Controlling Nonlinear Dynamics in Complex Networks

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Supported by

- **NSF Division of Mathematical Sciences**
  Grant DMS-0709212, CAREER Award DMS-1057128

- **Alfred P. Sloan Foundation**
  Sloan Research Fellowship Award BR-4990

- **National Cancer Institute**
  PSOC Grant 1U54CA143869-01

- **National Oceanic and Atmospheric Administration**
  Grant NA09NMF4630406

- **ISEN-ANL Early Career Award for Energy Research**

Preliminary Remarks on Linear Dynamics

Theory

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t)$$

$A_{ii} = 0$ Y. Y. Liu, J. J. Slotine, and A.-L. Barabási, Nature **473**, 167 (2011).

number of control inputs is determined mainly by ... degree distribution

$A_{ii} \neq 0$ N. J. Cowan, E. J. Chastain, D. A. Vilhena, J. S. Freudenberg, and C. T. Bergstrom, PLoS ONE **7**, e38398 (2012).

a single control input ... is all that is needed for structural controllability

C.-T. Lin, IEEE Trans. Autom. Control. **19**, 201 (1974).

K. Murota, *Systems Analysis by Graphs and Matroids: Structural Solvability and Controllability* (1st ed., Springer, Berlin, 1987).

Theory

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t)$$

Kalman's controllability matrix:

$$K = \left[B \; AB \cdots A^{n-1}B\right]$$

Can the control signal be constructed in practice?

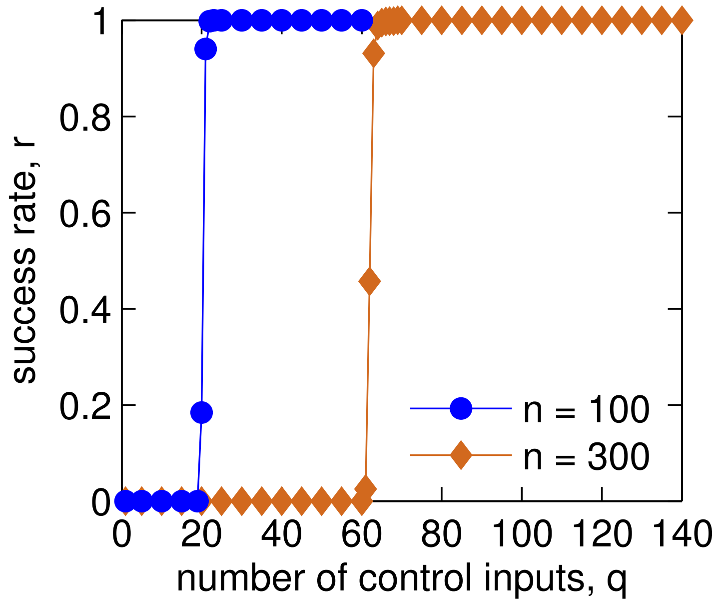
$$u(t) = B^T\Phi^T(t_0,t)W^{-1}(t_0,t_1)\left[\Phi(t_0,t_1)x^{(1)} - x^{(0)}\right]$$

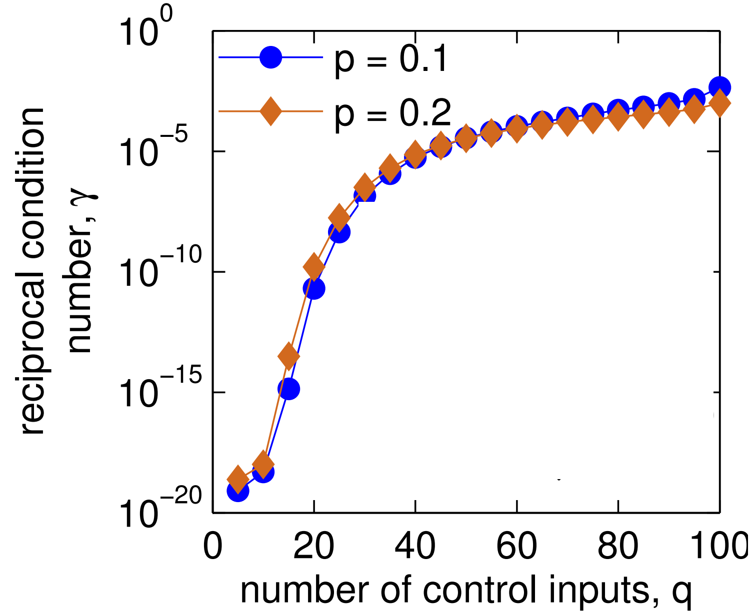
$$x(t) = \Phi(t,t_0)\left[x^{(0)} + W(t_0,t)W^{-1}(t_0,t_1)\left(\Phi(t_0,t_1)x^{(1)} - x^{(0)}\right)\right]$$

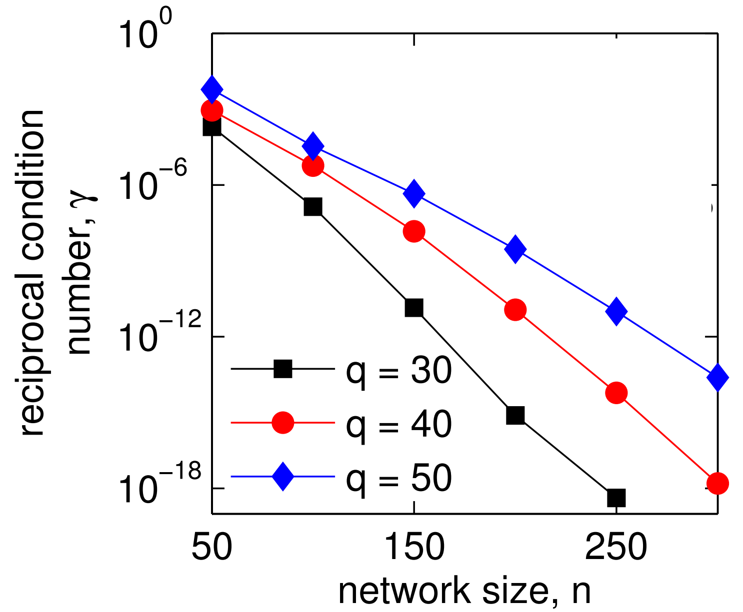
Controllability Gramian:

$$W(t_0,t_1) = \int_{t_0}^{t_1} \Phi(t_0,t)BB^T\Phi^T(t_0,t)dt, \quad \Phi(t',t) = e^{(t'-t)A}$$

Numerics







Sun and Motter, PRL 2013

Theory for numerics

Perturbation analysis:

$$\|\tilde{x}^{(1)} - x^{(1)}\| \lessapprox D\|W(\tilde{W}^{-1} - W^{-1})\|$$

Critical reciprocal condition number:

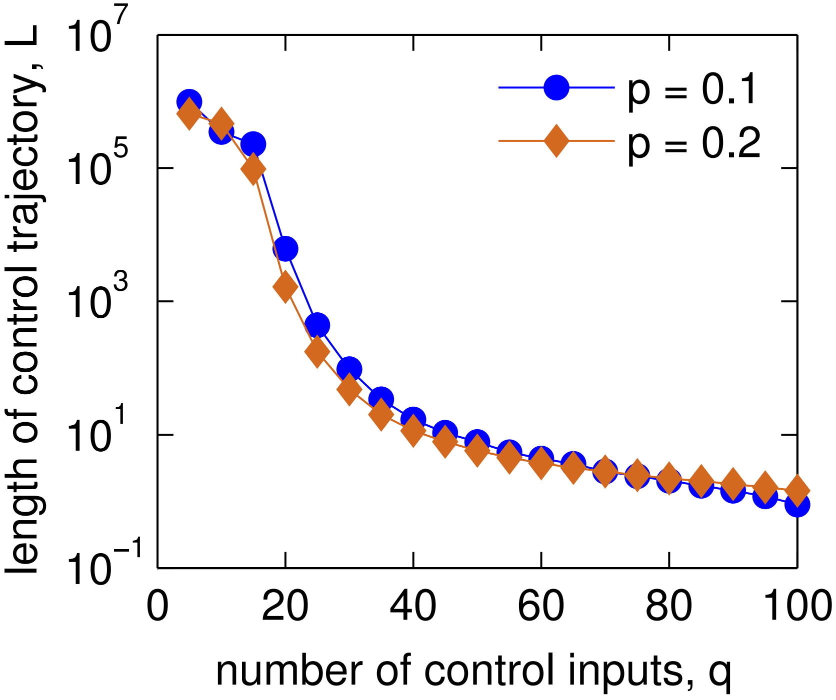
$$\gamma_c \approx D' \frac{\epsilon}{\eta}$$

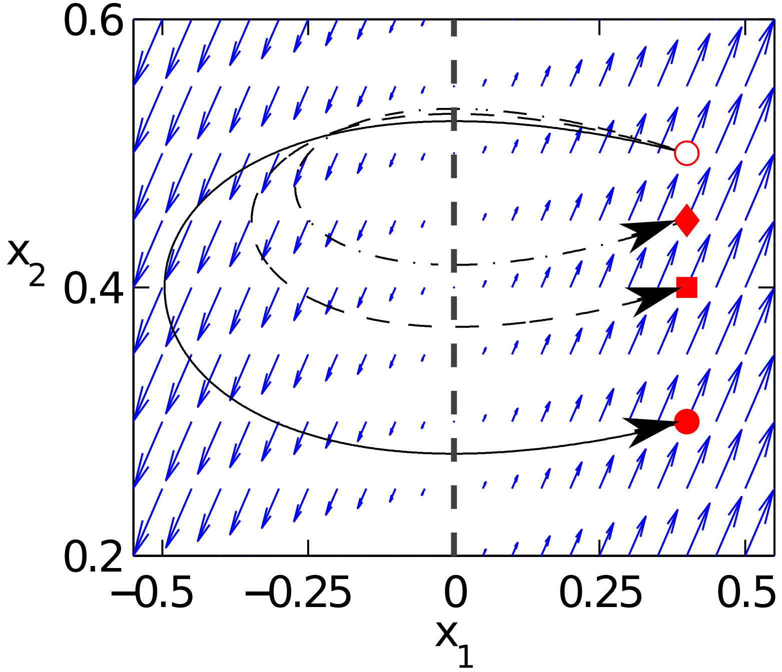
This leads to the notion of *<u>numerical rank condition</u>*

Sun and Motter, PRL 2013

Control trajectories

$$\dot{x}_1 = x_1 + u_1(t)$$
$$\dot{x}_2 = x_1$$

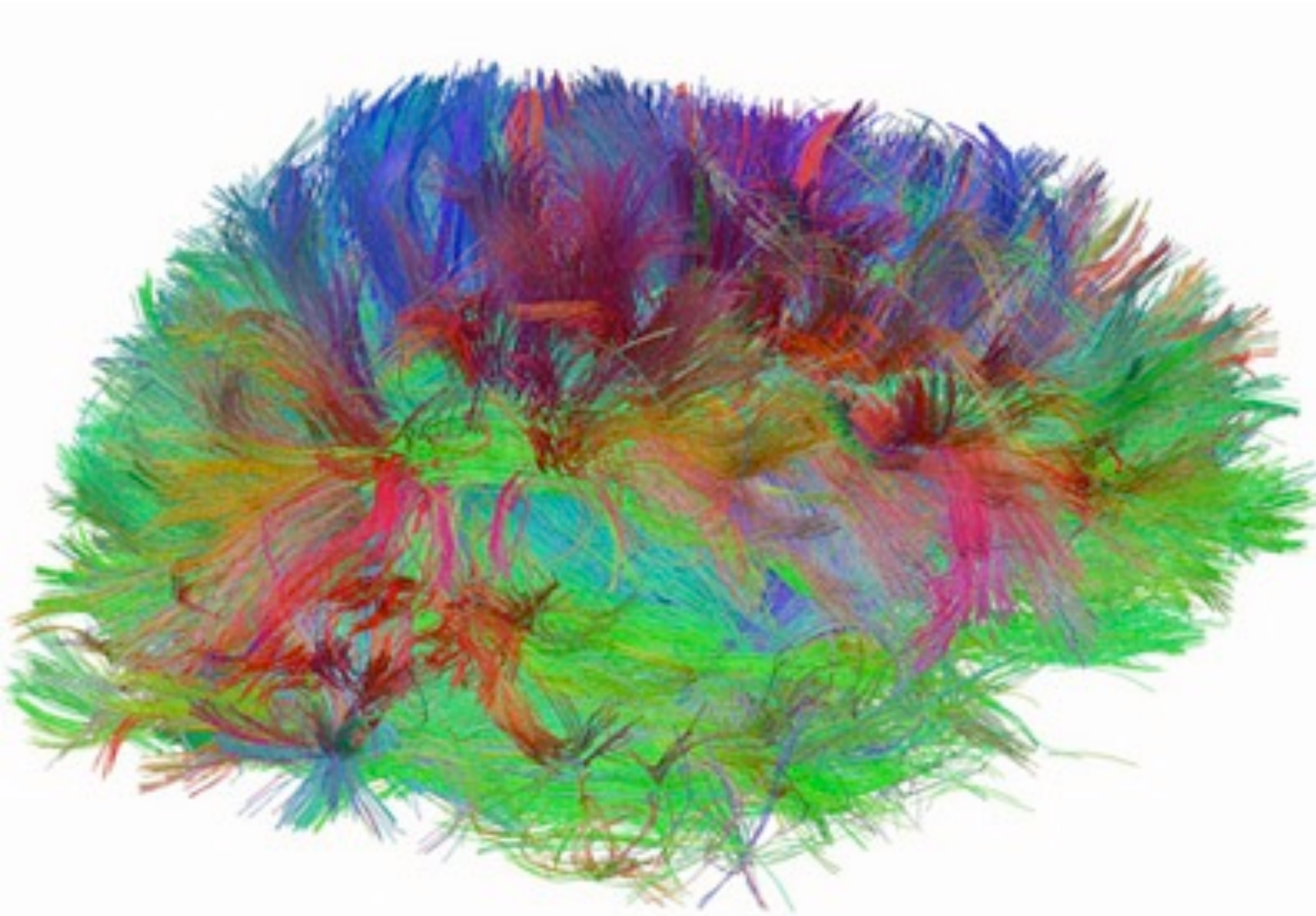




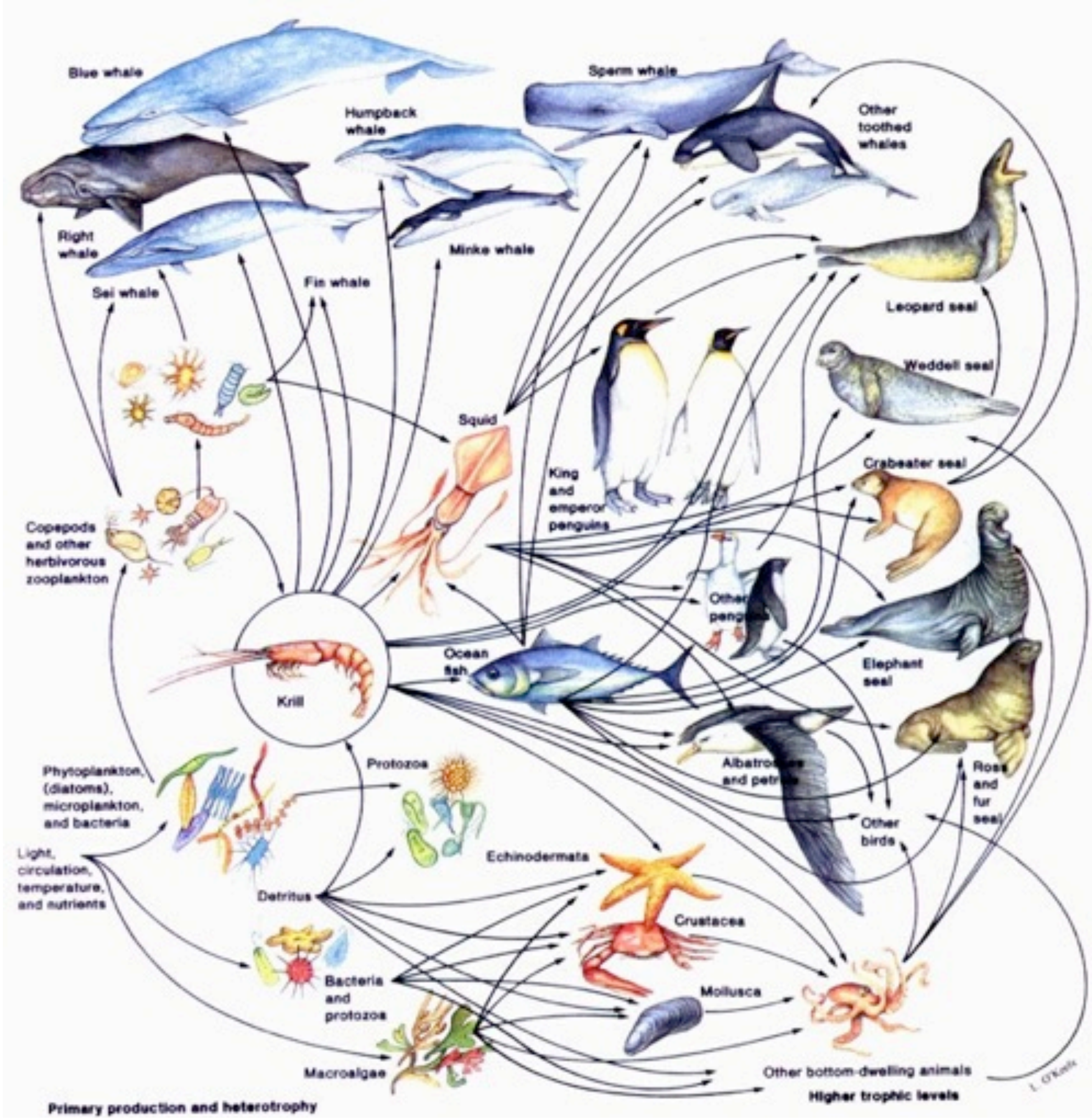
Control trajectories are nonlocal!

