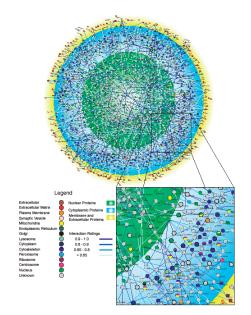
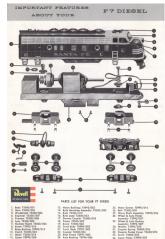
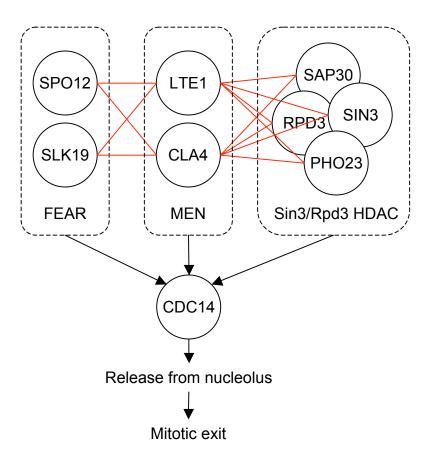
# Finding meaning in biological networks

#### Physical interaction network





#### Genetic interaction network



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Dec 3-5, 2008

### Main thanks

- Yan Qi (ABD): search on enemy networks
- Yongjin Park (CMU MS): extensions to CM, variational Bayes restaurant process
- Scott Patterson (stealth-mode biotech): degreecorrected stochastic block models



# The application PowerPoint quit unexpectedly.

Mac OS X and other applications are not affected.

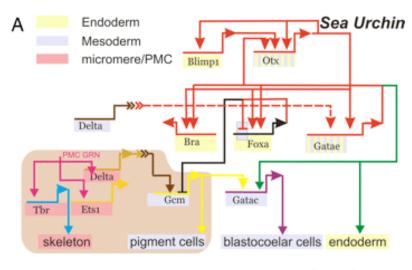
Click Reopen to open the application again. Click Report to see more details or send a report to Apple.

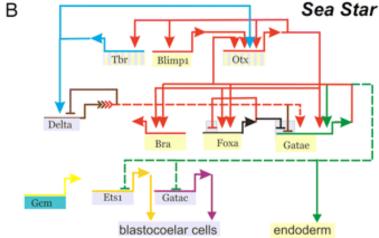
Close

Report...

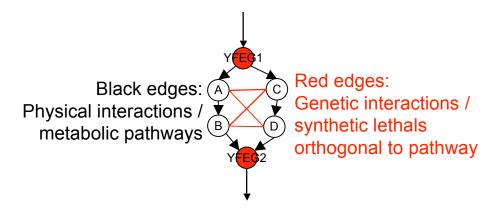
Reopen

# Gene regulatory vs. Genetic interaction





Physical reality: TF-DNA binding



Logical structure of network

Physical reality: phenotype of mutant

Relevant to network failure:

Yeast viability

Human disease

Communication failure

Current use: logical structure of yeast biological pathways

6000 genes

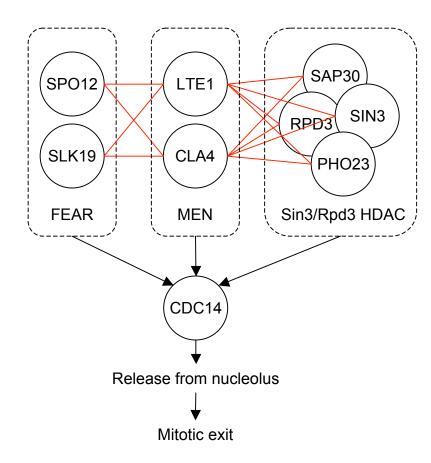
1000 essential

5000 non-essential, test all 12.5 M pairs

Higher-order combos: ask me

Leading groups at Toronto, UCSF, JHU

## Dual prediction problems



- Given red edges, predict genes/proteins in the same module.
   Share many red-edge neighbors Enriched for paths of length 2
- Given incomplete / noisy set of tested red edges, predict untested red edges (Similar to predator/parasite-prey prediction) Enriched for paths of length 3
  No path of length 1

Haven't been using knowledge of tested/untested Red edges are sparse (50/5000 = 1%)

Similar to spin-spin correlation in antiferromagnetic Ising lattice with network topology

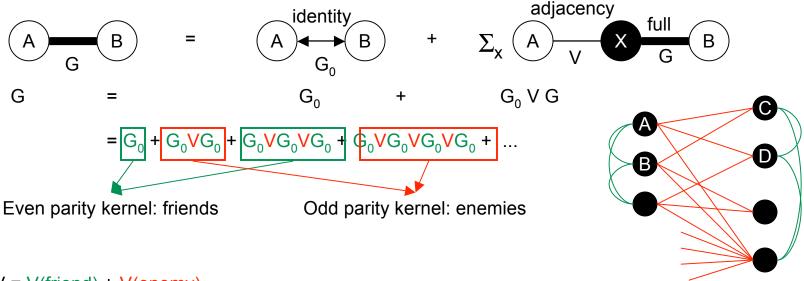
- + correlation = same module
- correlation = probable red edge

Linearized mean field theory = linear response theory = ...

