Anatomy of an Observation: A Rope of Sand

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Setting the Scene

$$\cdots \quad X_{-3} \; X_{-2} \; X_{-1} \quad X_0 \quad X_1 \quad X_2 \quad X_3 \quad \cdots$$

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Setting the Scene

$$X_0$$

$$\cdots \quad X_{-3} \quad X_{-2} \quad X_{-1} \quad X_0 \quad X_1 \quad X_2 \quad X_3 \quad \cdots$$

Setting the Scene

$X_{:0}$ X_0

$\cdots \; X_{-3} \; X_{-2} \; X_{-1} \; X_0 \; X_1 \; X_2 \; X_3 \; \cdots$

Setting the Scene

$X_{:0}$ $\quad X_0 \quad$ $X_{1:}$

$\cdots \; X_{-3} \; X_{-2} \; X_{-1} \; X_0 \; X_1 \; X_2 \; X_3 \; \cdots$

Setting the Scene

$X_{:0}$ X_0 $X_{1:}$

$$\cdots \; X_{-3} \; X_{-2} \; X_{-1} \; X_0 \; X_1 \; X_2 \; X_3 \; \cdots$$

We limit ourselves to timeseries which are:

Setting the Scene

$$X_{:0} \qquad X_0 \qquad X_{1:}$$

$$\cdots \; X_{-3} \; X_{-2} \; X_{-1} \; X_0 \; X_1 \; X_2 \; X_3 \; \cdots$$

We limit ourselves to timeseries which are:

- Ergodic

Setting the Scene

$X_{:0}$ X_0 $X_{1:}$

$\cdots \; X_{-3} \; X_{-2} \; X_{-1} \; X_0 \; X_1 \; X_2 \; X_3 \; \cdots$

We limit ourselves to timeseries which are:

- Ergodic
- Stationary

Setting the Scene

$X_{:0}$ $\quad$ X_0 $\quad$ $X_{1:}$

$$\cdots \; X_{-3} \; X_{-2} \; X_{-1} \; X_0 \; X_1 \; X_2 \; X_3 \; \cdots$$

We limit ourselves to timeseries which are:

- Ergodic
- Stationary
- Discrete

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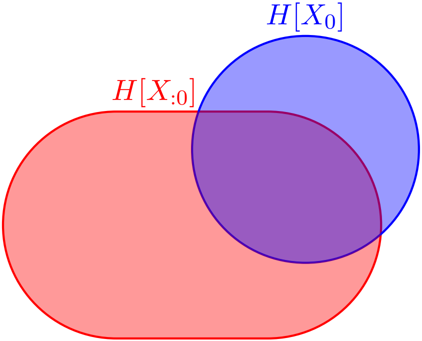
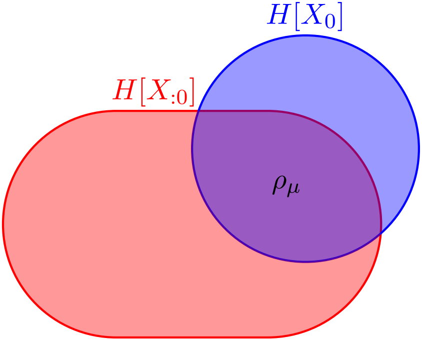
Past and Present

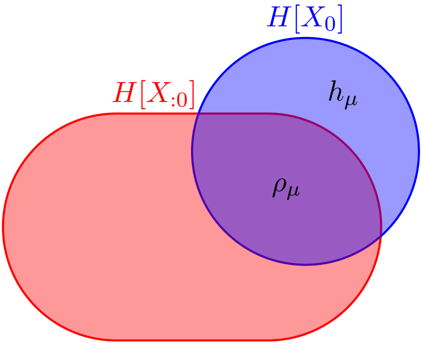


Past and Present



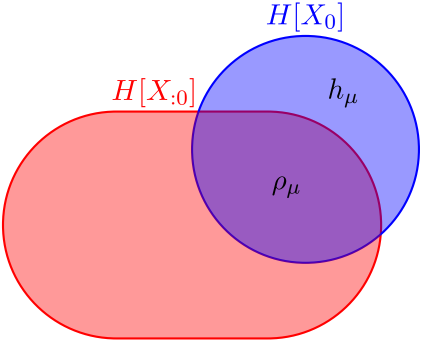
- $\rho_\mu = I[X_{:0}; X_0]$: redundant information