Sun and Motter, PRL 2013

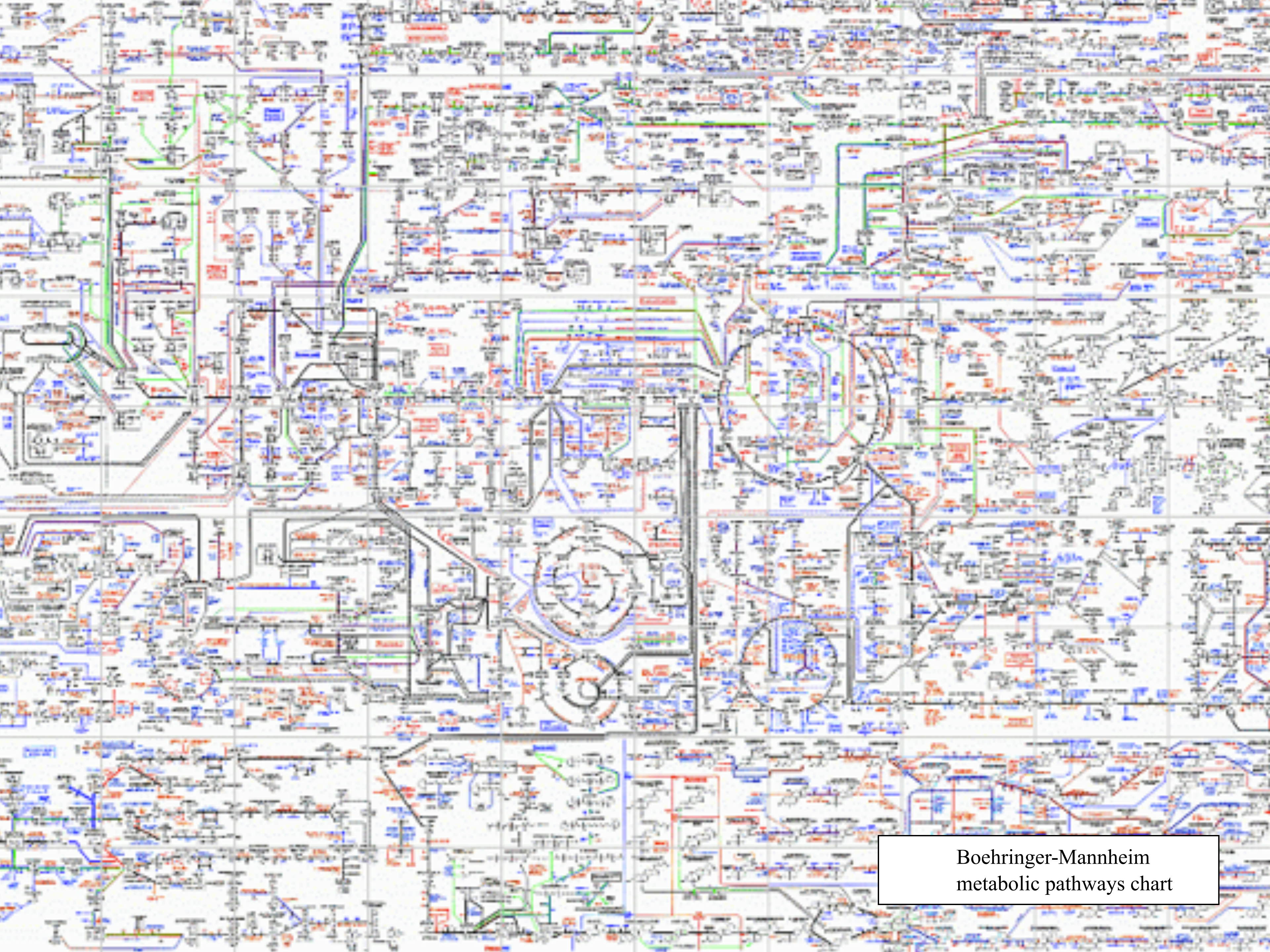
Now Turning to Real (Nonlinear) Systems



V. J. Wedeen/Harvard



S.A. Miller & P. Harley



Boehringer-Mannheim metabolic pathways chart

Common properties

(i) the dynamics is nonlinear

(ii) the system has multiple stable states (or attractors)

(iii) the system is described by a large number of dynamical variables

(iv) there are constraints on the physically feasible control interventions

(v) there might be noise and parameter uncertainty

(vi) decentralized (hence suboptimal) response to perturbations

Dynamical equations

Power grids:

$$M_i \frac{d\omega_i}{dt} = P_{\mathrm{m}i} - P_{\mathrm{e}i}, \quad \frac{d\delta_i}{dt} = \omega_i$$

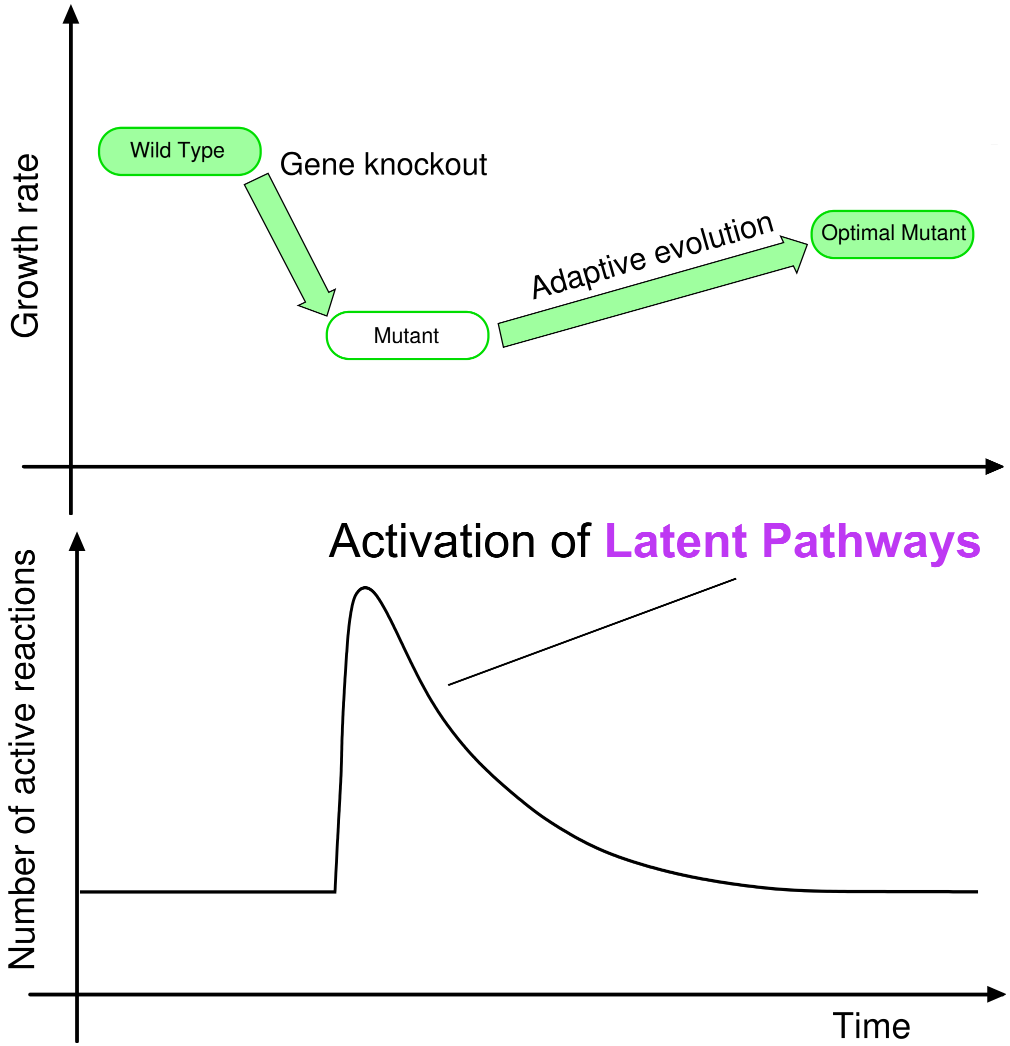
Metabolism:

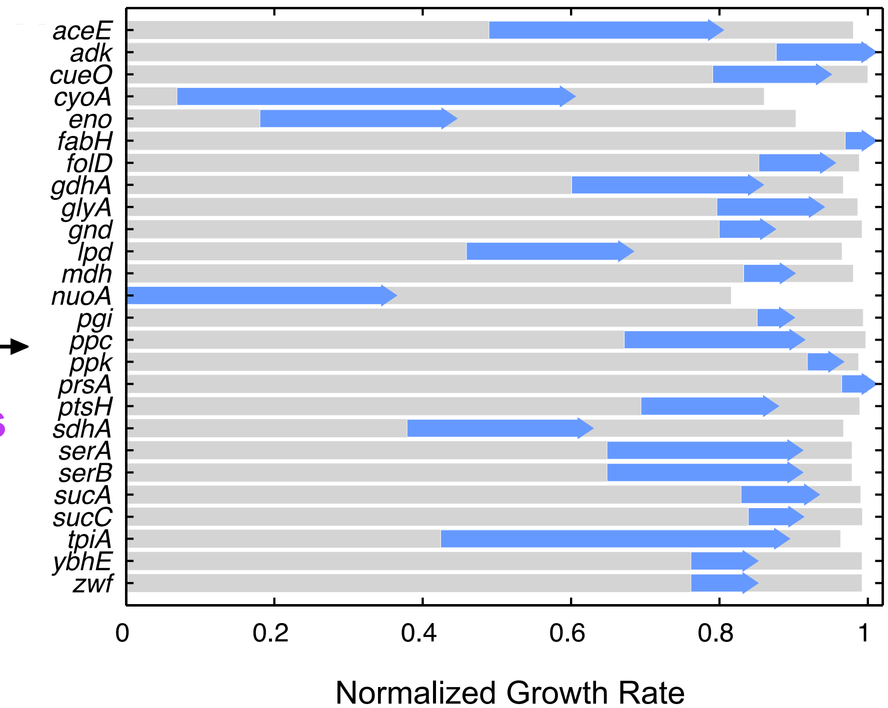
$$\frac{dX_i}{dt} = \sum_j S_{ij}\nu_j, \quad \nu_j = k_j \prod_i X_i^{\kappa_{ij}}$$