# Systematic path sums = Graph diffusion kernel

Recall Ornstein Zernicke / Poisson Boltzman:  $h = c + c \rho h$ ,  $c \sim -\beta u$ Recall path integrals:  $U(t) = \exp(H t) = \exp[(H_0 + V)t] = [I + (H_0 + V)t/n]^n$ kernel betw. A & B = (some constant if A = B) + (direct interactions all of its neighbors)

(direct interactions betw. A & all of its neighbors) times (kernel betw. neighbor & B)



V = V(friend) + V(enemy)

Early integration:  $G = [I - V(friend) - V(enemy)]^{-1}$ 

Late integration:  $G = [I - V(friend)]^{-1} + [I - V(enemy)^2]^{-1}$ 

Bad: scambles friends, enemies

Better, like naive Bayes

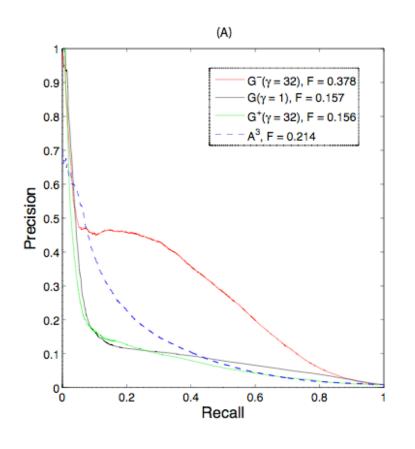
But what about predicting enemies?

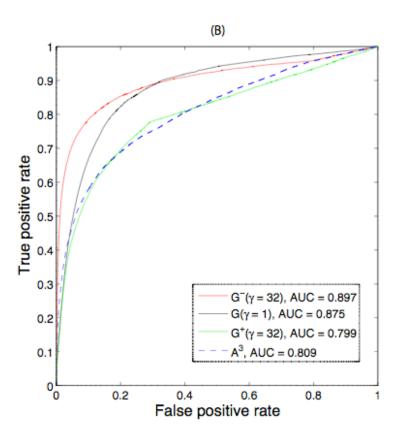
Renormalization:  $G_0 = [I - V(friend)]^{-1}$ 

 $G(friend) = [I - G_0 V(enemy) G_0 V(enemy)]^{-1}G_0$ 

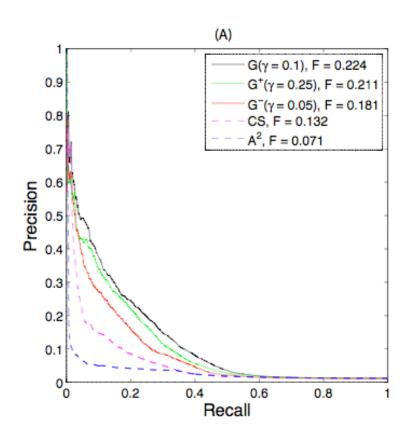
 $G(enemy) = [I - \mathring{G}_0 V(enemy) \mathring{G}_0 V(enemy)]^{-1} \mathring{G}_0 V(enemy) G_0$ 

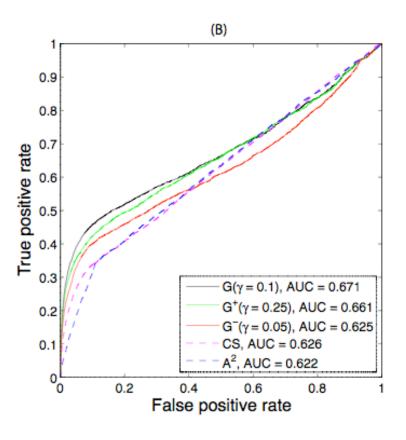
# Best method to date for predicting new edges





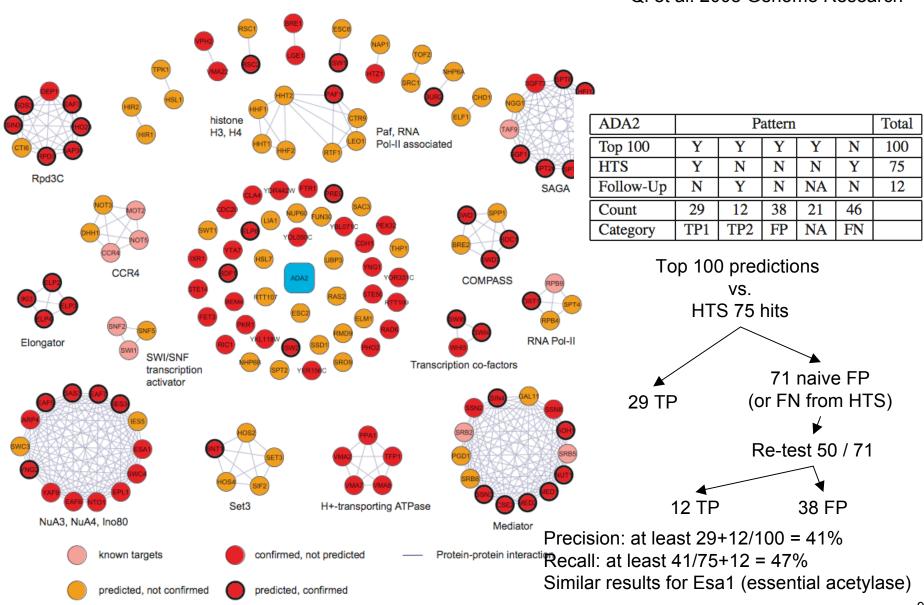
# Also good for dual problem (communities)



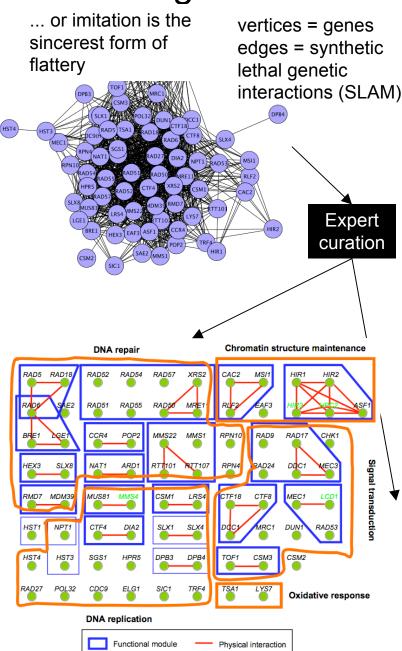


## Directed experimental search

Qi et al. 2008 Genome Research



# Adding to Clauset-Moore-Newman



Our Karate club:

SL screen of ~100 genes involved in mainting DNA integrity (DNA damage sensing and repair) (Pan et al. 2006 Cell) Network analyzed by a biological expert, segmented.

