Santa Fe Institute
2012 Complex Systems Summer School

Introduction to Nonlinear Dynamics

Instructor:

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Syllabus:

1. Introduction; Dynamics of Maps — chs 1 & 10 of [52]
   - a brief tour of nonlinear dynamics — [34] *(in [18])*
   - an extended example: the logistic map
     - how to plot its behavior
     - initial conditions, transients, and fixed points
     - bifurcations and attractors
     - chaos: sensitive dependence on initial conditions, λ, and all that
     - pitchforks, Feigenbaum, and universality — [23] *(in [18])*
     - the connection between chaos and fractals — [24], ch 11 of [52]
     - period-3, chaos, and the u-sequence — [33, 36] *(latter is in [18])*
     - *maybe:* unstable periodic orbits — [3, 26, 51]
2. Dynamics of Flows

   [52], sections 2.0-2.3, 2.8, 5, and 6 (except 6.6 and 6.8)

   - maps vs. flows
     - time: discrete vs. continuous
     - axes: state/phase space — [10]
   - an example: the simple harmonic oscillator
     - some math & physics review — [9]
     - portraying & visualizing the dynamics — [10]
   - trajectories, attractors, basins, and boundaries — [10]
   - dissipation and attractors — [44]
   - bifurcations

- how sensitive dependence and the Lyapunov exponent manifest in flows
- anatomy of a prototypical chaotic attractor: [24]
  - stretching/folding and the un/stable manifolds
  - fractal structure and the fractal dimension ch 11 of [52]
  - unstable periodic orbits [3, 26, 51]
  - shadowing
  - *maybe:* symbol dynamics [27] *(in [14])*; [29]
- *Lab:* (Joshua Garland) the logistic map and the driven pendulum

3. Tools [2, 10, 39, 42]

- ODE solvers and their dynamics [9, 35, 37, 46]
- Poincaré sections [28]
- stability, eigenstuff, un/stable manifolds and a bit of control theory
- embedology [30, 31, 32, 41, 48, 49, 47, 54] *([41] is in [39] and [47] is in [55];)*
- *Lab:* (Joshua Garland) nonlinear time series analysis with the TISEAN package[1, 32].

4. Applications [14, 39, 40]

- prediction [4, 5, 6, 15, 16, 55]
- filtering [21, 22, 25]
- control [8, 7, 12, 38, 50] *([38] is in [39])*
- communication [17, 43]
- classical mechanics [11, 45, 53, 56, 57]
- music, dance, and image [13, 19, 20]

References

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References [2, 4, 5, 14, 16, 18, 29, 39, 52, 55] above are in the CSSS library.

More Resources:

`www.cs.colorado.edu/~lizb/chaos-course.html`

`amath.colorado.edu/faculty/jdm/faq.html`

`www.mpipks-dresden.mpg.de/~tisean`