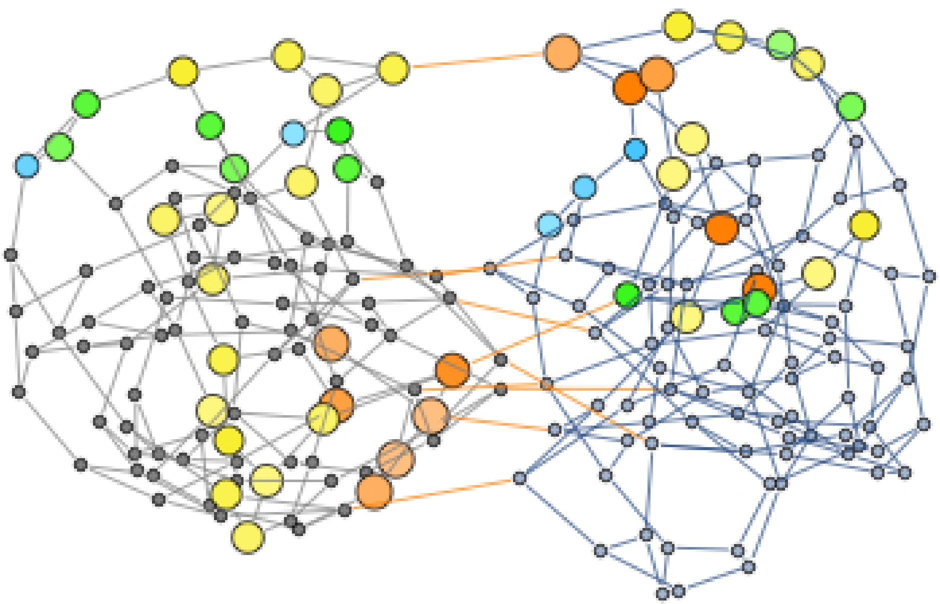
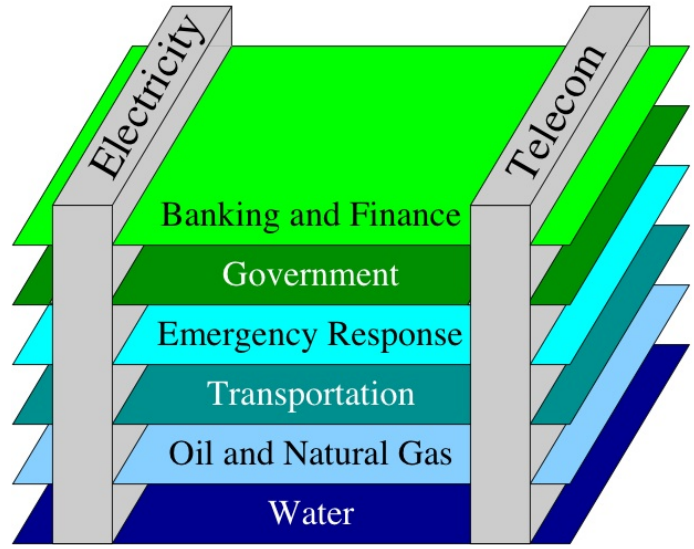
Is there a role for random graphs?
Models of percolation and cascades

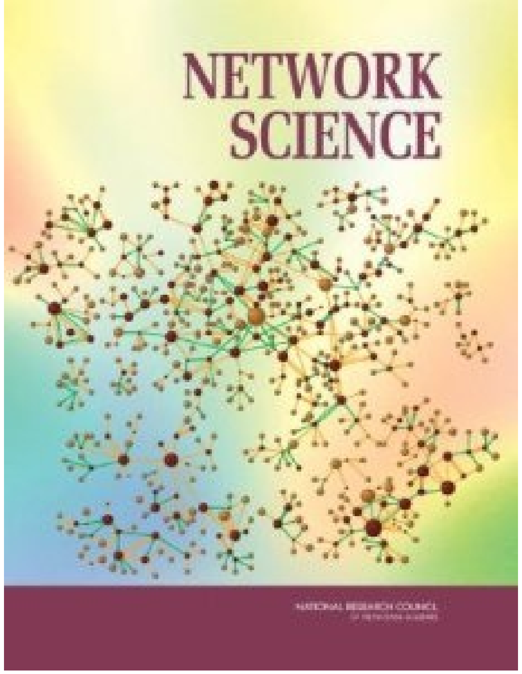




Raissa D'Souza
University of California, Davis
Dept of Mech. and Aero. Eng., Dept of CS
Complexity Sciences Center
External Professor, **Santa Fe Institute**