Can a clustering algorithm reproduce the truth (or was the expert wrong?)

Multiple edge types: SL (logical/emotional) PPI (physical/location)

|                | Components of   | Congruency <sup>b</sup> of | SFL within   | Protein-Protei                        |
|----------------|---|----------------------------|--|---------------------------------------|
| Name of Module | Module <sup>a</sup>   | SL Profiles                | Module <sup>c</sup>                                  | Interactiond                          |
| BRE1 <u>m</u>  | RAD6, BRE1, LGE1  | 22-109                     | No   | Yes                                   |
| CAF-I          | CAC2, MSI1, RLF2  | 33-34                      | No   | Yes                                   |
| CCR4m          | CCR4, POP2  | 133                        | No   | Yes                                   |
| CSM1m          | CSM1, LRS4  | 65                         | No   | Yes                                   |
| CTF18m         | CTF18, CTF8, DCC1   | 129-144                    | No   | Yes                                   |
| HEX3 <u>m</u>  | HEX3, SLX8  | 26                         | No   | Yes                                   |
| HIR            | ASF1, HIR1, HIR2,<br>HIR3, HPC2                             | 13–40                      | No   | Yes                                   |
| HR             | RAD50, MRE11, XRS2,<br>RAD51, RAD52,<br>RAD54, RAD55, RAD57 | 55-116                     | No   | Yes<br>(only among F<br>Mre11p, and I |
| MEC1m          | MEC1, LCD1, RAD53   | 10                         | No   | Yes (only between Mec1p and L         |
| MMS22 <u>m</u> | MMS22, MMS1,<br>RTT101, RTT107                              | 20–28                      | Yes <sup>f</sup> (only between<br>RTT101 and RTT107) | Yes<br>(only among I<br>Rtt101p, and  |
| MUS81 <u>m</u> | MMS4, MUS81   | 6°                         | No   | Yes                                   |
| NAT1 <u>m</u>  | NAT1, ARD1  | 184                        | No   | Yes                                   |
| PRR            | RAD6, RAD5, RAD18   | 19-50                      | No   | Yes                                   |
| RAD9m          | RAD9, DDC1, RAD17,<br>MEC3, RAD24                           | 27–38                      | No   | Yes<br>(only among I<br>Rad17p, and   |
| RMD7m          | RMD7, MDM39   | 187                        | No   | Yes                                   |
| TOF1m          | TOF1, CSM3  | 78                         | No   | Yes                                   |

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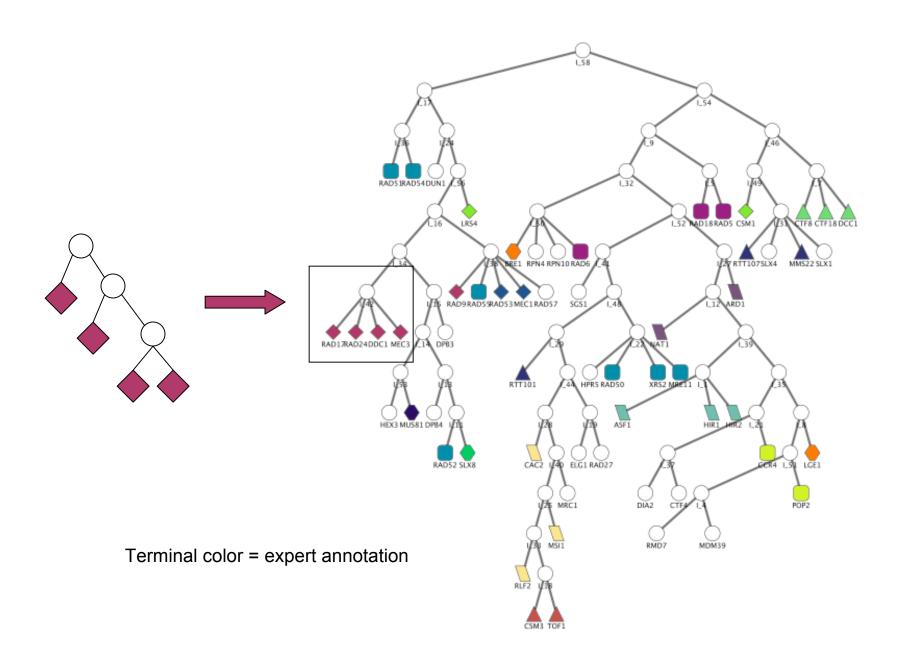
# Modification #1: Fully Bayes

$$\Pr(D \mid \{\rho_r\}) = \prod_{r \in D} \rho_r^{E_r} (1 - \rho_r)^{L_r R_r - E_r}$$

$$\Pr(\{\rho_r\} \mid a, b) = \prod_{r \in D} \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \rho_r^{a-1} (1 - \rho_r)^{b-1}$$

$$\Pr(D \mid a, b) = \left(\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}\right)^{|D|} \prod_{r \in D} \left[\frac{\Gamma(E_r + a)\Gamma(L_r R_r - E_r + b)}{\Gamma(L_r R_r + a + b)}\right]$$

# Model Selection: Left/Right vs. Center



# Modification #2: Left/Right vs. Center

$$\Pr(D \mid \{\rho_r\}) = \prod_{r \in D} \rho_r^{E_r} (1 - \rho_r)^{C(n,2) - E_r}$$

