Leverage, the act of borrowing money for investing, is a powerful force within financial markets. It allows investors to extract large gains from small movements in underlying prices. It can also force losses of equal magnitude. Because those who lend money to others for leverage place this money at risk, these banks and brokerage firms insist that traders maintain a minimum amount of equity, called margin. When the value of an investment drops, traders can exceed the maximum allowed leverage, leading to a margin call from the lender, which forces the trader to unwind positions to keep leverage within allowed levels. This selling effect prices, which can trigger a feedback loop and a downward spiral in prices. Conversely, rising prices increase trader’s equity, allowing them to take ever greater amounts of leverage, driving prices up.

Although this cycle may make sense conceptually, it remains to be fully explored by economists. In a recent paper, Thurner, Farmer, and Geanakoplos [1] presented an agent-based model of leveraged asset purchases with margin calls. Four different types of agents populate their model: noise traders buying and selling randomly around the fundamental value of a stock, hedge funds buying stock when its price is below the fundamental value and holding cash otherwise, investors investing in hedge funds or holding cash otherwise, and lastly a bank lending money to hedge funds at no interest. The bank extends their interest free loans whenever a hedge fund requests it and does not exceed a predetermined maximum leverage, defined as (total assets purchased / wealth of borrower). If fully leveraged hedge funds face falling stock prices, the bank observe that these funds exceed their maximum leverage and makes a margin call on the offending funds. Thurner et al. showed that this mechanism accelerates the decrease of stock prices, forcing previously safe funds into margin calls, as well, thereby increasing the volatility. Independent of any built-in non-standard rational behavior, the price distribution became heavy tailed and displayed clustered volatility. Consequently, system wide risk increases contrary to the intentions of individual leverage requirements.

This model touches upon two different, unusual, strands of research. First, it relies on exclusively agent-based modeling – a technique not widely used within the economics discipline in spite of strong recommendations from various researchers to adopt it more widely [2, 3]. A common critique of agent based models points to the plethora of parameters associated with often ad-hoc behaviors [4]. However, the standard rational or random behaviors are unable to capture a good deal of observed behavior [4]. The agent based model of Thurner et al. arrived, for the first time, to a heavy tailed and clustered volatility price distributions by a mechanism that exclusively relies on margin calls.

On the other hand, it relates to research on the leverage cycle that John Geanakoplos and others have undertaken over the last decade (see, for example Geanakoplos [5], Fostel & Geanakoplos [6]). Economic theory tends to focus on the interest rate that equilibrates demand and supply for credit while neglecting the highly variable leverage ratios, see Figure 1. Interest rates relates to the impatience of borrowers, while the leverage relates to the nervousness of lenders. If one examines the time-dependence of leverage ratios and assumes that investors, due to different degrees of sophistication or risk aversion[10], value assets differently, one has identified a mechanism that can create excessive asset price movements. In boom times, when lenders are not worried about repayment, leverage is readily given, expanding demand for assets and inflating prices. During crises, fear leads to a contraction of leverage, depressing prices. For example, the leverage ratio in the US mortgage market during the years 1999-2006 was 20:1, whereas it has fallen to or even below 5:1 in 2007-2009. This points to the necessity to limit swings in leverage and the possibility

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*DRAFT: do not quote.
† We would like to thank the H. Doyne Farmer for guidance and sharing his work with us. We would also like to thank Corinne Teeter and J.P Gonzales for helpful discussions. Of course, many other CSSS 2009 participants lent too many thoughts and ideas to acknowledge individually, so we thank you all. Especially Tom Carter.
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that government regulation might by the only means to achieve this restraint [7].

This paper builds on the Thurner, Farmer, and Geanakoplos [1] model of leveraged asset purchases with margin calls. It relies on the agent-based approach and aims to test the robustness of the Thurner et al. results by adding features that bring the model closer to reality. The features are independent of each other and, therefore, do not obfuscate the analysis. The features of more realistic models can be compared to the baseline model, although this as not been done in detail yet. The more realistic features are:

- a greater variety of trading strategies, including trend reinforcing momentum traders and risk-neutral arbitrageurs.
- adaptive behavior of hedge funds where more successful hedge funds are imitated by less successful ones.
- endogenously determined interest rates
- a more realistic banking sector with a network of various competing banks

We have currently implemented, to some degree, all of these items using NetLogo 4.0.4[11]. However, we have not implemented all of the features of Thurner et al. Specifically, we do not have independent investors that pile into successful funds. Without these, we cannot fully observe market domination that is observed by [1]. This appears to be an essential feature to market crashes in their model.

**II. METHODS**

Here we document the details of our model, and the source code will be available on the CSSS 2009 website.

**A. Traders**

Each trader belongs to one of multiple behavioral classes. We have deadheads, momentum traders, value traders, and risk-neutral traders. Deadheads buy, sell, and borrow randomly. Momentum traders estimate the return of the equities market by extrapolating from past returns. Value traders know the fundamental mean-reverting price, and use leverage to aggressively invest when the price falls below this fundamental. Risk-neutral traders only compare the instantaneous return on investments between money saved in the bank and money in the market. Many other types of investors could exist, and the simulation allows for extending the family of traders.

At the beginning of the simulation, each trader is endowed with a fixed amount of cash and shares. All cash is kept in the bank and receives daily interest at the posted annual rate. The interest payment per day is the annual deposit interest rate plus a spread, divided by the number of trading days, multiplied by the size of the agent’s loan.

Loan interest charged by the bank is deducted from the deposited cash. If the trader doesn’t have sufficient cash, the bank sells some of the trader’s shares. If the trader still does not have enough to pay the interest, the trader must declare bankruptcy. Once bankrupt, a new trader
pulling money out of the market as it declines. 
Notably, no trader is able to sell short.

1. Deadheads

Deadheads serve as explicit sources of randomness for the bank. These traders act without any coherent strategy. At each turn, the agent randomly determines to buy or sell, and to borrow or to repay. If the trader chooses to buy, it spends a random fraction of its available cash. If the trader chooses to sell, it sells a random fraction of its shares. If desired by the user, the deadhead can also use leverage by randomly choosing each turn to borrow or repay. If the trader chooses to borrow, it borrows a random fraction up to the maximum possible leverage. If it chooses to repay, it repays a random fraction of its loan.

2. Momentums

Momentum traders, like many of us, are trend followers. They attempt to catch the market’s up-swings and avoid the market’s down-swings. They possess sufficient reason to compare their expected returns in the market with their expected returns by keeping their money in the bank[12]. They calculate the expected return by first calculating the market price return over a trader-specific time-window. This return is then multiplied by an exaggeration factor, which is also specific to each trader. The exaggeration represents the psychological hysteria that causes people to over estimate ups and downs [8]. If the expected return exceeds the deposit-rate offered by the bank, the momentum trader takes all of its money out of the bank and puts it in the market. The variation in parameters for estimating the return means that, although any one trader may over-react, putting their life savings in the market, the group of momentum traders as a whole will act more progressively. When the expected return exceeds the rate for borrowing a loan (the deposit-rate plus the spread), the momentum trader will borrow money from the bank to invest. The amount borrowed equals the maximum possible loan times a scaling factor, which is

\[
\text{expected return} - \text{current borrow rate} \over \text{expected return}
\]

Momentum traders can cause a self-fulfilling prophecy by pushing more money into the market as it goes up and pulling money out of the market as it declines.

3. Values

A value trader is someone who buys an equity because they believe it is underpriced. In reality, this is an imperfect estimation, at best. Value investors such as Warren Buffett and Ben Graham claim their goal is not to estimate a break-even price, but to classify a company as grossly over- or under-valued. On the other hand, investment analysts frequently quote “target prices” for stocks, which can have significant influence on short-term price movements.

In our model, value traders do know the precise mean-reverting price of the equity. The amount to buy is then determined by the potential gain resulting when the price returns to the fundamental. The actual algorithm is copied as closely as possible from [1].

In many ways, value traders are the compliment to momentum traders. Value traders attempt to follow the maxim of buying high and selling low. Because momentum traders sell as the price drops, they are frequently the counter-party to trades with value traders. Momentum traders, as a whole, tend to buy high and sell low.

4. Risk-Neutrals

Arbitrage is a major force in determining prices. In this model, arbitrage takes place between the return of the market and the interest from the banks. Unlike momentum traders, which estimate returns using extrapolation, the model’s risk-neutral traders know the true annual return on the fundamental price. To achieve the best possible investment gains, the risk-neutral trader moves money into the best investment vehicle.

Each turn, the trader compares the interest on deposits with the return of the market. If the interest for deposits exceeds the fundamental return, the trader moves all of its assets into the bank, selling all held shares. Alternatively, when the interest rate is smaller than the fundamental return, the trader uses its cash to buy shares. When the return on equity exceeds even the interest on a loan, the risk neutral trader uses leverage to put as much money as possible into the stock market. To avoid monolithic behavior, each risk neutral trader estimates the true fundamental return with a small error.

In this way, the risk neutral trader helps to synchronize the market return with interest rates. When the bank has an excess of cash, and interest rates are low, the risk neutral traders will borrow, bringing up interest rates. If interest rates are excessively high, risk neutral traders are an explicit mechanism from moving cash from the market into the bank, depressing interest rates.

5. Evolutionary Dynamics

The current simulation is also a first step towards a full-fledged implementation of “evolutionary” dynamics among agents. In particular, we want to investigate how different leverage choices give rise to different global outcomes in the market. Thurner et al. observed that high leverage led to an increase in volatility and crashes, while
no leverage also increased volatility [1]. Evolutionary Dynamics may help uncover an "optimal" leverage level.

The current model features imitation dynamics and replicator dynamics. The former is a function that tries to model the spread of good strategies in the market. Agents are allowed to look into their neighborhood and copy, with a certain probability, agents that prove to be more successful in the market[13]. The latter dynamic is the well-known standard dynamics of evolutionary game theory, where the proportion at $t+1$ of a given breed, $A$, in a population, $P$, is given by the proportion at time $t$ times the ratio between $B$'s fitness and $P$'s average fitness.

Although further developments are still needed to critically assess the importance of evolutionary processes in the market, strategic interactions among breeds appear to be a particularly promising line of enquiry. For example, it is well known that mutual funds that advertise different strategies frequent hold very similar portfolios because fund managers fear underperforming their competitors.

### B. Banking

In our model, interest rates frequently determine the actions of traders who weigh returns in the stock market with interest from deposits. Intuitively, rising interest rates depress market prices by increasing the cost of borrowing and competing for investing money. Conversely, low interest rates encourage loans and make risky investments more attractive.

One way to create a model economy is to fix interest rates, allowing the experimenter to observe causal effects of interest rate perturbations. However, when demand for stock is high, so is demand for loans. Fixing the interest rate could lead to fundamental imbalances. Buying frenzies that encourage borrowing deplete banks of cash reserves, driving up interest rates. Some equilibrium between the market and the banks should emerge, if it exists.

Thus, the central question in integrating a banking sector is how to faithfully determine interest rates endogenously. In reality, the return on risk-free government bonds and interbank lender rates play a large role. Furthermore, there is not one unique interest rate; they vary both from bank to bank and depend on loan terms. Ultimately, many of the factors that determine these rates are political or ideological, such as "breaking the back of inflations." These would be beyond the scope of our model.

#### 1. Interest Rates

How can we endogenously determine interest rates without predetermining some of the most common interest rate driving forces? In our model, we use a common feature of banking regulations, the minimum reserve requirement [9]. This is, banks tune interest rates to stay as close as possible to the minimum reserve level.

The minimum reserve is a set fraction of deposits the bank must keep on hand (10%). A bank in our model obtains its revenue from interest payments on loans. A bank can maximize its profits by lending as much as possible while still maintaining the needed cash minimum. To determine interest rates that maximize profit, instead of solving a simultaneous equation to balance demand and supply of cash between the traders and the bank, banks can use the interest rate as a control parameter to attract loans or deposits.

Multiple times each turn, the bank adjusts its rate to stay as close as possible to the target reserve level via an open feedback loop. When called upon, the bank adjusts rates via

$$
\Delta r = \alpha \sqrt{r}(\beta \text{Deposits} - \text{Cash on Hand})
$$

where $r$ is the interest rate the bank pays for deposits, $\Delta r$ is the change in the rate, $\alpha$ is an arbitrary sensitivity parameter, and $\beta$ is the reserve fraction, 10%.

The factor of $\sqrt{r}$ was chosen to draw analogy with the Cox-Ingersoll-Ross model of interest rate fluctuations. The excess reserve term multiplying the $\sqrt{r}$ is therefore analogous to the source of noise in the CIR model, and $\alpha$ becomes a volatility. More specifically, $\alpha$ determines how severely the bank should adjust interest rates, and it is usually a very small number. In Figure 2, we show how overreaction leads to instability in interest rates and, by extension, the stock market. For the same population of traders (1000 risk-neutral, 100 deadheads, 1000 value, 1000 momentums), the choice of $\alpha$ determines the stability of interest rates. The difference between $\alpha = 0.001$ (Fig. 2(a)) and $\alpha = 0.002$ (Fig. 2(b)) is sufficient to destabilize interest rates.

The feedback interest rate model is sufficient to produce a stable equilibrium between the fundamental return of the market and deposit interest rates, when the trader population is dominated by risk-neutral traders. No excess returns are possible. However, this validation alone is not sufficient for banks to survive.