Past and Present



- $\rho_\mu = I[X_{:0}; X_0]$: redundant information
- $h_\mu = H[X_0 | X_{:0}]$: unanticipated information

Past, Present and Future



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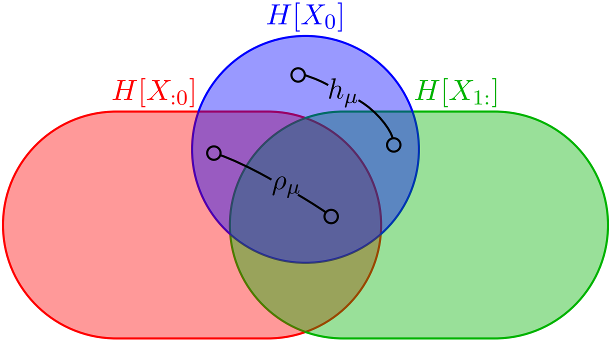
Past, Present and Future



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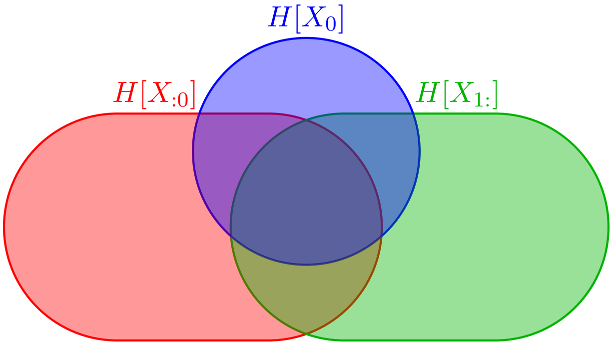
Past, Present and Future



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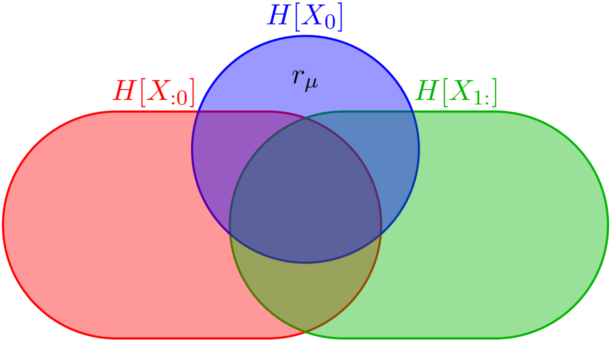
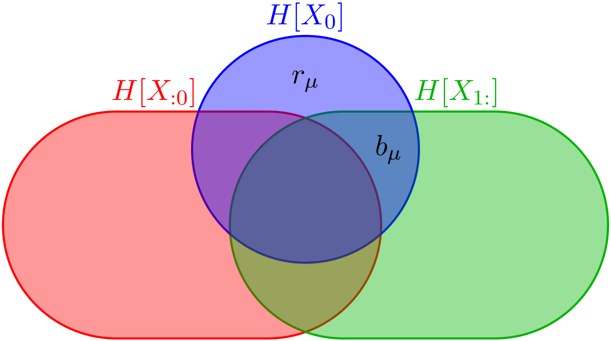
Past, Present and Future



- $r_\mu = H[X_0|X_{:0}, X_{1:}]$: ephemeral information

Past, Present and Future



- $r_\mu = H[X_0 | X_{:0}, X_{1:}]$: ephemeral information
- $b_\mu = I[X_0; X_{1:} | X_{:0}]$: structural information