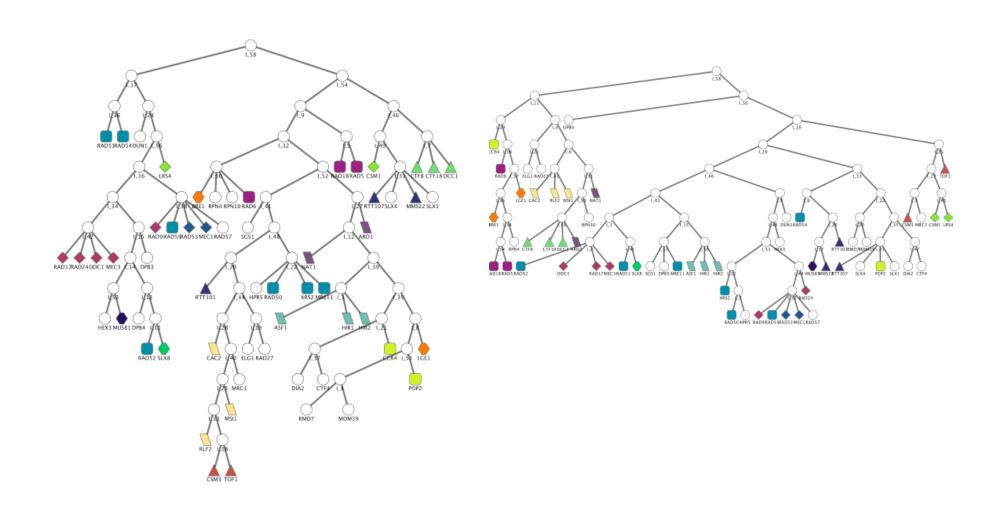
$$\Pr(\{\rho_r\} \mid a, b) = \prod_{r \in D} \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \rho_r^{a-1} (1 - \rho_r)^{b-1}$$

$$\Pr(D \mid a, b) = \left(\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}\right)^{D} \prod_{r \in D} \left[\frac{\Gamma(E_r + a)\Gamma(C(n,2) - E_r + b)}{\Gamma(C(n,2) + a + b)}\right]$$

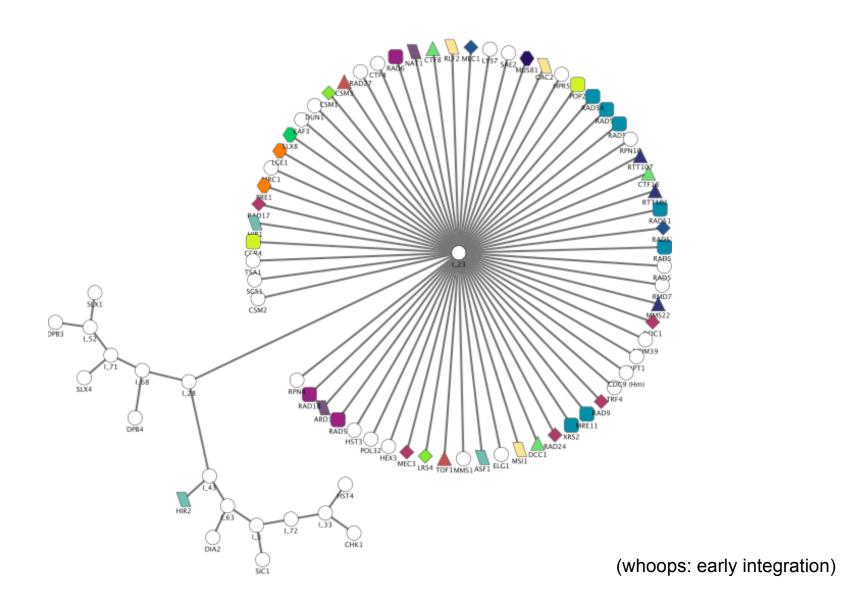
# Modification #3: Multiple edge types

Just PPI/Physical/Location

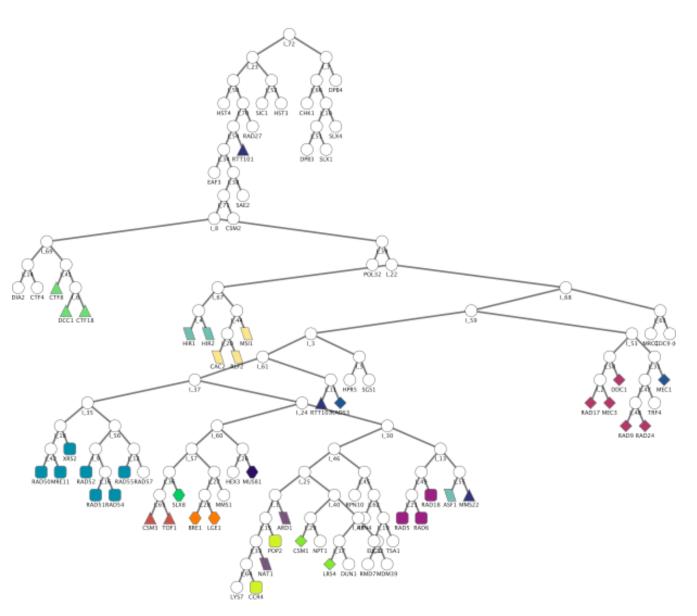
Just SL/logical/social preference



# Both



# **Both**

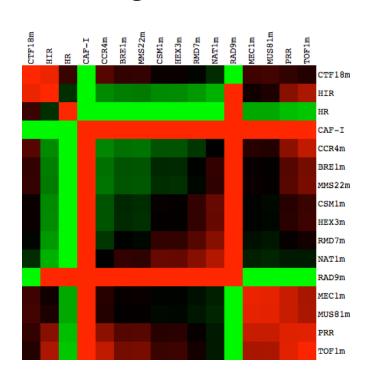


White terminal nodes: unannotated by expert

Problem: long run time

Fix with LR/C collapsing? Have to implement detailed balance

# Degree-corrected block models



Problems to address:

Long branch attraction High-degree vertices grouped together

Multiple edge types

Model selection

How many clusters? Form of prob. distribution?

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Color = edge probability within/between blocks
Red = depleted
Green = enriched

**Parameters** 

# of groups:

fixed K

sum over all K (restaurant process, Hofman/Wiggins, Jordan)

block-block probabilities:

2 parameters (within/between)

K(K+1)/2 parameters (each unique block pair)

# Trial #1: Newman's asymmetric block model

- Parameters are groups x vertices
- 1. For each v in V, sample v's membership according to prior distribution of mixture (e.g., DPM)
- 2. Given v's membership, sample adjacency profile (a column vector) from Multinomial distribution

$$\mathbf{z}_{\mathsf{i}}^{\mathsf{k}} = \left\{ \begin{array}{ll} 1 & v_i \in k \\ 0 & otherwise \end{array} \right.$$

# Asymmetric mixture model

$$\Pr(\overrightarrow{A}_{i} \mid \{\rho_{kj}\}, z_{i}^{k}) = \prod_{k'=1}^{K} \left[\prod_{j=1}^{n} \rho_{k'j}^{A_{ij}}\right]^{z_{i}^{k'}}$$

$$\Pr(\{\rho_{kj}\} \mid \lambda) = \operatorname{Dir}(\{\rho_{kj}\} \mid \lambda \mid n, ..., \lambda \mid n) = \frac{\Gamma(\lambda)}{\Gamma(\lambda \mid n)^{n}} \prod_{j=1}^{n} \rho_{kj}^{(\lambda/n)-1}$$

If "K" is fixed and with a uniform prior, we can derive the marginal as:

$$\Pr(A \mid \{z_i^k\}, \lambda) = \frac{\Gamma(\lambda)}{\Gamma(\lambda/n)^n} \prod_{k=1}^K \left[ \frac{\prod_{j=1}^n \Gamma(\lambda/n + \sum_{i=1}^n A_{ij} \cdot z_i^k)}{\Gamma(\lambda + \sum_{i=1}^n \sum_{j=1}^n A_{ij} \cdot z_i^k)} \right]$$

# CRP for the asymmetric mixture

CRP enables to sample latent membership from Dirichlet Process Mixture by Gibbs sampling

More efficiently, we can simulate the mixture using the collapsed Gibbs sampling (Neal 2000), where parameters are integrated out beforehand.

# CRP: prior distribution

If "k" is a new cluster,

$$\Pr(z_i^{k^*} \mid z_{\neg i}) = \frac{\alpha}{n - 1 + \alpha}$$

where  $\mathcal{Z}_{\neg i}$  denotes all the fixed membership except for the i-th one.

If "k" is one of clusters that already exist,

$$\Pr(z_{i}^{k} \mid z_{\neg i}) = \frac{\mid C_{k} \mid}{n - 1 + \alpha} = \frac{\sum_{j=1}^{n} z_{j}^{k}}{n - 1 + \alpha}$$

CRP: predictive distributions for a collapsed Gibbs sampling

If "k" is a new cluster,

$$\Pr(A_i \mid z_i^k) = \frac{\Gamma(N\lambda)}{\Gamma(\lambda)^N} \frac{\prod_{j=1}^n \Gamma(\lambda + A_{ij})}{\Gamma(\sum_{j=1}^n \lambda + A_{ij})}$$

If "k" is one of clusters that already exist,

$$\Pr(A_{i} \mid z_{i}^{k}, z_{\neg i}, A_{\neg i}) = \frac{\Gamma(N\lambda + \sum_{j=1}^{n} \sum_{l:l \neq i} A_{lj} z_{l}^{k})}{\Gamma(N\lambda + \sum_{j=1}^{n} \sum_{l:l \neq i} A_{lj} z_{l}^{k} + \sum_{j=1}^{n} A_{ij})} \prod_{j=1}^{n} \left[\lambda + \sum_{l:l \neq i} A_{lj} z_{l}^{k}\right]^{A_{ij}}$$

# CRP: collapsed Gibbs sampling

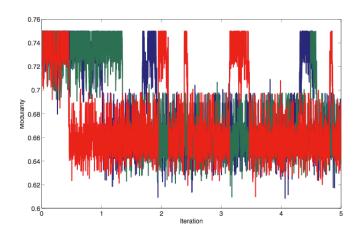
From previous distributions, we can either assign to a new cluster with a probability of

$$\Pr(z_i^{k^*} \mid A_i, \alpha) \propto \Pr(z_i^{k^*} \mid \alpha) \Pr(A_i \mid z_i^{k^*})$$

Or assign to one of existing cluster with a probability of

$$\Pr(z_i^k \mid A_i, z_{\neg i}, A_{\neg i}) \propto \Pr(z_i^k \mid z_{\neg i}, A_{\neg i}) \Pr(A_i \mid z_i^k, z_{\neg i}, A_{\neg i})$$

Non-ergodic?



# DPM: Stick-breaking process

Another realization of DPM is the stick-breaking process (Sethuraman 1994), which provides a more explicit framework for a variational calculation (Blei and Jordan 2006).

## **DPM: Variational inference**

$$Pr(V \mid \alpha) = \prod_{k=1}^{\infty} Beta(v_k \mid 1, \alpha)$$

 $v_k$  is the amount of mass in group k

$$\Pr(z_i \mid V) = \prod_{k=1}^{\infty} v_k^{z_i^k} (1 - v_k)^{\sum_{l>k} z_i^l}$$

$$\Pr(\overrightarrow{A}_i \mid \{\rho_k\}, z_i^k) = \prod_{k=1}^{\infty} \left[ \prod_{j=1}^n \rho_k^{A_{ij}} \right]^{z_i^k}$$

$$Pr(\rho_k \mid \lambda) = Dir(\rho_k \mid (\lambda \mid n, ..., \lambda \mid n))$$

$$q(V, Z, \rho) = \prod_{k=1}^{K_{Tr}-1} \text{Beta}(v_k \mid \gamma_{k1}, \gamma_{k2}) \prod_{k=1}^{K_{Tr}} \text{Dir}(\rho_k \mid \tau_k) \prod_{i=1}^n \text{Mult}(z_i \mid 1, \phi_i)$$

# **DPM**: Variational updates

1.

$$\langle \ln(v_i) \rangle = \psi(\gamma_{k1}) - \psi(\gamma_{k1} + \gamma_{k2})$$

$$\gamma_{k1} = 1 + \sum_{i=1}^{n} \langle z_i^k \rangle$$

$$\langle \ln(1 - v_i) \rangle = \psi(\gamma_{k2}) - \psi(\gamma_{k1} + \gamma_{k2})$$

$$\gamma_{k2} = \alpha + \sum_{i=1}^{n} \sum_{l>k}^{K_{Tr}} \langle z_i^l \rangle$$

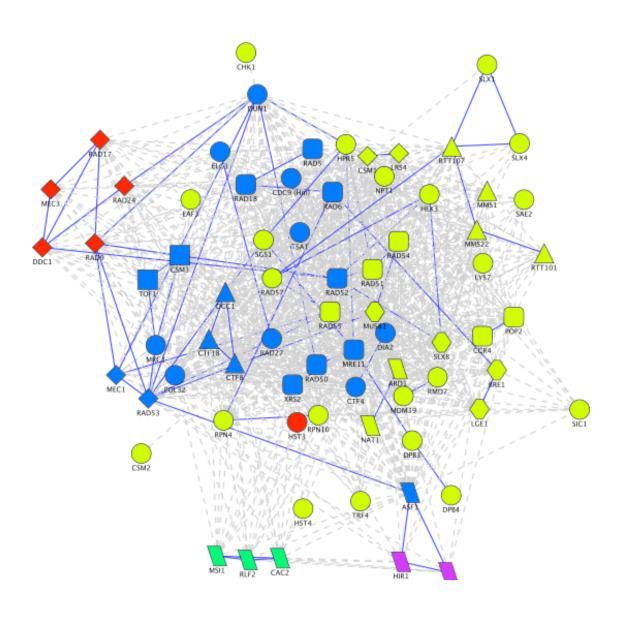
2.

$$\langle \ln(\rho_{kj}) \rangle = \psi(\tau_{kj}) - \psi(\sum_{j=1}^{n} \tau_{kj}) \qquad \tau_{kj} = \frac{\lambda}{n} + \sum_{i=1}^{n} \langle z_i^k \rangle A_{ij}$$

3.

$$\langle z_{i}^{k} \rangle \propto \frac{\exp\{\langle \ln(v_{k}) \rangle + \sum_{k'=1}^{k-1} \langle \ln(1-v_{k'}) \rangle + \sum_{j=1}^{n} \langle \ln(1-v_{k'}) \rangle + \sum_{j=1}^{n} \langle \ln(\rho_{kj}) \rangle + \sum_{j=1}^{n} \langle \ln(1-v_{k'}) \rangle$$

# Too much lumping



### Trial #2: Boltzmann Machine

Construction of the probability distribution

by symmetric (vertex-vertex) metric: 
$$\Pr(\{w_{ij}\} \mid \{z_i^k\}) \propto \prod_{k}^{K} \exp\left\{\sum_{i < j} w_{ij} z_i^k z_j^k\right\}$$

Extension using DPM prior

$$\Pr(z_i \mid V) = \prod_{k=1}^{\infty} v_k^{z_i^k} (1 - v_k)^{\sum_{l>k} z_i^l}$$

$$\Pr(V \mid \alpha) = \prod_{i=1}^{\infty} \operatorname{Beta}(v_i \mid 1, \alpha)$$

## DPM-BM: Newman-Girvan Modularity

Negative sign for disassortative (SL) edges

$$\sum_{i,j} w_{ij} z_i^k z_j^k = \sum_{ij} (A_{ij} - E_{H_0}[A_{ij}]) z_i^k z_j^k$$

where

$$E_{H_0}[A_{ij}] = \frac{\deg(i)}{|E|} \times \frac{\deg(j)}{|E|} \cdot |E|$$

Newman and Girvan, Phys.Rev.E (2004) Clauset *et al.* Phys.Rev.E (2004)

## DPM-BM: variational inference

$$z \approx \arg\min \frac{1}{T} \langle -\ln \Pr(\{w\} \mid \{z\}) \rangle + \langle \ln \Pr(\{z\}, \{v\} \mid \alpha) \rangle$$
$$\{z\} + \langle \ln q(\{z\} \mid \{\mu\}) \rangle + \langle \ln q(\{v\} \mid \{\gamma\}) \rangle$$

using the following mean-field distribution

$$q(V,Z) = \prod_{k=1}^{K_{Tr}-1} \text{Beta}(v_k \mid \gamma_{k1}, \gamma_{k2}) \prod_{i=1}^{n} \prod_{k=1}^{K_{Tr}} \mu_{ik}^{z_i^k}$$

# DPM-BM: variational updates

1.

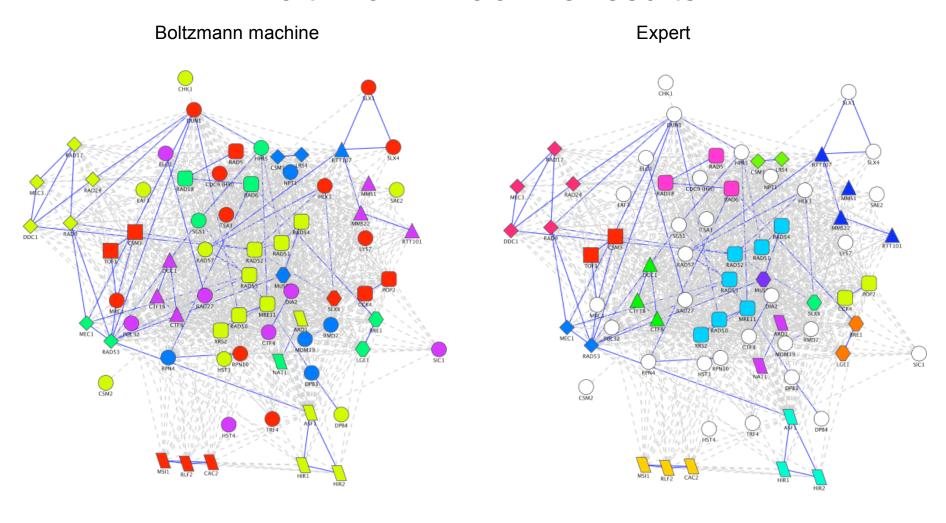
$$\langle \ln(v_i) \rangle = \psi(\gamma_{k1}) - \psi(\gamma_{k1} + \gamma_{k2}) \qquad \gamma_{k1} = 1 + \sum_{i=1}^{n} \langle z_i^k \rangle$$

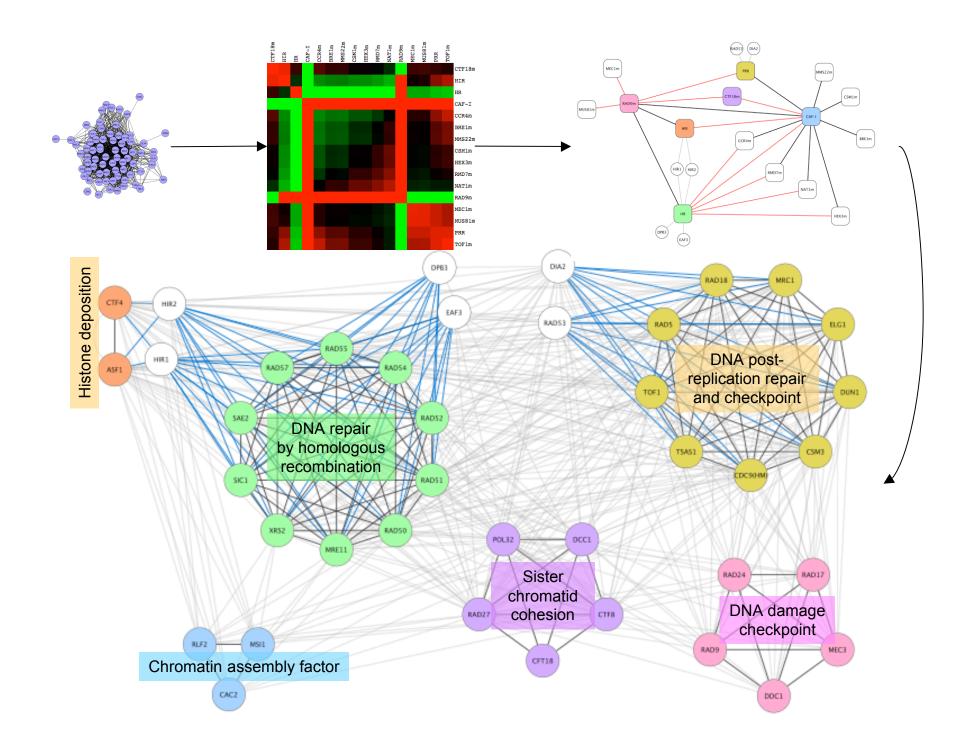
$$\langle \ln(1 - v_i) \rangle = \psi(\gamma_{k2}) - \psi(\gamma_{k1} + \gamma_{k2}) \qquad \gamma_{k2} = \alpha + \sum_{i=1}^{n} \sum_{l>k}^{K_{Tr}} \langle z_i^l \rangle$$

2.

$$\langle z_i^k \rangle = \mu_i^k \propto \exp \left\{ \langle \ln(v_k) \rangle + \sum_{k'=1}^{k-1} \langle \ln(1 - v_{k'}) \rangle + \frac{1}{T} \sum_{j: j \neq i}^n w_{ij} \mu_j^k \right\}$$

# Boltzmann machine results





# Missing nodes and edges

### The Unknown

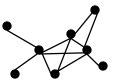
As we know, There are known knowns. There are things we know we know. We also know There are known unknowns. That is to say We know there are some things We do not know. But there are also unknown unknowns, The ones we don't know We don't know.

Donald Rumsfeld, Feb. 12, 2002, Department of Defense news briefing

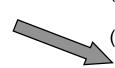
# Capture-Recapture for Networks Edges

Sample edges with replacement

True Network (relevant connections)



Draw from true network (true positive)  $Prob = 1 - \alpha$ 



Complete Graph (red herrings)



(Could improve, have to start somewhere)

Draw spurious edge (false-positive) Prob =  $\alpha$ 

### Can we estimate k and f from {n, w, s}?

Chicken-and-egg problem:

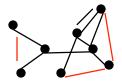
If we knew parameters, we could estimate hiddens. If we knew hiddens, we could fit parameters.

