimura's Neutrality Inequality

*Condition for Neutrality*      $sN < 1$

# Evolution's Arrow

*Eigen-Kimura*

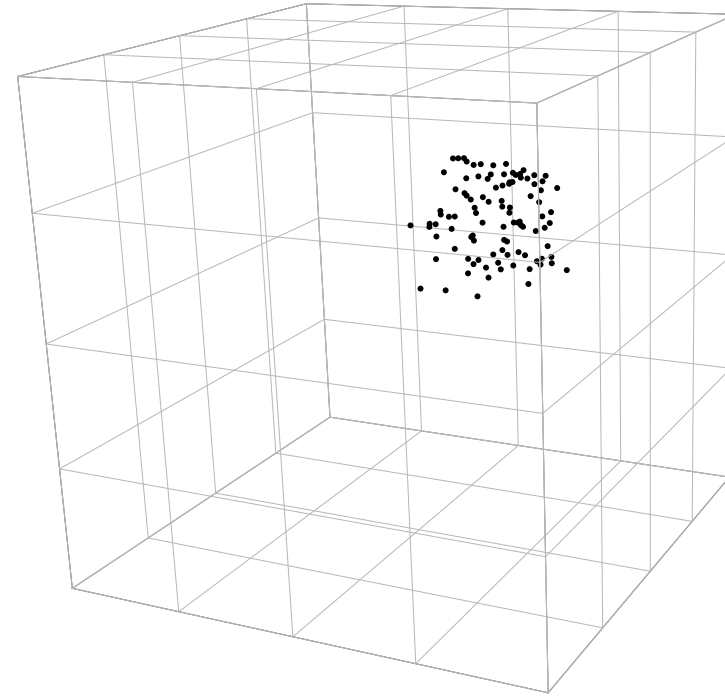
*Information Storage*

*Principle*

# Evolutionary Information Storage

*Information Conserved*

$$\mu L < 1 < sN$$

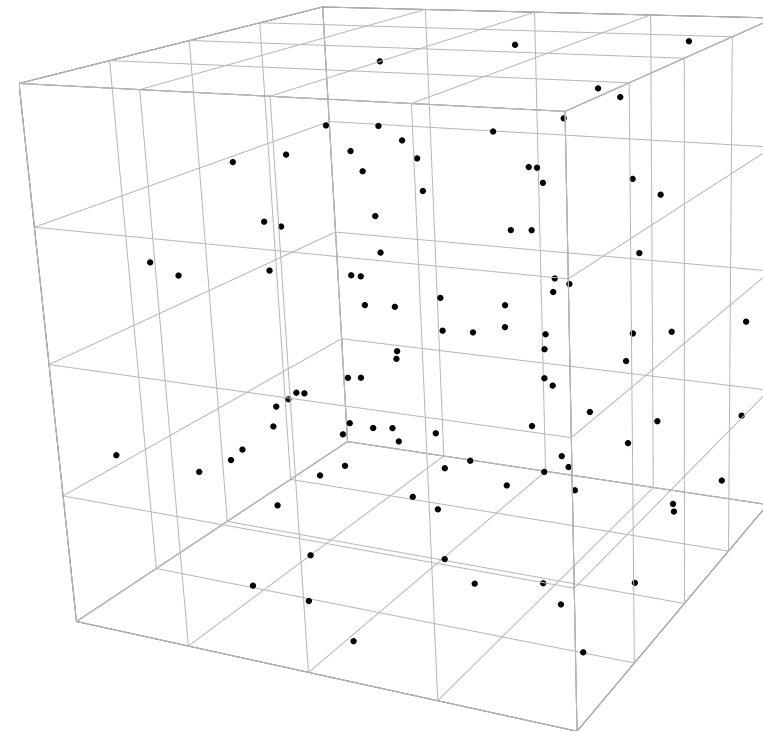


*Information Lost*

$$\mu L > 1 < sN$$

$$\mu L < 1 > sN$$

$$\mu L > 1 > sN$$



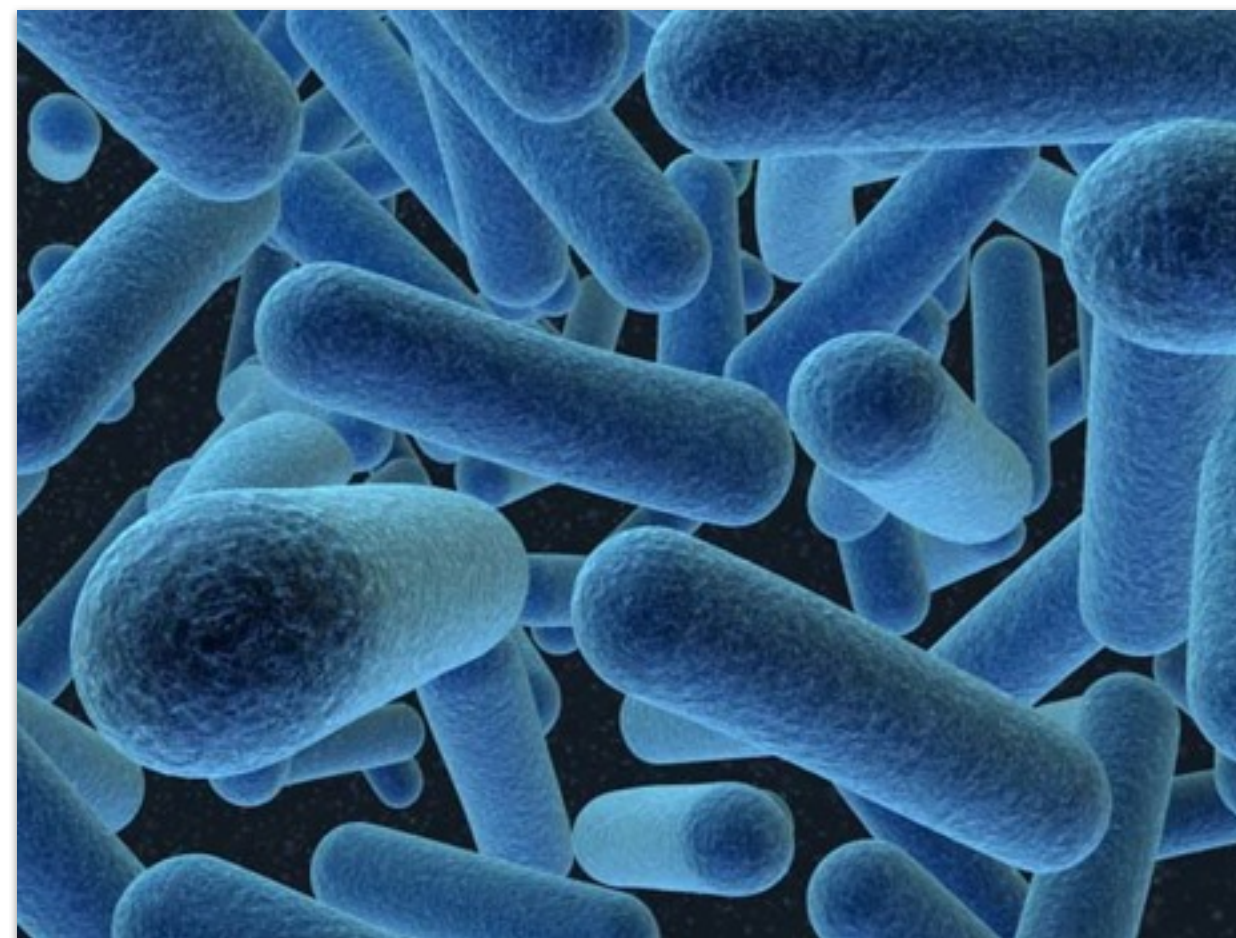
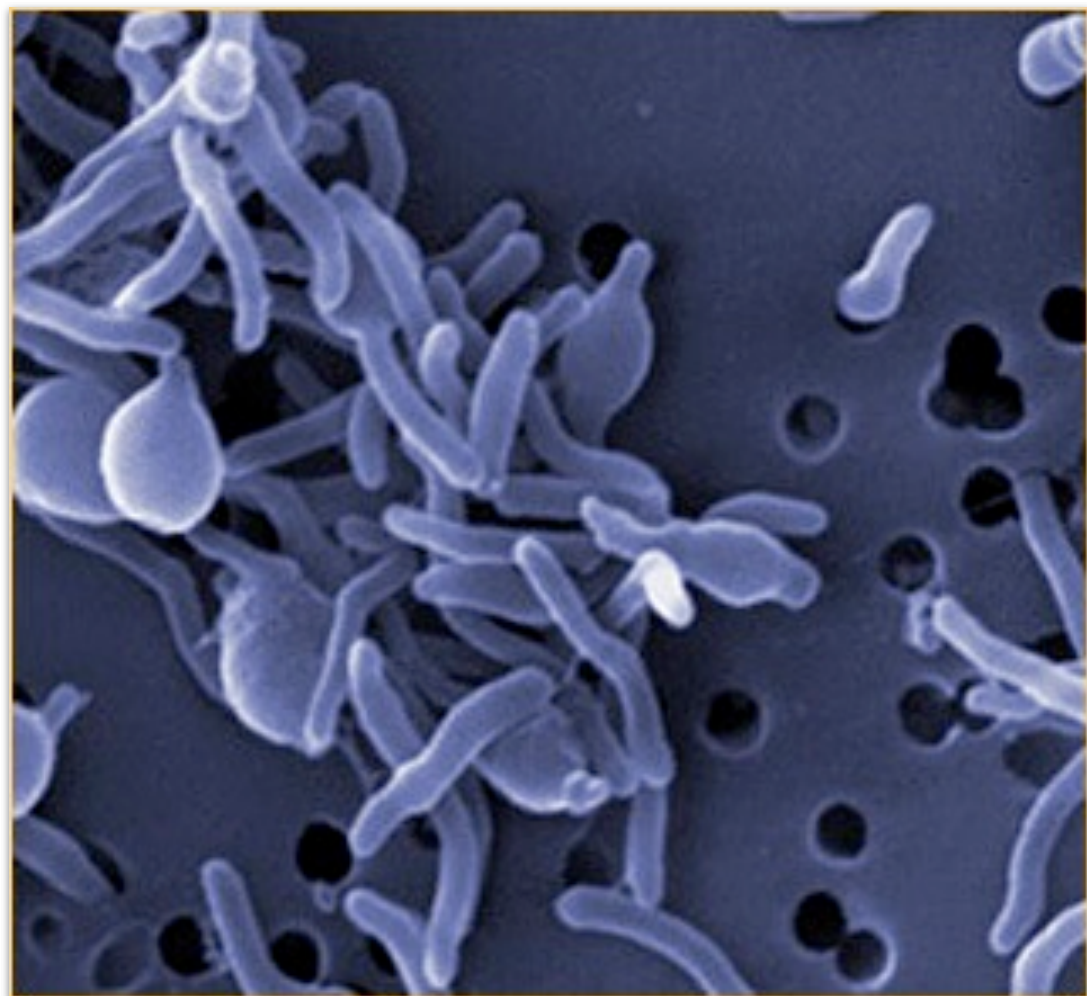
# “Darwinian” Selection for Compression

$$\mu L < 1 < sN$$


$$sN > \mu L$$

The Preference for  
Population Multiplicity,  
Individual Minimality &  
High Diversity.