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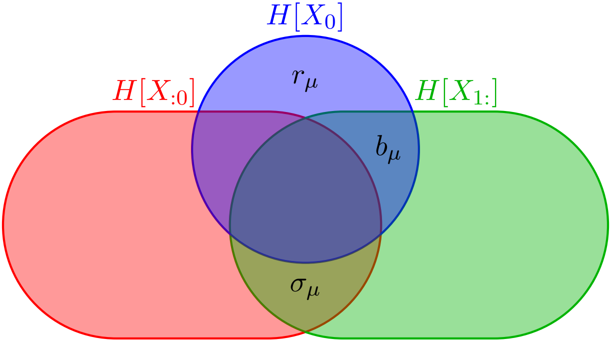
Past, Present and Future



- $r_\mu = H[X_0|X_{:0}, X_{1:}]$: ephemeral information
- $b_\mu = I[X_0; X_{1:}|X_{:0}]$: structural information
- $\sigma_\mu = I[X_{:0}; X_{1:}|X_0]$: evidence of internal states

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Entropy Rate: h_μ

$$H(\ell) = H[X_{0:\ell}]$$

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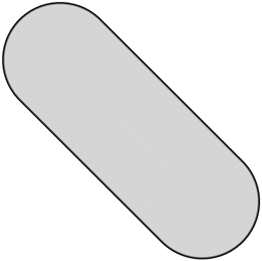
$$H(\ell) = H[X_{0:\ell}]$$



$H[X_0]$

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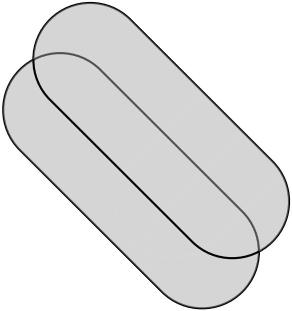
Entropy Rate: h_μ

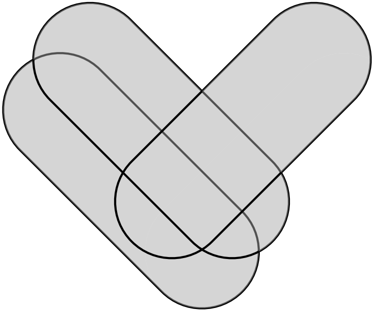
$$H(\ell) = H[X_{0:\ell}]$$



$$H[X_0, X_1]$$

Entropy Rate: h_μ

$$H(\ell) = H[X_{0:\ell}]$$



$$H[X_0, X_1, X_2]$$

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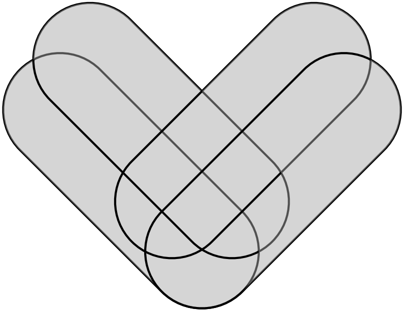
Entropy Rate: h_μ

$$H(\ell) = H[X_{0:\ell}]$$



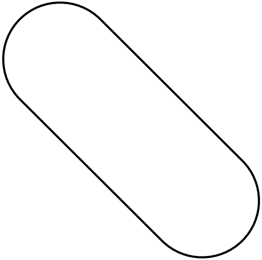
$$H[X_0, X_1, X_2, X_3]$$

Total Correlation Rate: ρ_μ

$$T(\ell) = \sum_{i=0}^{\ell} H[X_i] - H[X_{0:\ell}]$$

Total Correlation Rate: ρ_μ

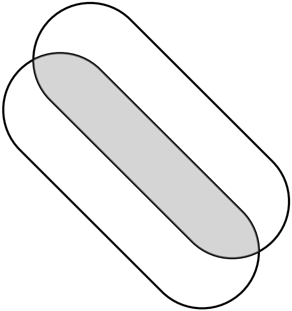
$$T(\ell) = \sum_{i=0}^{\ell} H[X_i] - H[X_{0:\ell}]$$