Food webs:

$$\frac{dX_i}{dt} = X_i(b_i + \sum_j a_{ij} X_j)$$

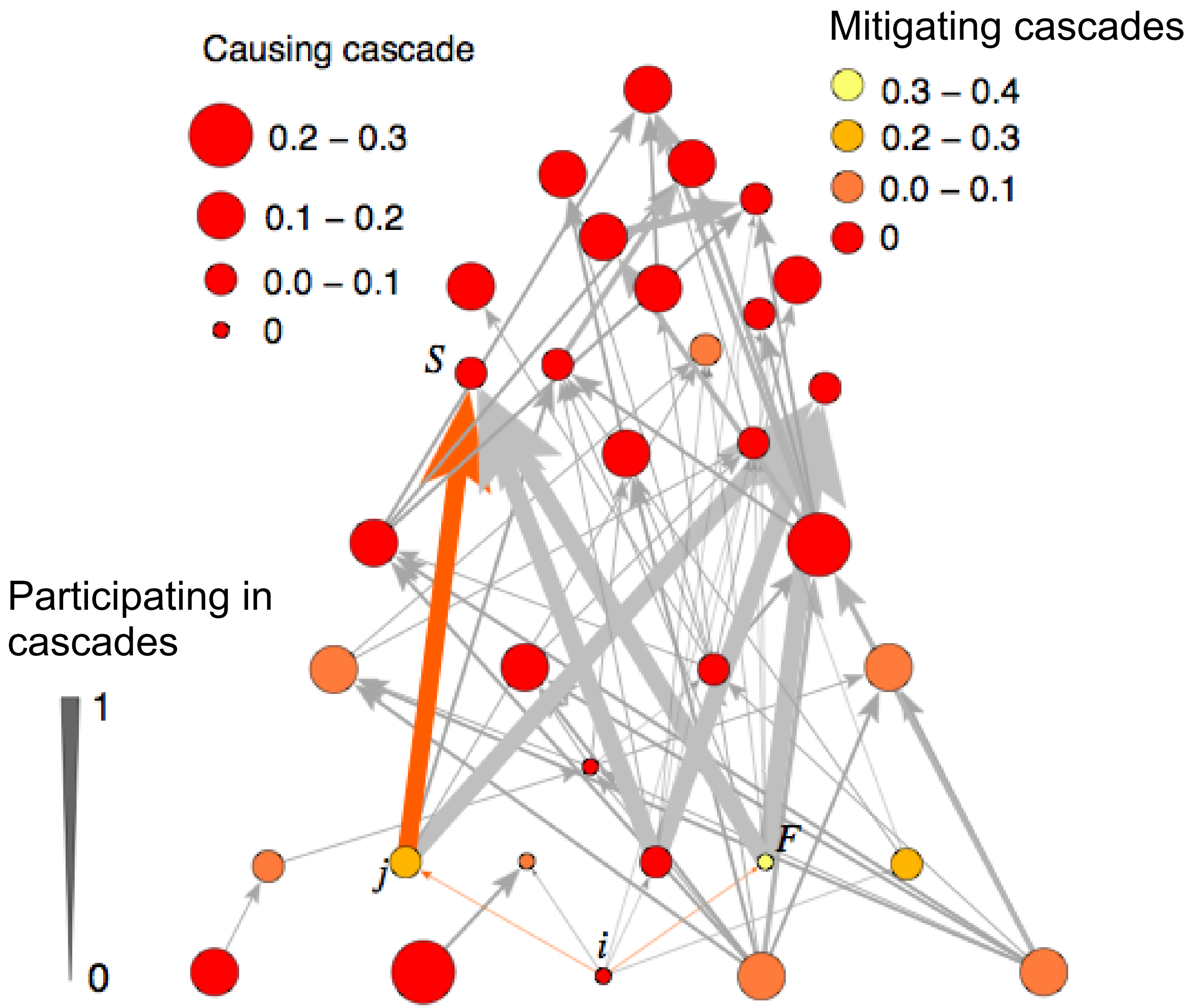
Synthetic rescues in metabolic networks





Cornelius, Lee, Motter, PNAS 2011

Mitigation of extinctions in food webs



Sahasrabudhe, Motter, Nature Comm 2011

General Problem

$$\frac{d\mathbf{x}}{dt} = \mathbf{F}(\mathbf{x}; \boldsymbol{\beta})$$

state: $\mathbf{x} \in D \subset \mathbb{R}^m$

parameters: $\boldsymbol{\beta} \in S \subset \mathbb{R}^M$

vector field: $\mathbf{F} : D \times S \to \mathbb{R}^m$

attraction basin: $\Omega_{\boldsymbol{\beta}}(A) \subset D$

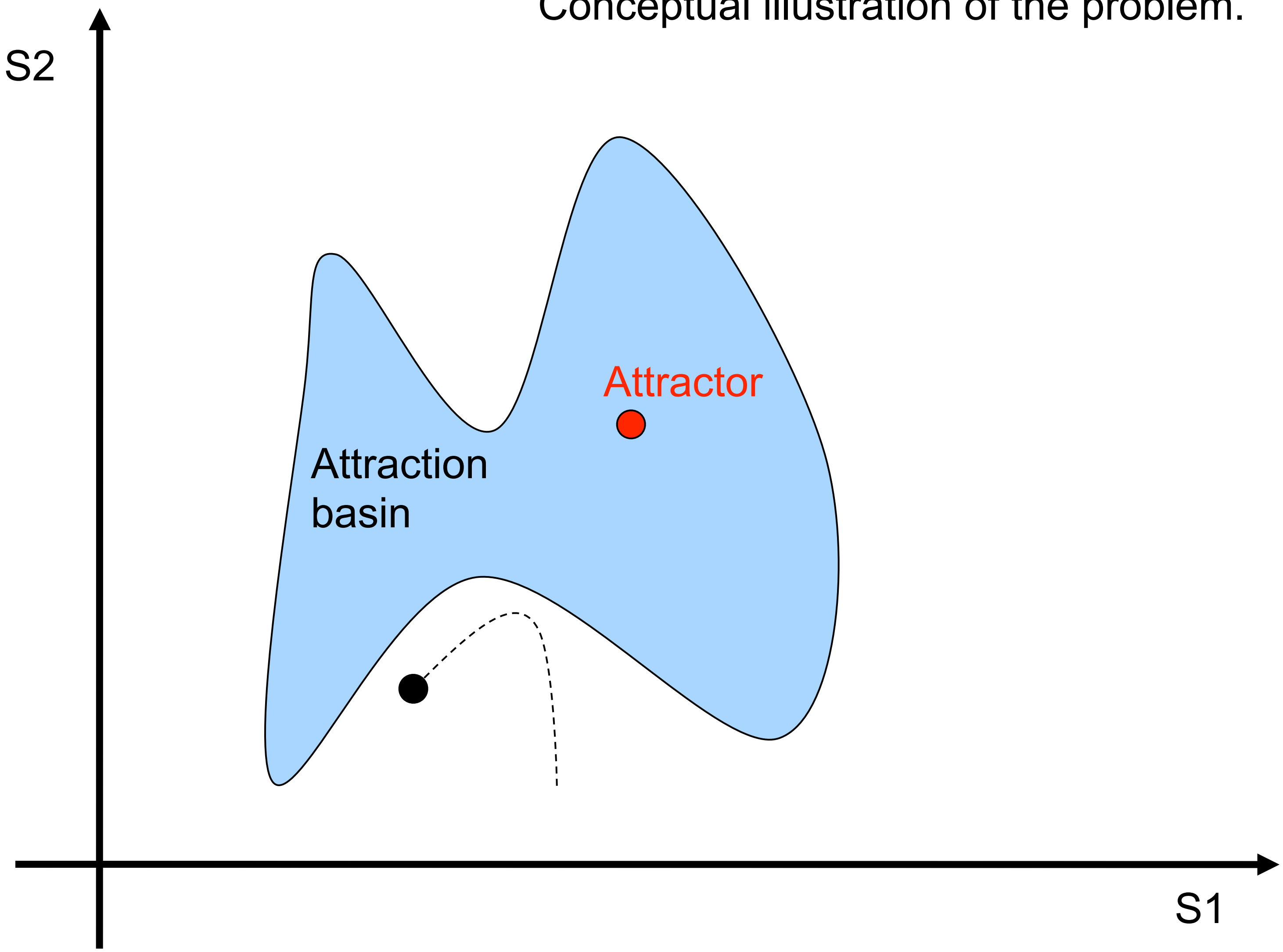
$(\mathbf{x}_0 \in \Omega_{\boldsymbol{\beta}}(A)$ iff $\boldsymbol{\phi}_{\boldsymbol{\beta}}(t, \mathbf{x}_0) \to A$ as $t \to +\infty)$

Given $\mathbf{x}_0 \notin \Omega_{\boldsymbol{\beta}}(A)$, find $\Delta\mathbf{x}_0^A$ such that

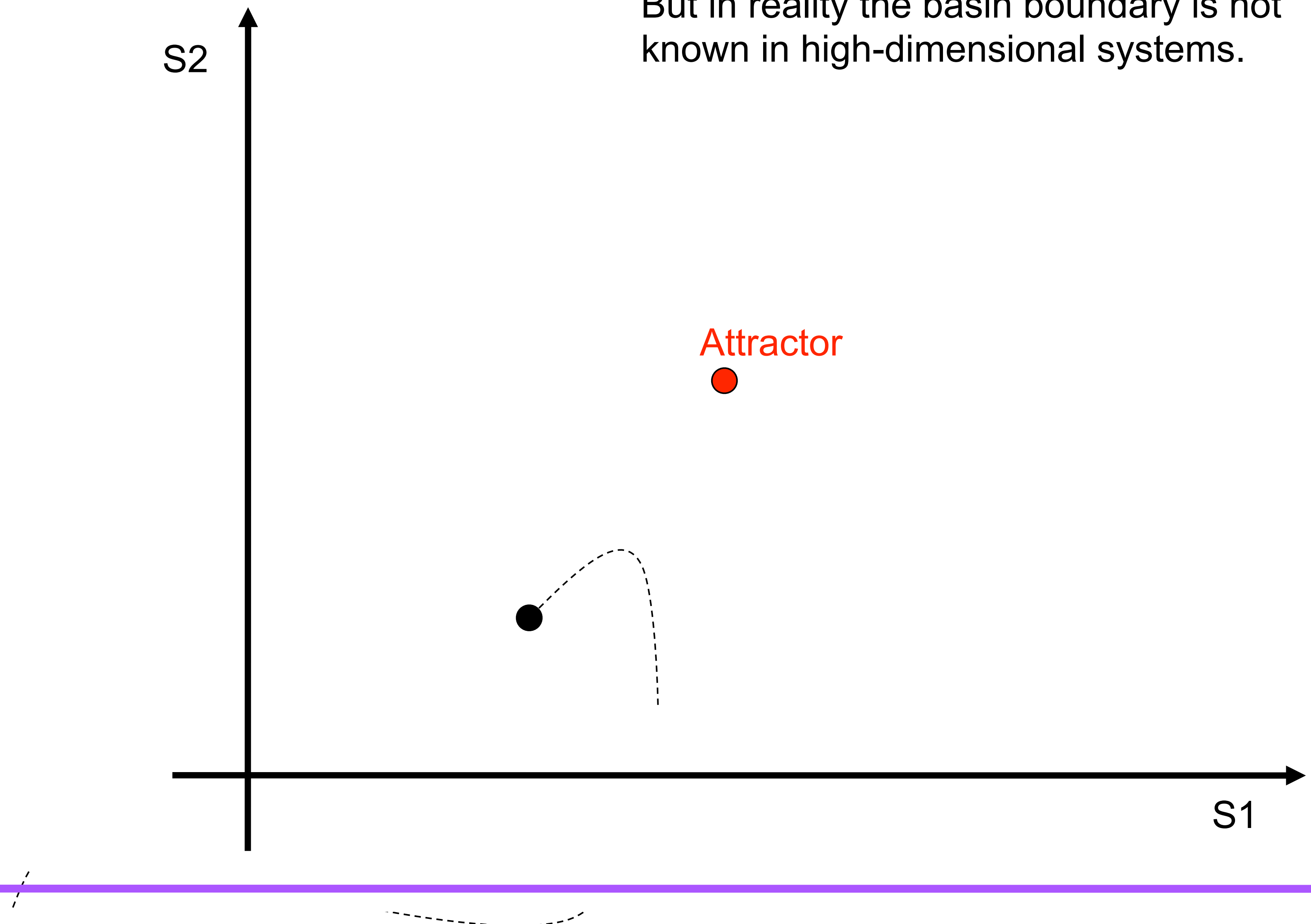
$$\mathbf{x}_0' \equiv \mathbf{x}_0 + \Delta\mathbf{x}_0^A \in \Omega_{\boldsymbol{\beta}}(A)$$
$$g_i^x(\mathbf{x}_0', \mathbf{x}_0) \leq 0, \quad i = 1, ..., p$$
$$h_j^x(\mathbf{x}_0', \mathbf{x}_0) = 0, \quad j = 1, ..., q, \text{ where } g_i^x, h_j^x : D \times D \to \mathbb{R}$$

Cornelius, Kath, Motter, Nature Comm 2013

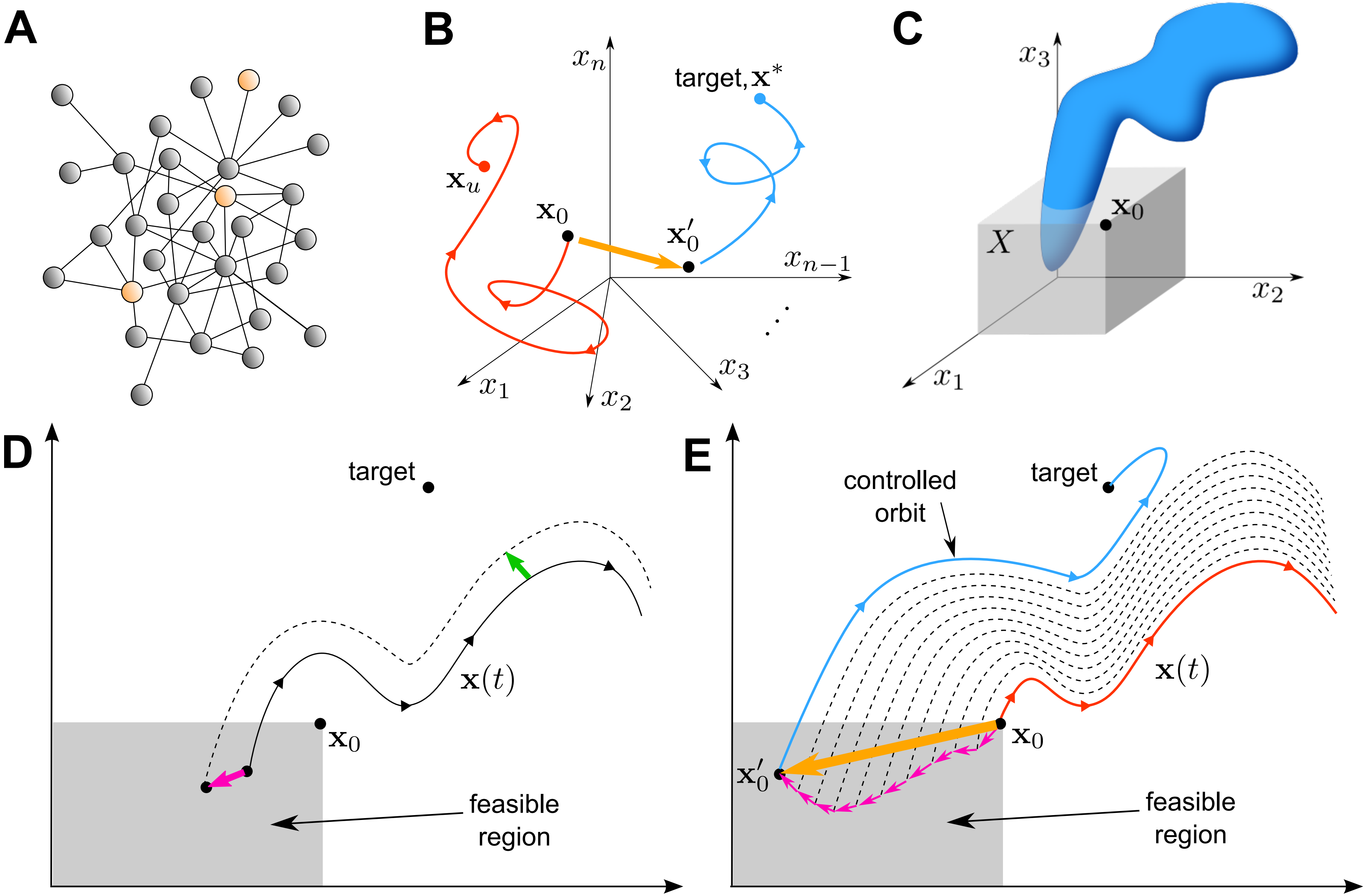
Conceptual illustration of the problem.



But in reality the basin boundary is not known in high-dimensional systems.



Identifying Control Interventions



Cornelius, Kath, Motter, Nature Comm 2013

Effectiveness and Efficiency

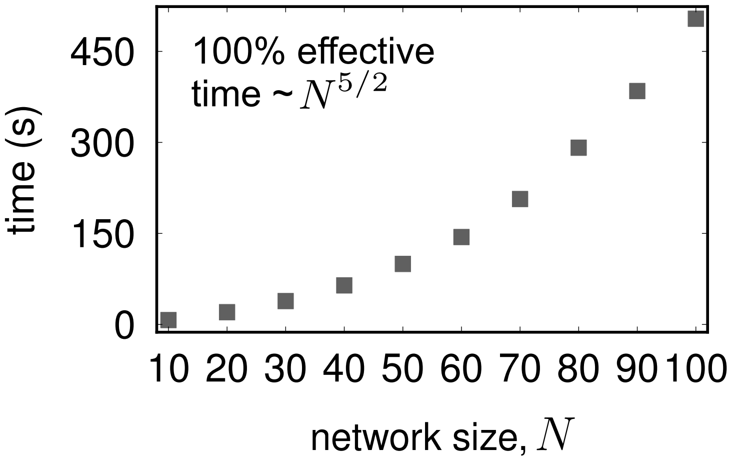
Units

$$\frac{dx_1}{dt} = a_1 \frac{x_1^4}{x_1^4 + S^4} + b_1 \frac{S^4}{x_2^4 + S^4} - k_1 x_1 + f_1$$

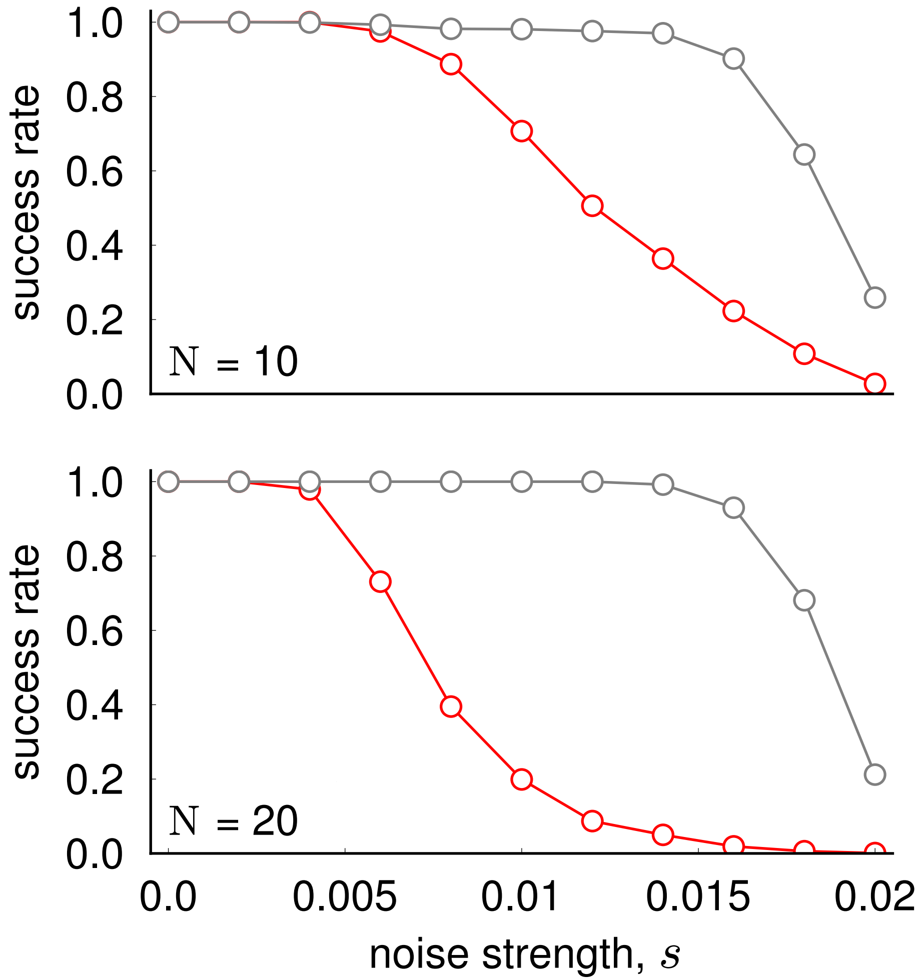
$$\frac{dx_2}{dt} = a_2 \frac{x_2^4}{x_2^4 + S^4} + b_2 \frac{S^4}{x_1^4 + S^4} - k_2 x_2 + f_2$$

Network

$$\frac{d\mathbf{x}_i}{dt} = \mathbf{f}(\mathbf{x}_i) + \frac{c}{d_i} \sum_j A_{ij} [\mathbf{x}_j - \mathbf{x}_i]$$

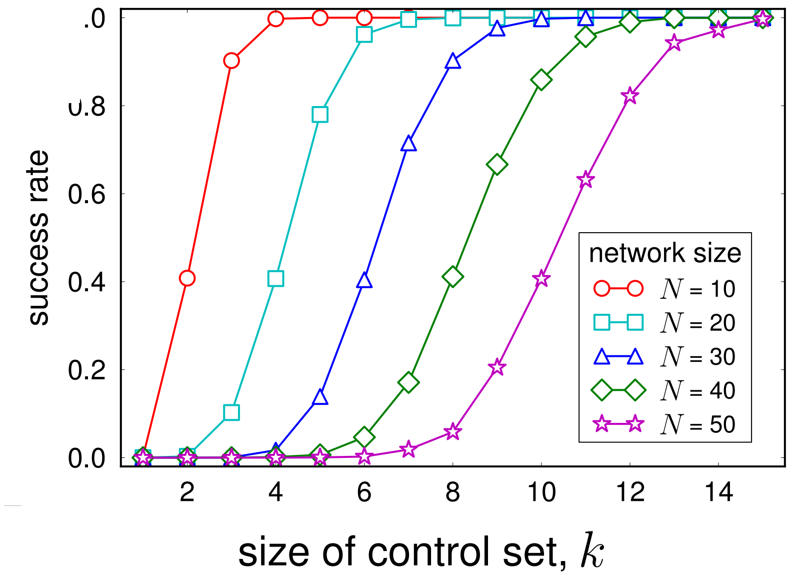


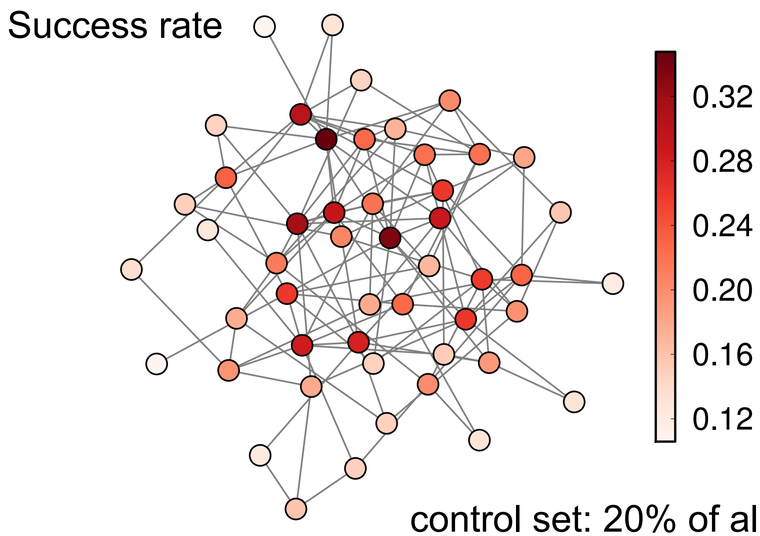
Robust against Stochasticity and Parameter Uncertainty



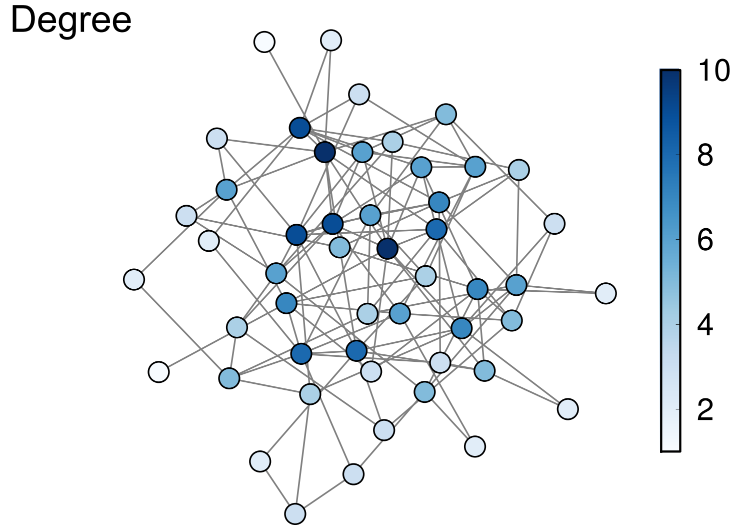
Small Number of Accessible Nodes

Randomly selected nodes

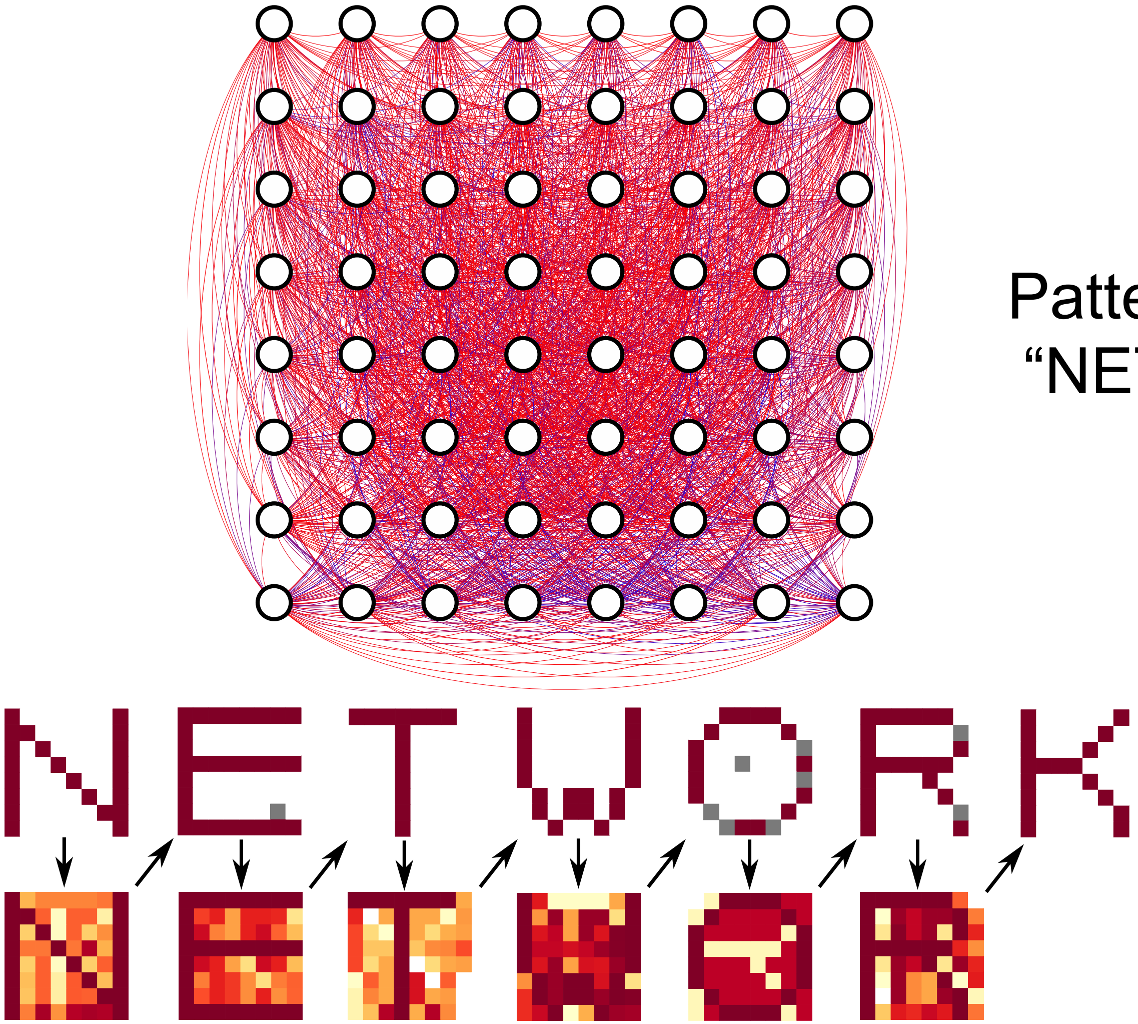




control set: 20% of all nodes



Example
Transitions in associative-memory networks

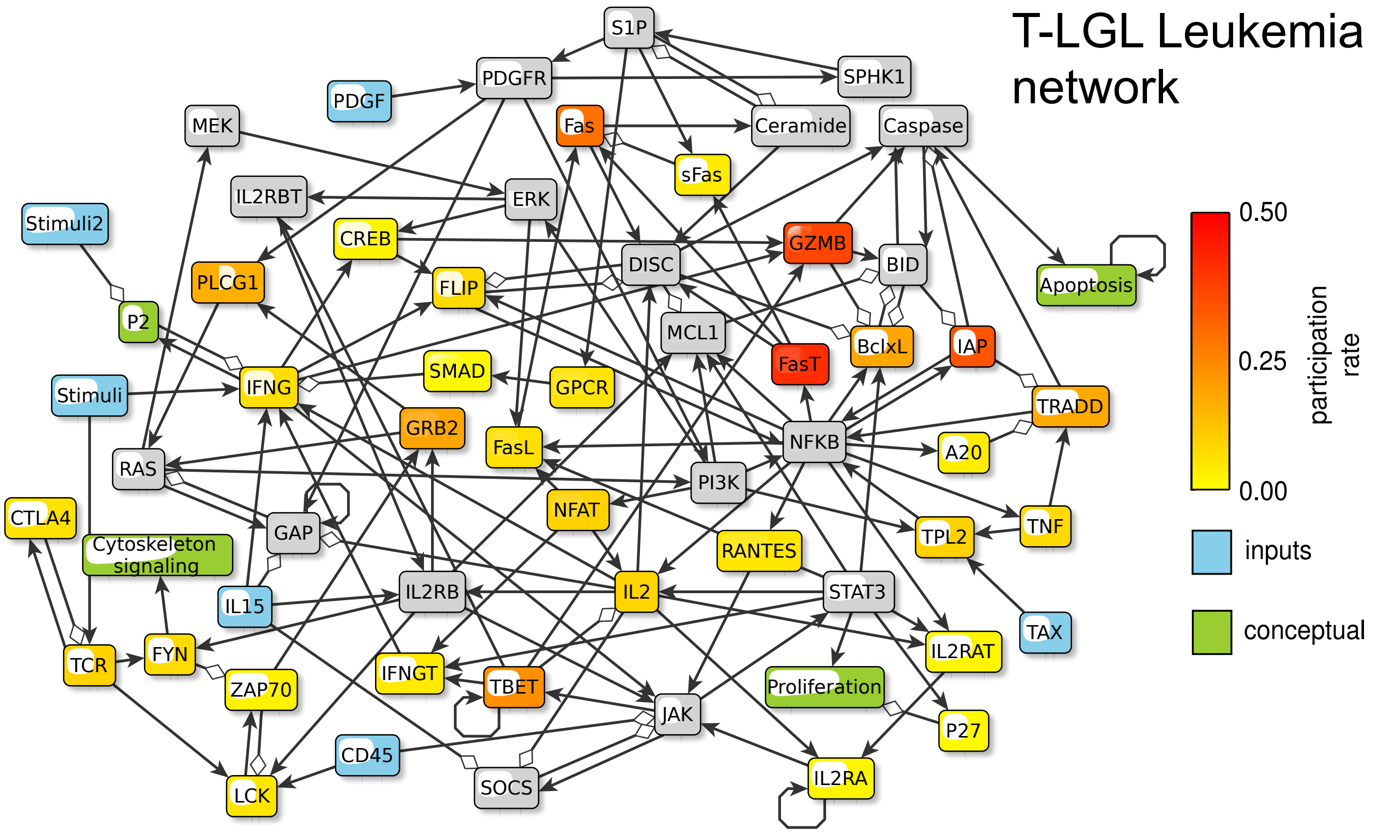


Pattern stored
"NETWORK"

Cornelius, Kath, Motter, Nature Comm 2013

Example
Identification of therapeutic Interventions

T-LGL Leukemia signaling network



Cornelius, Kath, Motter, Nature Comm 2013

Control by Manipulating Stability

Synchronization in power grid networks

Starting from the swing equation

$$\frac{2H_i}{\omega_\mathrm{R}}\frac{\mathrm{d}^2\delta_i}{\mathrm{d}t^2} = P_{\mathrm{m}i} - P_{\mathrm{e}i}$$