A bank must pay interest on deposits from income from loans. If the bank were to charge the same interest rate for loans and deposits, a bank can only sustainably operate when loans happen to exceed deposits. To ensure the bank can meet its obligations, it charges borrowers an additional spread on top of the base deposit rate. In our model, spread is calculated by determining the amount the bank needs to charge its borrowers to cover its expenses, and the bank gradually adjusts the spread up or down to cover expenses:

$$
\Delta s \propto \frac{(\text{Expenses} - \text{Income})}{\text{Loans}}
$$

Because an increase in spread discourages loans, it has the knock-on effect of depressing the deposit rate. As
Figure 2(a) shows, deposit rates can effectively be driven to zero, while the borrowing rate can be a more common number, say 4 or 5%. This disparity in interest rates should be familiar to anyone with a checking or margin account, especially in recent months.

Various variations of the spread and deposit rate processes can alter the character of the interest rate dynamics, but that does not mean any single model is correct or incorrect. Comparing a simulated single spot rate with a real-world spot-rate remains to be done. Furthermore, implementing a term-structure would add significantly to the complexity of any agent based model, because it requires agents to distinguish expectations of returns at different times in the future and develop a strategy that accounts for money being sequestered in CD’s, bonds, etc. for different, often overlapping, periods.

2. Bank Leverage

Just like individuals, banks frequently borrow to invest or cover short-term liabilities. The minimum reserve requirement applies to cash in a “vault,” whether that money belongs to the bank or not[14] [9]. A bank may try its best to lend out no more than regulations allow, but unexpected withdrawals can cause it to suddenly be in violation of the minimum reserve. In such a case, banks must borrow from other banks on the overnight market. This money is intended to be repaid the next day, along with a nominal interest fee.

Our model has been extended to include multiple banks, but at the moment it is restricted only one. When a bank needs to borrow, it borrows from the equivalent of the Federal Reserve at a user-set rate. Although the user chooses this Federal Funds Rate, the underlying interest rate is determined by market dynamics, endogenously. NetLogo allows this rate to be changed mid-simulation, so we can understand how the target rate effects interest rate dynamics.

Along with cash from deposits, the bank has cash assets that it starts with and earns from loan interest payments. This cash counts toward the reserve requirements, and has the effect of depressing interest rates. The bank can borrow money to invest in loans, but it must pay interest at the target rate. This interest is deducted from the cash at hand, so it can threaten the reserve balance of the bank. Hence, borrowing for making loans can inflate the interest rate.
The bank's leverage is capped, and it can extend its lending power by increasing its assets or obtaining more deposits. Formally, the bank's leverage is

\[ \text{debt} \over \text{assets} + \beta \text{ deposits} \]

The bank cannot borrow to exceed this leverage. However, in the case when the bank has insufficient cash to cover a withdrawal, the bank is forced to borrow to ensure customers can recover their deposits[15], no matter the leverage required. If a bank does not have sufficient cash to pay interest on its deposits, it will attempt to borrow the necessary funds. In the case when the bank is unable to borrow this money, it does not pay the interest. It alters its published deposit rate to reflect the amount it is able to pay in deposits. Unlike withdrawing deposits, interest payments are not guaranteed.

The bank is not subject to margin calls, because there is no mechanism for the bank to force traders to repay their loans. The bank attempts to repay its debts as quickly as possible, in line with the notion of an overnight market. If it can not afford to repay its debts, interest continues to accrue and compound, but the bank does not declare bankruptcy. However, the bank may not be able to make new loans or pay interest on deposits; when the bank exceeds maximum leverage due to poor financial performance it may become a "zombie" bank. How this situation plays out in the simulation is not always clear, but we have certainly experienced similar credit freezes during 2008/2009.

C. Equity Market

Our equity market contains only a single stock. In this way, it represents the equity sector as a whole. The dynamics of the market are modeled after [1], with a number of modifications to account for the diversity of strategies by the agents.

Because we do not employ an order book to determine prices, agents do not explicitly trade with each other. Instead, the counterparty to every trade is an implicit noise trader, which soaks up excess demand. As with Thurner et al., the total number of shares is split between the agents and the noise traders. Because all shares are held either by an agent or an implicit noise trader, we have

\[ \sum_i D_i(P) + D_{nt}(P) = N, \]

where \( D_i \) is the demand of trader \( i \) given price \( P \), and \( D_{nt} \) is the demand held by the noise traders. To set the price, we define, as with Thurner et al.

\[ D_{nt}(P) = \frac{\zeta_t}{P}, \]

where \( \zeta_t \) is a mean reversion geometric brownian motion, which weakly reverts to a fundamental price and is updated once per turn. This fundamental price can gradually grow over time, giving the risk-neutral traders something to compare to deposit rates.

