Is there a role for random graphs?
Models of percolation and cascades

Raissa D’Souza
University of California, Davis
Dept of Mech. and Aero. Eng., Dept of CS
Complexity Sciences Center
External Professor, Santa Fe Institute
The past decade, a “Science of Networks”:
(Physical, Biological, Social)

- **Geometric** versus **virtual** (Internet versus WWW).
- **Natural** /spontaneously arising versus **engineered** /built.
- Each network may **optimize** something unique.
- Fundamental **similarities** and **differences** to guide design/understanding/control.
- Interplay of **topology** and **function**?
- Up until now, **studied largely** as individual networks in isolation.

NRC, 2005
Single Network View

- **Broad scale** degree distributions ubiquitous.
- **Small world** effect (small diameter and local clusters).
- **Vulnerability** to “hub” removal / **resilience** to random removal.
- **Percolation**, spreading and epidemics (phase transitions)
- **Cascades**.
- **Synchronization**.
- **Random walks** / **Page rank**.
- **Communities** / subnetworks.
- **Structural roles** of nodes.
A collection of interacting networks:

- **Transportation Networks/Power grid** (distribution/collection networks)
- **Biological networks** - protein interaction - genetic regulation - drug design
- **Computer networks**
- **Social networks** - Immunology - Information - Commerce

- E-commerce → WWW → Internet → Power grid → River networks.
- Biological virus → Social contact network → Transportation networks → Communication networks → Power grid → River networks.
Interdependent networks
What are the simplest, useful, abstracted models?

• What are the emergent new properties?
  – Host-pathogen interactions
  – Phase transition thresholds

• Interactions: Cooperative, competitive, neutral?

• How do demands in one system shape the performance of the others? (e.g., demand informed by social patterns of communication)

• How do constraints on one system manifest in others?
  – (River networks shape placement of power plants)
  – (Overlay networks)

• Coupling of scales across space and time / co-evolution.
Models of interacting networks

- **Random graphs & branching processes**
  (“Typical” graph consistent with specified parameters)

- **Phase transitions**
  (The surprising consequences of interactions)

- **Motifs** (distinguishing real systems from random graphs)
Modeling networks as random graphs


- Configuration models (Bollobás 1980, Molloy and Reed RSA 1995). Enumerating over all networks with specified \( \{p_i\} \).

- Preferential attachment (Barbási-Albert 1999, etc.)

- Growth by copying (Kumar, Raghavan, Rajagopalan, Sivakumar, Tomkins, Upfal FOCS 2000), including duplication/mutation (Vazquez, Flammini, Maritan, Vespignani, ComPlexUs 2003)

- Random graphs analysis considers the ensemble of all graphs that can be constructed consistent with specified properties.
Cautions for use of random graphs

- Ensemble not necessarily representative

- Degree distribution is often not enough:

  Doyle, et. al., PNAS 102 (4)2005.

All these have same deg dist, $p_i$:

- Graph distance ... complicated to build in Euclidean space
Opportunities for random graphs?

- Enhance/delay onset of percolation

- Local optimization models:
  tradeoffs between Euclidean and tree metrics

- Epidemic spreading: SIS/SIR

- Socio-technical models (“Task oriented social networks”)
  - Wen, R.D, Devanbu, Filkov (under review): OSS systems: shared ownership of tasks good; but need a project lead.

- Signatures for onset of phase transitions
Wiring which respects group structures percolates earlier!

(Also tradeoffs between sparser and denser subnetworks.)

- Probability distribution for node degrees: \( \{ p_{k_a k_b}^a, p_{k_a k_b}^b \} \)
- Generating functions to calculate properties of the ensemble of such networks.
Calculating optimal interconnectivity

(Author Summary may be of particular interest)

- Branching process on multi-type random-regular graphs
  \[ z_a = 3 \]
  \[ z_b = 4 \]
  \[ p = 0.1 \]

- Simulations of sandpiles on real power grid topologies
A view from the UK’s Chief Science Advisor

(Source: Prof. Brian Collins, Chief Science Advisor, UK Dept of Transport)

Thanks to: