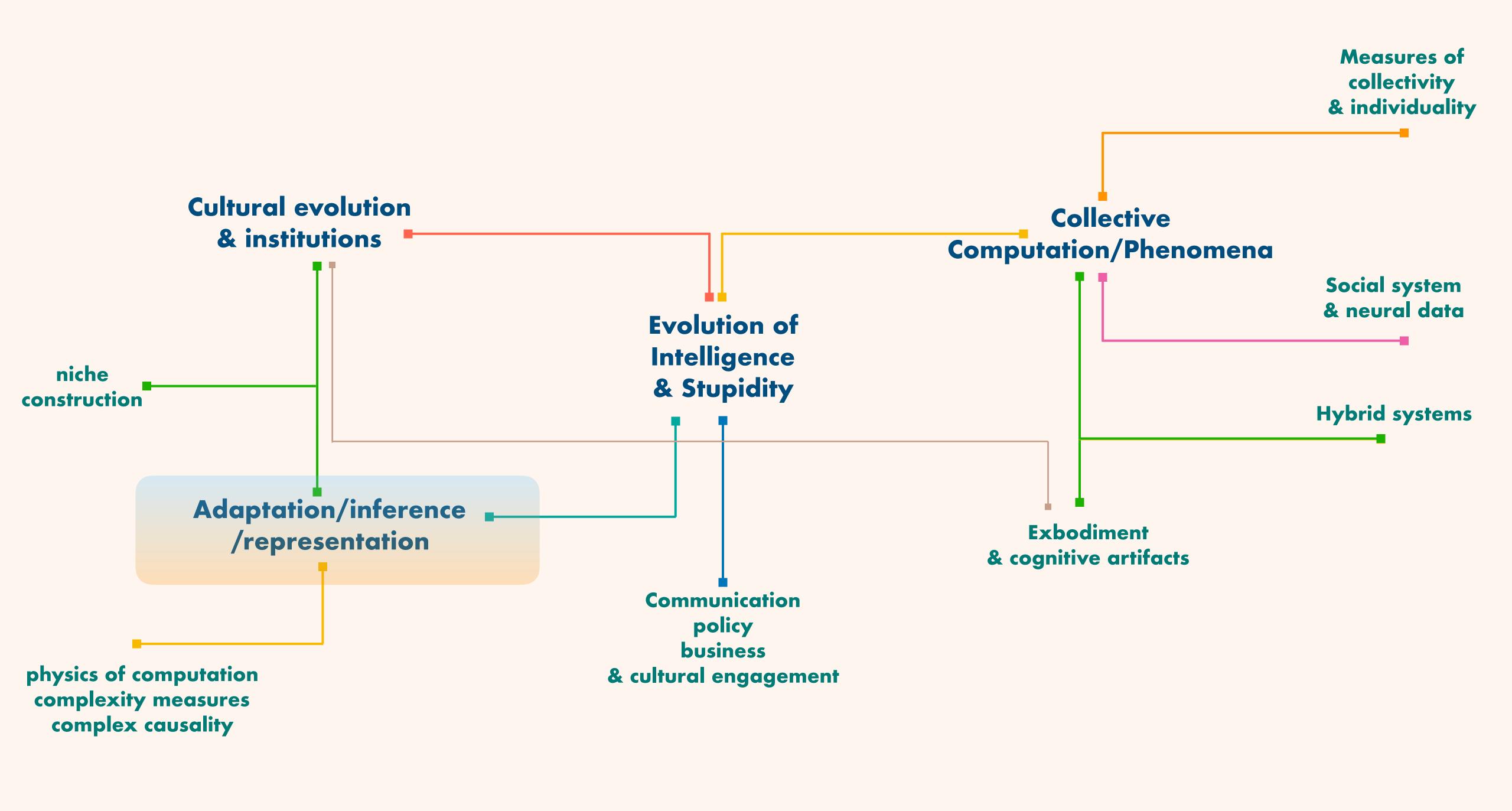
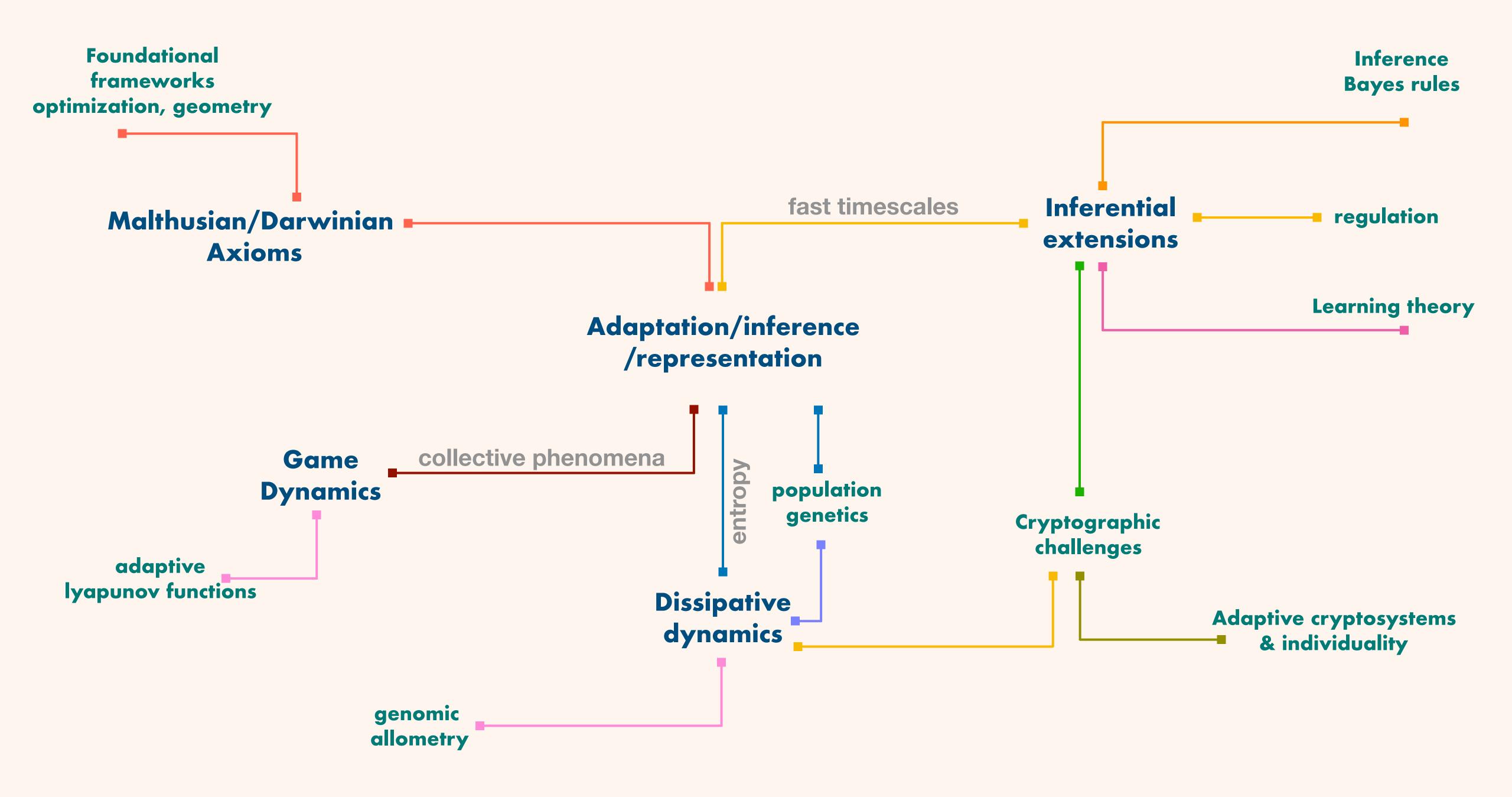


Figure 1.b: ADAPTATION
WWW.TONYCLIFF.COM

# Central Concerns in Complexity

- Noise versus Determinism
- Energy (Symmetry) & Information (Symmetry breaking)
- Dynamics versus Algorithms
- Adaptation, Learning, Inference & Selection
- Levels & Units of Causation, Coarse-graining, & Representation
- Universality & Materiality
- Description, Causation, Prediction & Explanation
- Models, Simulation, & Theory





- Krakauer, D.C. 2011. Darwinian demons, evolutionary complexity and information maximization. CHAOS 21, 037110-1-12
- Krakauer, D.C. Rockmore, D. 2015 Adaption or the ten Avatars of Vishnu. Princeton Companion to Applied Mathematics. Ed. Nicholas Higham. pp 591-599
- Krakauer, D.C. 2017 Cryptographic Nature. IN: From Matter to Life. Eds. Sara Walker, Paul Davies, George Ellis. 155-174 (and preprint ArXiV)
- ▶ Krakauer, D.C. Bertschinger, N. Olbrich, E. Ay, N. and Flack, J.C. 2017 The information theory of individuality. Theor. in Bioscience (Sub) (and preprint ArXiV).
- Krakauer DC, Müller L, Prohaska SJ, Stadler PF. 2017 Design specifications for cellular regulation. Theory in Biosciences. PMID 27864730 DOI: 10.1007/s12064-016-0239-5 and (and preprint bioaRxiv)



Malthus 1

"Yet in all societies, even those that are most vicious, the tendency to a virtuous attachment is so strong, that there is a constant effort towards an increase of population.... This constant effort tends to subject the lower classes of the society to

- Malthus T.R. 1798. An Essay on the Principle of Population. Chapter II

distress and to prevent any great permanent amelioration of their condition".

replication/growth

$$g_i \xrightarrow{r_i} 2g_i$$
Energy + Resources

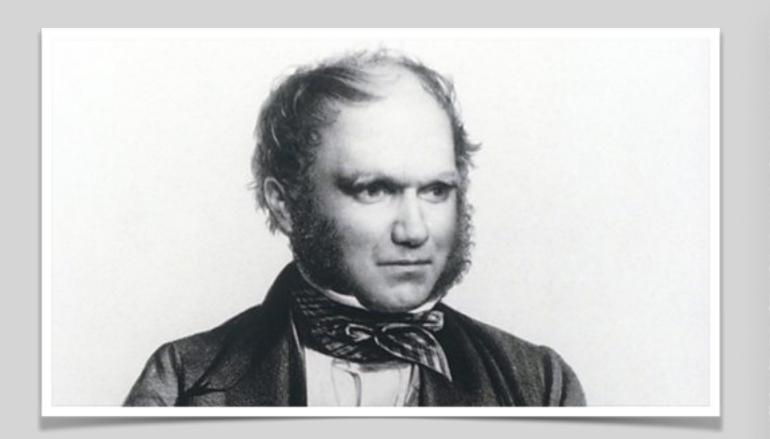


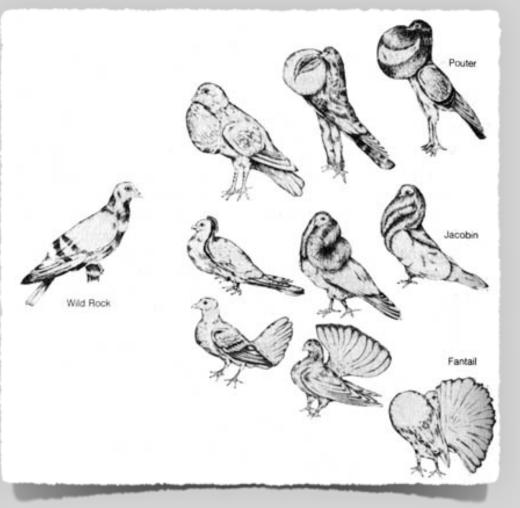
Malthus 2

- "Yet in all societies, even those that are most vicious, the tendency to a virtuous attachment is so strong, that there is a constant effort towards an increase of
- population. This constant effort tends to subject the lower classes of the society to distress and to prevent any great permanent amelioration of their condition".
  - Malthus T.R. 1798. An Essay on the Principle of Population. Chapter II

competition/scarcity

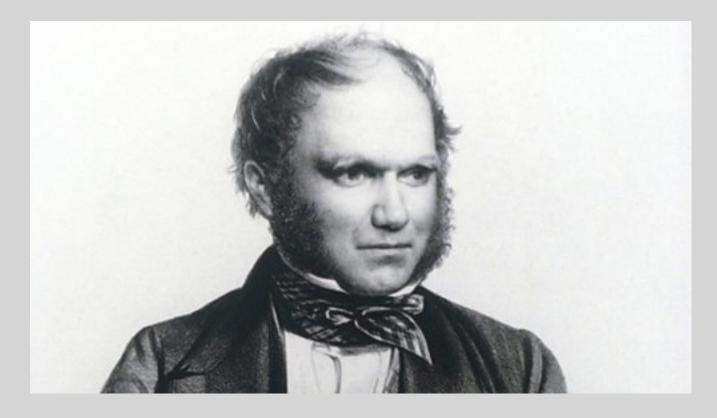
$$g_i + g_j \xrightarrow{c_{ij}} g_j$$





Can the principle of selection, which we have seen is so potent in the hands of man, apply in nature?... that individuals having any advantage, however slight, over others, would have the best chance of surviving and of procreating their kind? On the other hand, we may feel sure that any variation in the least degree injurious would be rigidly destroyed. This preservation of favorable variations and the rejection of injurious variations, I call Natural Selection. Variations neither useful nor injurious would not be affected by natural selection, and would be left a fluctuating element, as perhaps we see in the species called polymorphic.

Chapter 4. Natural Selection. The Origin of Species. 1856



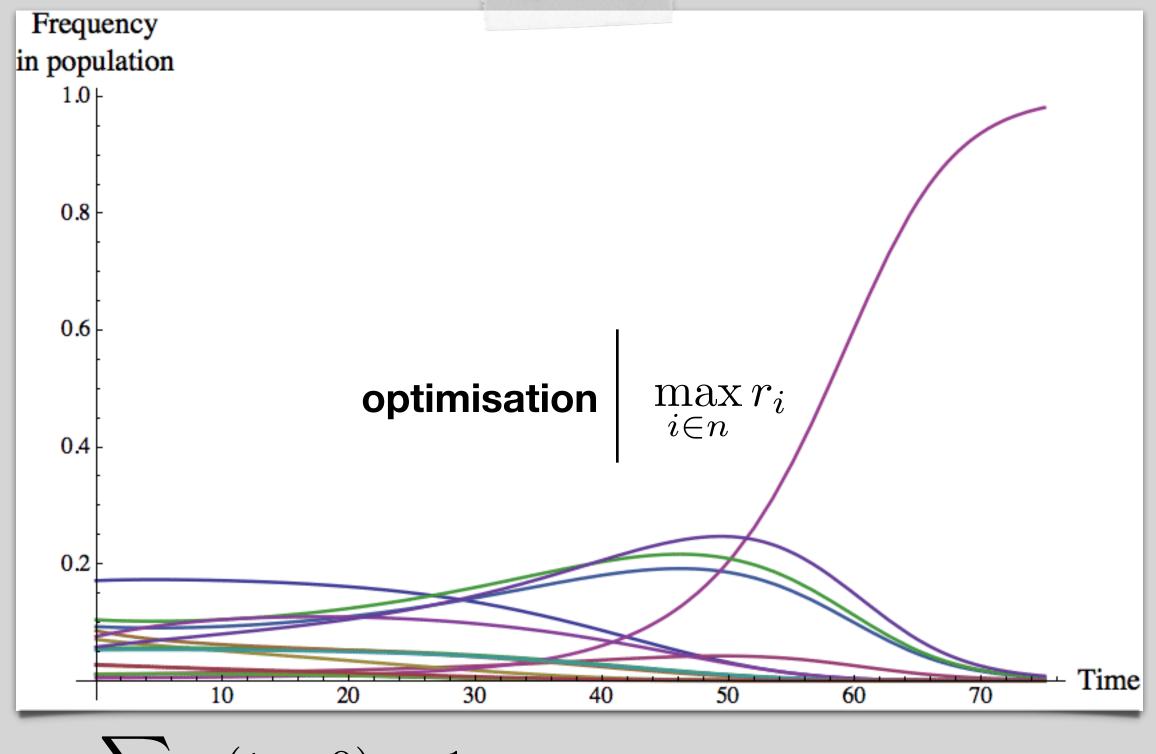
Darwin = Malthus 1 + Malthus 2

$$g_i \xrightarrow{r_i} 2g_i$$

$$g_i + g_j \xrightarrow{c_{ij}} g_j$$

$$\frac{dg_i}{dt} = g_i(r_i - \bar{f})$$
where  $\bar{f} = \sum_i r_i g_i$  and  $c_{ij} = 1$ 

$$r_i = s(g_i, E)$$



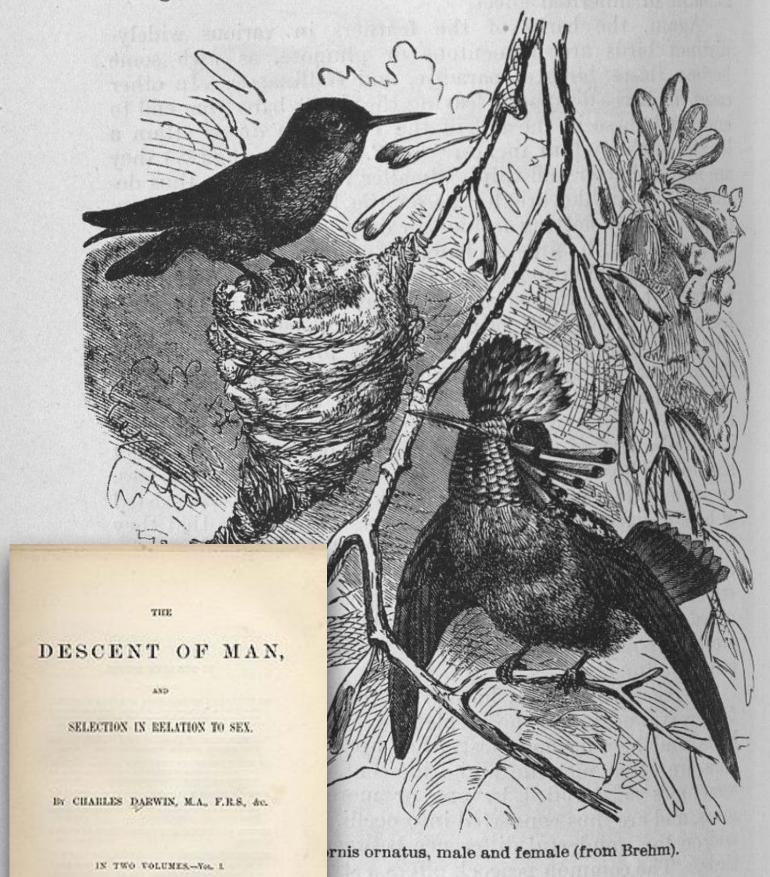
$$\sum_{i} g_i(t=0) = 1$$

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PART IL

CHAP. XIII.

highly decorated of all birds, and in so many different ways that they must be seen to be appreciated. The elongated and golden-orange plumes which spring from beneath the



disea apoda, when vertically erected and e described as forming a sort of halo, in h the head "looks like a little emerald rmed by the two plumes." 73 In another

afresnaye in xiii. 1854, p. 157: see also Mr. Walat. Hist.' vol. lace's much fuller account in vol. xx

most beautiful species the head is bald, "and of a rich cobalt blue, crossed by several lines of black velvety feathers." 14

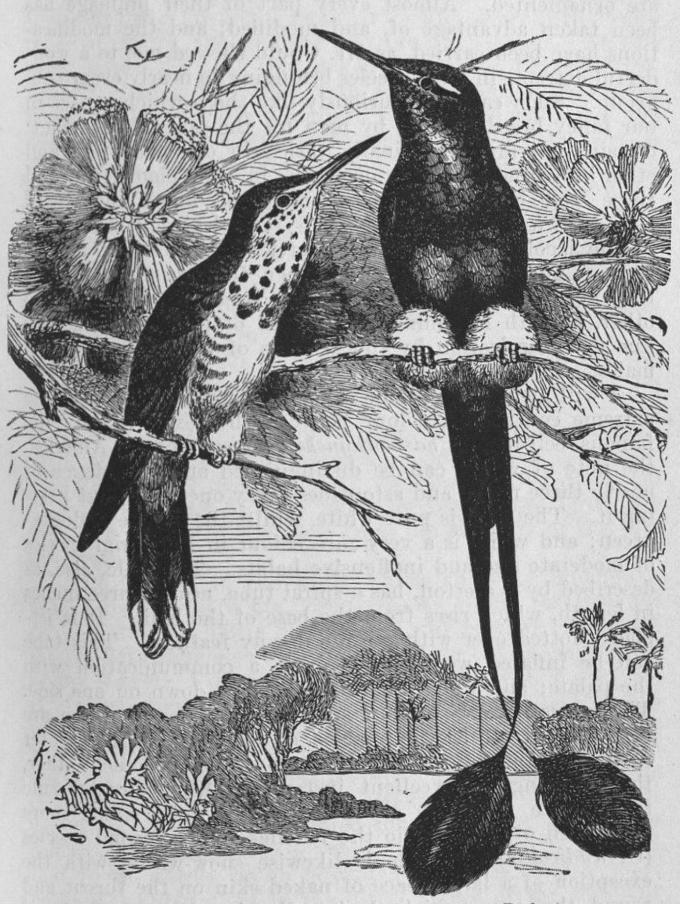


Fig. 49.—Spathura underwoodi, male and female (from Brehm).

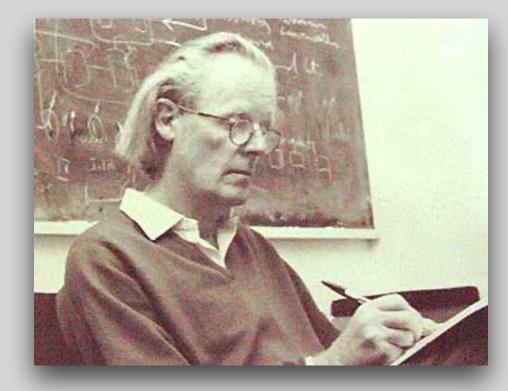
Male humming-birds (figs. 48 and 49) almost vie with birds of paradise in their beauty, as every one will admit who

1857, p. 412, and in his 'Malay Archipelago,' Wallace, 'The Malay Archipelago,' vol. ii. 1869, p. 405.

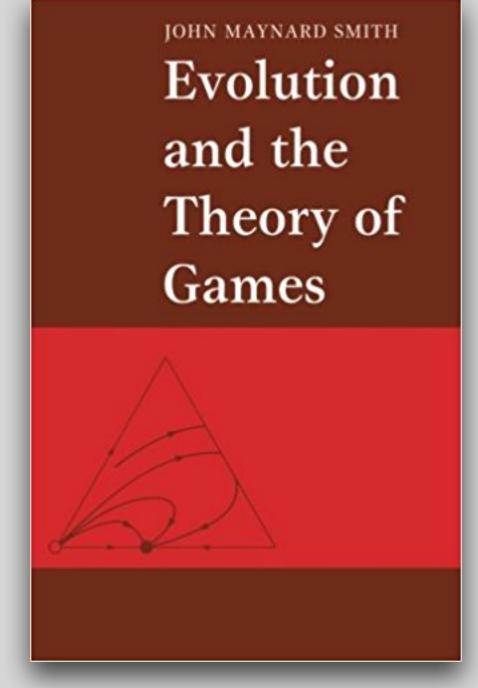
JOHN MURRAY, ALBEMARLE STREET.

1871.

(The right of Proseletion is reserved.)

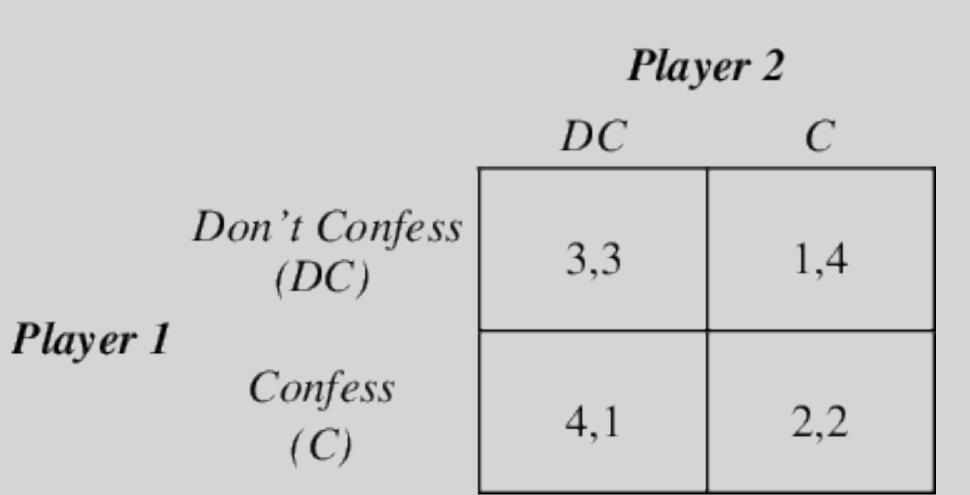


# John Maynard Smith (1920-2004)



# Collective Phenomena

$$r_i = s(g_i, E) \qquad r_{ij} = s(g_i, g_j)$$



1982

# Collective Phenomena

#### **Game Dynamics**

Payoff Matrix 
$$P = [p_{ij}]$$

$$\dot{g}_i = g_i(\sum_{j}^n g_j p_{ij} - \sum_{j}^n g_j \sum_{k}^n g_k p_{jk})$$

$$\dot{g}_i = g_i((Pg)_i - g \cdot Pg)$$

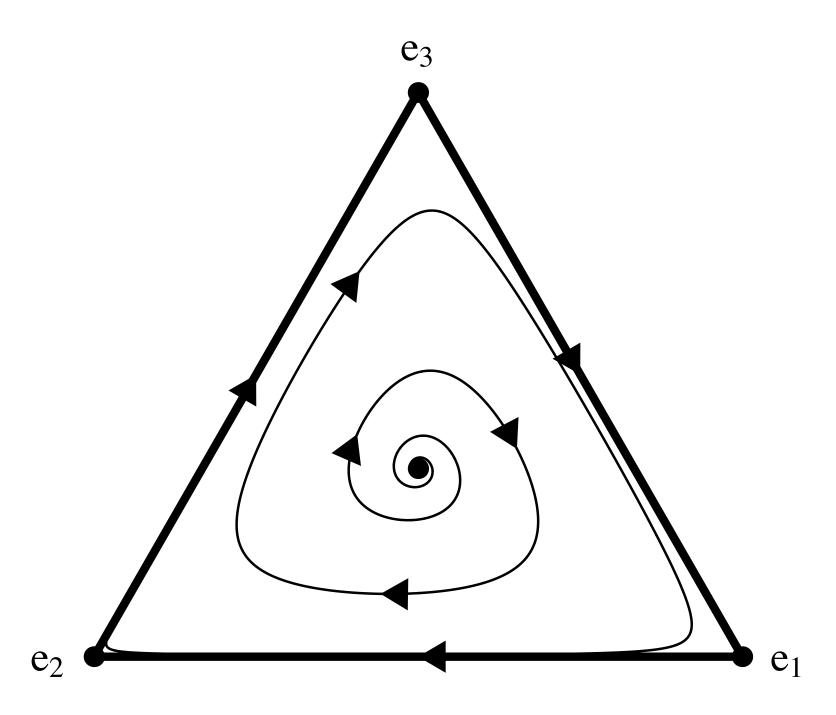
#### Which is of the replicator form:

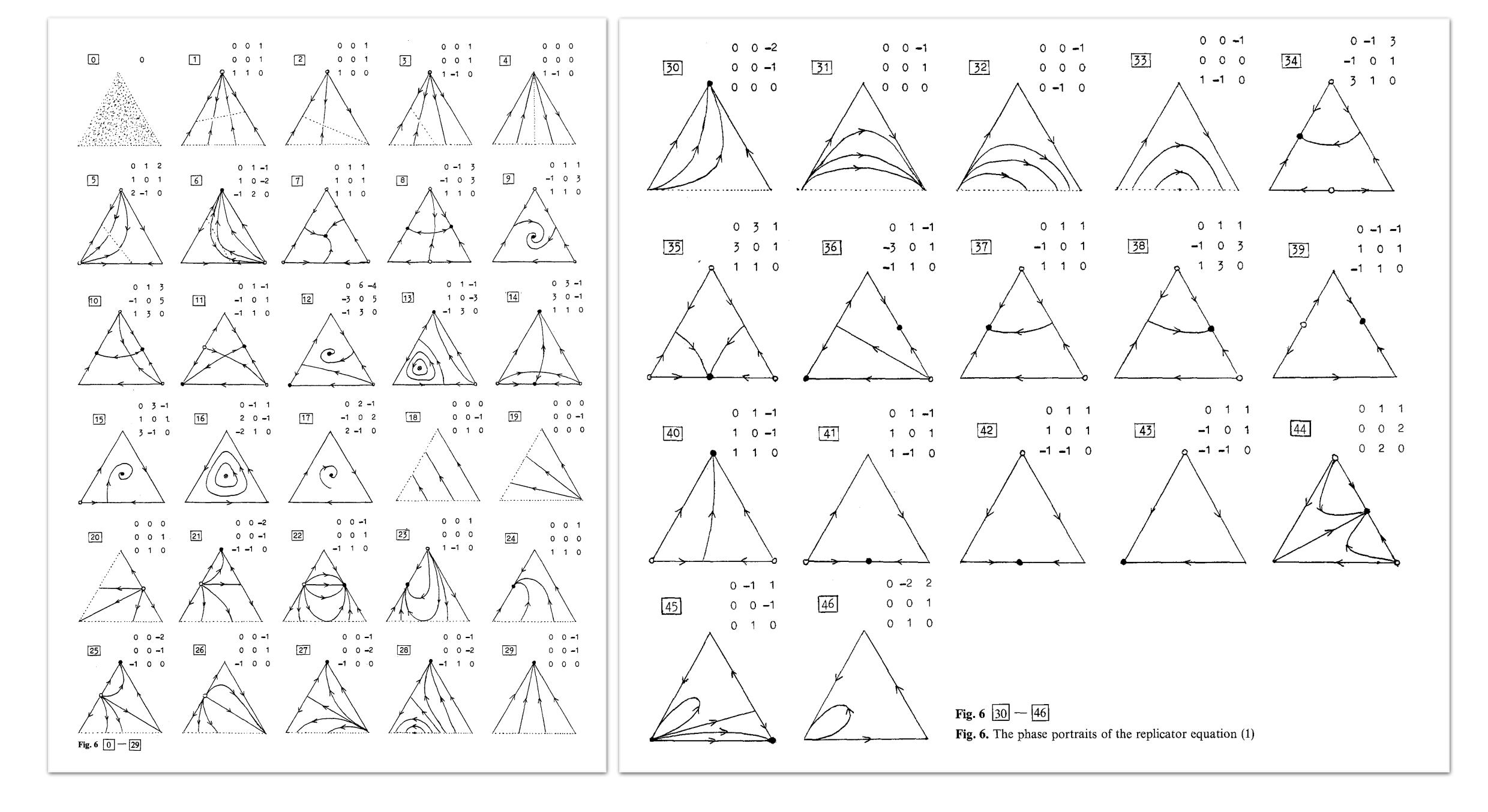
$$\frac{dg_i}{dt} = g_i(r_i - \bar{f})$$



# Payoff Matrix

|   | R | Р | S |
|---|---|---|---|
| R | 1 | 0 | a |
| Р | a | 1 | 0 |
| S | 0 | a | 1 |



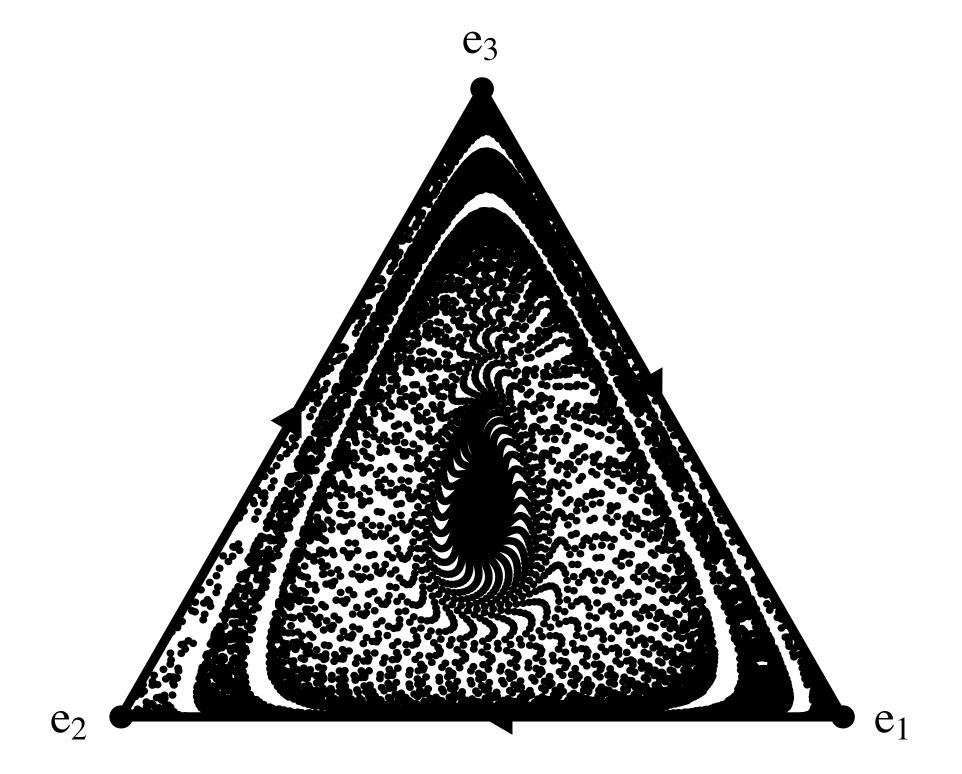


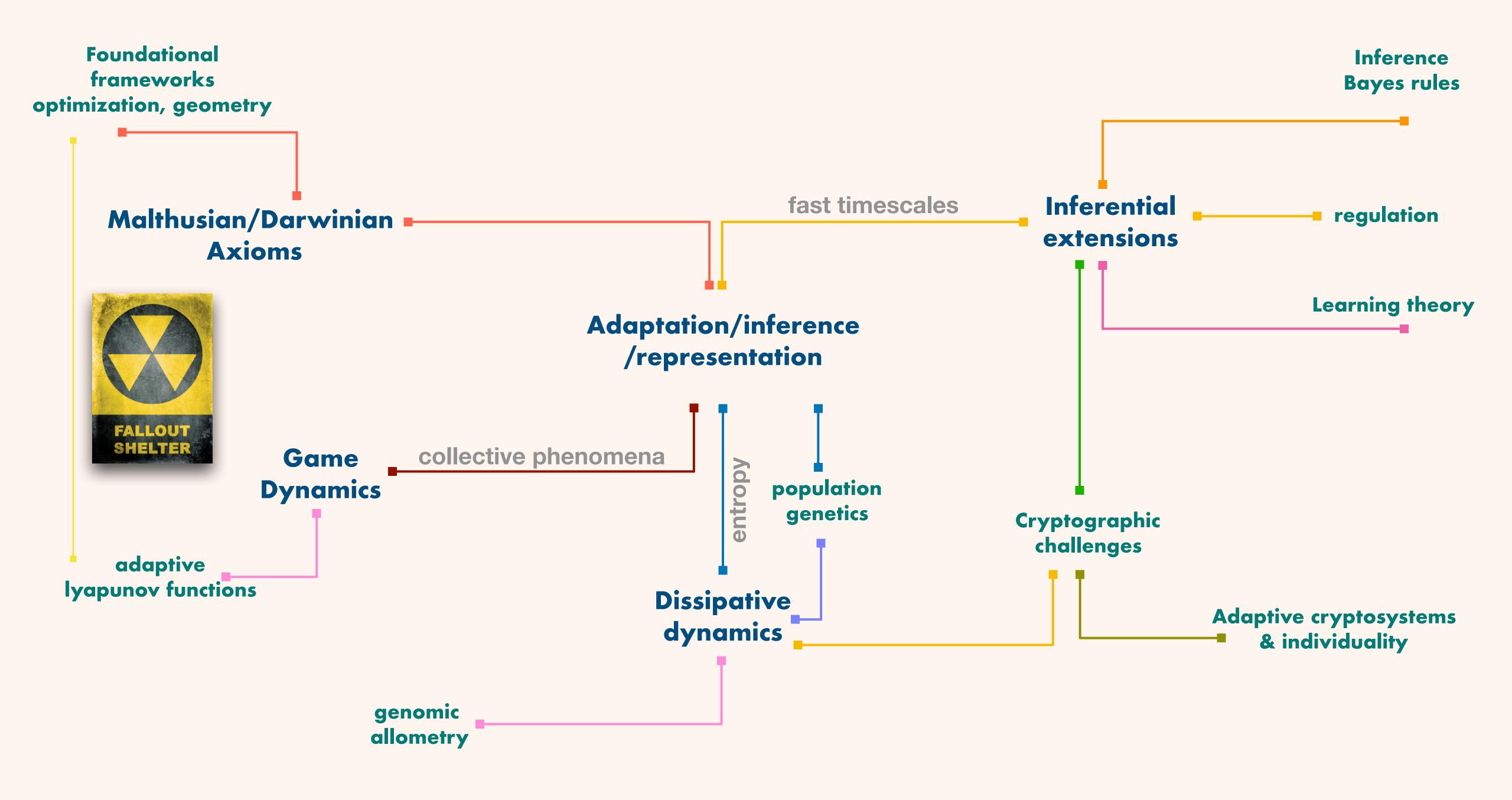
Bomze, Biol. Cybernetics (1983)

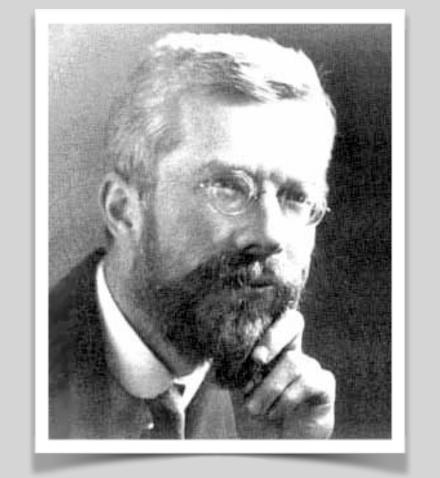
$$g_i(t+h) = g_i(t) \frac{(Pg(t))_i + c}{g(t) \cdot Pg(t) + c}$$

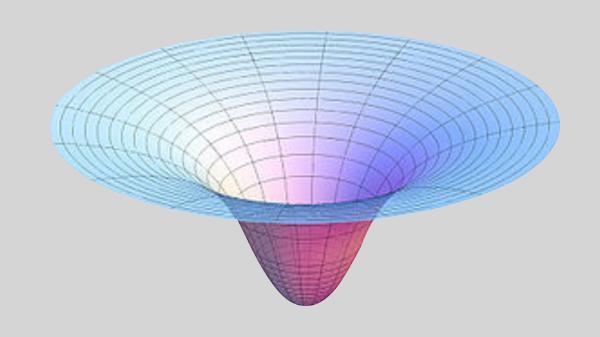
$$c \to \infty$$
  $\dot{g}_i = g_i((Pg)_i - g \cdot Pg)$ 

|   | R     | P      | S  |
|---|-------|--------|----|
| R | 0     | 4      | -2 |
| Р | -2    | 0      | 4  |
| S | 13.75 | -11.75 | 0  |
|   |       |        |    |









$$\frac{dx_i}{dt} = \frac{\partial V}{\partial x_i}$$

the Euclidean gradient vector field on  $\mathbb{R}^n$ 

$$\dot{V}(\mathbf{x}) = \sum_{i=1}^{n} \frac{\text{shashahani gradient}}{x_i \left(r_i(\mathbf{x}) - \bar{f}(\mathbf{x})\right)^2}$$

The Fundamental Theorem of Natural Selection

R.A. Fisher, 1930

#### Fisher curvature metric

#### distribution of organisms

distribution of environments

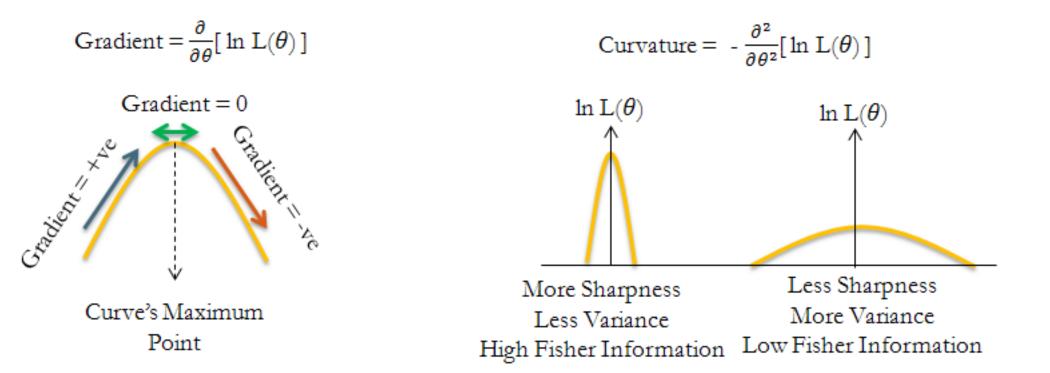
$$D_{KL}(X||Y) = \sum_{i} x_i \ln\left(\frac{x_i}{y_i}\right) \approx \frac{1}{2} \int x(\partial_{\epsilon} \ln x)(\partial_{\epsilon} \ln x) dx$$

for derivation see:

Krakauer, D.C. Rockmore, D. 2015 Adaption or the ten Avatars of Vishnu. Princeton Companion to Applied Mathematics. Ed. Nicholas Higham. pp 591-599

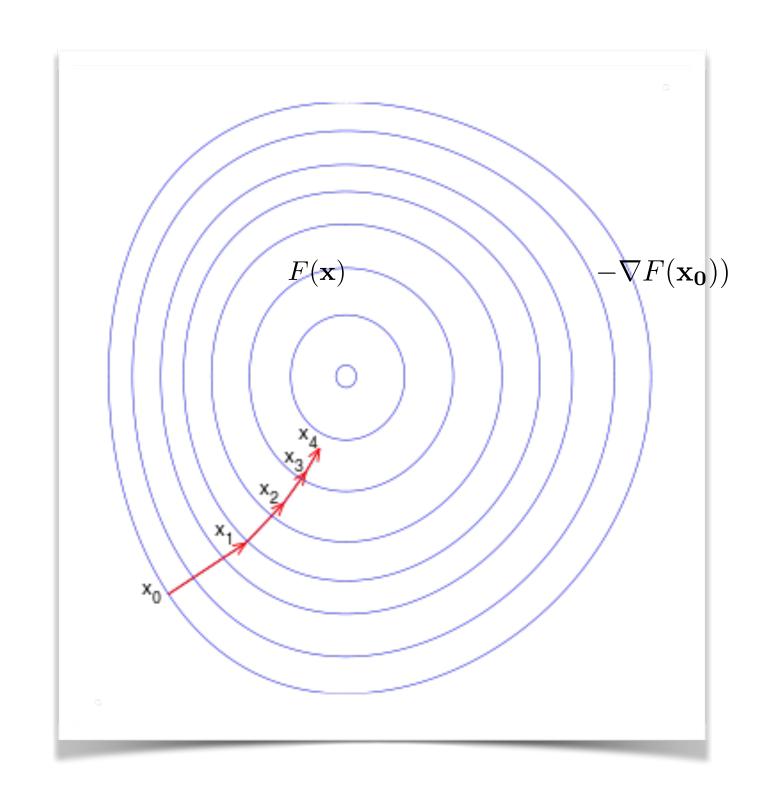


# Estimating parameters for a pdf - the maximum likelihood estimate (inverse in the variance)

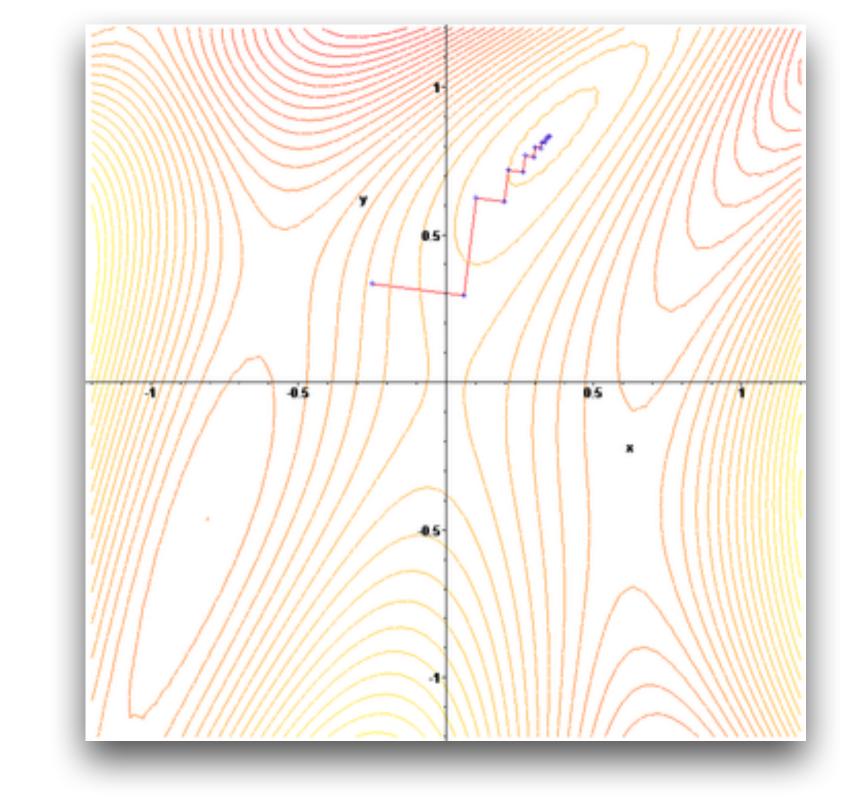


#### Adaptation as Non-linear Optimization

ouch...!



$$x(t+1) = x(t) - k\nabla F(x(t))$$



#### Gradient Descent

$$x(t+1) = x(t) - k\nabla F(x(t))$$

Natural Gradient Descent (use the Fisher Information metric)

$$x(t+1) = x(t) - kG_{x(t)}^{-1}\nabla F(x(t))$$

rescaling using the curvature metric

#### distribution of organisms

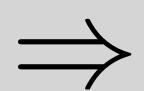
#### distribution of environments

$$D_{KL}(X||Y) = \sum_{i} x_{i} \ln\left(\frac{x_{i}}{y_{i}}\right) \approx \frac{1}{2} \int x(\partial_{\epsilon} \ln x)(\partial_{\epsilon} \ln x) dx$$

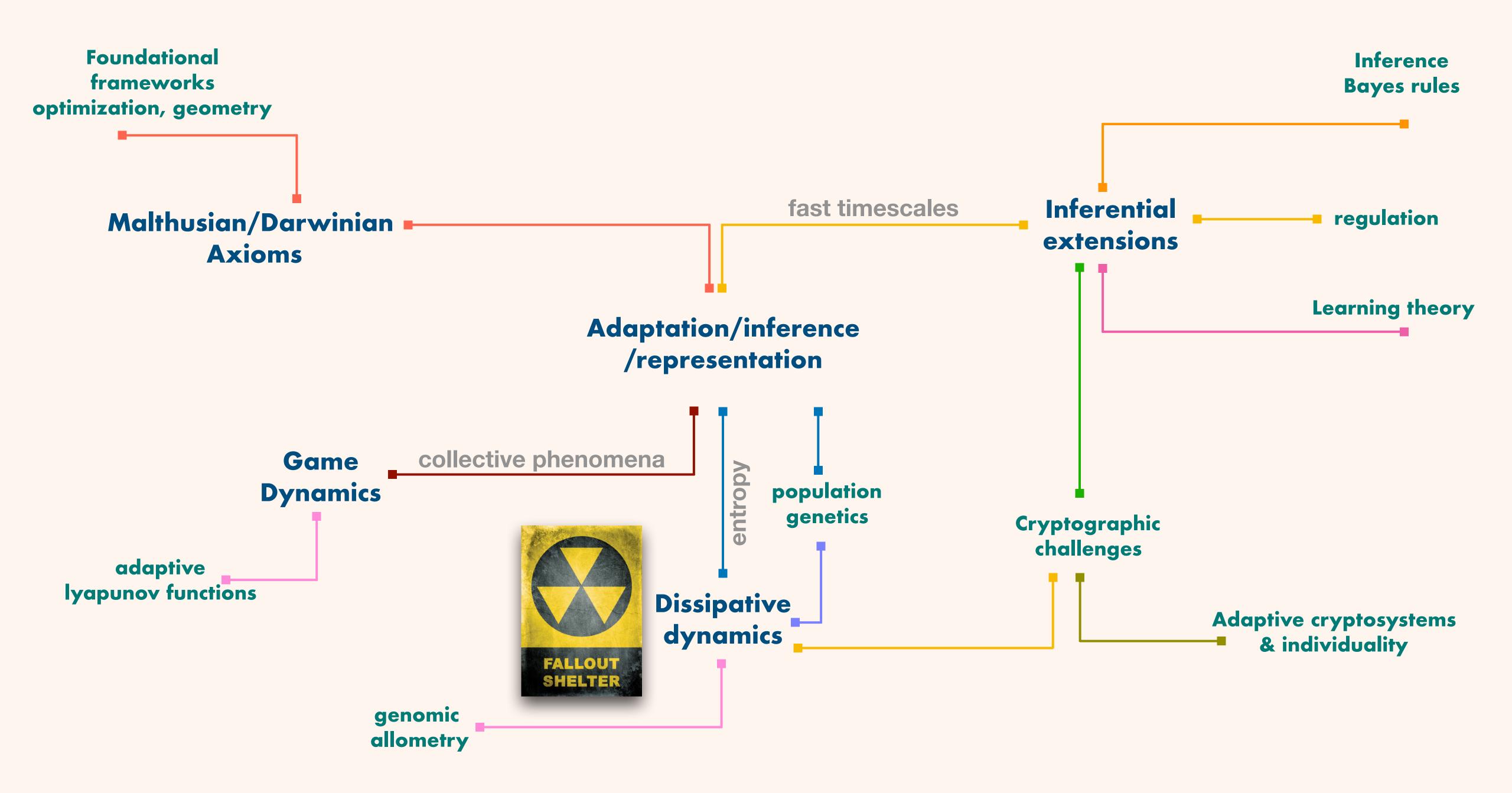
$$D_{KL}(X,Y||XY) = \sum_{y} p(y) \sum_{x} p(x|y) \ln\frac{p(x|y)}{p(x)} = I(X;Y)$$

$$I(X;Y) = H(X) + H(Y) - H(X,Y)$$

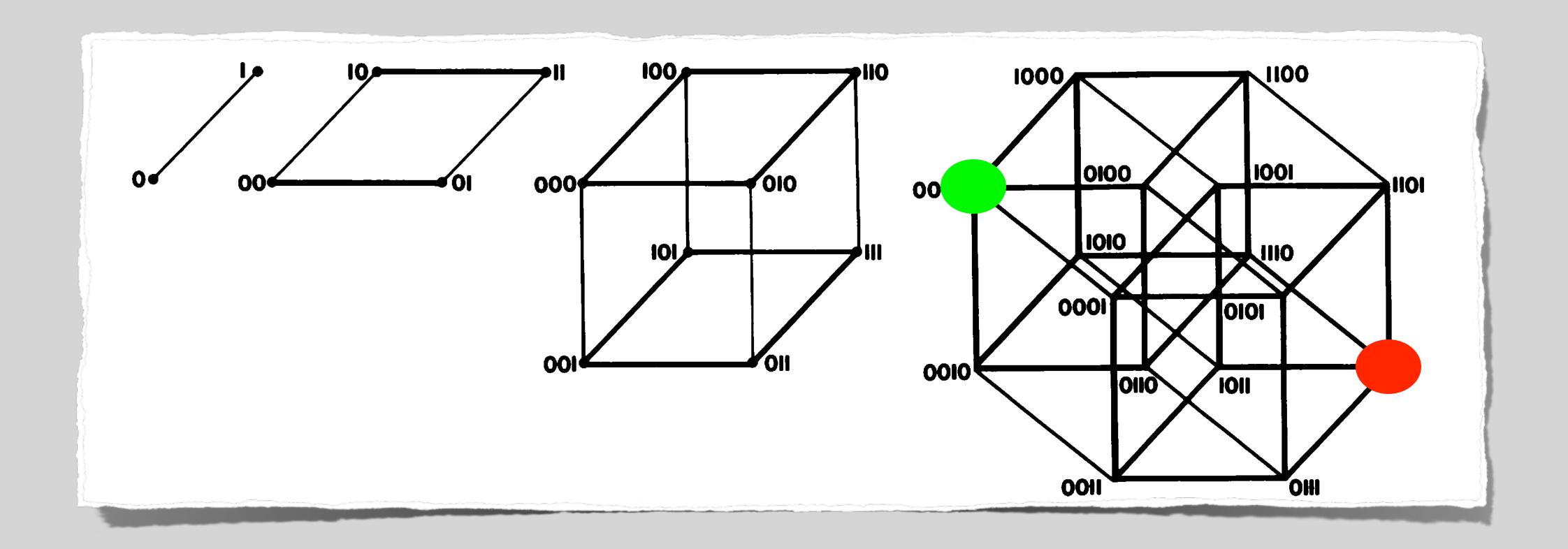
$$= H(Y) - H(Y|X)$$



adaptation is an optimization dynamics generating ecological compression



## An evolutionary configuration space: sequence Space



## mutation

$$g_i \xrightarrow{m_{ij}} g_j$$
Radiation

$$m_{ij} = \mu^{H(i,j)} (1 - \mu)^{L-H(i,j)}$$

# REPLICATOR-MUTATOR EQUATION

$$\dot{g}_i = \sum_{j}^{2^n} g_j r_j(\mathbf{g}) m_{ij} - g_i \bar{f}$$

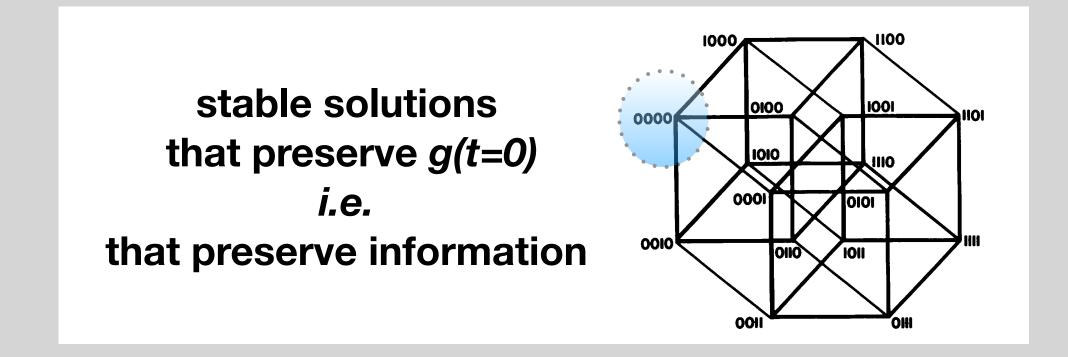
$$m_{ij} = \mu^{H(i,j)} (1 - \mu)^{L-H(i,j)}$$

### Equilibrium Solutions

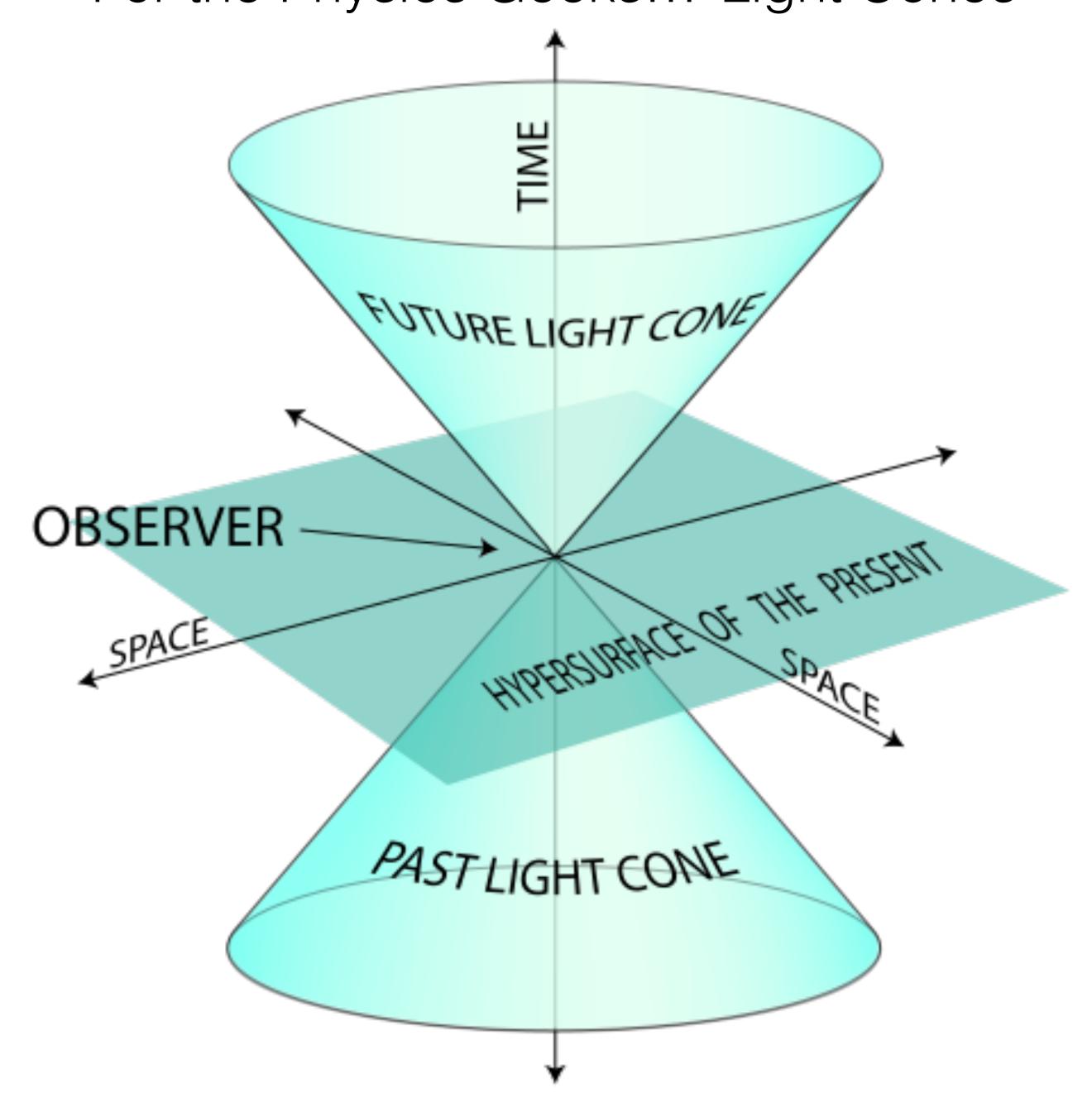
$$\dot{g} = RMg - Fg$$

$$RM\hat{g} = F\hat{g}$$

Solve for dominant right eigenvector of RM

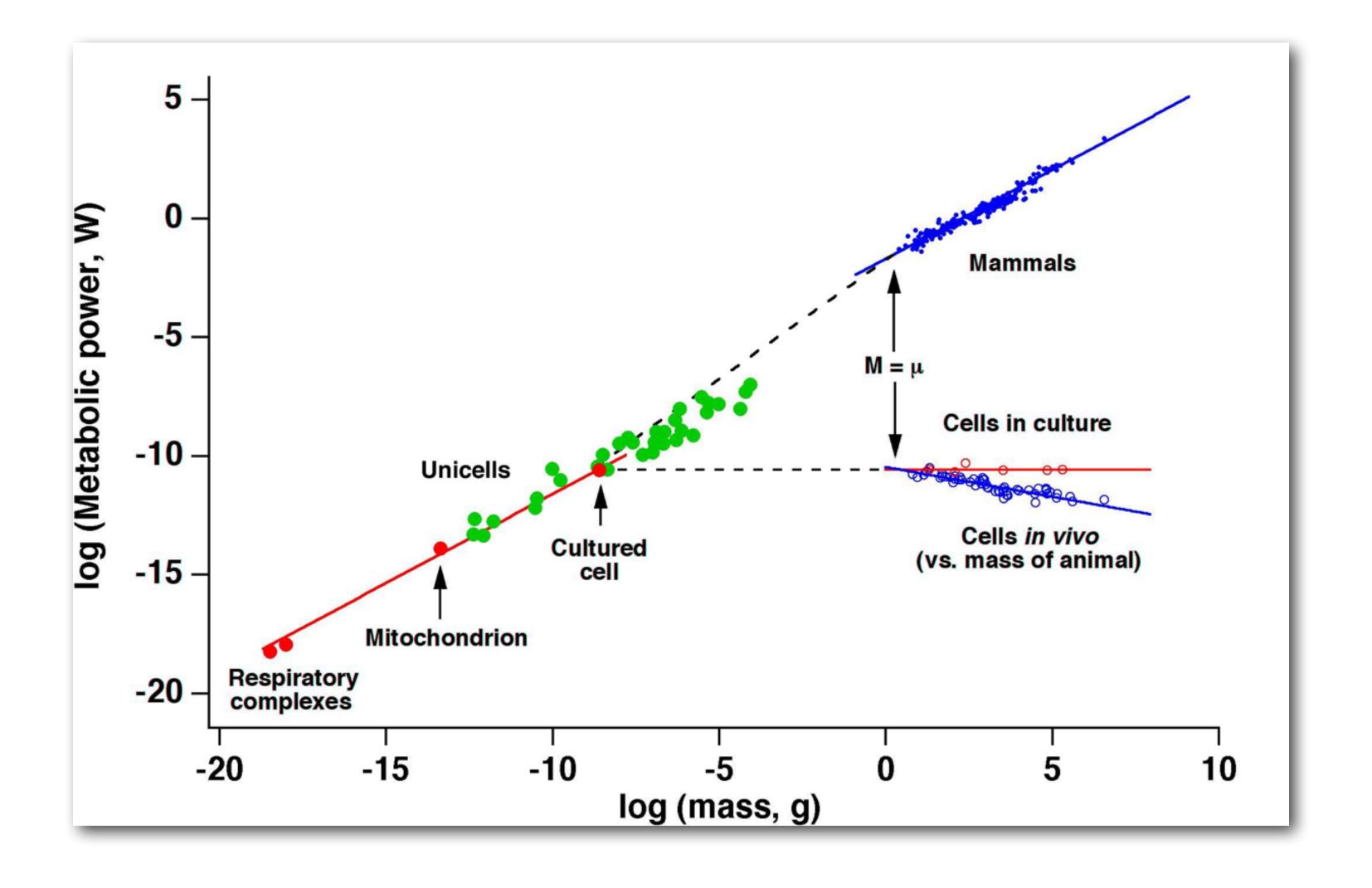


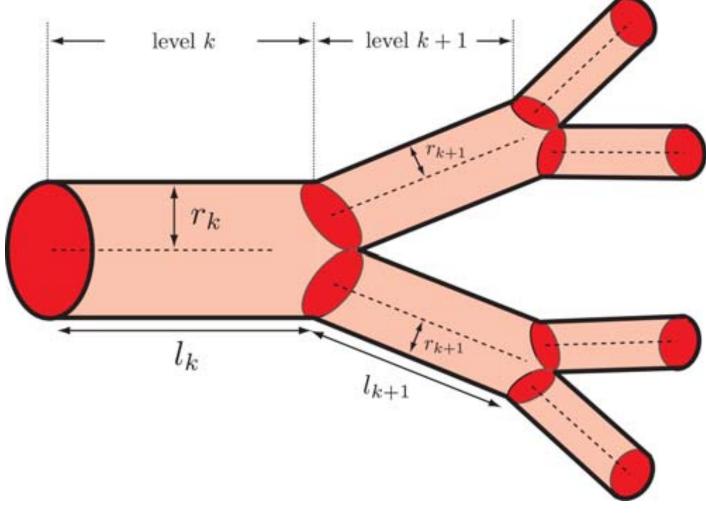
For the Physics Geeks..."Light Cones"



For the Evolution Geeks. ... "Life Cones" MAXIMUM VELOCITY OF GENETIC INFORMATION TRANSFER 1 mutation per genome per generation Space = mutations per generation **The Present** 

# $B = kM^{3/4}$





#### Minimize reflection [0, k]

$$b_2 = r_{k+1}/r_k = n^{-1/2}$$

#### Minimize dissipation [k+1, N]

$$b_1 = r_{k+1}/r_k = n^{-1/3}$$

West, Brown & Enquist, 1997

Metabolic free Energy is required for sustained propagation of genetic information:

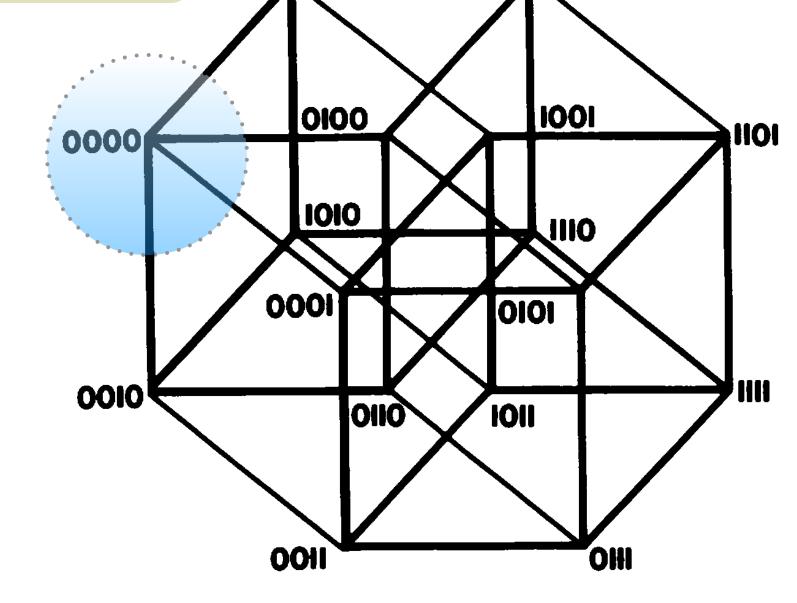
mutation is upper bounded by

$$\mu = qM^{-\frac{1}{4}}$$

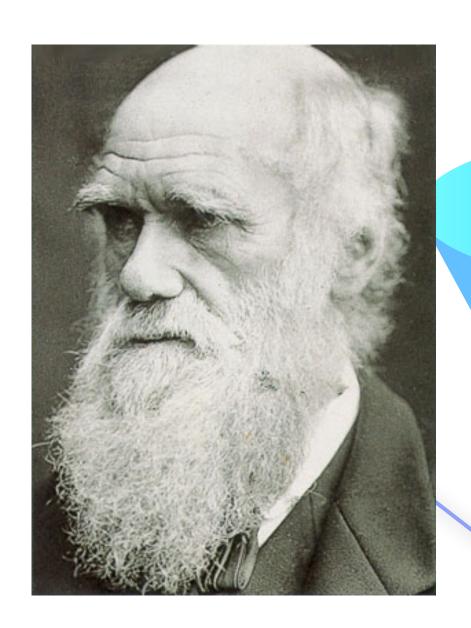
(empirical scaling law)

$$\dot{g}_i = \sum_{j}^{2^n} g_j r_j(\mathbf{g}) m_{ij} - g_i \bar{f}$$

$$m_{ij} = \mu^{H(i,j)} (1 - \mu)^{L-H(i,j)}$$



$$\mu = qM^{-\frac{1}{4}}$$



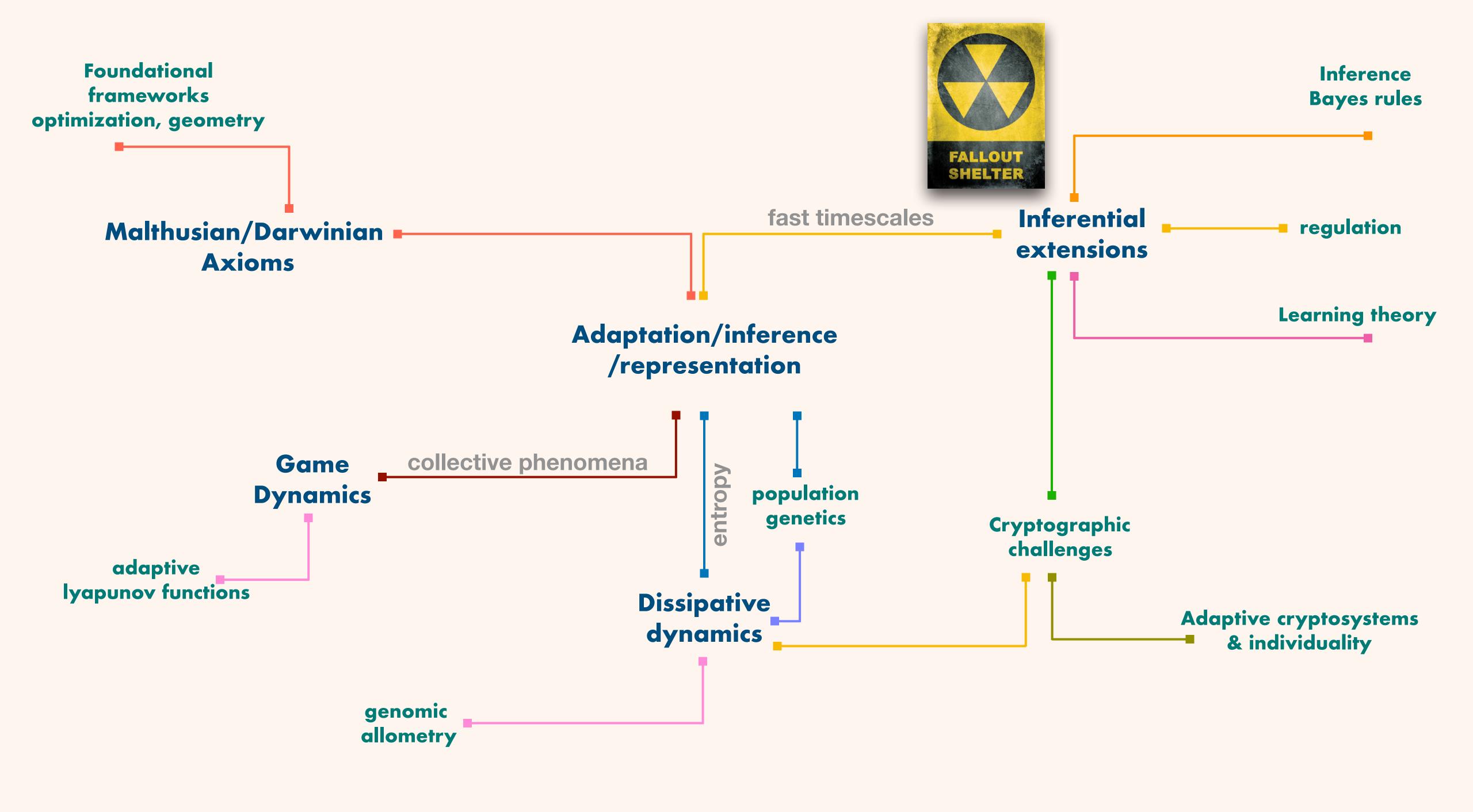
**The Present** 

Optimal relationship between Stored Information and Energy

$$L(M) = \frac{1}{\mu} = kM^{1/4} = k'B^{1/3}$$
 observation theory 1 theory 2

Space = mutations per generation

Time = generation



# An Essay towards solving a Problem in the Doctrine of Chances. By the late Rev. Mr. Bayes, communicated by Mr. Price, in a letter to John Canton, M. A. and F. R. S. (1763)



#### PROBLEM.

Given the number of times ion which an unknown event has happende and failed: Required the chance that the probability of its happening in a single trial lies somewhere between any two degrees of probability that can be named.

#### SECTION I.

DEFINITION 1. Several events are *inconsistent*, when if one of them happens, none of the rest can.

- 2. Two events are *contrary* when one, or other of them must; and both together cannot happen.
- 3. An event is said to *fail*, when it cannot happen; or, which comes to the same thing, when its contrary has happened.
  - 4. An event is said to be determined when it has either happened or failed.
- 5. The *probability of any event* is the ratio between the value at which an expectation depending on the happening of the event ought to be computed, and the chance of the thing expected upon it's happening.
  - 6. By *chance* I mean the same as probability.
- 7. Events are independent when the happening of any one of them does neither increase nor abate the probability of the rest.

$$P(H|E) = \frac{P(E|H)P(H)}{P(E)}$$

$$P_x(t) = P_x(t-1) \frac{L_x(t-1)}{\bar{L}(t-1)}$$

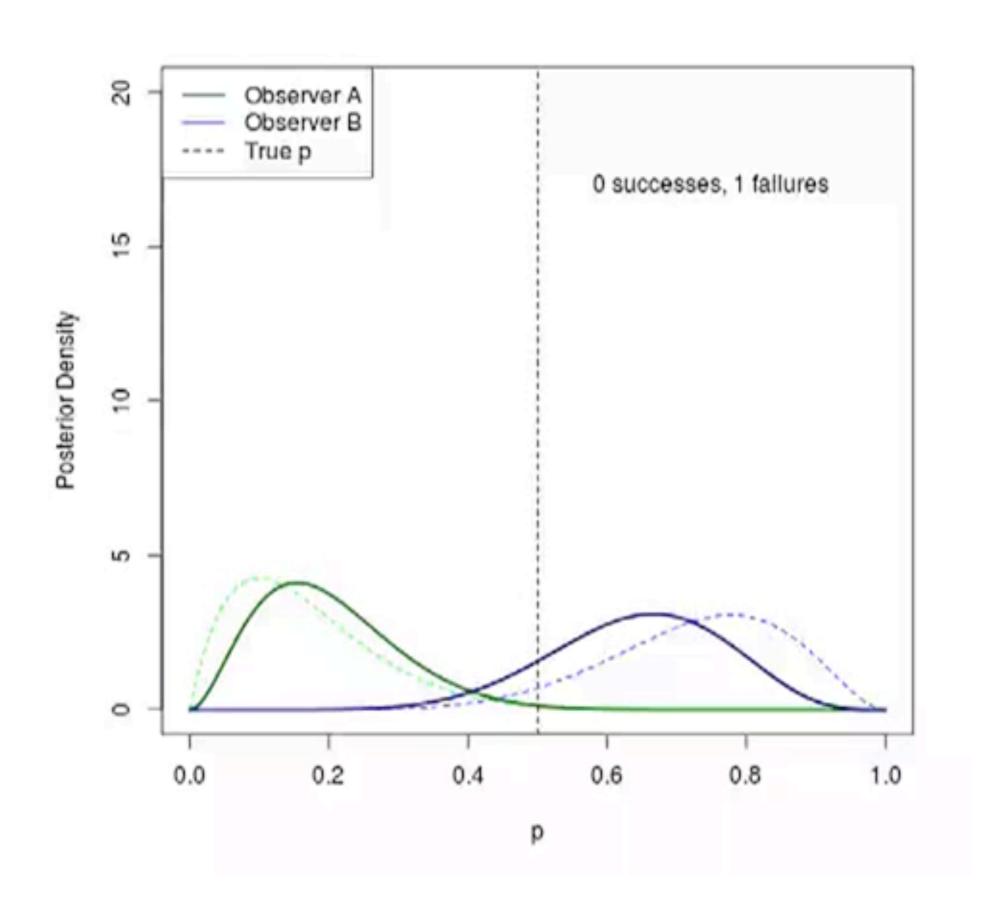
$$\Delta P_x(t) = P_x(t-1) \frac{L_x(t-1)}{\bar{L}(t-1)} - P_x(t-1)$$

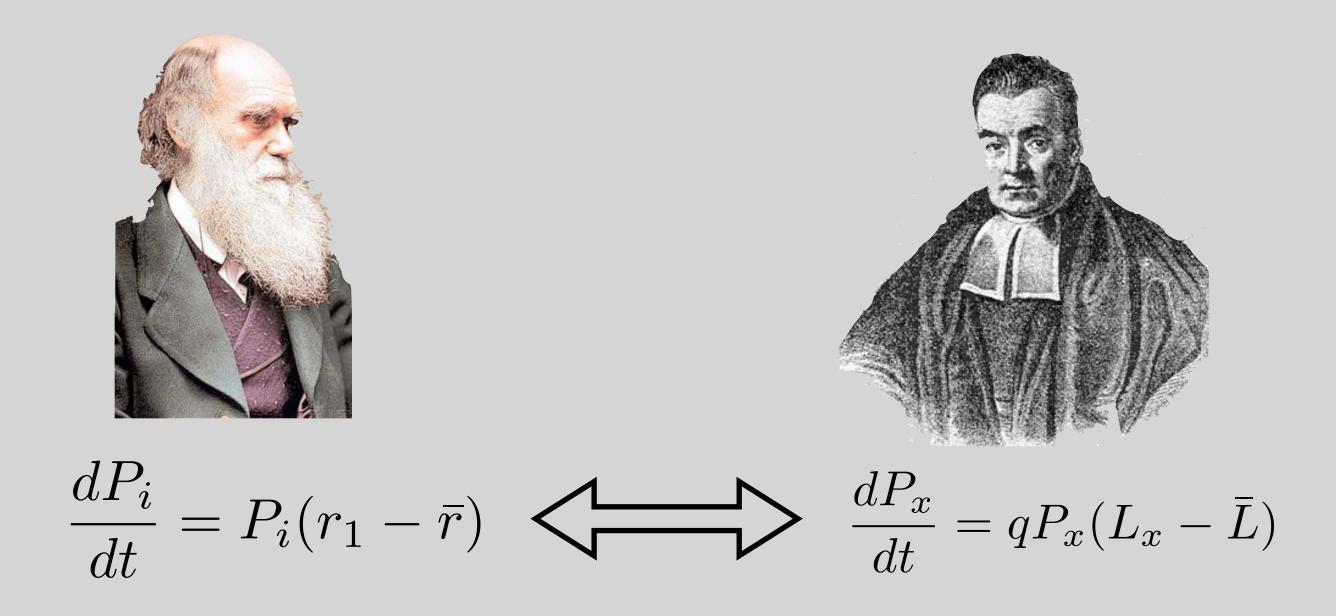
$$\Delta P_x(t) = P_x(t-1)(\frac{L_x(t-1)}{\bar{L}(t-1)} - 1) = \frac{1}{\bar{L}(t-1)}P_x(t-1)(L_x(t-1) - \bar{L}(t-1))$$

$$t \to 0$$

$$\frac{dP_x}{dt} = qP_x(L_x - \bar{L})$$

# Bayes rhymes with Darwin





**Bayesian Darwinism** 

Offspring are the likely hypotheses of parents, formulated through the success of ancestors at predicting the current state of the world

**Darwinian Bayesianism** 

Successful hypotheses are the surviving ideas in the battle of existence in the environment of experiments

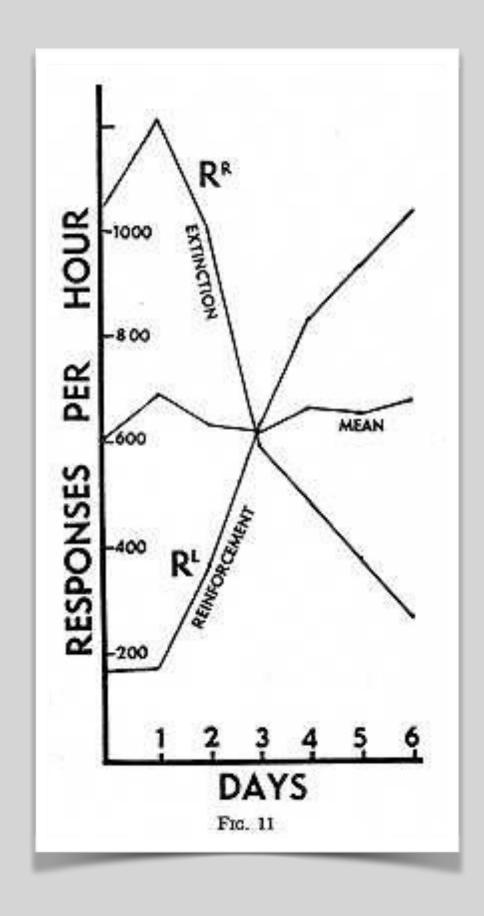


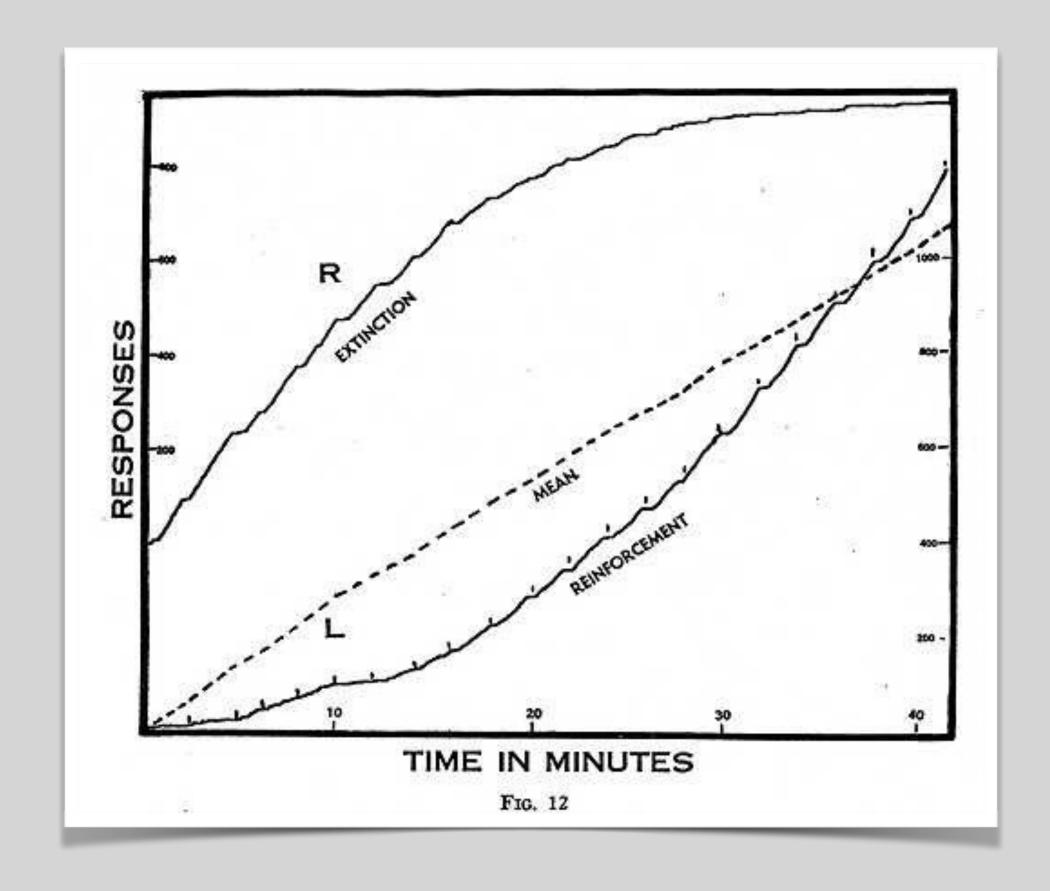


B.F. Skinner (1904-1990)

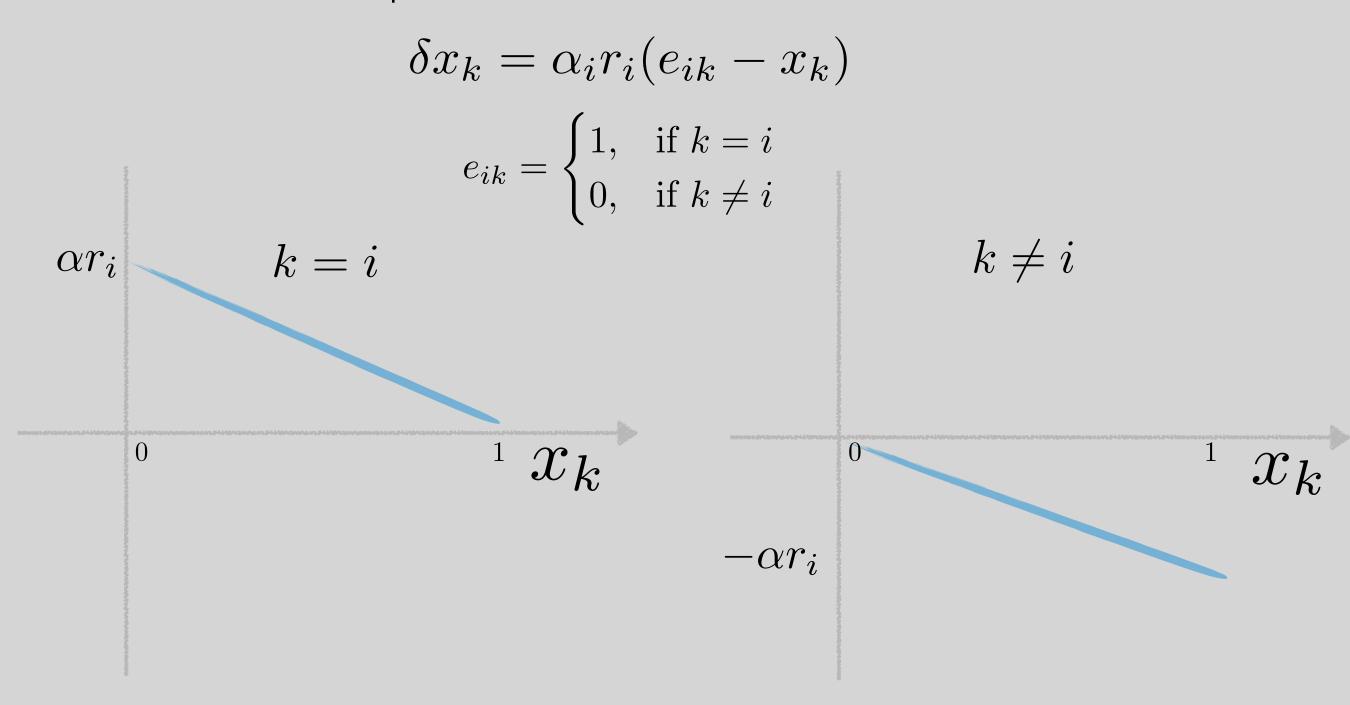
"Education is what survives when what has been learnt has been forgotten"

## Are Theories of Learning Necessary? B.F. Skinner. Psych Rev. 1950





Linear Operator Model: choice i is rewarded



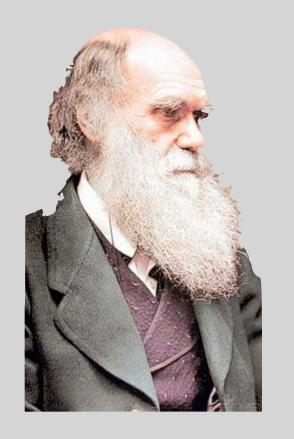
$$\Delta x_k = \sum_i x_i [\alpha r_i (e_{ik} - x_k)] \tag{1}$$

$$= \alpha \sum_{i} x_{i} r_{i} e_{ik} - \alpha x_{k} \sum_{i} x_{i} r_{i}$$
 (2)

$$= \alpha x_k r_k - \alpha x_k \sum_i x_i r_i \tag{3}$$

$$t \to 0$$

$$\frac{dx_k}{dt} = \alpha x_k (r_k - \bar{f})$$



$$\frac{dP_i}{dt} = P_i(r_1 - \bar{r})$$

Replication & competition

**Evolution** 



$$\frac{dP_x}{dt} = qP_x(L_x - \bar{L})$$

Estimating Probabilities

Inference



$$\frac{dP_i}{dt} = P_i(r_1 - \bar{r}) \qquad \frac{dP_x}{dt} = qP_x(L_x - \bar{L}) \qquad \frac{dx_k}{dt} = \alpha x_k(r_k - \bar{f})$$

Law of effect

Learning

### distribution of organisms

### distribution of environments

$$D_{KL}(X||Y) = \sum_{i} x_{i} \ln\left(\frac{x_{i}}{y_{i}}\right) \approx \frac{1}{2} \int x(\partial_{\epsilon} \ln x)(\partial_{\epsilon} \ln x) dx$$

$$D_{KL}(X,Y||XY) = \sum_{y} p(y) \sum_{x} p(x|y) \ln\frac{p(x|y)}{p(x)} = I(X;Y)$$

$$I(X;Y) = H(X) + H(Y) - H(X,Y)$$

$$= H(Y) - H(Y|X)$$

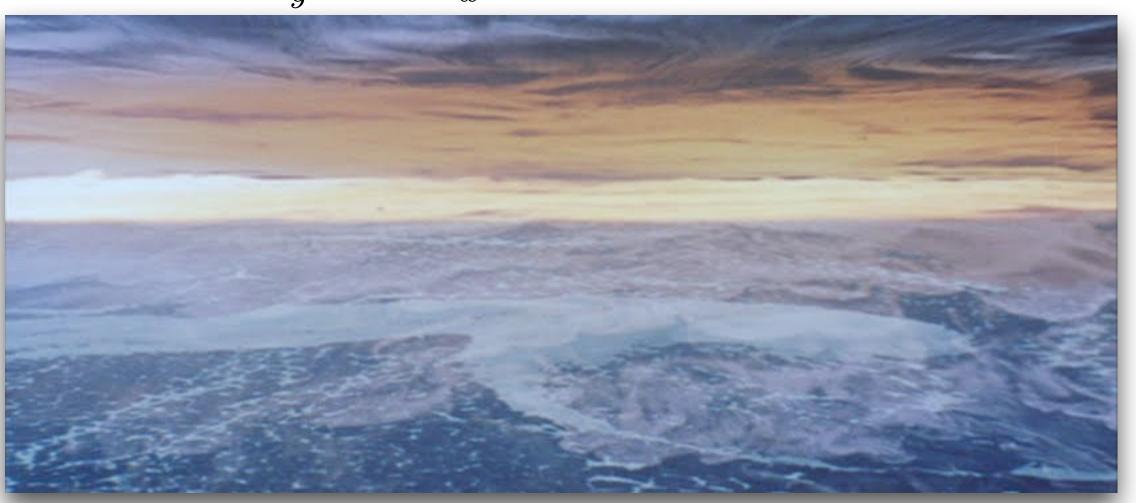


adaptation is an optimization dynamics generating global information

### The Solaris Paradox

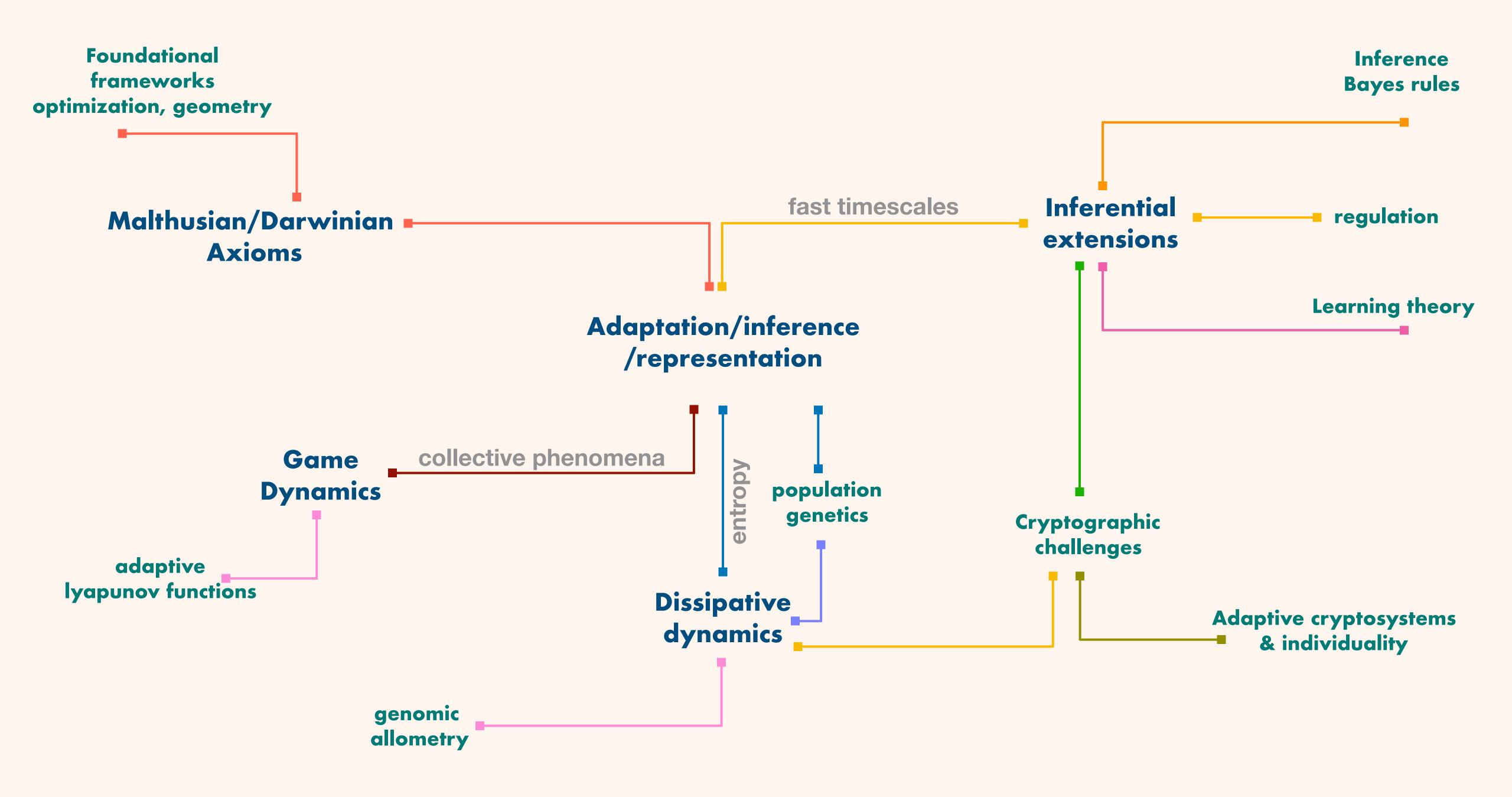
$$D_{KL}(X,Y||XY) = \sum_{y} p(y) \sum_{x} p(x|y) \ln \frac{p(x|y)}{p(x)} = I(X;Y)$$

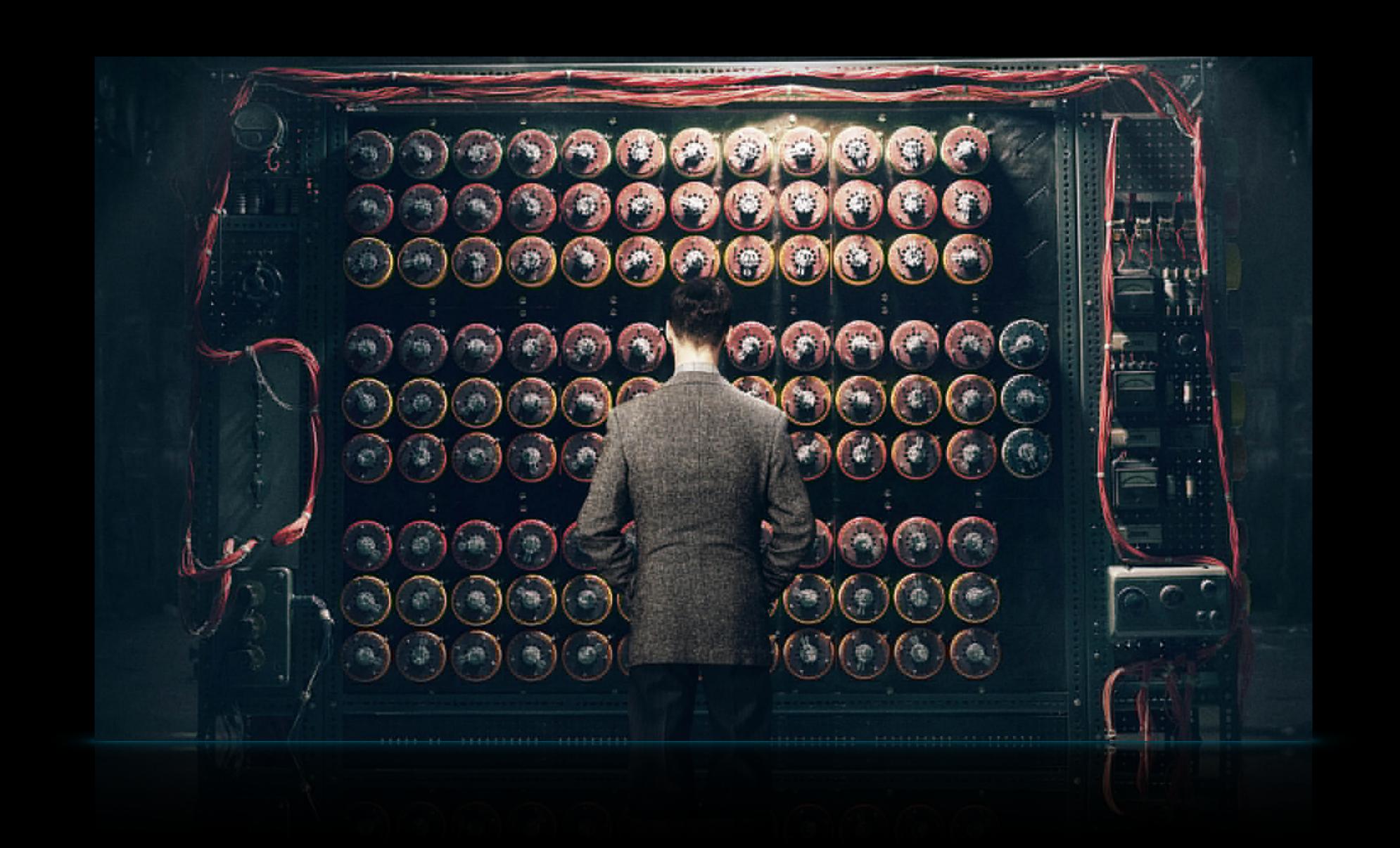






images from: Solaris. DIR Andrey Tarkovsky (1972)





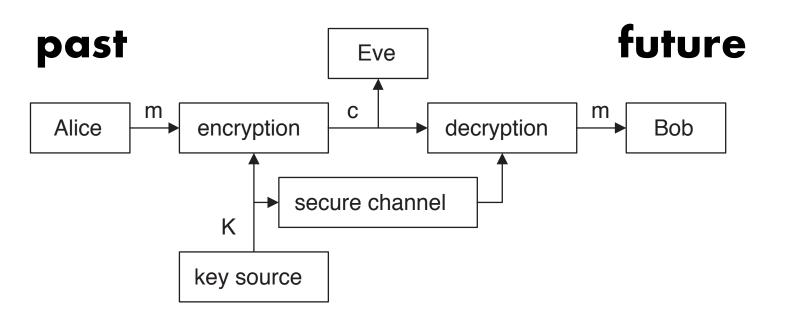
$$\dot{g}_i = \sum_{j}^{2^n} g_j r_j(\mathbf{g}) m_{ij} - g_i \bar{f}$$

imitating adaptation

$$\dot{r}_i = g_i \sum_j c_{ij} g_j (\frac{1}{2} \tanh(f(r_j - r_i) + 1) - er_i)$$

$$\hat{g}_i o 1/n \ \ orall i i i mitation terminates optimization$$

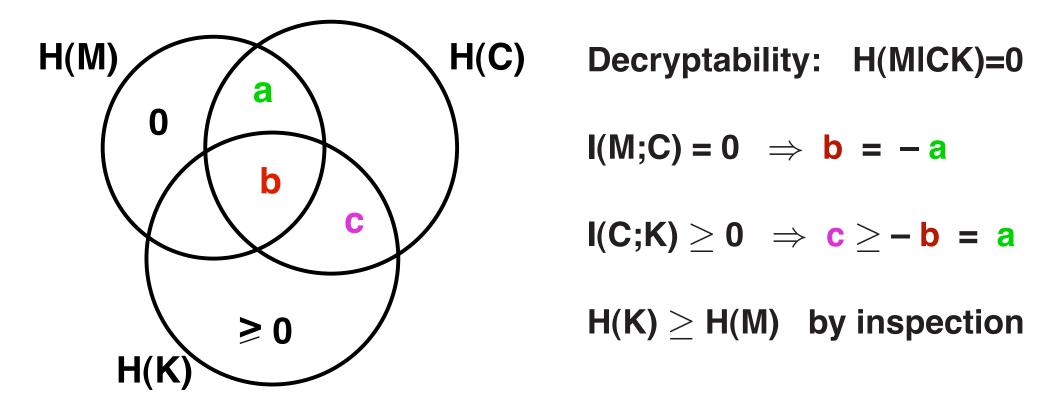
need to encrypt *r* by minimizing communication



### Shannon's theorem

Theorem:  $H(K) \ge H(M)$  for every perfect cipher.

#### **Proof:**



i.e. Eve does not know everything about Alice: they are distinct *individuals* 

# Individuals are persistently distinct informational aggregates that reliably transmit useful information from the past into the future

- Krakauer, D.C. 2017 Cryptographic Nature. IN: From Matter to Life. Eds. Sara Walker, Paul Davies, George Ellis. 155-174 and ArXiV
- ▶ Krakauer, D.C. Bertschinger, N. Olbrich, E. Ay, N. and Flack, J.C. 2017 The information theory of individuality. Theor. in Bioscience (Sub) and ArXiV.



not a battery



you are a history book

you are