The past decade, a "Science of Networks": (Physical, Biological, Social)

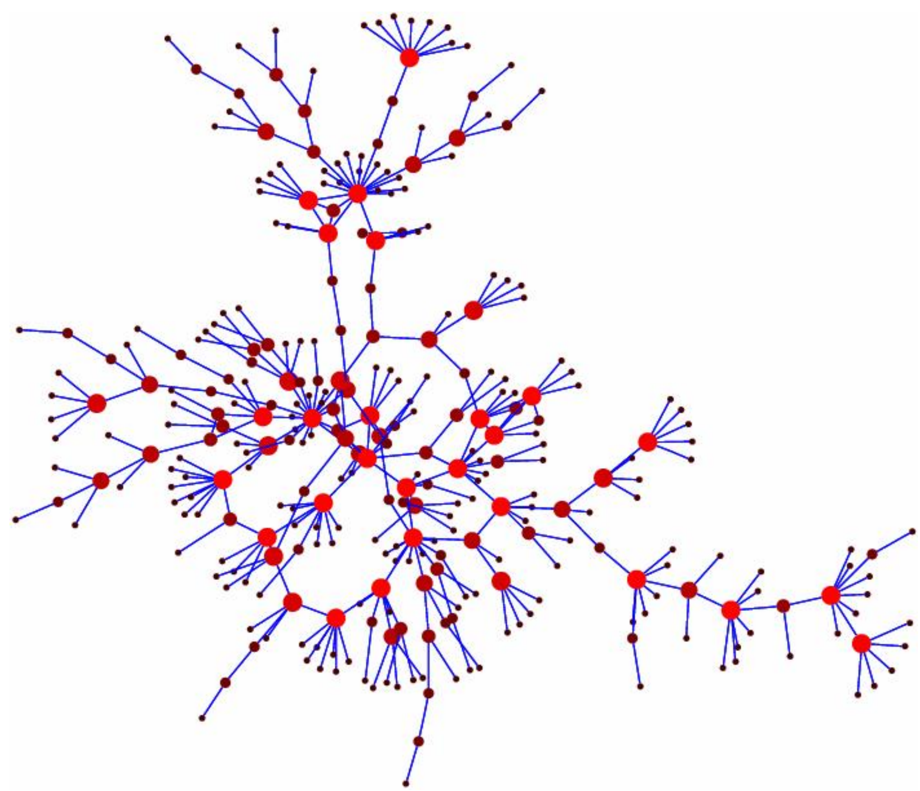
- **Geometric** versus **virtual** (Internet versus WWW).
- **Natural** /spontaneously arising versus **engineered** /built.
- Each network may **optimize** something unique.
- Fundamental **similarities** and **differences** to guide design/understanding/control.
- Interplay of **topology** and **function** ?
- Up until now, **studied largely as individual networks in isolation .**



NRC, 2005

Single Network View

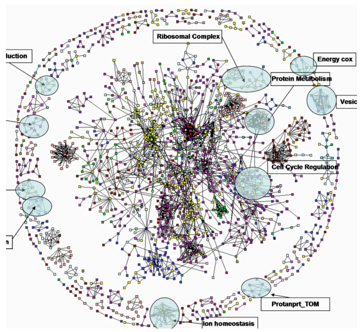
- **Broad scale** degree distributions ubiquitous.
- **Small world** effect (small diameter and local clusters).
- **Vulnerability** to "hub" removal / **resilience** to random removal.
- **Percolation**, spreading and epidemics (phase transitions)
- **Cascades**.
- **Synchronization**.
- Random walks / **Page rank**.
- **Communities** / subnetworks.
- **Structural roles** of nodes.



A collection of interacting networks:

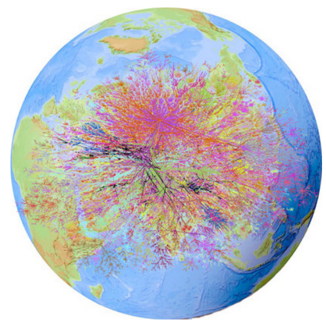
Networks:

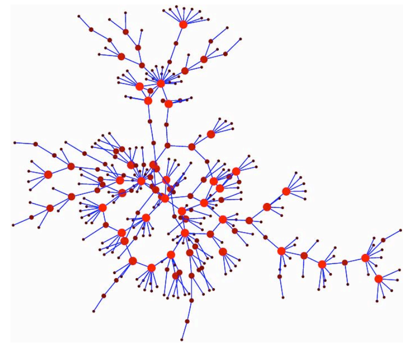
Transportation Networks/ Power grid
(distribution/ collection networks)



Biological networks
- protein interaction
- genetic regulation
- drug design

Computer networks





Social networks
- Immunology
- Information
- Commerce

- E-commerce $\rightarrow$ WWW $\rightarrow$ Internet $\rightarrow$ Power grid $\rightarrow$ River networks.
- Biological virus $\rightarrow$ Social contact network $\rightarrow$ Transportation networks $\rightarrow$ Communication networks $\rightarrow$ Power grid $\rightarrow$ River networks.