we can show that stability of synchronous states is enhanced when

$$\beta_i = (D_i + 1/R_i)/2H_i$$

for all generators is equal to $2\sqrt{\alpha_2}$

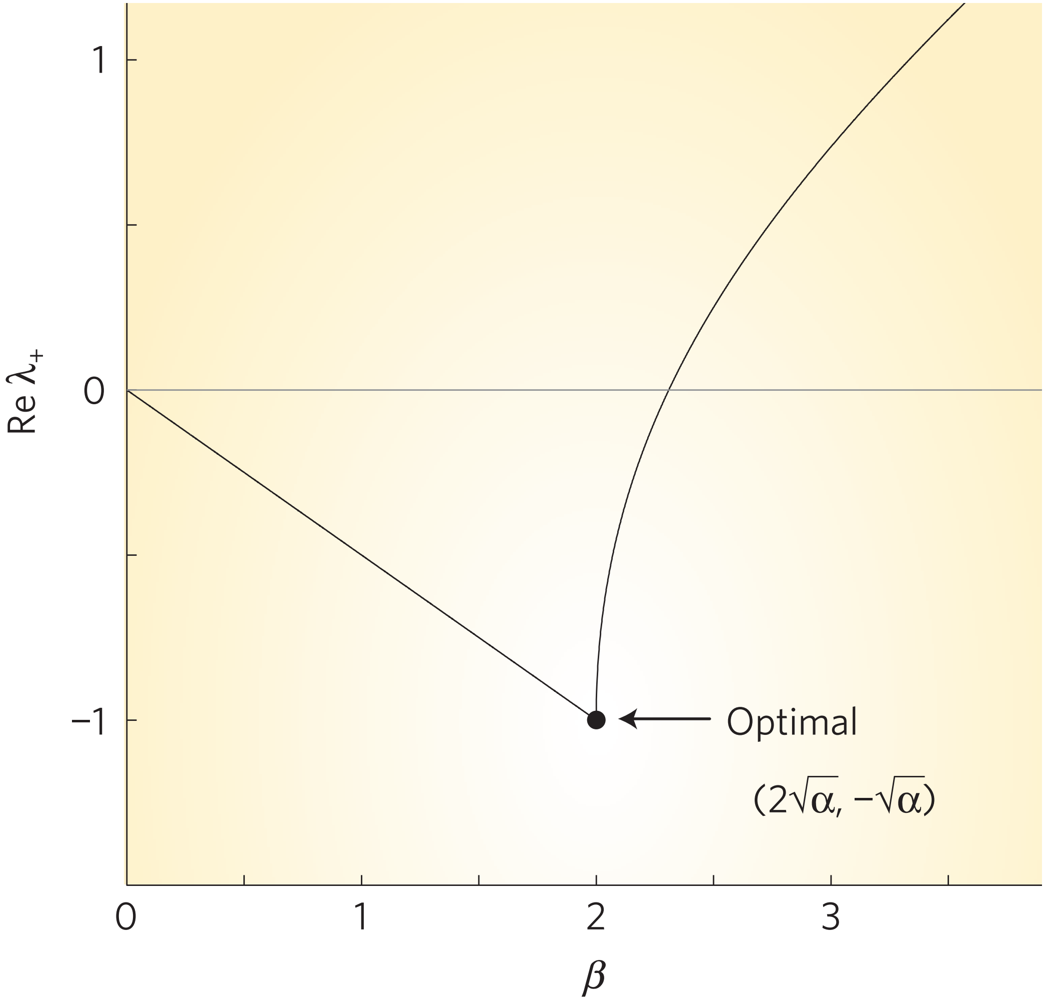
where α_2 is the smallest nonzero eigenvalue of the “coupling” matrix

$$P_{ij} = \begin{cases} \frac{\omega_\mathrm{R} E_i E_j}{2H_i}(G_{ij}\sin\delta_{ij}^* - B_{ij}\cos\delta_{ij}^*), & i \neq j \\ -\sum_{k\neq i} P_{ik}, & i = j \end{cases}$$

Motter, Myers, Anghel, Nishikawa, Nature Phys. 2013

Control by Manipulating Stability

Synchronization in power grid networks



Motter, Myers, Anghel, Nishikawa, Nature Phys. 2013

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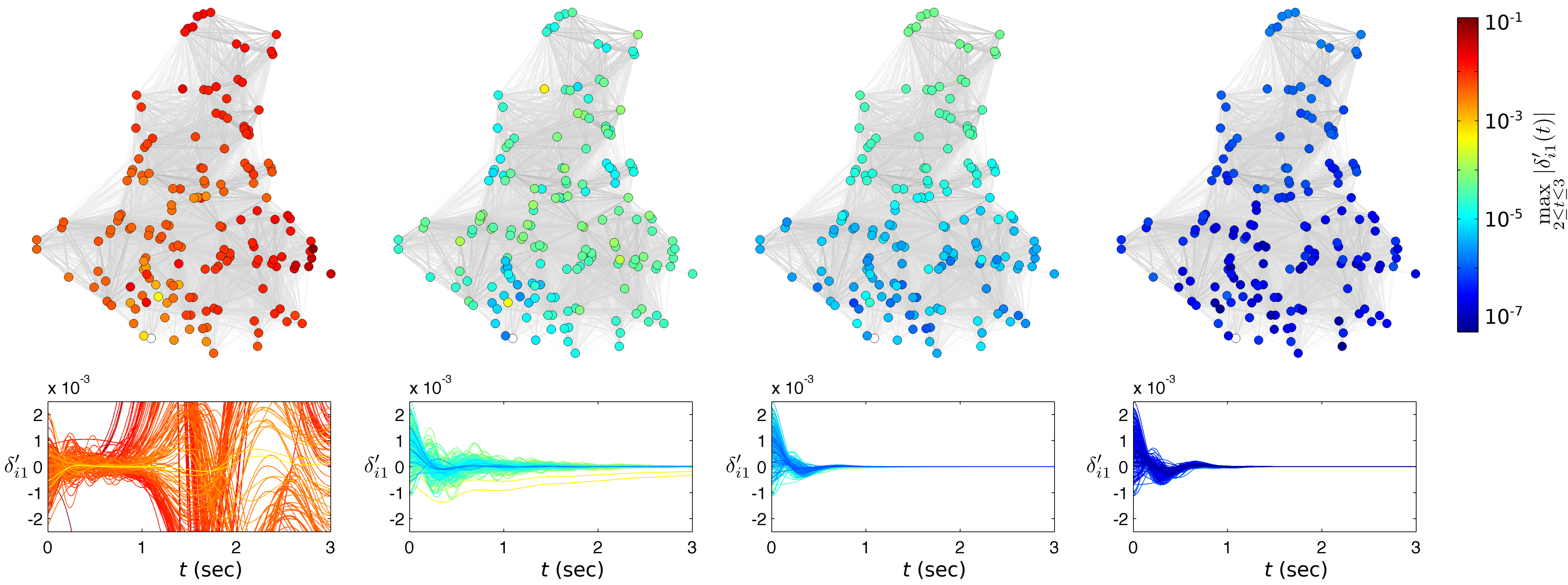
Synchronization in power grid networks



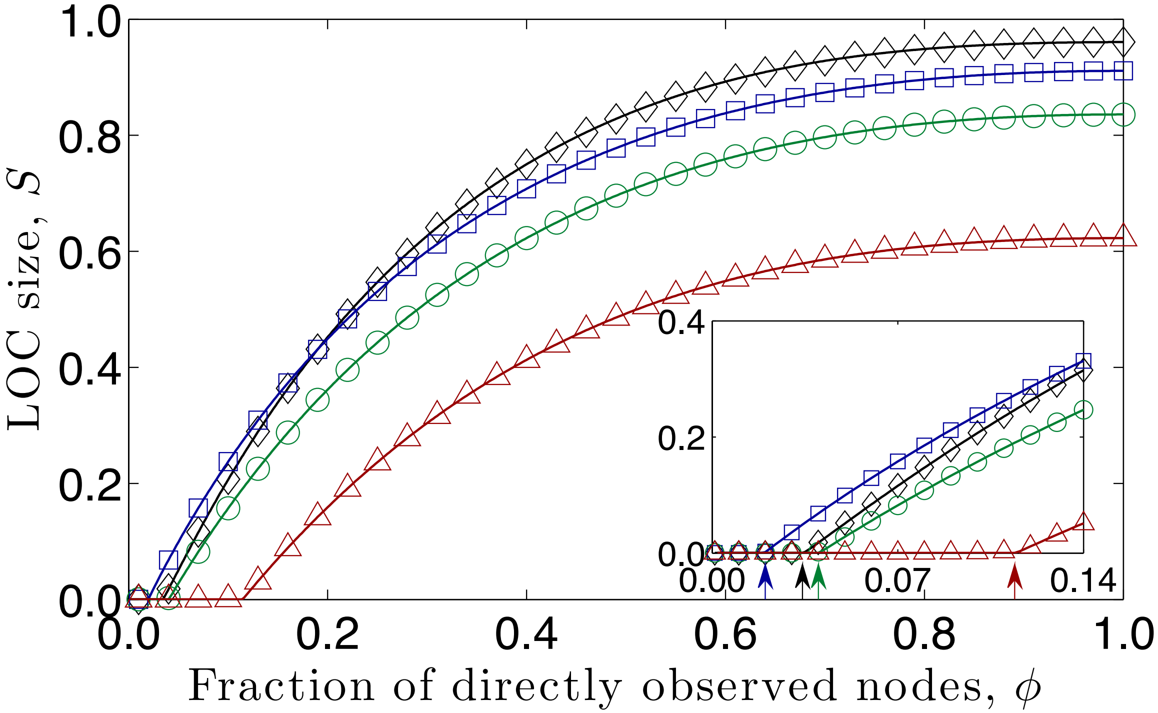
Motter, Myers, Anghel, Nishikawa, Nature Phys. 2013

Network Observability Transitions

Example: phasor measurement units in power grids

will allow real time wide area monitoring

new type of percolation transition



Yang, Wang, Motter, PRL 2012

Outlook

1. Network response to perturbations can be largely controlled by *eligible* interventions
2. The control interventions can be identified without any *a priori* information about the basins of attraction
3. These interventions can be effective even when the target stable state *cannot* be reached directly
4. It is often the case that globally advantageous interventions are (apparently or in fact) *locally deleterious*
5. We can take advantage of the *nonlinear* (and possibly *noisy*) nature of the dynamics

For more information, please go to the group's webpage: http://dyn.phys.northwestern.edu/