$$H[X_0] - H[X_0]$$

Total Correlation Rate: ρ_μ

$$T(\ell) = \sum_{i=0}^{\ell} H[X_i] - H[X_{0:\ell}]$$



$$H[X_0] + H[X_1] - H[X_0, X_1]$$

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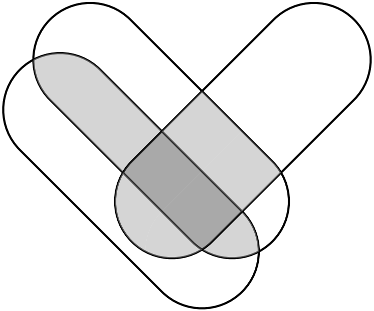
Total Correlation Rate: ρ_μ

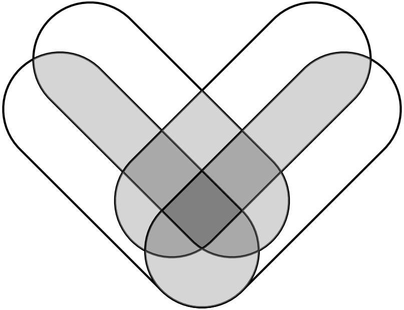
$$T(\ell) = \sum_{i=0}^{\ell} H[X_i] - H[X_{0:\ell}]$$



$$H[X_0] + H[X_1] + H[X_2] - H[X_0, X_1, X_2]$$

Total Correlation Rate: ρ_μ

$$T(\ell) = \sum_{i=0}^{\ell} H[X_i] - H[X_{0:\ell}]$$



$$H[X_0] + H[X_1] + H[X_2] + H[X_3] - H[X_0, X_1, X_2, X_3]$$

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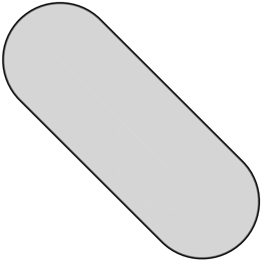
Residual Entropy Rate: r_μ

$$R(\ell) = \sum_{i=0}^{\ell} H[X_i | X_{0:\ell \setminus i}]$$

Residual Entropy Rate: r_μ

$$R(\ell) = \sum_{i=0}^{\ell} H[X_i | X_{0:\ell \setminus i}]$$



$H[X_0]$

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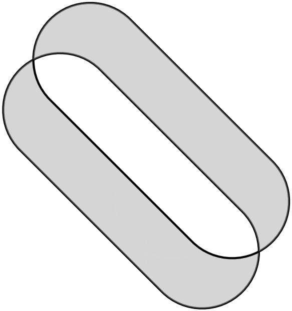
Residual Entropy Rate: r_μ

$$R(\ell) = \sum_{i=0}^{\ell} H[X_i | X_{0:\ell \setminus i}]$$



$$H[X_0|X_1] + H[X_1|X_0]$$

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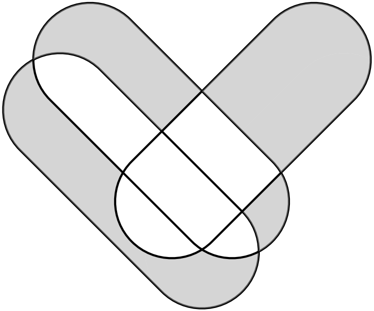
Residual Entropy Rate: r_μ

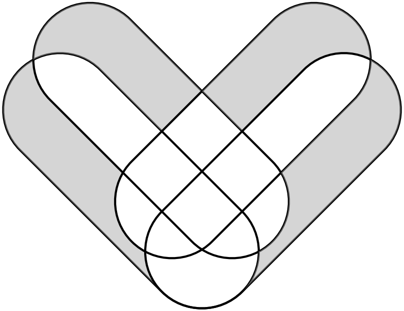
$$R(\ell) = \sum_{i=0}^{\ell} H[X_i | X_{0:\ell \setminus i}]$$



$$H[X_0|X_1, X_2] + H[X_1|X_0, X_2] + H[X_2|X_0, X_1]$$

Residual Entropy Rate: r_μ

$$R(\ell) = \sum_{i=0}^{\ell} H[X_i | X_{0:\ell \setminus i}]$$



$$H[X_0|X_1, X_2, X_3] + H[X_1|X_0, X_2, X_3] + H[X_2|X_0, X_1, X_3] + H[X_3|X_0, X_1, X_2]$$

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Binding Information Rate: b_μ

$$B(\ell) = H[X_{0:\ell}] - \sum_{i=0}^{\ell} H[X_i | X_{0:\ell \setminus i}]$$

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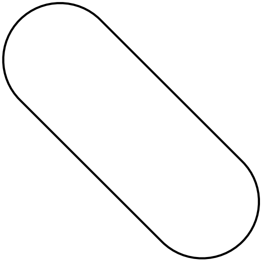
Binding Information Rate: b_μ

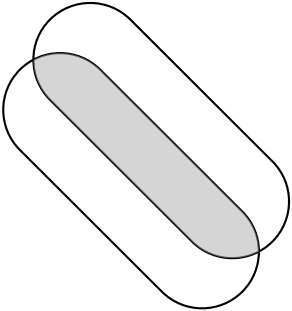
$$B(\ell) = H[X_{0:\ell}] - \sum_{i=0}^{\ell} H[X_i | X_{0:\ell \setminus i}]$$



$$H[X_0] - H[X_0]$$

Binding Information Rate: b_μ

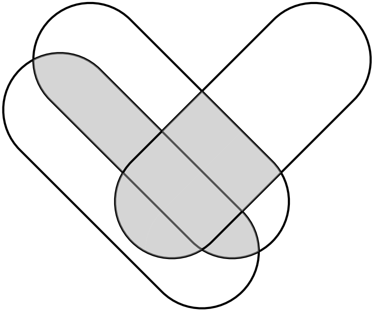
$$B(\ell) = H[X_{0:\ell}] - \sum_{i=0}^{\ell} H[X_i | X_{0:\ell \setminus i}]$$



$$H[X_0, X_1] - H[X_0|X_1] - H[X_1|X_0]$$

Binding Information Rate: b_μ

$$B(\ell) = H[X_{0:\ell}] - \sum_{i=0}^{\ell} H[X_i | X_{0:\ell \setminus i}]$$



$$H[X_0, X_1, X_2] - H[X_0 | X_1, X_2] - H[X_1 | X_0, X_2] - H[X_2 | X_0, X_1]$$

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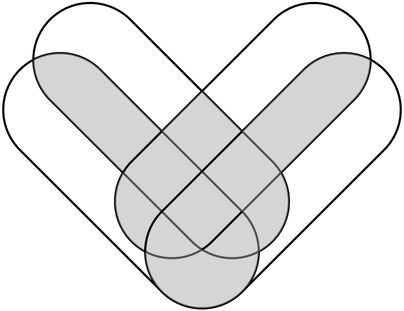
Binding Information Rate: b_μ

$$B(\ell) = H[X_{0:\ell}] - \sum_{i=0}^{\ell} H[X_i | X_{0:\ell \setminus i}]$$



$$H[X_0, X_1, X_2, X_3] - H[X_0|X_1, X_2, X_3] - H[X_1|X_0, X_2, X_3] - H[X_2|X_0, X_1, X_3] - H[X_3|X_0, X_1, X_2]$$