# So What's Missing?

Complex multicellularity

# The parameters we are going to investigate

$$g_i \xrightarrow[\text{Energy} + \text{Resources}]{r_i} 2g_i$$

.....origins of replicative advantages

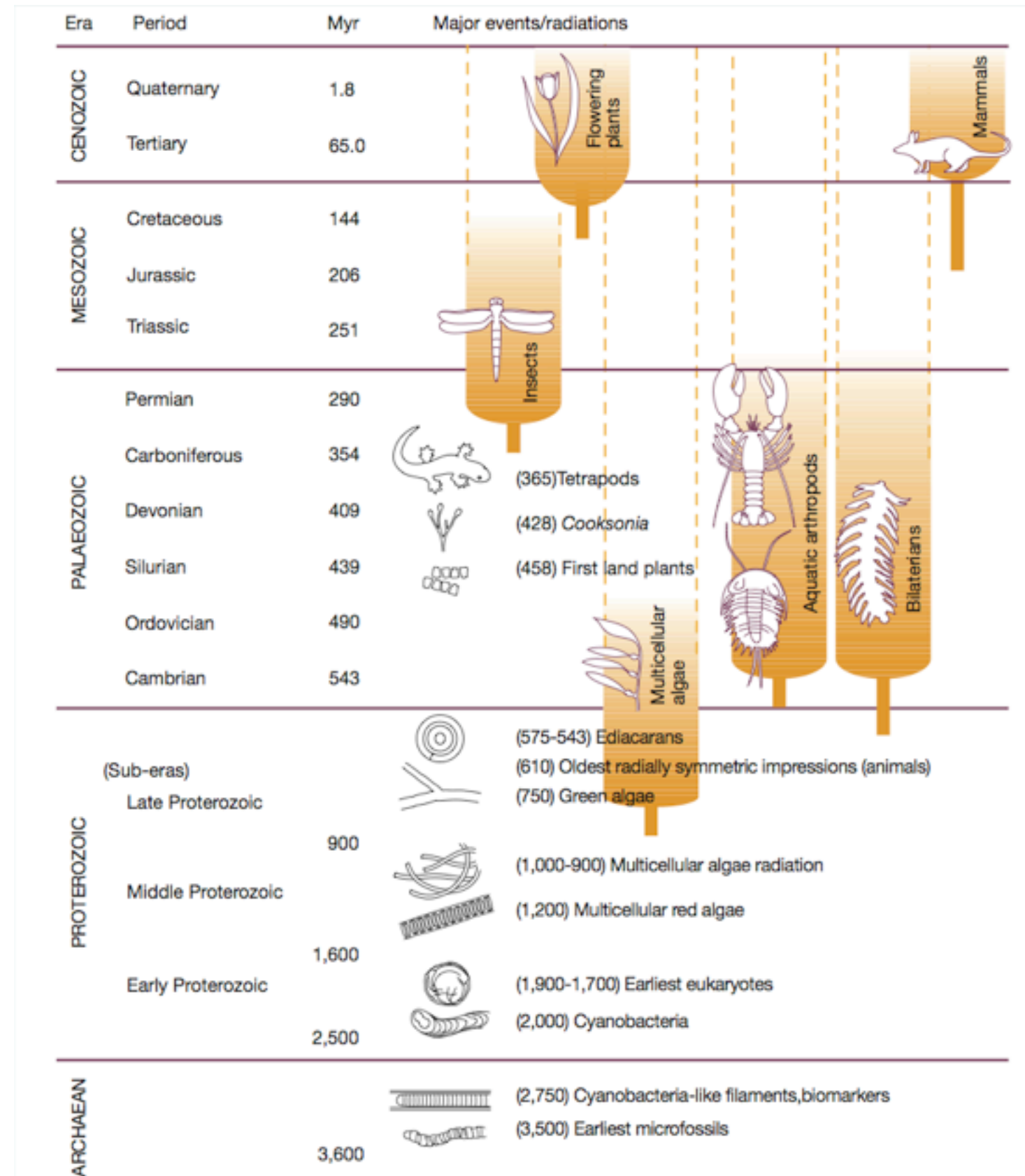
$$g_i + g_j \xrightarrow{c_{ij}} g_j$$

..... sources of density dependence

$$g_i \xrightarrow{d_i} p_i$$

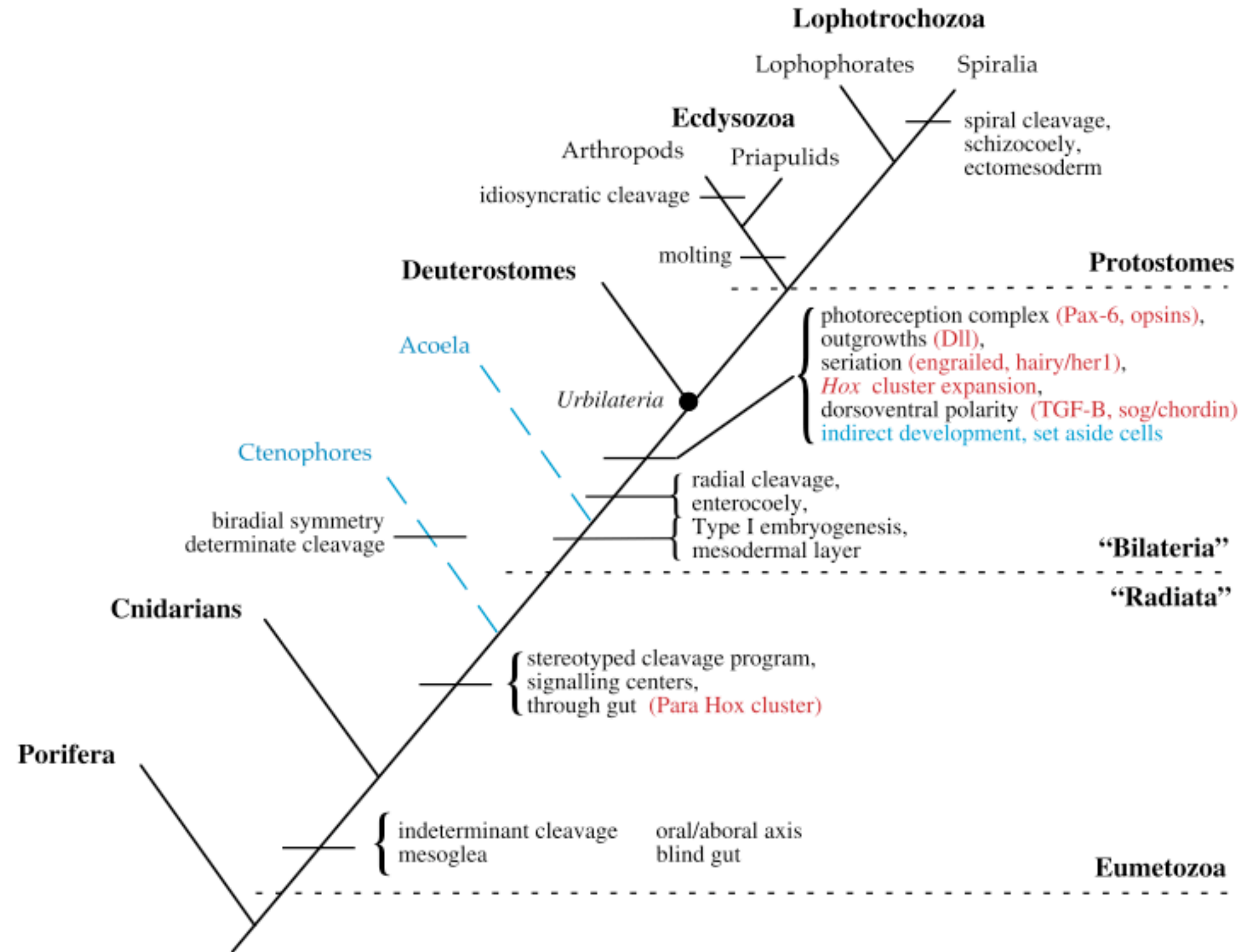
.....limitations of genetic models





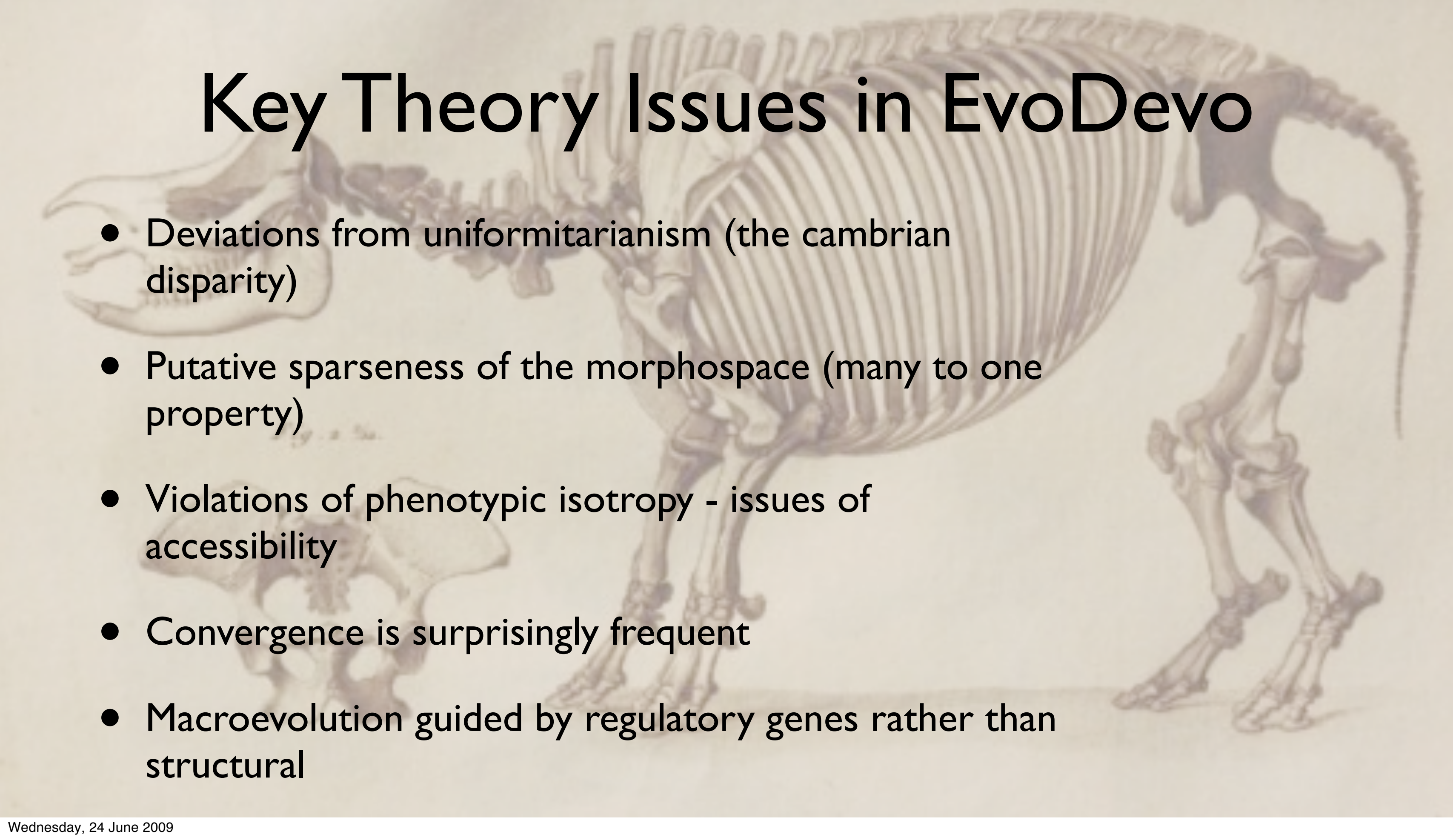
Carroll, S. Nature 409. 2001

# Developmental Innovations



Knoll, A. & Carroll, S. Science 284. 1999

# Key Theory Issues in EvoDevo

A faint, sepia-toned illustration of a dinosaur skeleton, likely a T-Rex, is visible in the background. The skeleton is shown in profile, facing left, with its head lowered and its long tail extending to the right. The ribcage and spine are clearly visible, and the legs are positioned as if the dinosaur is standing or walking.

- Deviations from uniformitarianism (the cambrian disparity)
- Putative sparseness of the morphospace (many to one property)
- Violations of phenotypic isotropy - issues of accessibility
- Convergence is surprisingly frequent
- Macroevolution guided by regulatory genes rather than structural



[illegible]



## Placentals



Flying squirrel  
(*Glaucomys*)



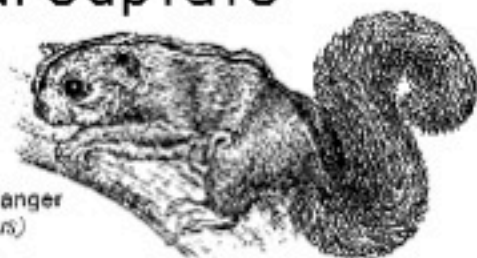
Ground hog  
(*Marmota*)



Mole  
(*Talpa*)



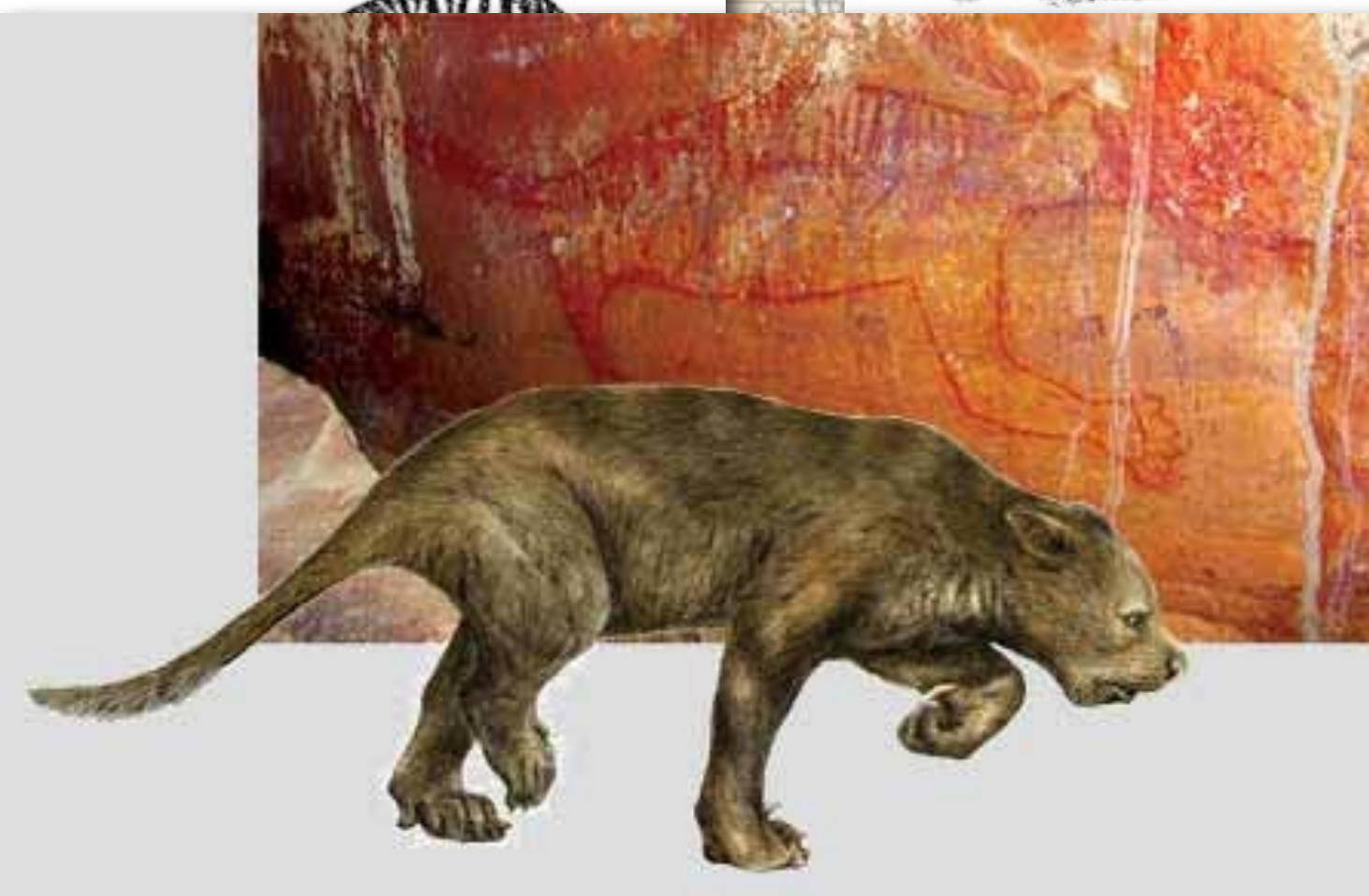
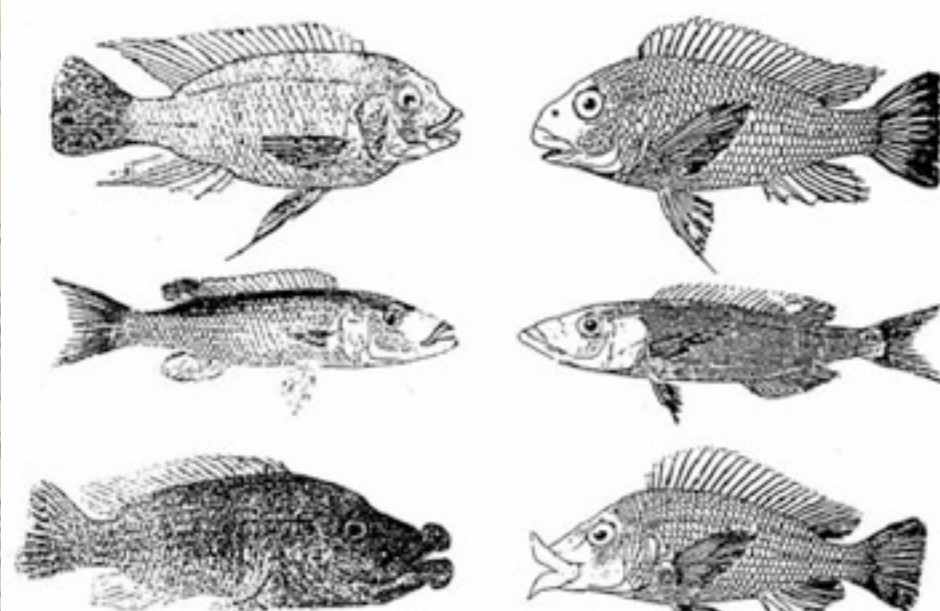
## Marsupials



Flying phalanger  
(*Petaurus*)



Wombat  
(*Phascogale*)







appearance of dwarf and giant varieties	all
piebald coat color	all
wavy or curly hair	sheep, poodles, donkeys, horses, pigs goats, mice, guinea pigs
rolled tails	dogs, pigs
shortened tails, fewer vertebrae	dogs, cats, sheep
floppy ears	dogs, cats, pigs, horses, sheep, goats, cattle
changes in reproductive cycle	all except sheep

increase in frequency
(percent)
+1,646
+423
+400
+35
+6,900
+1,033



brown mottling	450	86
gray hairs	500	100
floppy ears	230	170
short tail	140	2
tail rolled in circle	9,400	830



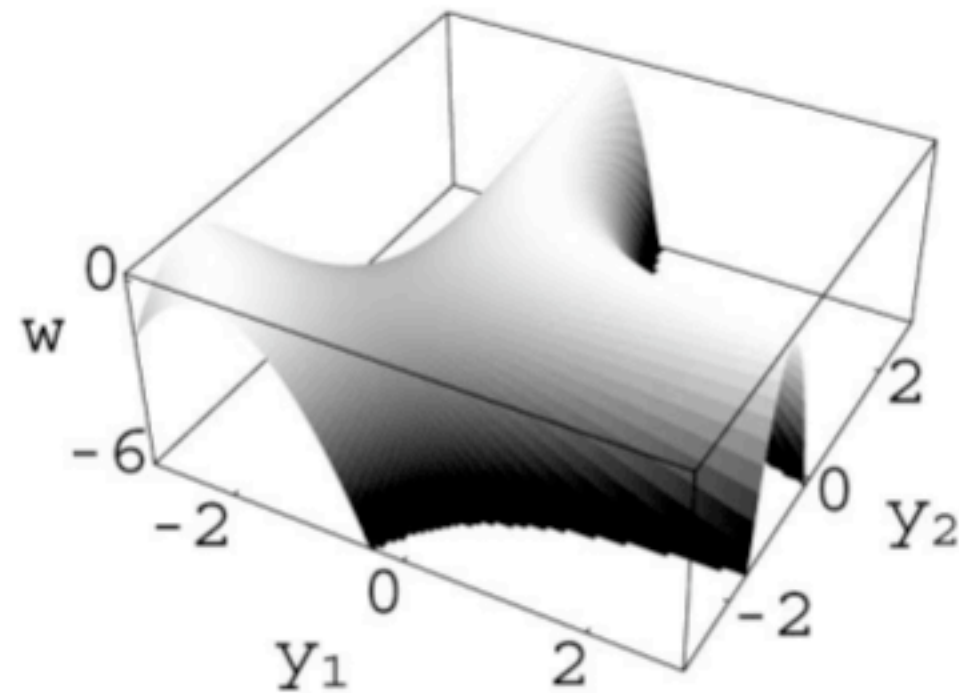
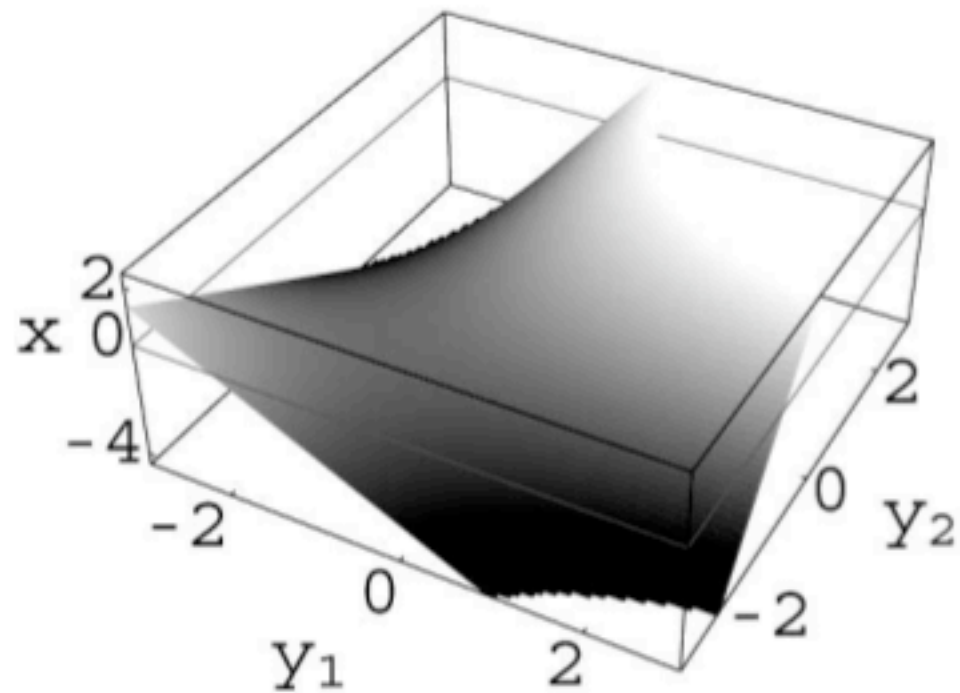


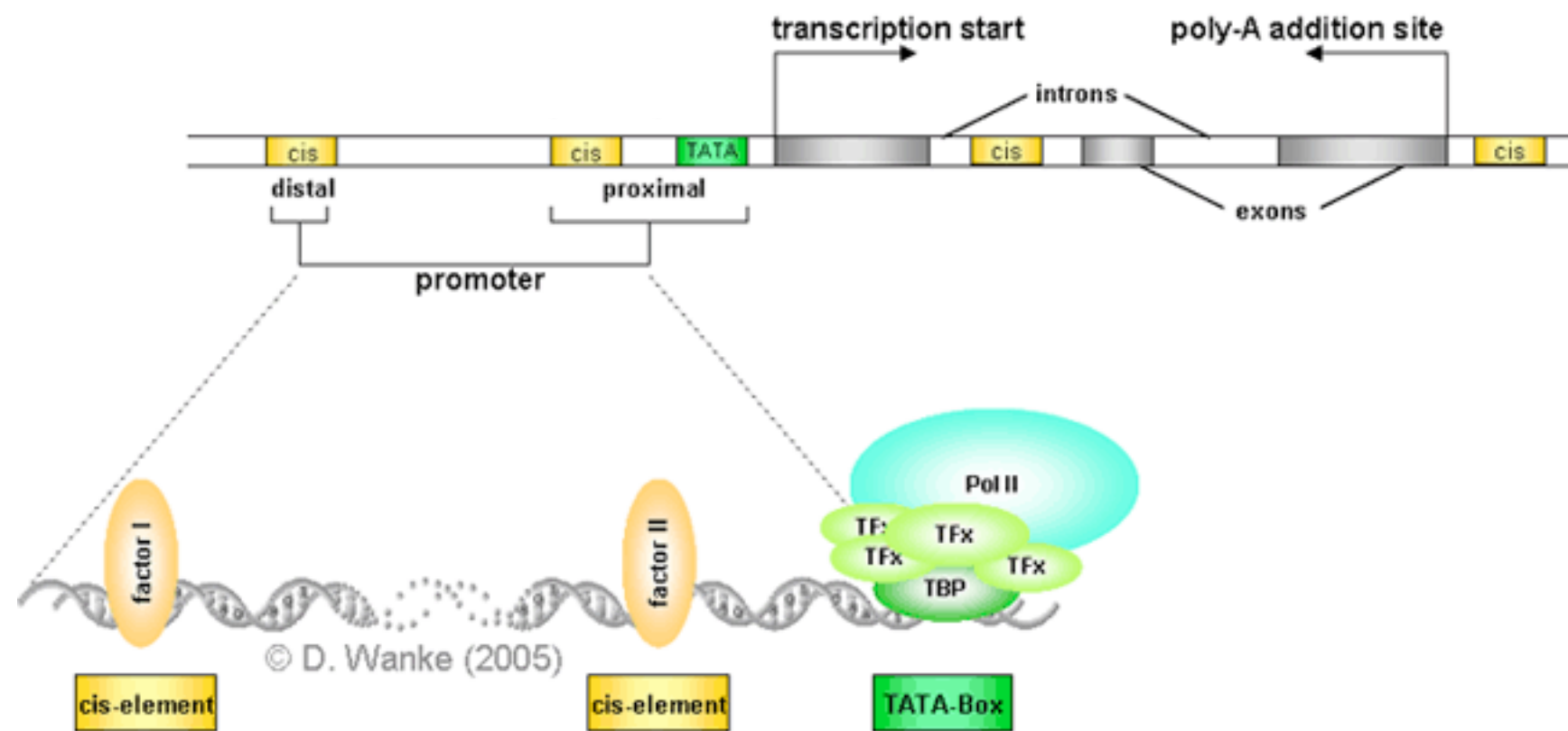
# Epistasis

$$z = x + e$$

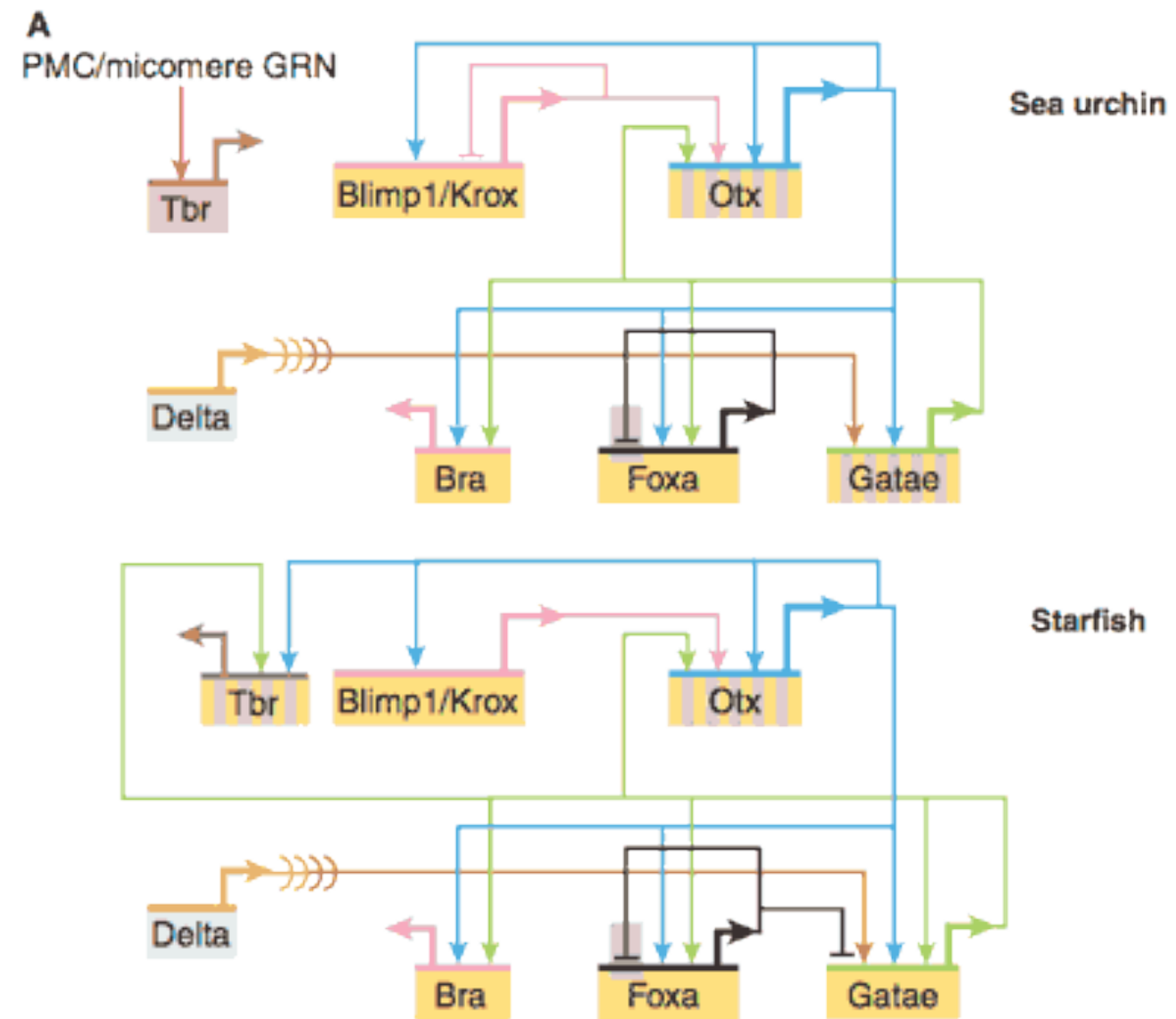
$$x = x_r + \sum_i y_i + \sum_{i,j>i} \epsilon_{ij} y_i y_j$$

$$f_i = \frac{\partial}{\partial y_i} x|_y = 1 + \sum_{j \neq i} \epsilon_{ij} y_j$$



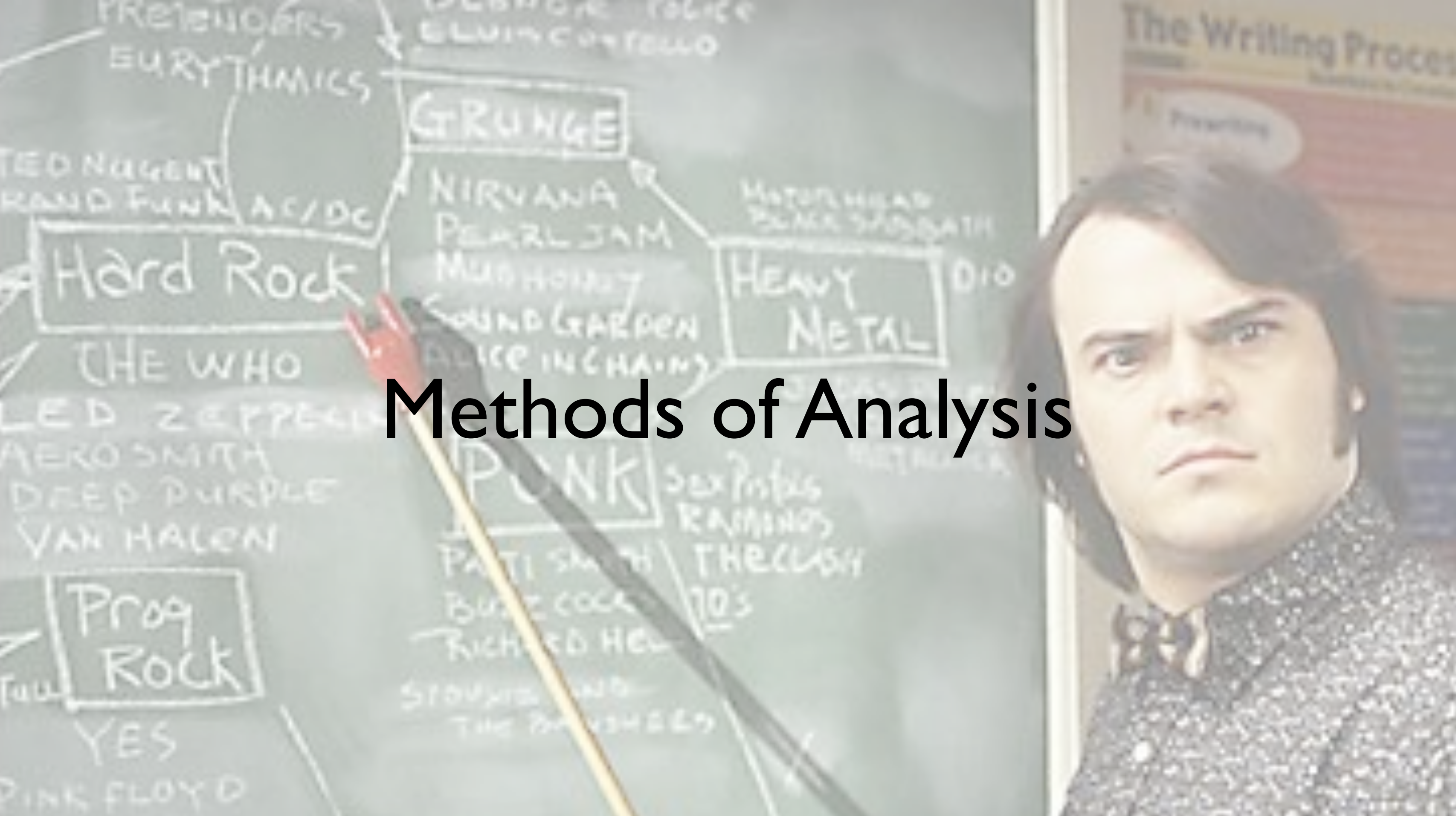








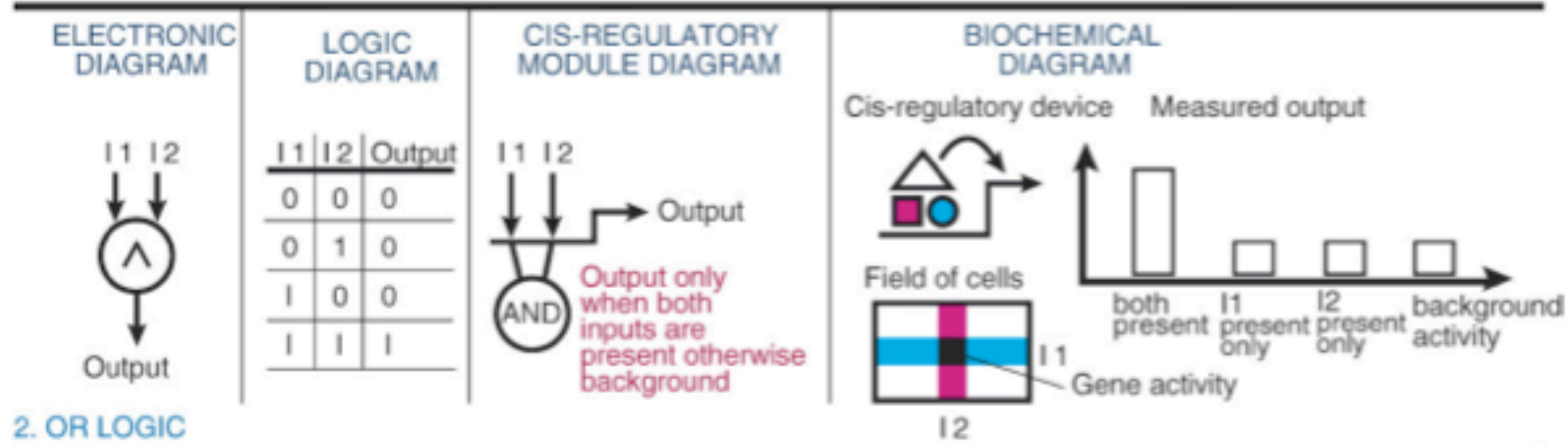




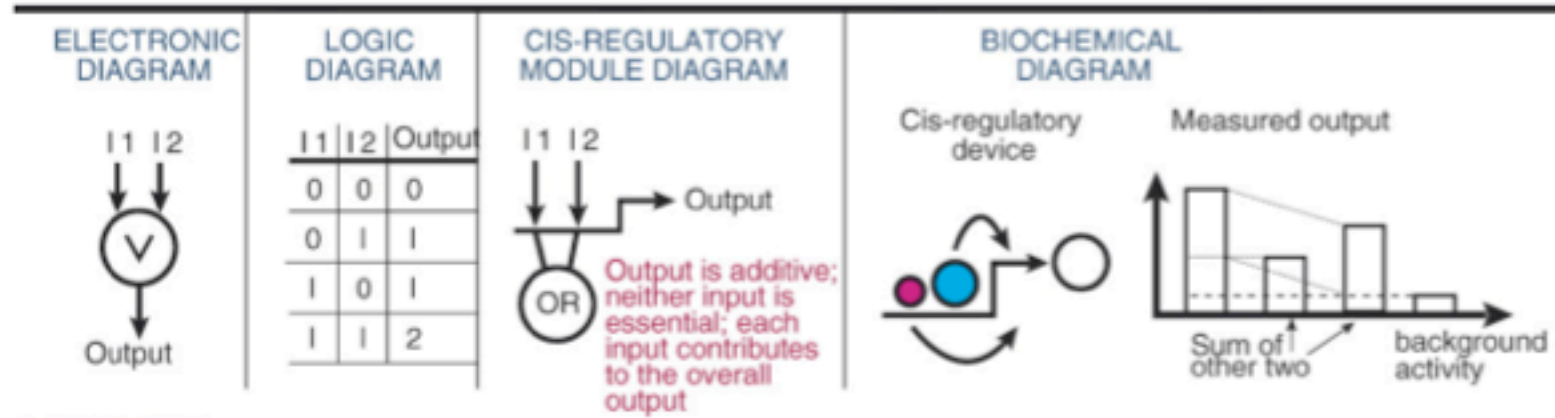
# Methods of Analysis



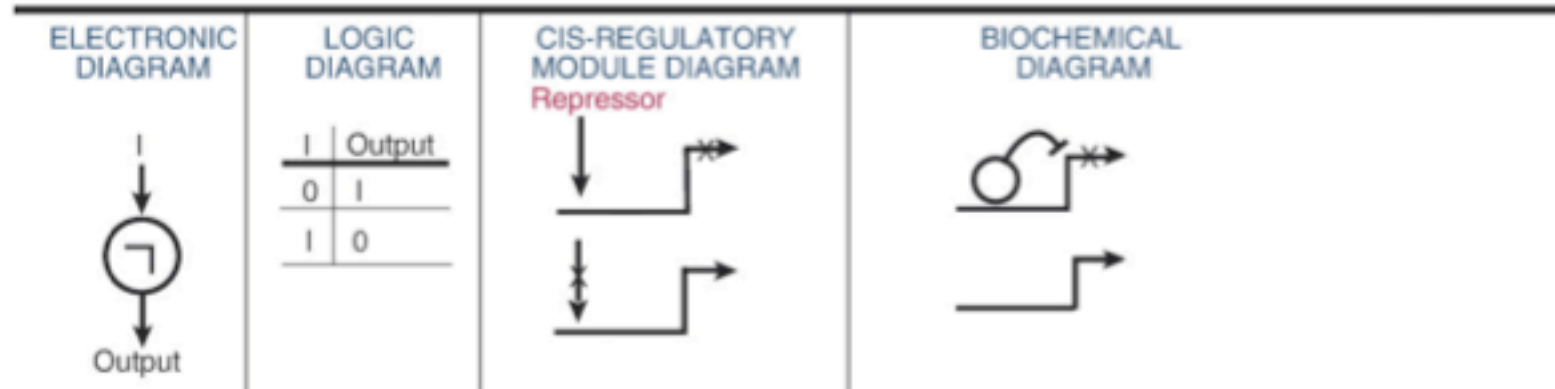
### 1. AND LOGIC



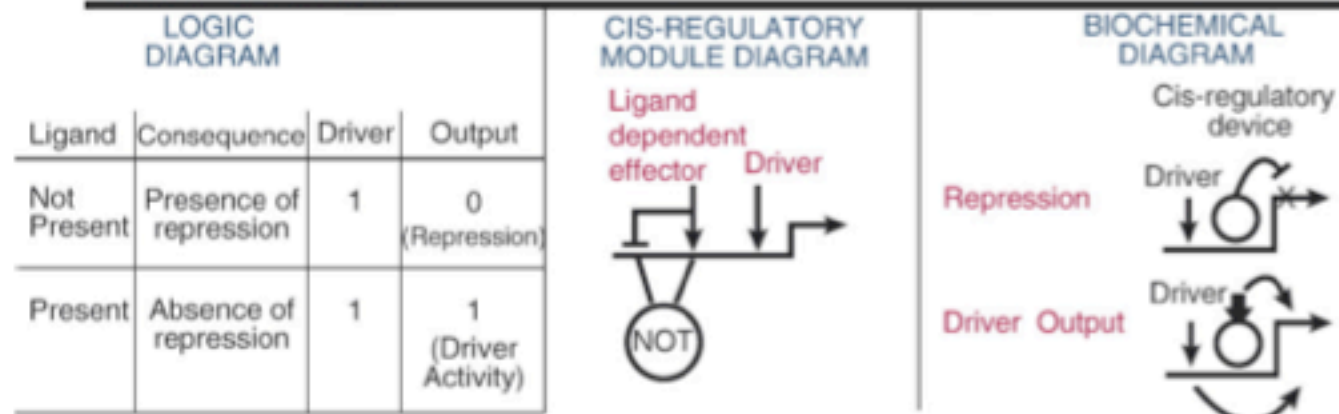
### 2. OR LOGIC



### 3. NOT LOGIC



### 4. COMPOUND LOGIC



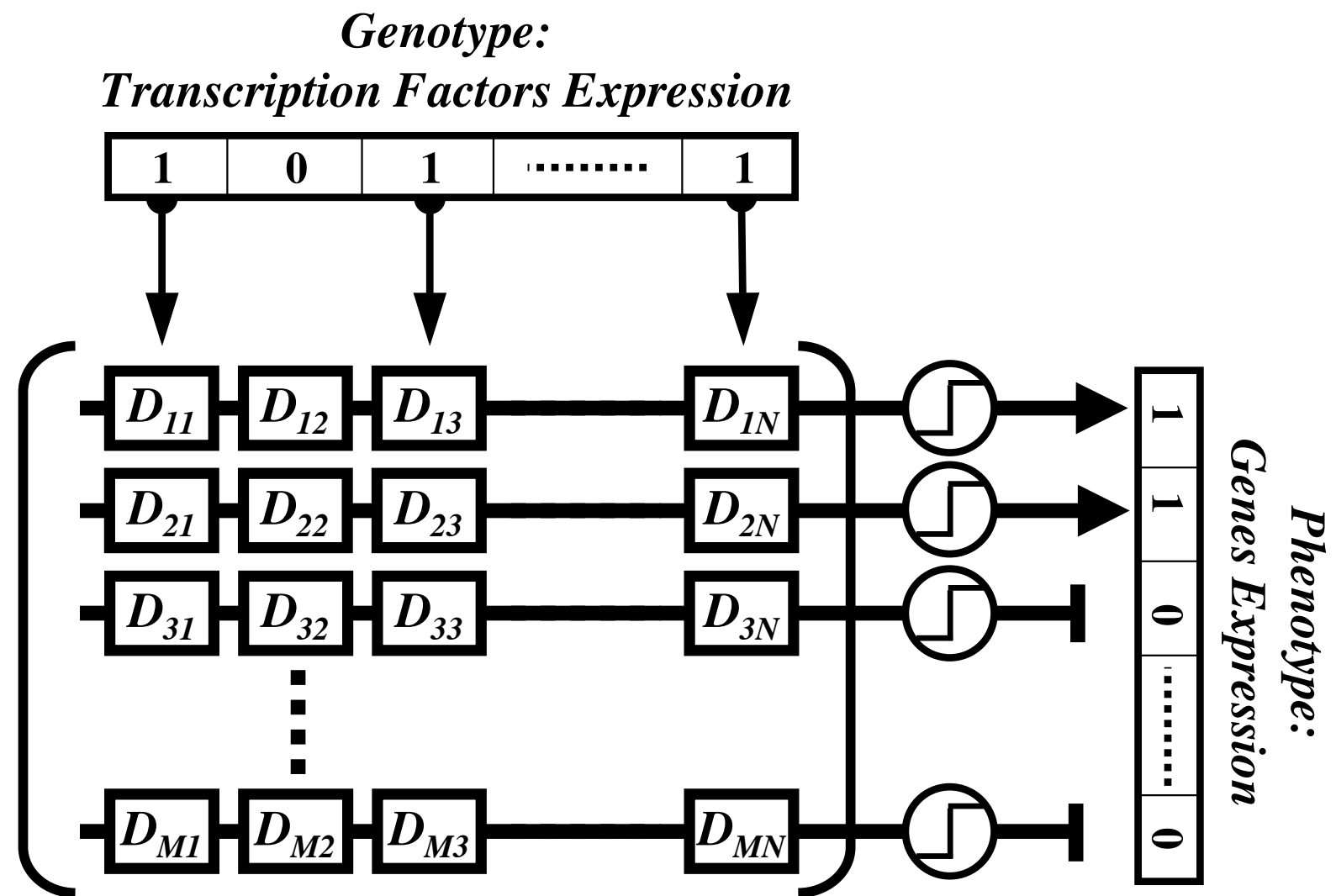




# A Statistical Analysis of cis-maps

Borenstein & Krakauer Plos. Comp. Biol. 2008





$$p(D_{ij} = +1 | c_{ij} = 1) = q \quad \vec{p} = H(D\vec{g})$$

## Single Projection

$$\vec{p} = H(D\vec{g})$$

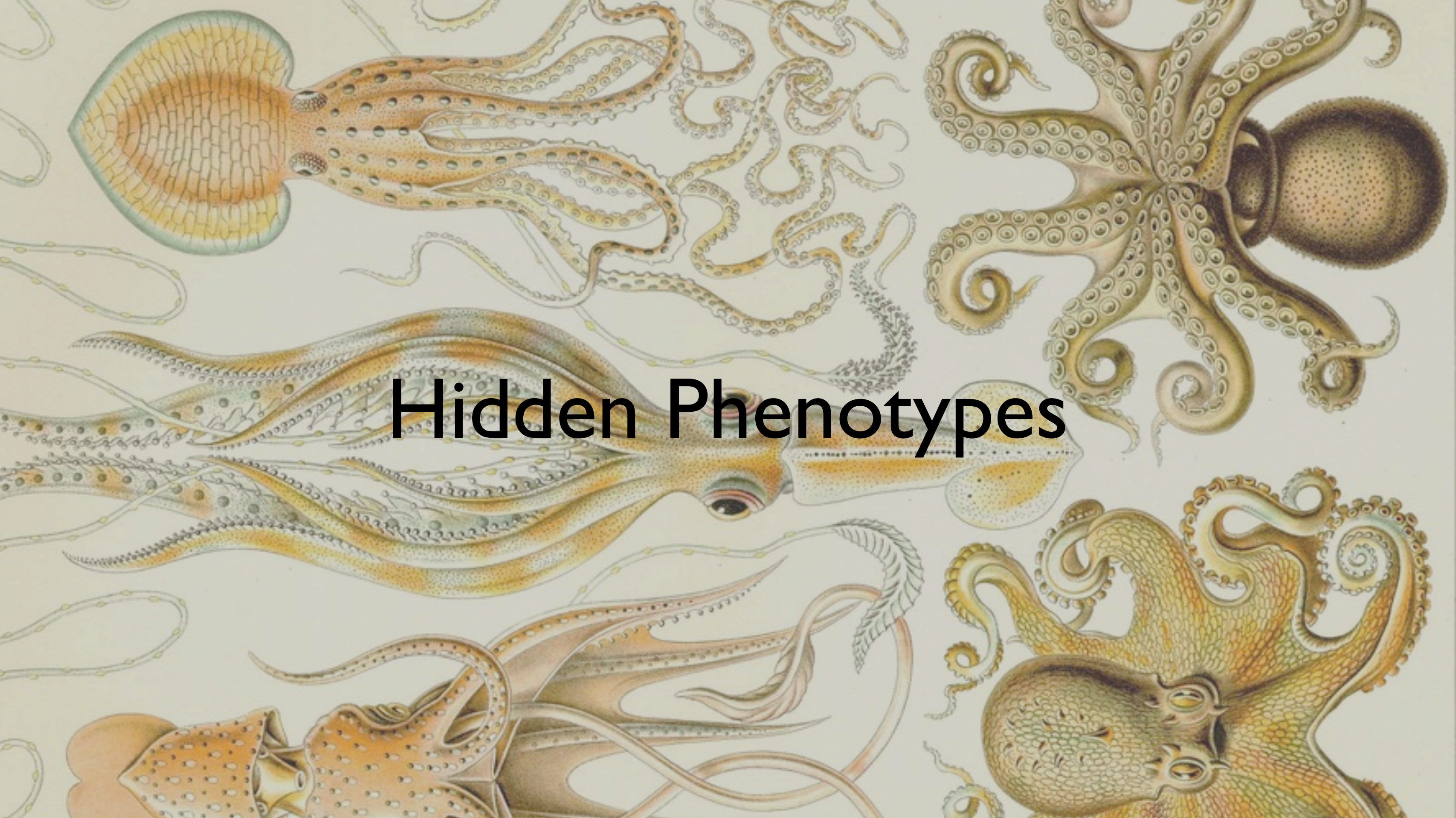
## Recurrent Projection

$$\vec{p}_t = H(D\vec{p}_{t-1})$$

## Recurrent Random Projection

$$\vec{p}_t = H(D_t\vec{p}_{t-1})$$

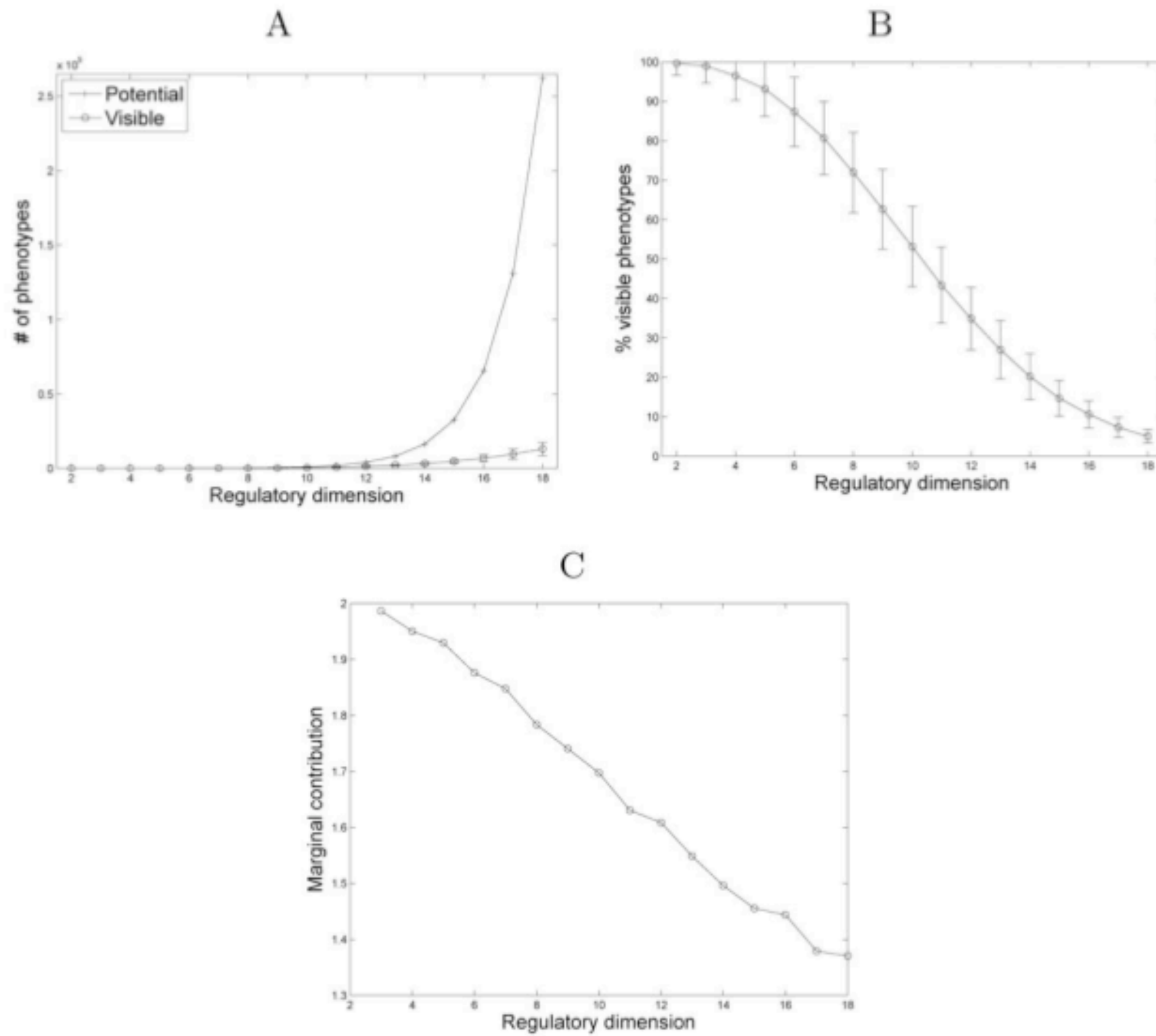




# Hidden Phenotypes



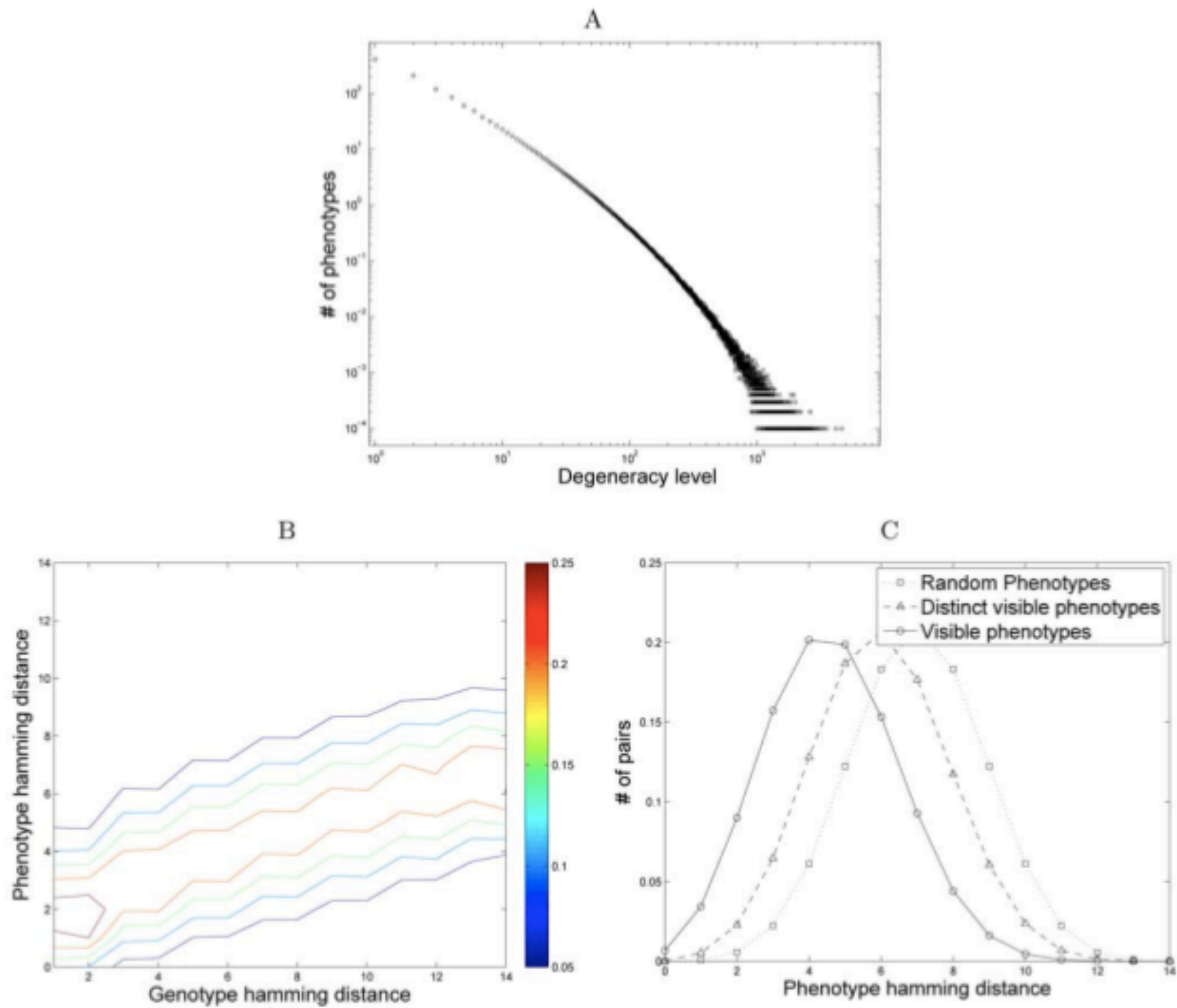
# Diversity

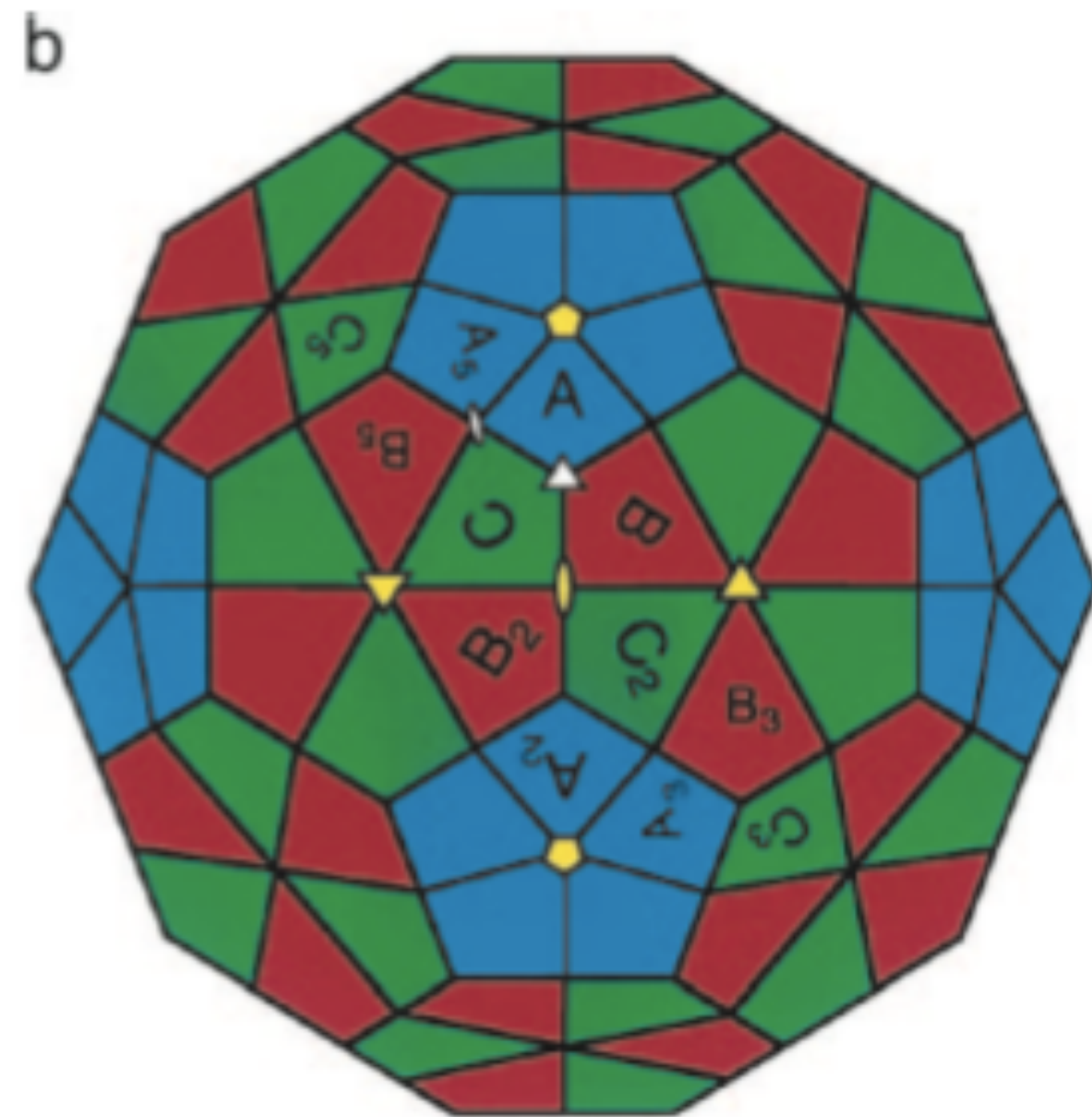
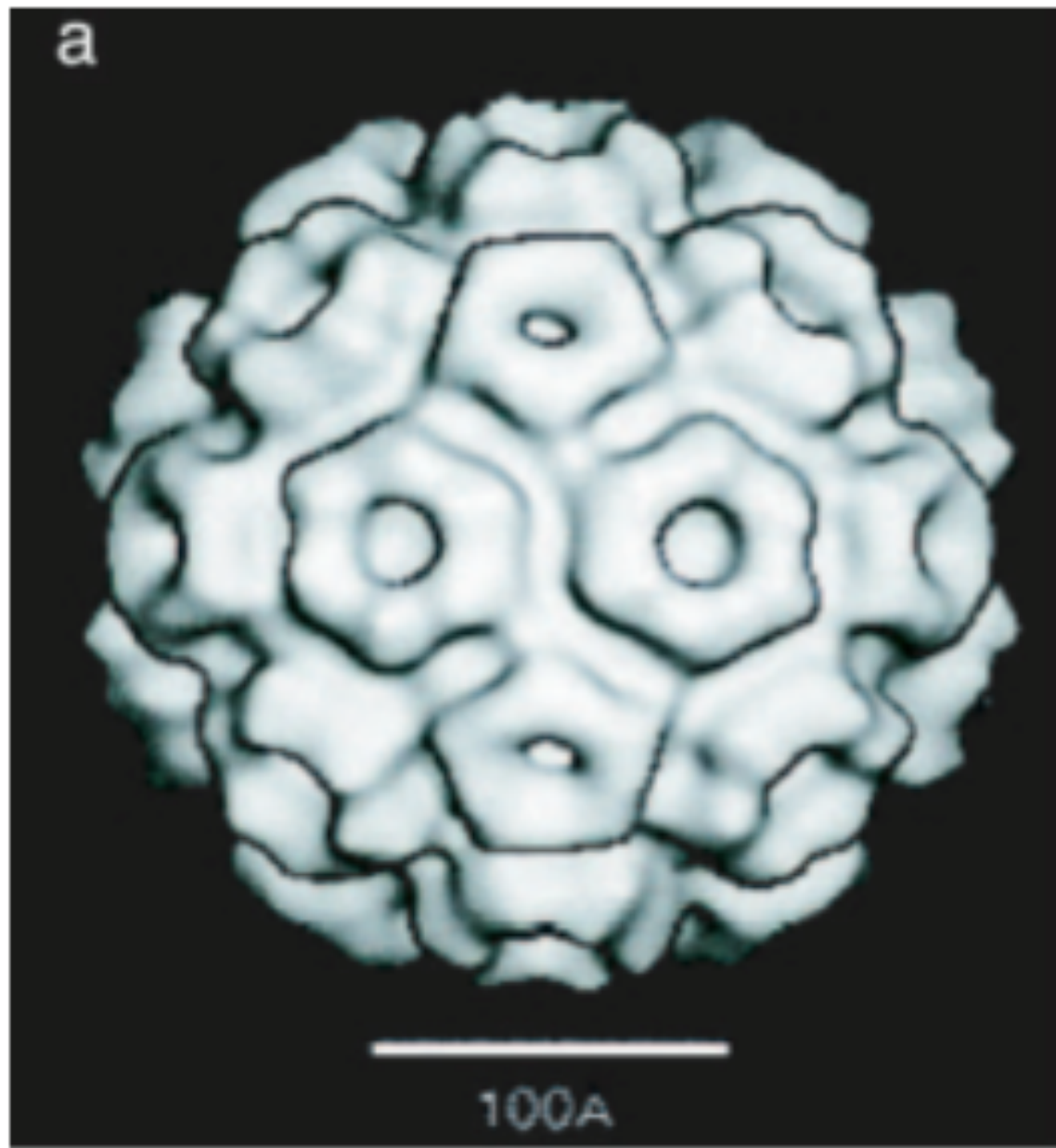