Because our model has only one agent trading at a time, the demand of all other agents is constant of the course of a trade. As a result, for a purchase initiated trade, the agent sets the amount of money they are going to spend (as opposed to buying a fixed number of shares). The new price is

\[ \frac{\zeta_t + S}{N - D}, \]

where \( S \) is the cash amount the trader is committing, and \( D \) is that total shares held by all other agents.

When an agent initiates a sell order, they put up a fixed number of shares (as opposed to selling a fixed dollar amount). Again, no agents trade simultaneously, so the new price is

\[ \frac{\zeta_t}{N - D + S}, \]

where \( S \) is the number of shares to be sold. The larger the total available shares, \( N \), the smaller the price impact of any one trader, making the choice of \( N \) an important parameter in the model. The pricing algorithm has not been thoroughly checked to ensure zero-sum trades, meaning the amount of money in the market may not be constant. It seems to work well enough.

III. RESULTS

The first stage in our investigation is to simply validate the model and understand its components. This takes two forms, if we hope to draw analogy with real market dynamics, we must explicitly compare known features of real markets with those produced by our model. Second, we have modeled much of our work after Thurner et al, so we should recreate their behavior as a check. In fact, as mentioned earlier, the latter is not quite possible, because we have not implemented independent investors who put their money directly into the trading agents, a key feature of Thurner et al.

Here is a brief list of notable results gained so far:

1. Risk-neutral traders are by far the most vulnerable to bankruptcy
2. Margin calls actually decrease bankruptcy rates, especially among risk-neutral traders.
3. Margin calls decrease interest rates, See Figure 3.
4. Momentum traders cause interest rates to be correlated with equity price returns, not anti-correlated as normally assumed.
5. Value traders, as expected, are the most likely to earn profits, but are rarely leveraged more than 2 or 3 times. In good conditions, their earnings decrease interest rates.
6. Insufficient maximum leverage increases the interest spread, because the bank cannot make enough loans to cover deposits. It also decreases the deposit interest rate, so both depositors and lenders lose out.

Our research into evolutionary dynamics is still at its beginning. We have yet to really fully integrate the complete system within the model. We look forward to observing how market dynamics are affected by changes in strategy. We will try different measures of “fitness,” such as trying to minimize volatility, maximize Sharpe ratio, or perhaps another metric of excess return. Our current implementation of evolutionary dynamics involves competition between deadheads and momentum traders. In this, we see that the final population is path dependent, with either all deadheads or all momentums. This suggests this research may yield some very interesting, context dependent, results.

We hope to migrate our code from NetLogo into a faster language. NetLogo has been very effective at prototyping and inspecting our market, but lack of speed, and particularly debugging features, prevent us from expanding it into more complex realms. For example, we also have a credit network between multiple banks, but this increased complexity slows down the simulation significantly. It also increases the difficulty in debugging and uncovering accounting errors. We will be better able to add traders who execute more complex behavioral strategies, which will allow us to investigate the effects of psychology on market behavior.

To fully compare our model with Thurner et al., we need to incorporate independent investors who place their money into the accounts of traders who are performing well. This increases the market representation of that particular strategy. If the independent investors switch funds, this can cause a sell-off, furthering any downward price movements. We must also fully validate our market dynamics by comparing our price and interest-rate statistics to those observed in reality. We do not expect to be able to reproduce reality exactly, but we hope to observe gross features.

IV. CONCLUSIONS

We have constructed an agent-based model of an equities market that endogenously determines both stock price and interest rates. We have the ability to observe the effects of different trading strategies on price and interest rates. Although interest rates are influenced by many factors, of which only a small part is demand for leverage, we have a system that can explore how banks are coupled to the stock-market.

With this tool, and a few more modifications, we can follow the lead of Thurner et al., and investigate the effect of leverage on market stability. Our evolutionary dynamic will allow us to search for an ideal level of leverage, which a question of great importance in the wake of the recent credit crisis. A great deal of finger pointing and recrimination has emerged after the credit-crisis. Because our agents play pure strategies, and not greed, obfuscation, or cheating, we have a window into how credit shocks can form and, possibly, how they can be avoided.

[10] this assumption is contrary to the standard economic theory that ascribes the same fundamental value to everyone
[12] Most features, such as momentum traders comparing the market return to savings returns, can easily be modified to understand complete effects of a behavior, with and with-out specific sub-behaviors
Success could be high returns, low volatility, or high Sharpe ratio, etc. This is for future work.

Technically, all deposits are liabilities. Cash is frequently apt to be called away. Thus, the reserve requirement can be met by keeping cash on hand or by short term borrowing.

Think of this as an FDIC insurance. We do not allow the bank to entirely fail.