Solution: Expectation Maximization

The Problem:

Given a sampled network of strong and weak edges for a graph with an unknown strong-edge degree distribution

- (1) Estimate the number of strong edges we've missed
  - Estimate the probability that an edge observed once is strong



### Observed variables

n = total number of draws (12)

w = number unique (11)

s = number of singletons (10)

#### Hidden variables

k = number of true edges (11)

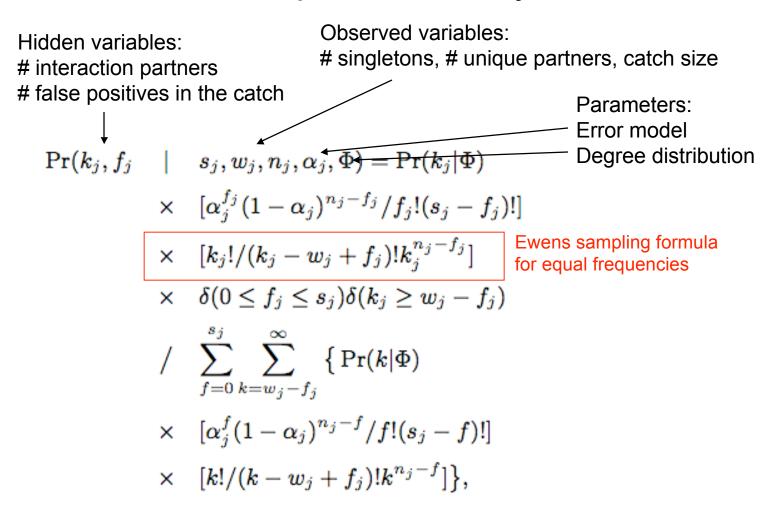
f = number of FP's (3)

#### <u>Parameters</u>

 $\alpha$  = FP rate (expect f ~  $\alpha$ n)

Pr(k) (or uniform)

# **Expectation: Bayes Rule**



Huang, Jedynak, Bader, PLoS Comp Bio 2007 Improved method: Beta distribution for strong/weak edge mixing parameter (in review)

|                                      | Yeast   | Worm      | Fly     |                                |  |
|--------------------------------------|---------|-----------|---------|--------------------------------|--|
| Screen properties                    |         |           |         |                                |  |
| Total # proteins                     | 6,697   | 20,069    | 14,086  |                                |  |
| Total # baits                        | 1,532   | 729       | 3,639   |                                |  |
| Total # preys                        | 2,520   | 2,116     | 5,479   |                                |  |
| Total # used as bait and as prey     | 772     | 212       | 2,109   |                                |  |
| Fraction screened per bait           | 0.376   | 0.105     | 0.389   |                                |  |
| Fraction screened overall            | 0.086   | 0.004     | 0.100   | Only 5 to 10% of pairs tested  |  |
| False-pos. rates                     |         |           |         |                                |  |
| Per prey $(\overline{\alpha})$       | 0.093   | 0.122     | 0.157   |                                |  |
| Per unique interaction               | 0.24    | 0.44      | 0.41    |                                |  |
| Per singleton interaction            | 0.36    | 0.66      | 0.65    |                                |  |
| True-pos. rates                      |         |           |         |                                |  |
| Systematic ( $p_{\text{syst}}$ )     | 0.31(2) | 0.45(4)   | 0.15(1) |                                |  |
| Sampling ( $p_{\text{samp}}$ )       | 0.47    | 0.53      | 0.67    | 0.0                            |  |
| Total                                | 0.15    | 0.24      | 0.10    | Of these,                      |  |
| Mean # partner s                     |         |           |         | only 10 to 20% of TPs captured |  |
| Unique preys per bait, full          | 3.0     | 5.6       | 5.7     |                                |  |
| Unique preys per bait, core          | 1.8     | 4.3       | 1.8     |                                |  |
| Corrected for false positives        | 2.3     | 3.1       | 3.4     |                                |  |
| and sampling loss                    | 4.8     | 5.9       | 5.0     |                                |  |
| and systematic loss                  | 15.4    | 13.1      | 33.9    |                                |  |
| and fraction screened                | 40.8    | 124.4     | 87.0    |                                |  |
| Median # partner s                   |         |           |         |                                |  |
| Corrected for FP's and sampling loss | 1.0     | 2.9       | 2.7     |                                |  |
| and systematic loss                  | 3.3     | 6.4       | 18      |                                |  |
| and fraction screened                | 8.8     | 61        | 46      |                                |  |
| Total # protein interactions         |         |           |         | Huang Jadynak Rador            |  |
| based on mean                        | 137,000 | 1,250,000 | 613,000 | Huang, Jedynak, Bader          |  |
| based on median                      | 30,000  | 610,000   | 325,000 | 2007 PLoS Comp Bio             |  |

# JHU CS Faculty Search

#### **Faculty Applications**

The Department of Computer Science at Johns Hopkins University is seeking applications for a tenure-track faculty position. Our primary interest is hiring at the Assistant Professor level, but candidates of all ranks will be considered. All areas will be considered, but candidates with research agendas in **security**, applied algorithms, computer systems, or **bioinformatics** will receive special attention. All applicants must have a Ph.D. in computer science or a related field and are expected to show evidence of ability to establish a strong, independent, multidisciplinary, internationally recognized research program.

Commitment to quality teaching at the undergraduate and graduate levels will be required of all candidates considered. The department webpage at <a href="http://www.cs.jhu.edu">http://www.cs.jhu.edu</a> provides information about the department, including links to research laboratories and centers.

For full consideration, applicants should apply online before January 5 2009. Questions should be directed to fsearch@cs.jhu.edu. The Department is committed to building a diverse educational environment: women and minorities are strongly encouraged to apply. The Johns Hopkins University is an EEO/AA employer.

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