# Localization of visible sub-space



# Disparity

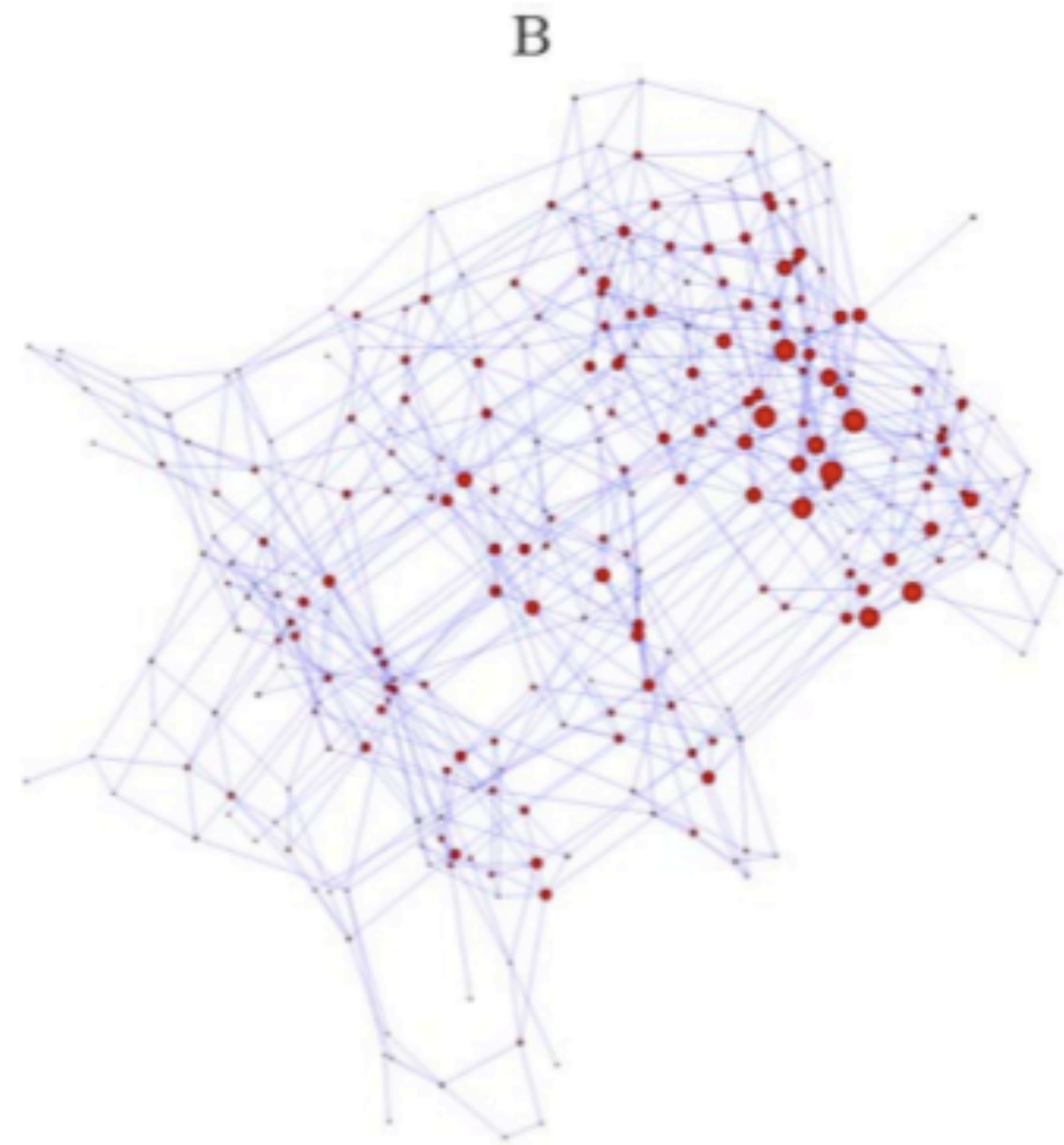
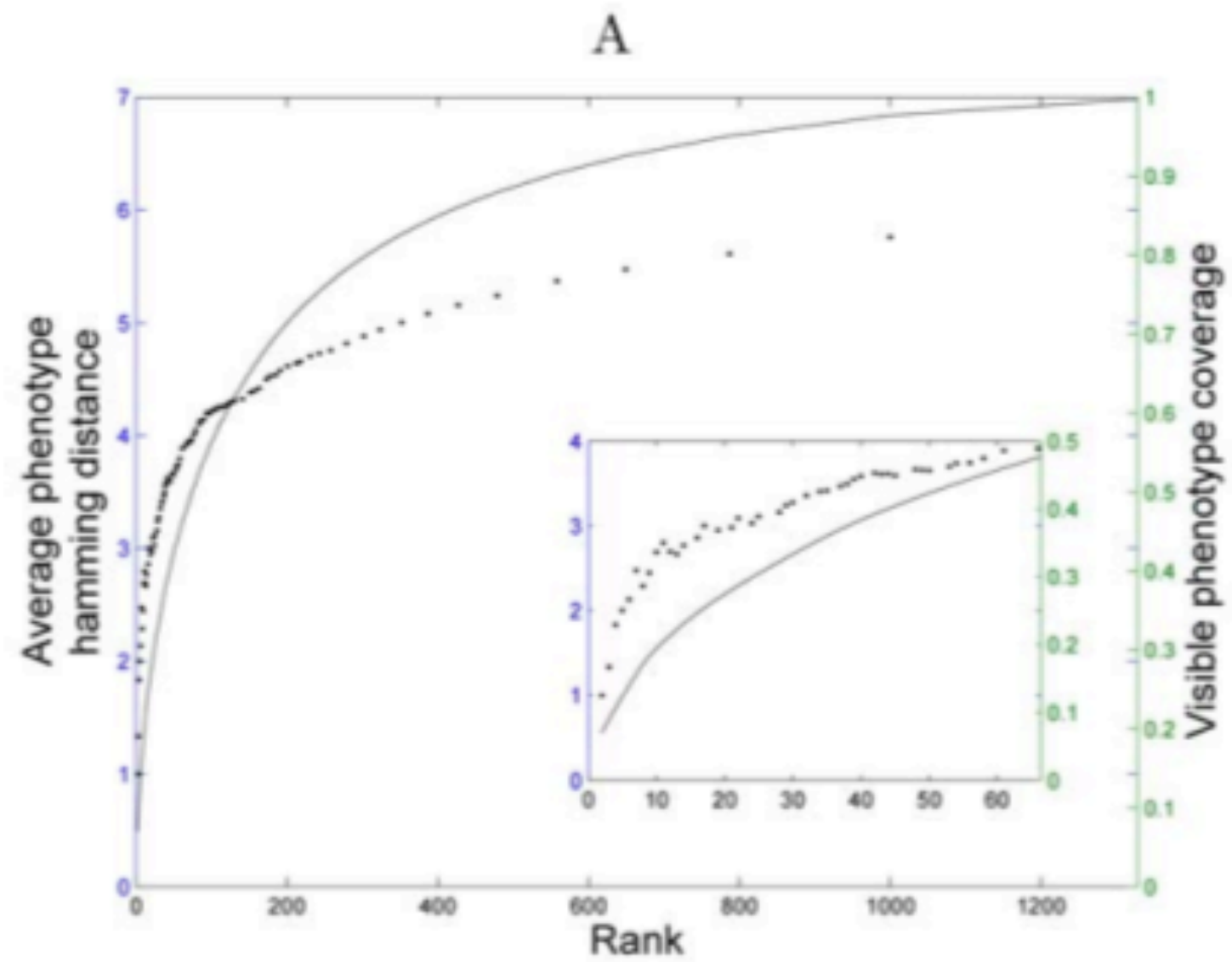






# Structure of visible sub-space

# Common Phenotypes are Similar





# Hierarchical Projections:

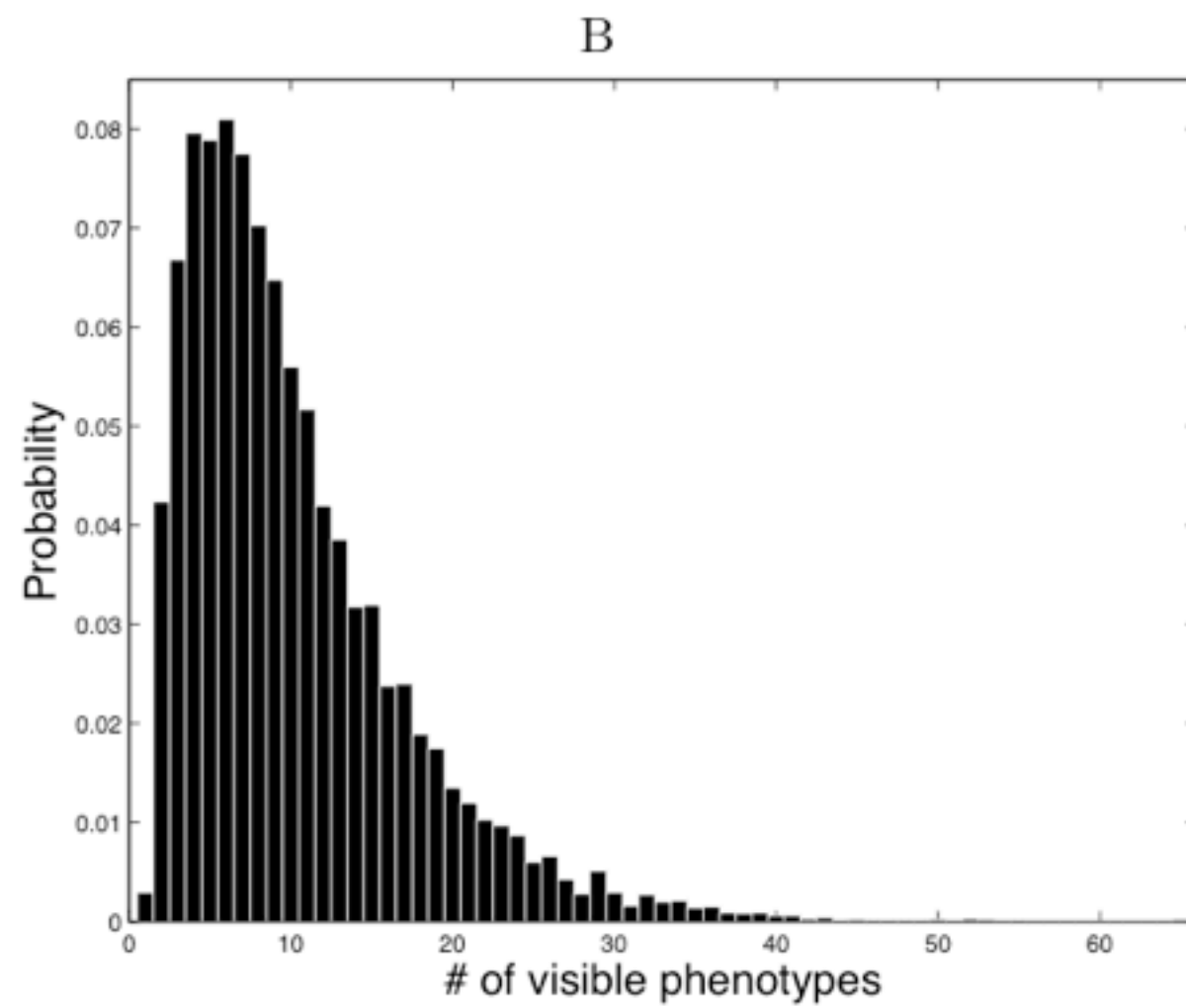
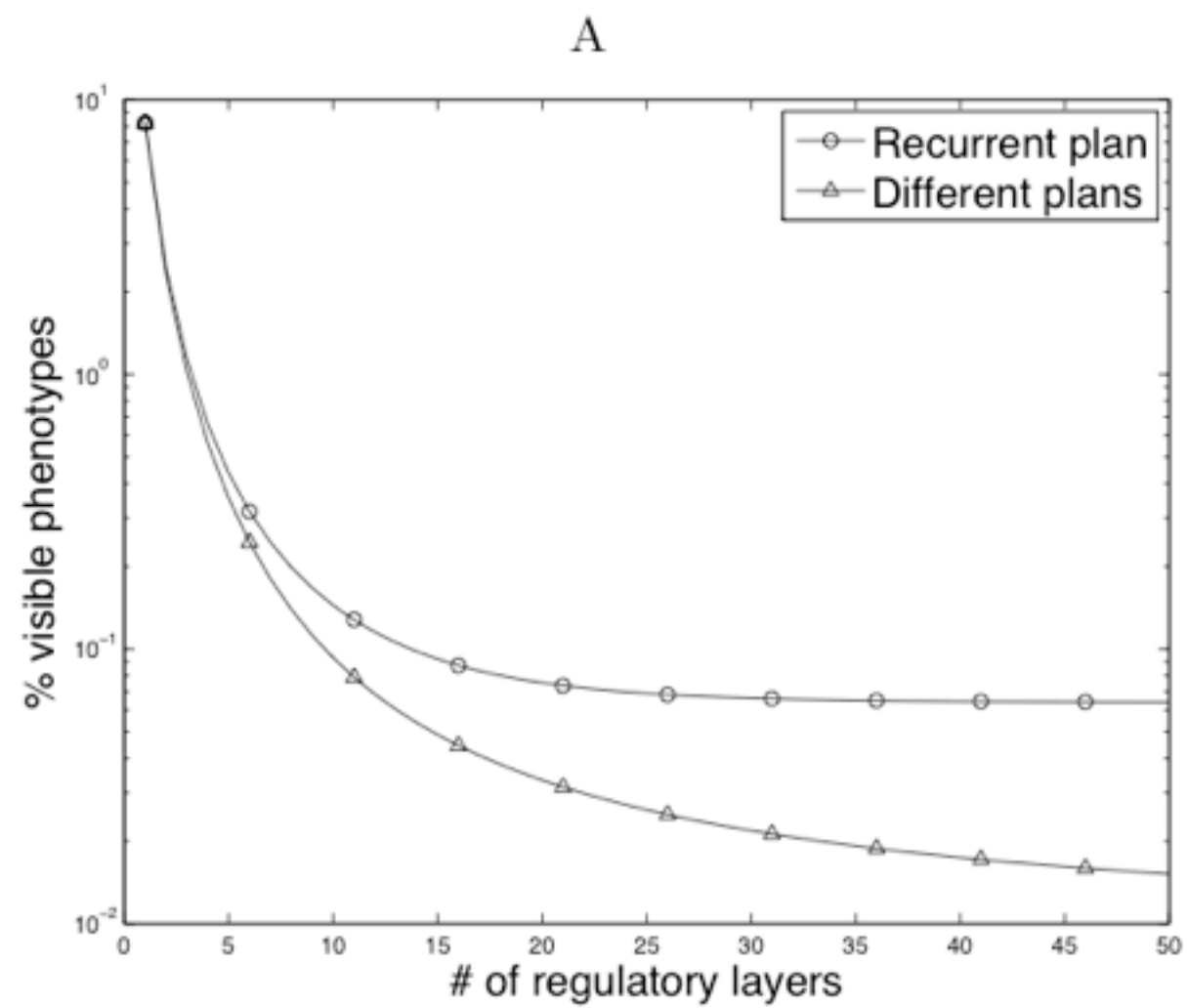
## Recurrent Projection

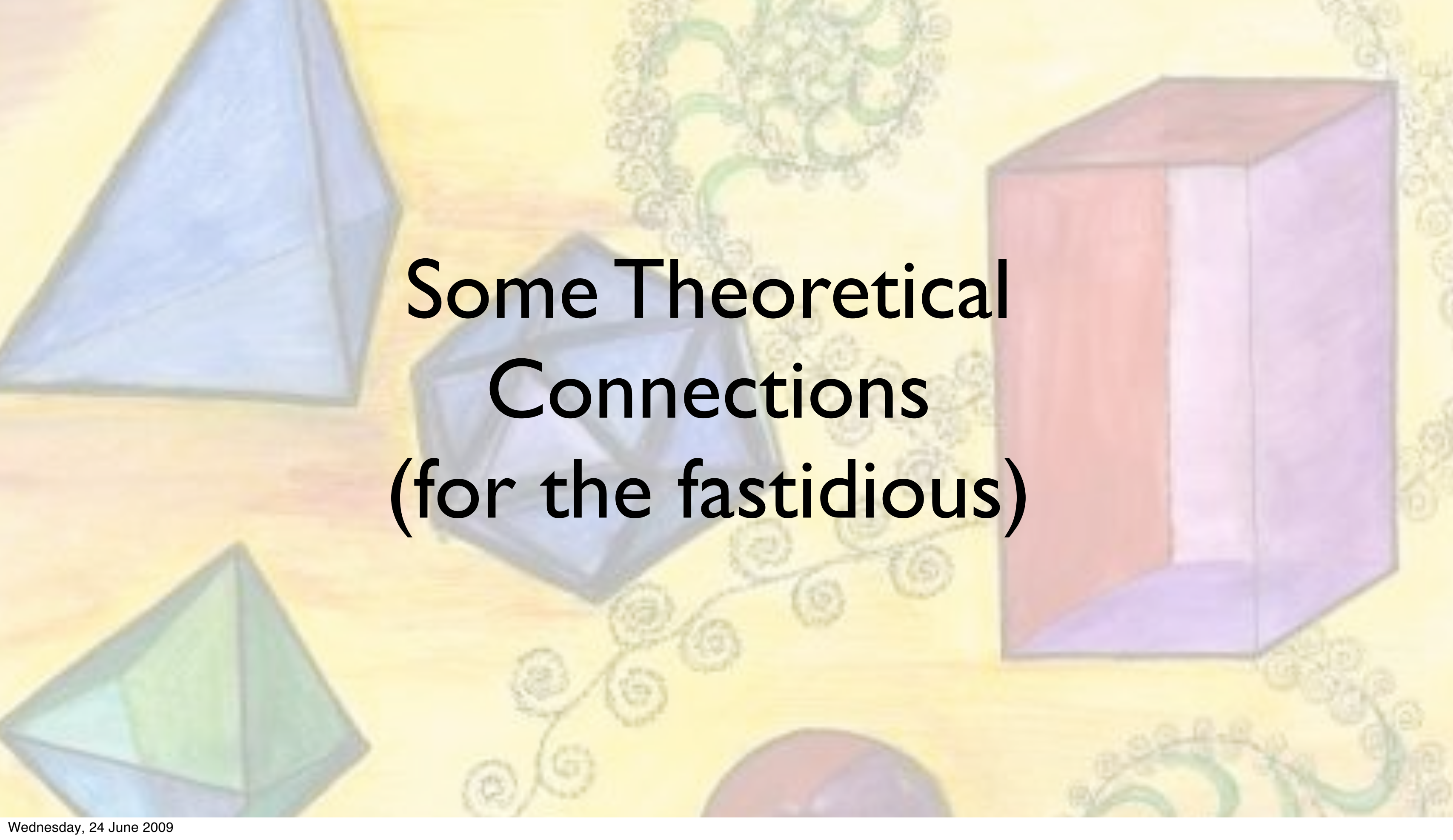
$$\vec{p}_t = H(D\vec{p}_{t-1})$$

## Recurrent Random Projection

$$\vec{p}_t = H(D_t\vec{p}_{t-1})$$



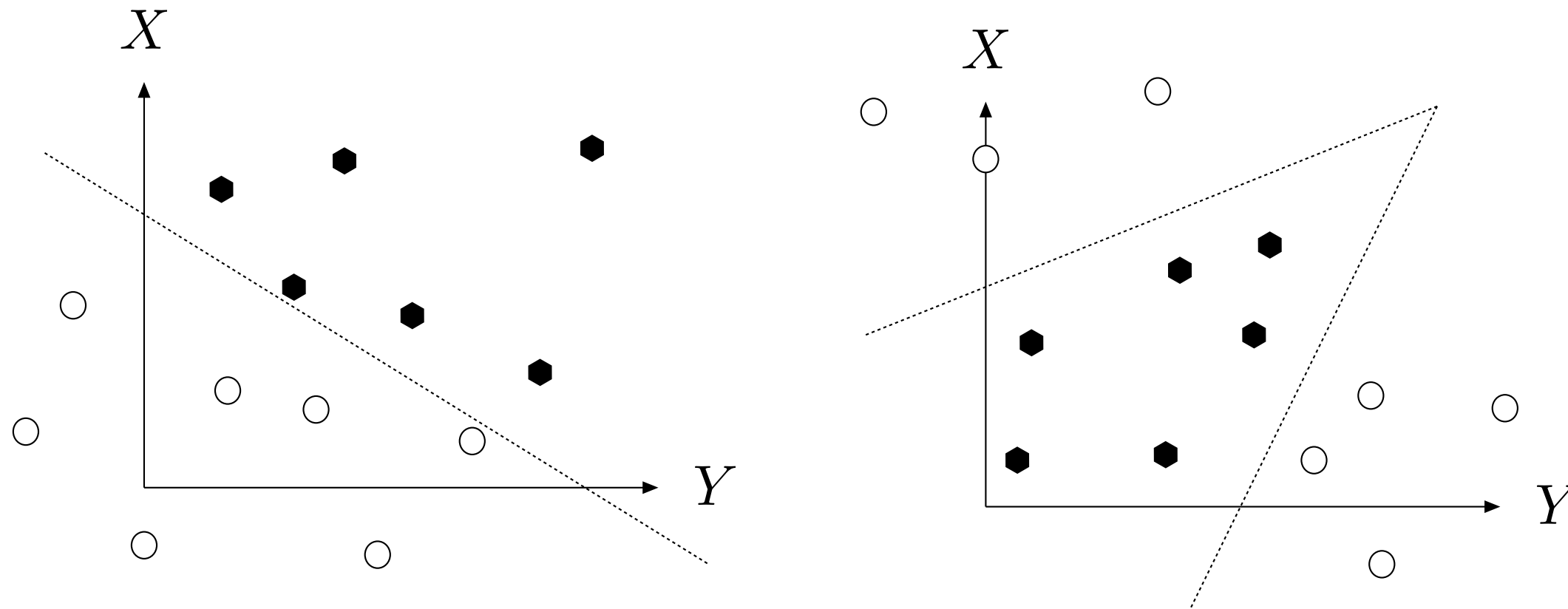




# Some Theoretical Connections (for the fastidious)



# Linear & Polyhedral separability of activation patterns in phenotypic feature space



- NP-complete to determine polyhedral separability of 2 sets of points
- NP-complete to determine whether  $k$ -lines can separate 2 sets
- For  $k$  lines polynomial time to determine whether sets are separable



Cybenko Theorem:  
universal function approximation  
with superpositions of sigmoids

$$G(\mathbf{x}, \mathbf{w}, \alpha, \theta) = \sum_{i=1}^N \alpha_i \sigma(\mathbf{w}_i^T \mathbf{x} + \theta_i)$$

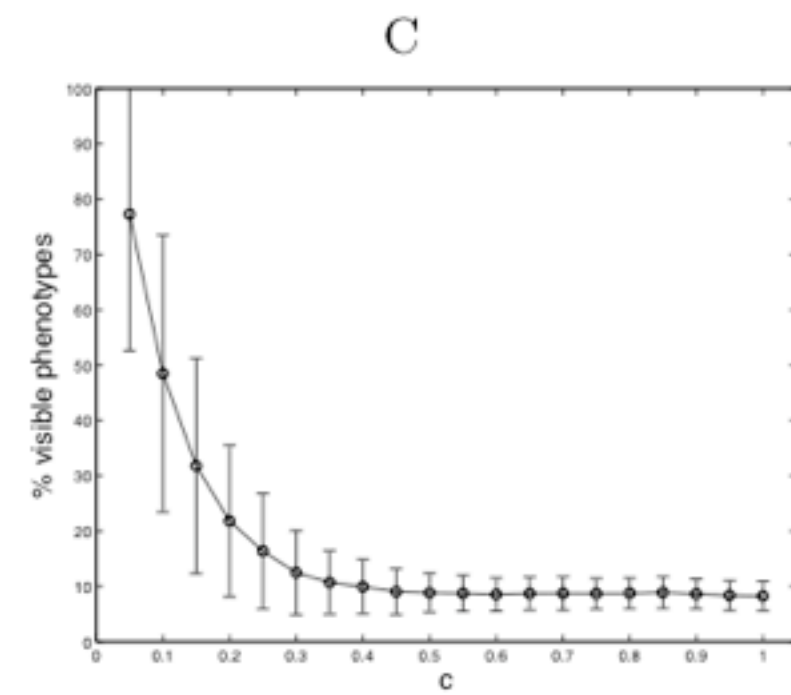
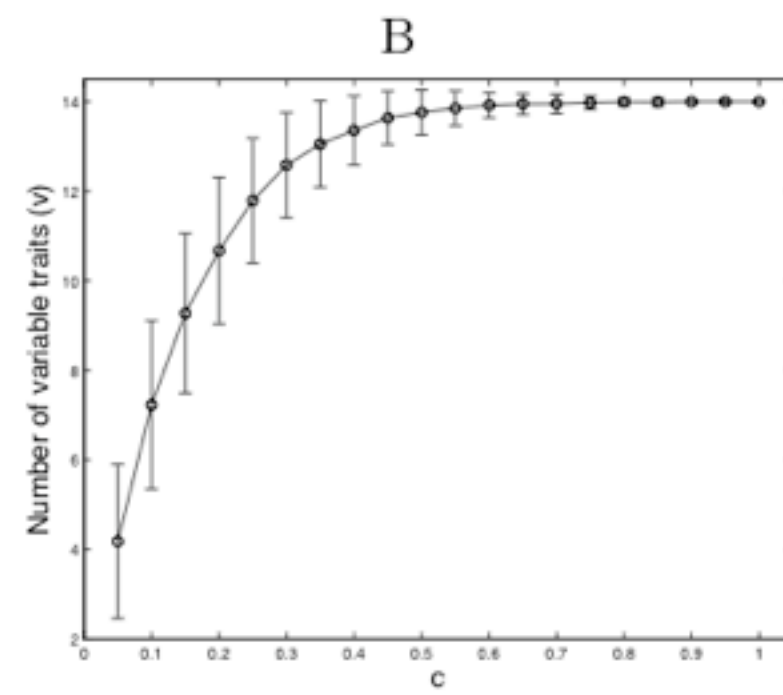
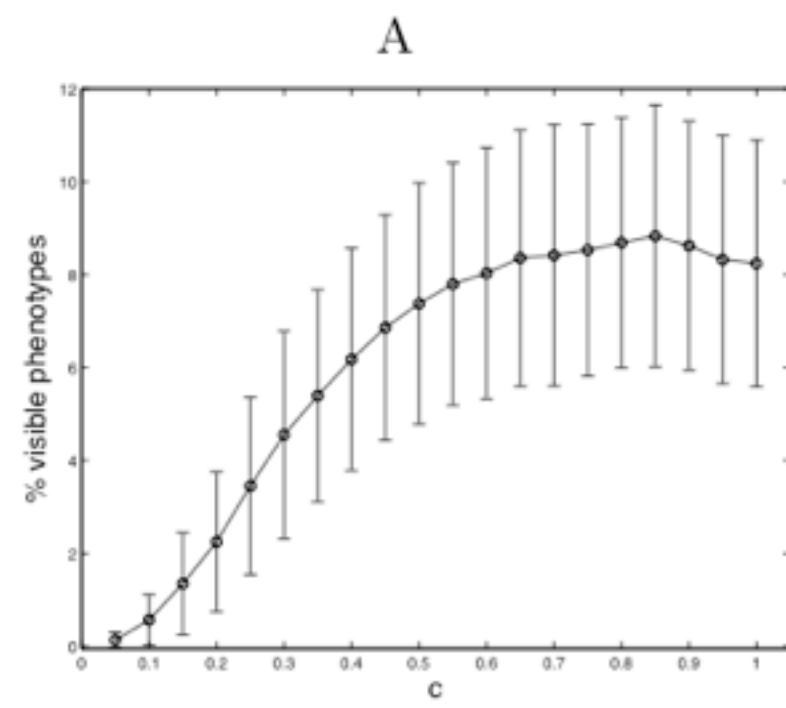
$$|G(\mathbf{x}, \mathbf{w}, \alpha, \theta) - f(x)| < |\epsilon|, \mathbf{x} \in [\mathbf{0}, \mathbf{1}]^n$$

- Products of random matrices essential in chaotic dynamics
- In  $d=1$  case, limiting distribution of random products,  $\log(X)$
- Problems of non-commutativity of matrix
- Infinite products of 2 by 2 linear projections is a solved problem.



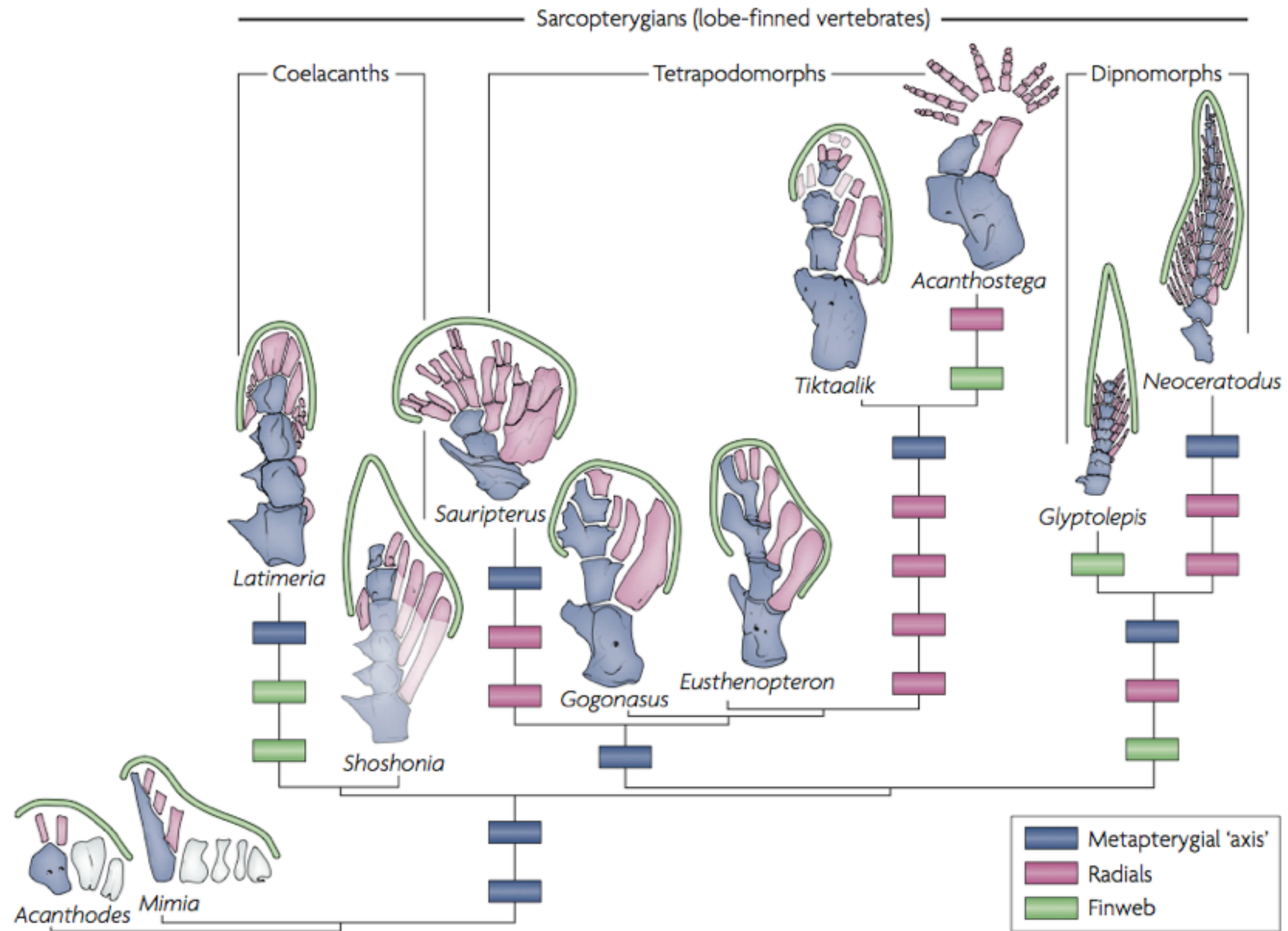
# Sparseness: freedom or constraint?

The Origins of Regulatory Freedom

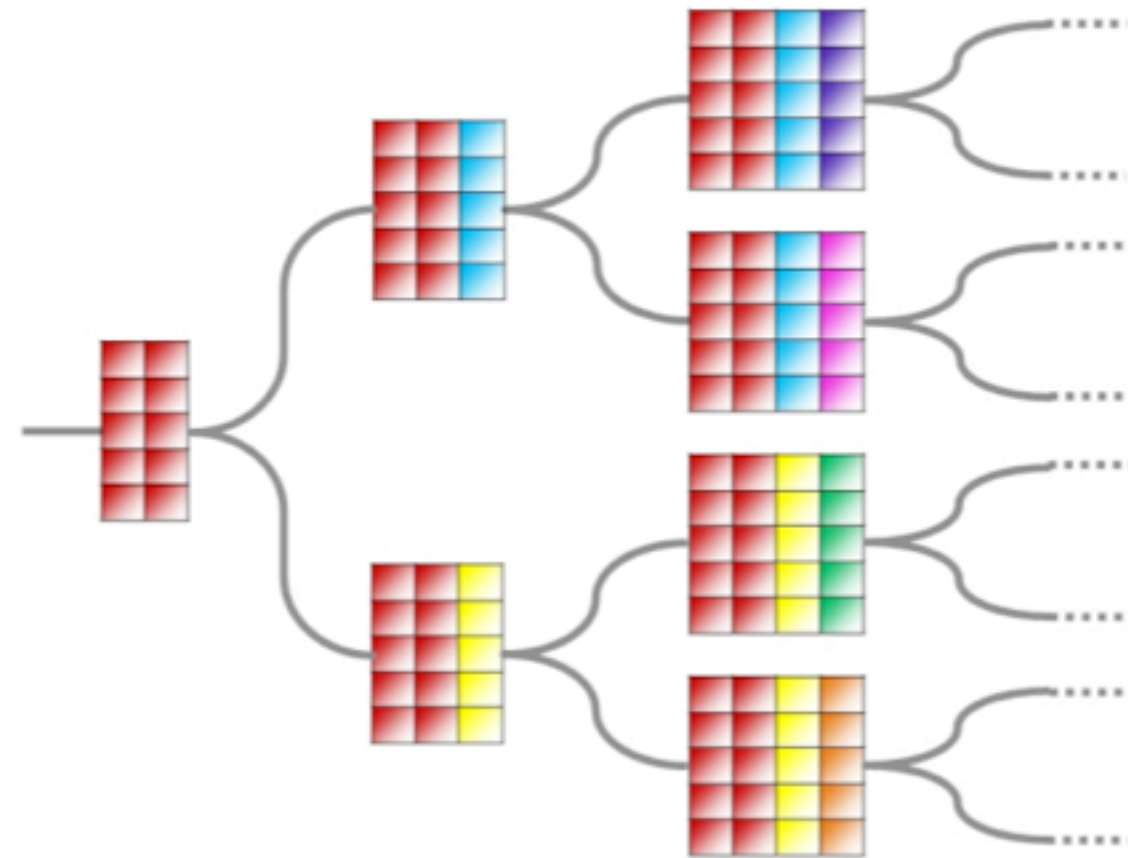




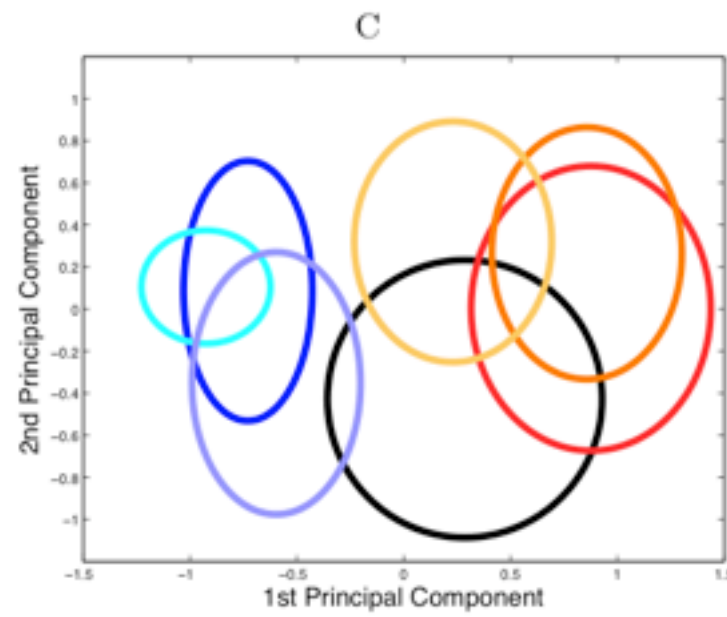
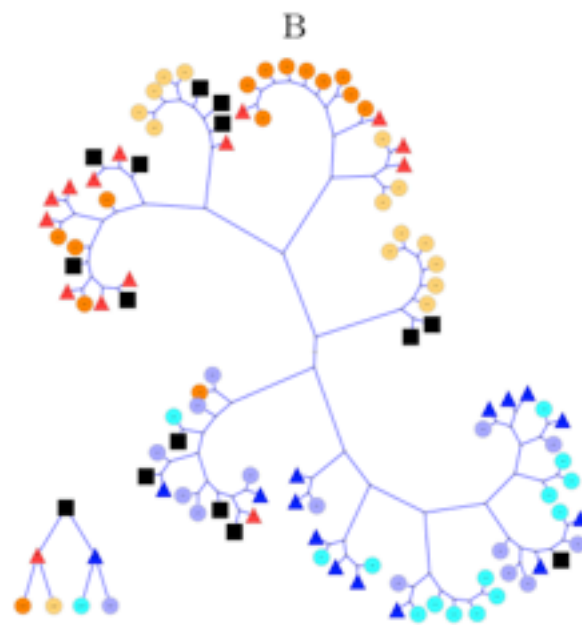
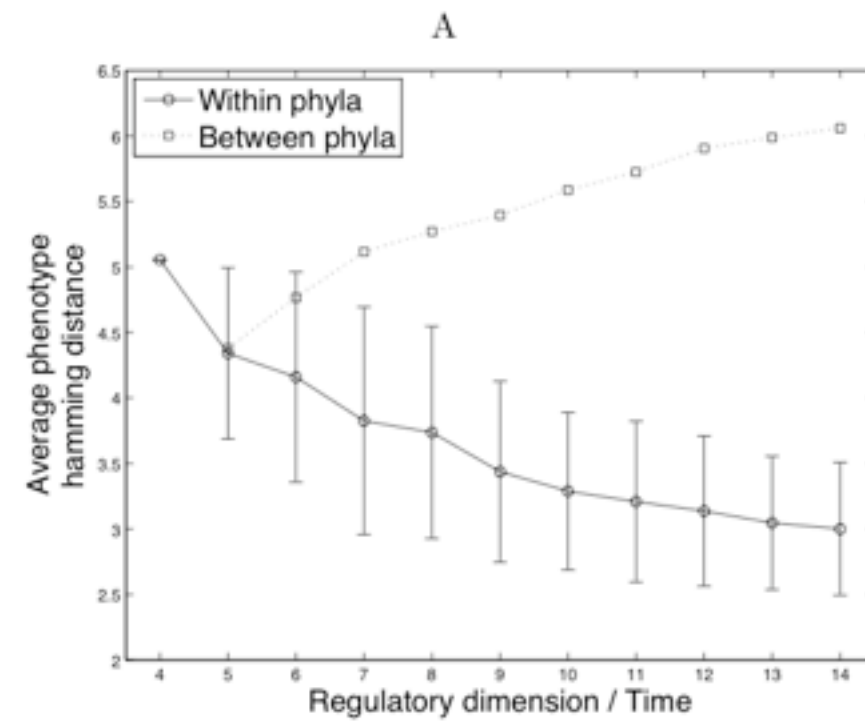
# Developmental Evolution



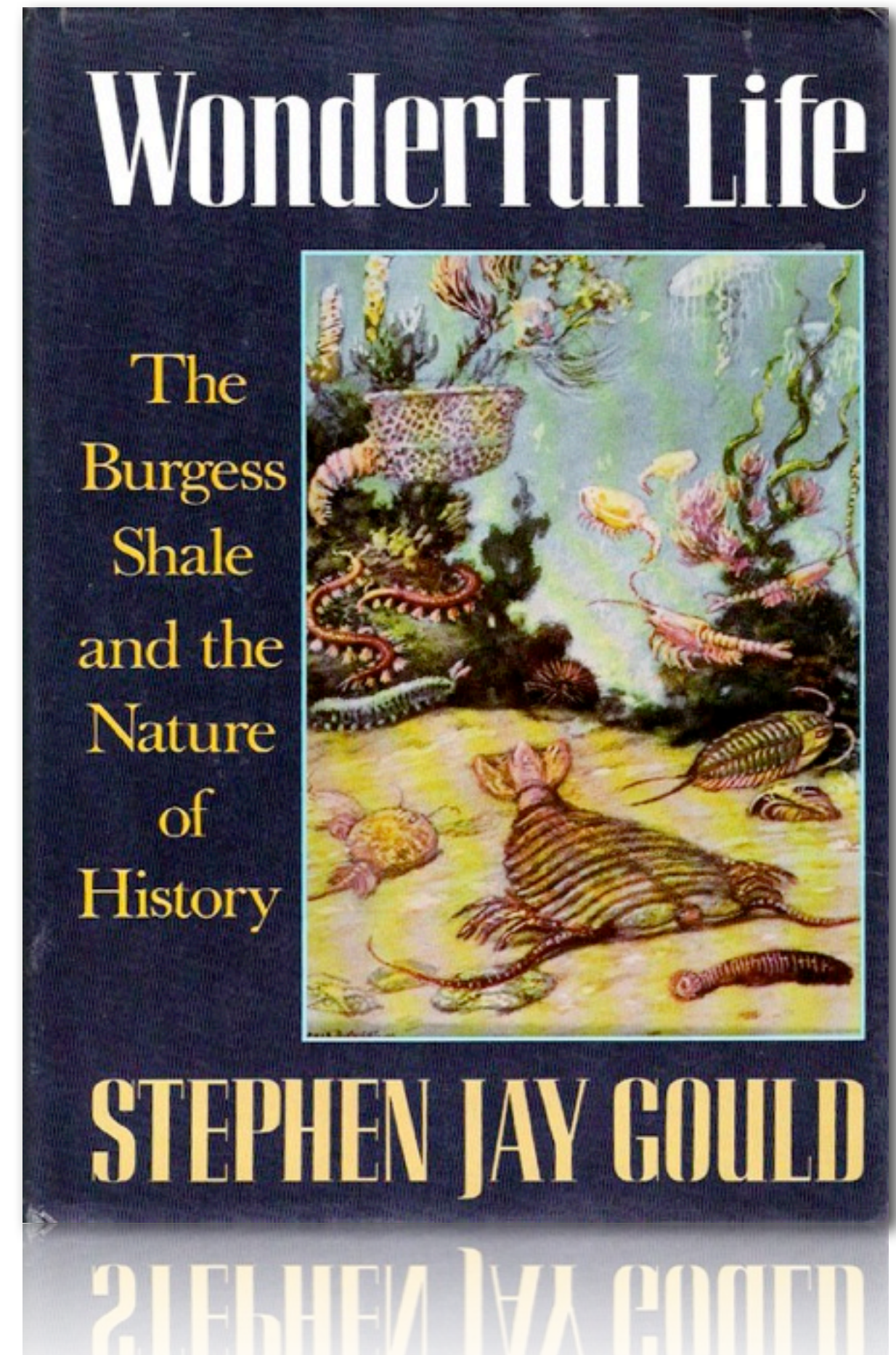
R. Raff. Nature Reviews Genetics. 8. (2007)

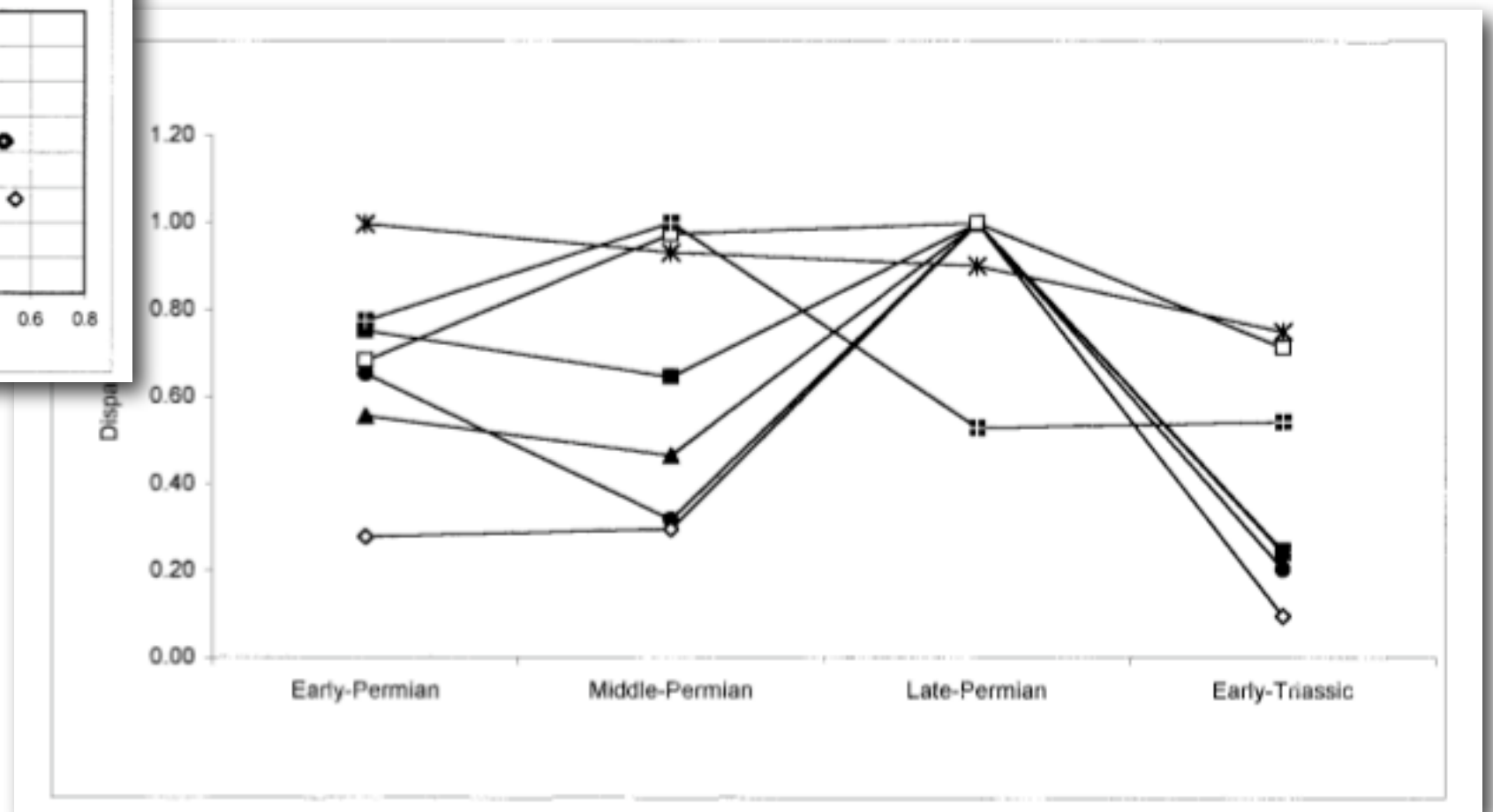
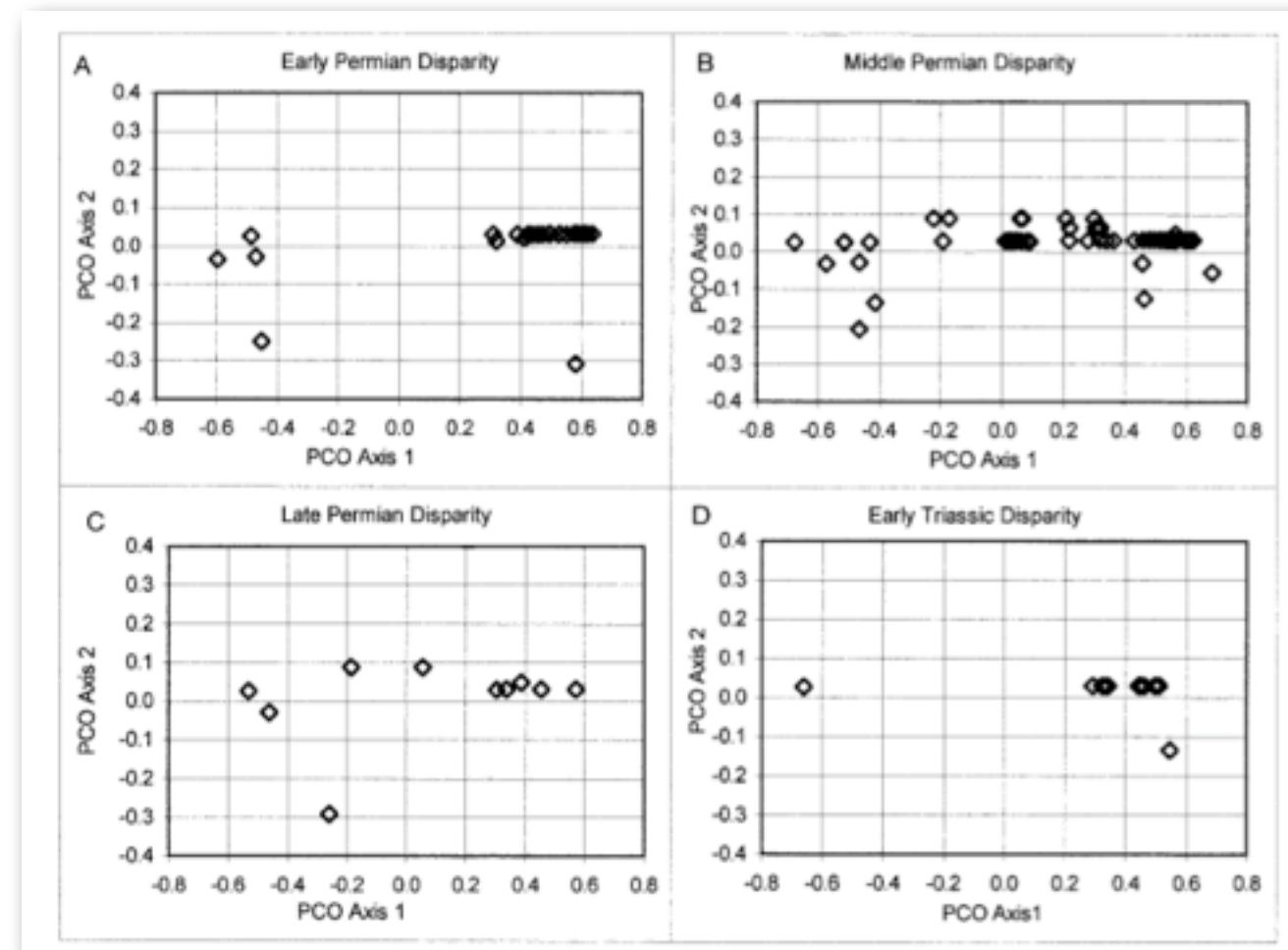






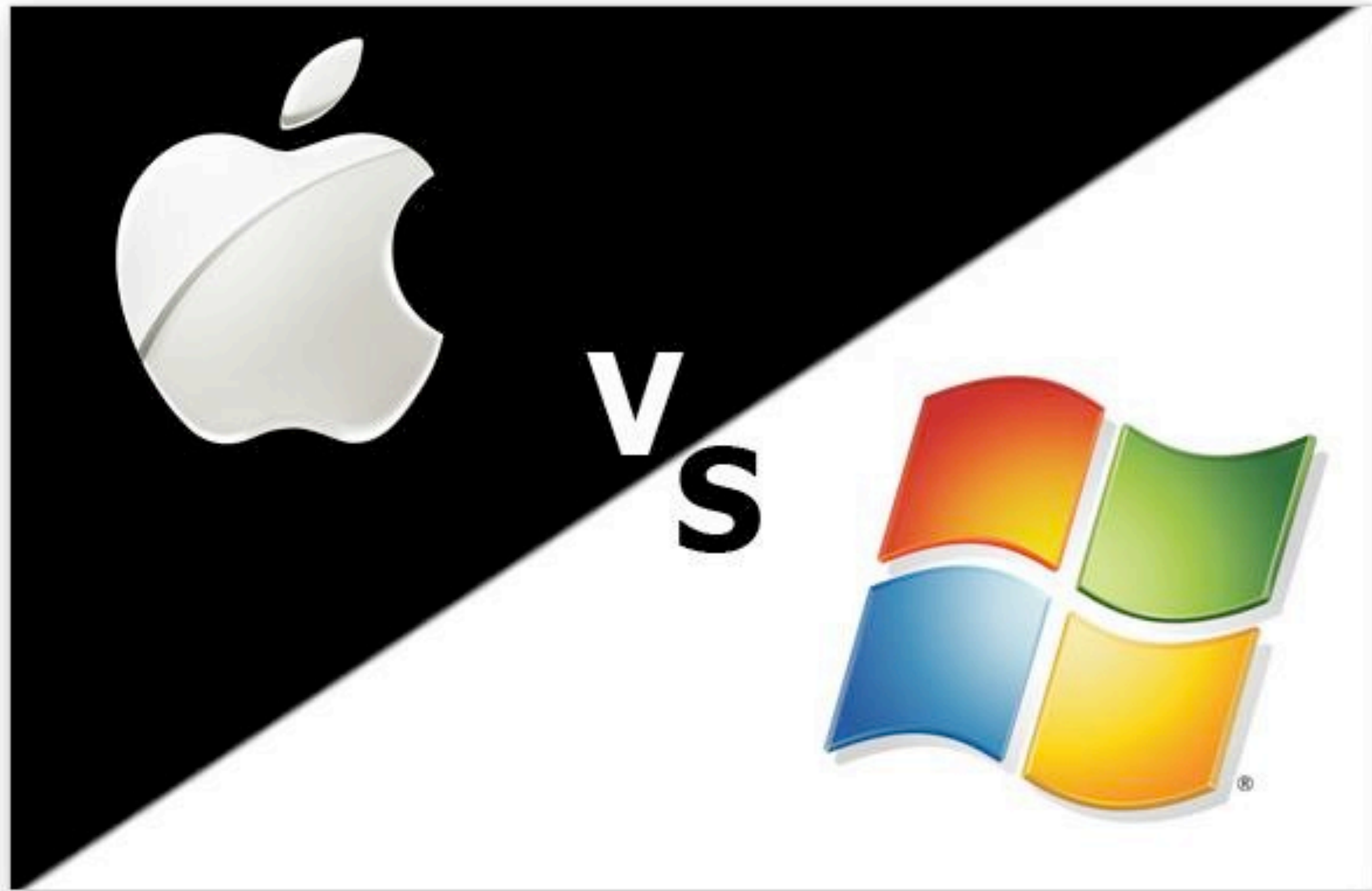
- Ancestral diversity low (low dimension)
- Ancestral Taxa more disparate than derived taxa
- Ancestral taxa sample a large space of phenotypes more uniformly



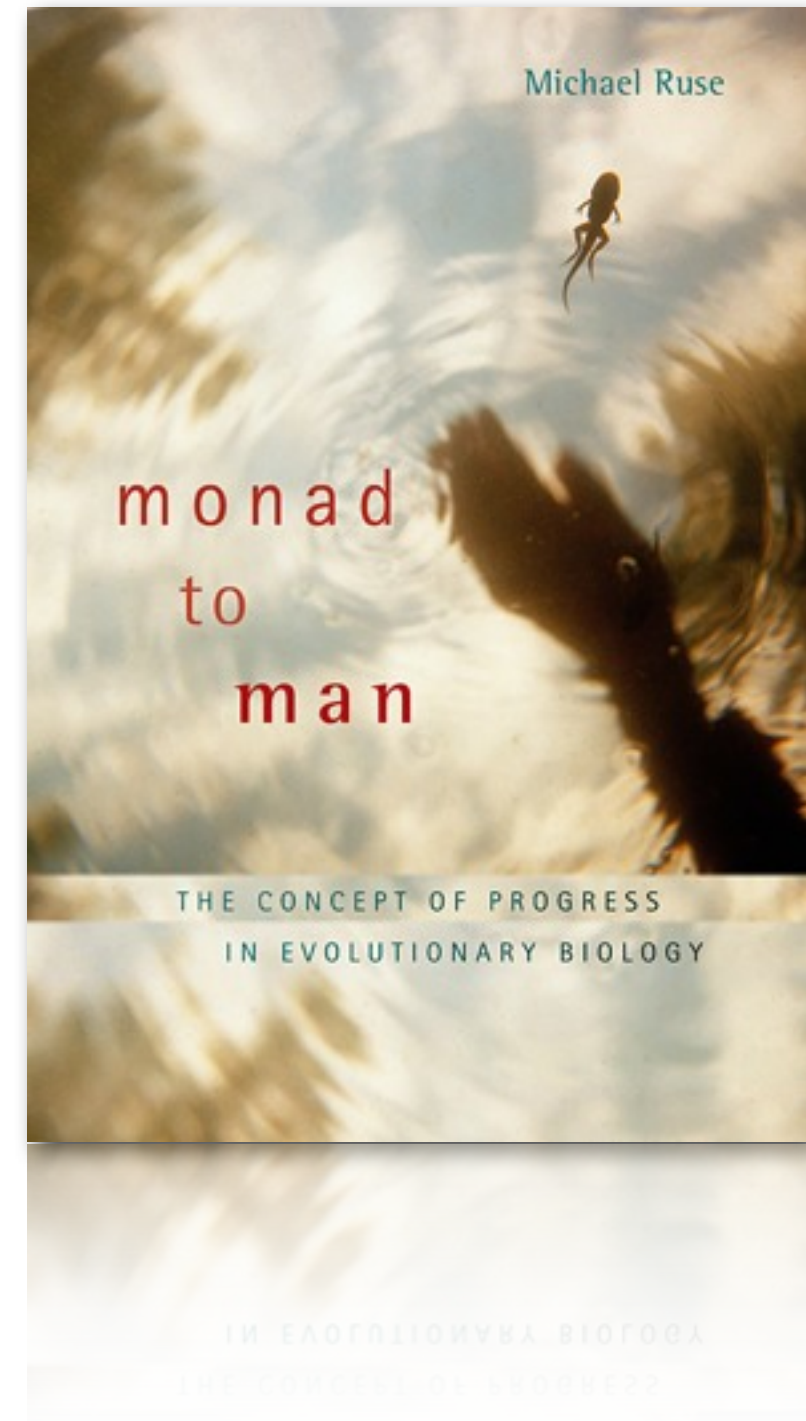
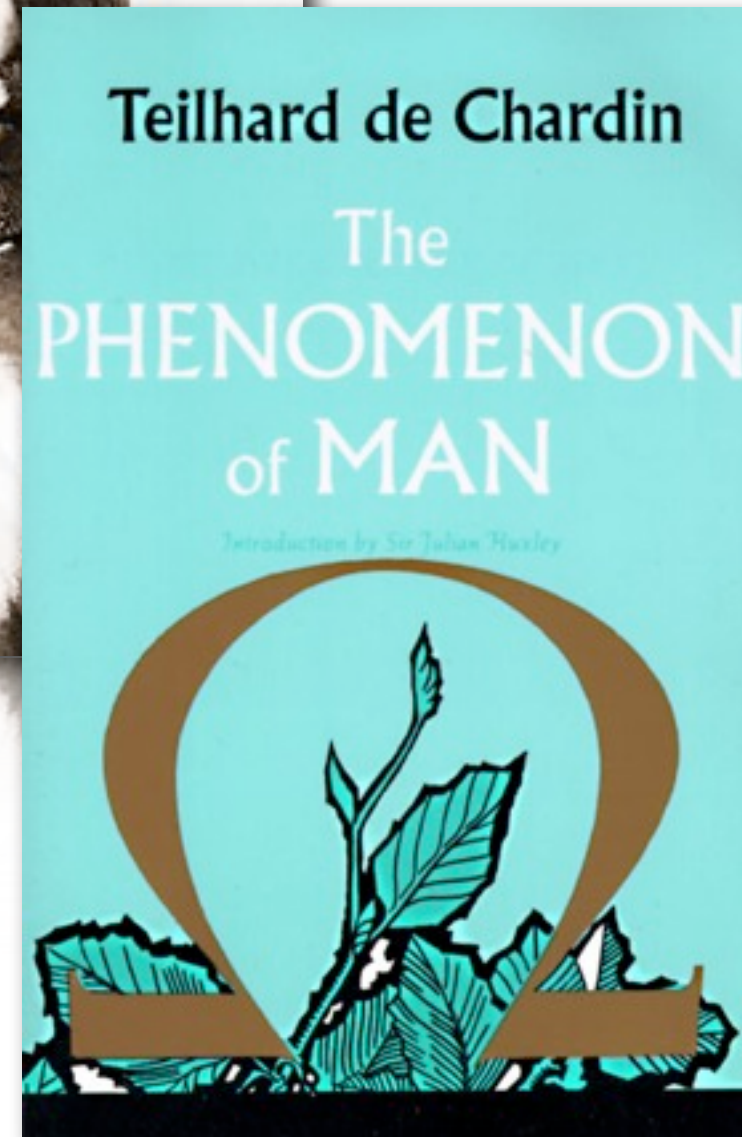
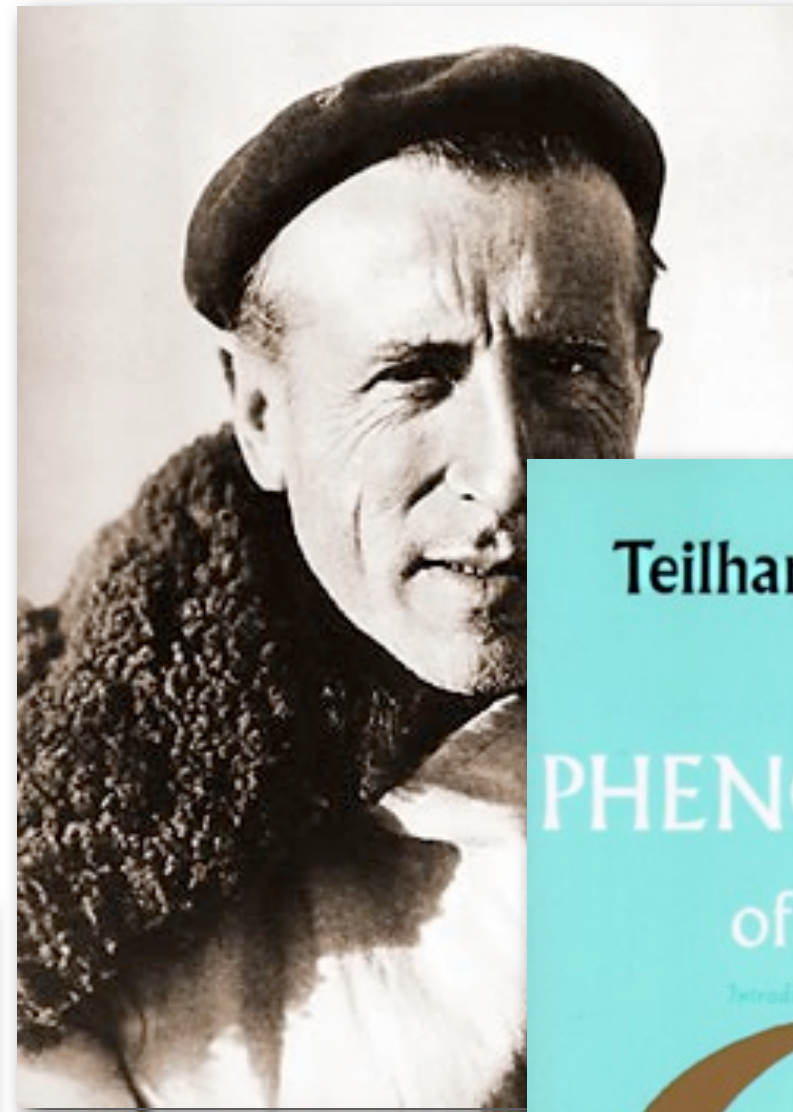
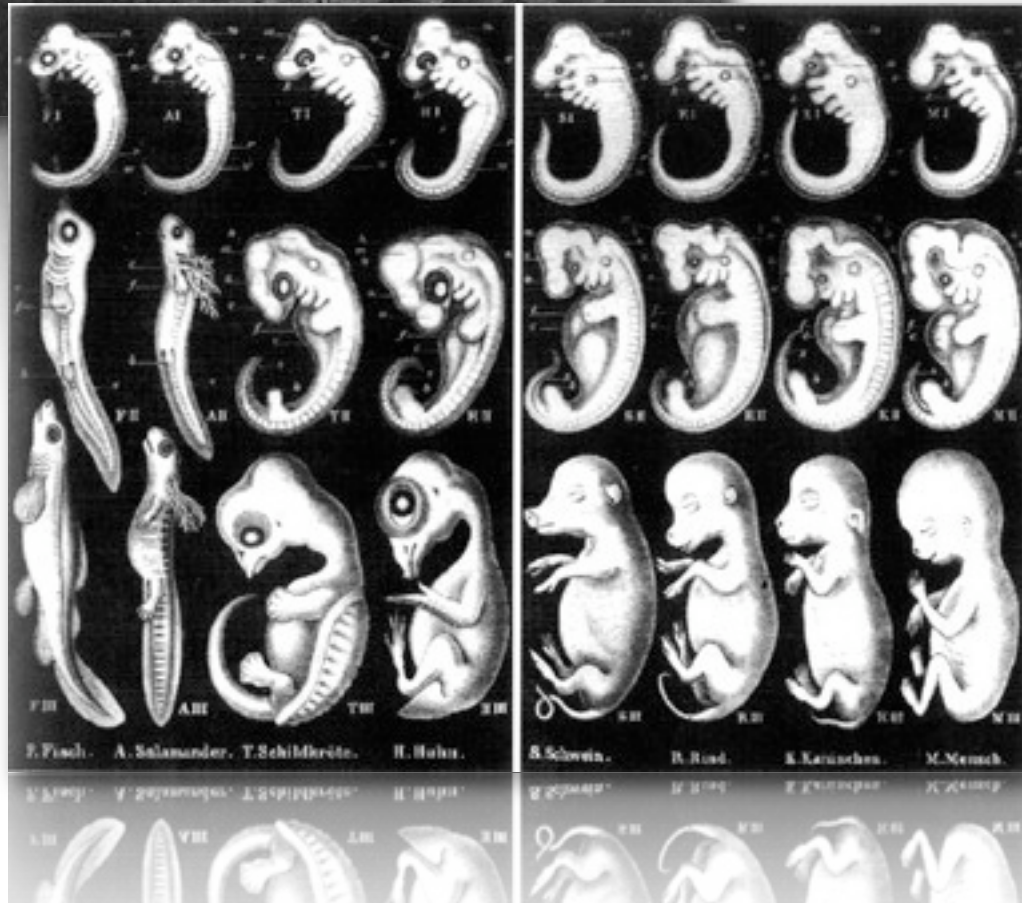
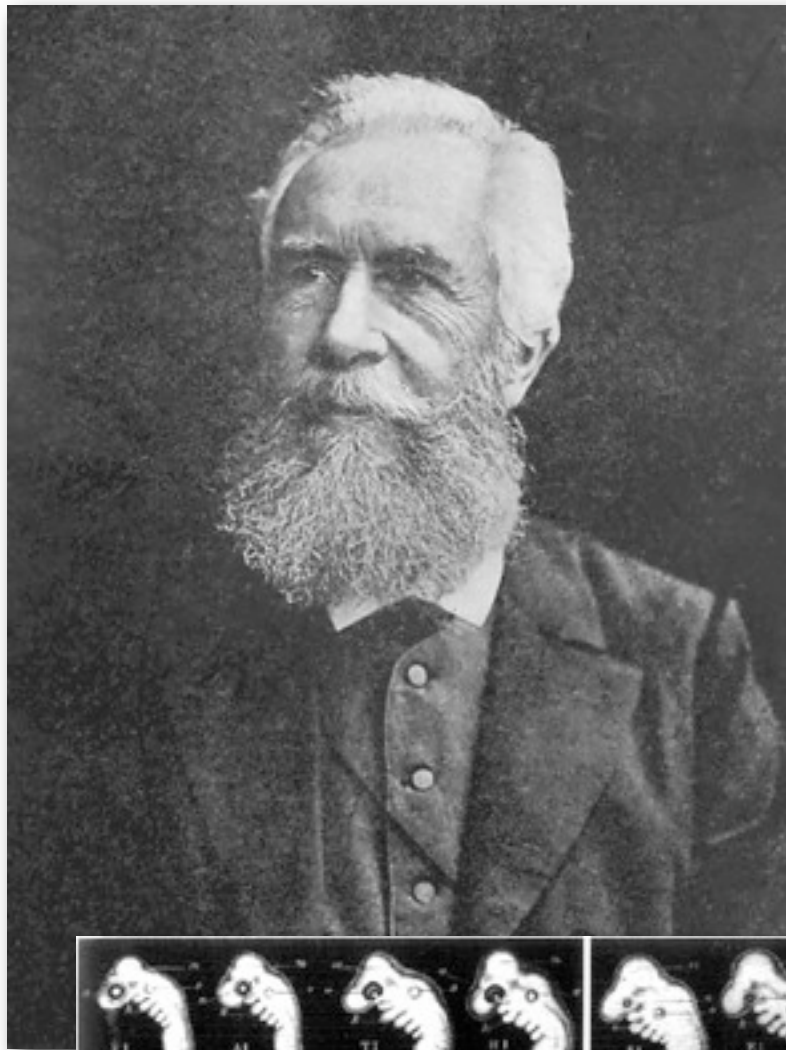


*Ciampaglio et al. Paleobiology, 27(4), 2001, pp. 695–715*









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