Interdependent networks
What are the simplest, useful, abstracted models ?

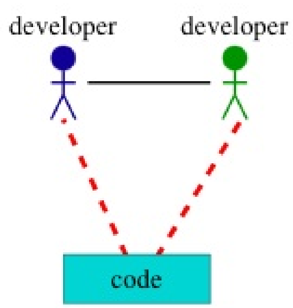
- What are the **emergent new properties**?
  - Host-pathogen interactions
  - Phase transition thresholds
- Iteractions: **Cooperative**, **competitive**, **neutral** ?
- How do **demands** in one system shape the performance of the others? (e.g., demand informed by social patterns of communication)
- How do **constraints** on one system manifest in others?
  - (River networks shape placement of power plants)
  - (Overlay networks)
- **Coupling of scales across space and time** / co-evolution.

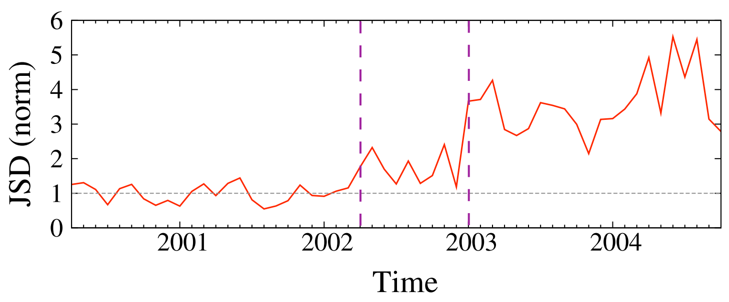
Models of interacting networks

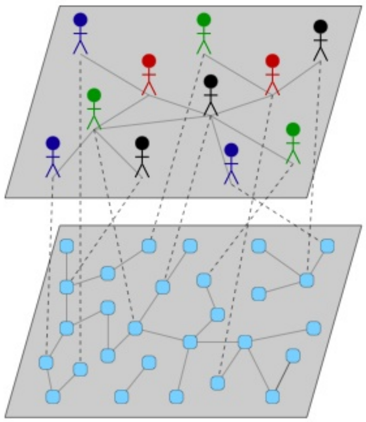
- **Random graphs & branching processes**
  (“Typical” graph consistent with specified parameters)

- **Phase transitions**
  (The surprising consequences of interactions)

- **Motifs** (distinguishing real systems from random graphs)



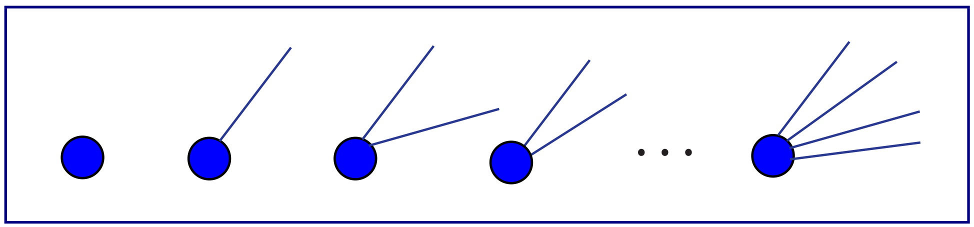




Modeling networks as random graphs

- Erdős and Rényi random graphs (1959, 1960).
  Phase transition in large-scale connectivity.

- Configuration models (Bollobás 1980, Molloy and Reed *RSA* 1995). Enumerating over all networks with specified $\{p_i\}$.



- Preferential attachment (Barbási-Albert 1999, etc.)

- Growth by copying (Kumar, Raghavan, Rajagopalan, Sivakumar, Tomkins, Upfal *FOCS* 2000), including duplication/mutation (Vazquez, Flammini, Maritan, Vespignani, *ComPlexUs* 2003)

- **Random graphs analysis considers the <u>ensemble</u> of all graphs that can be constructed consistent with specified properties.**

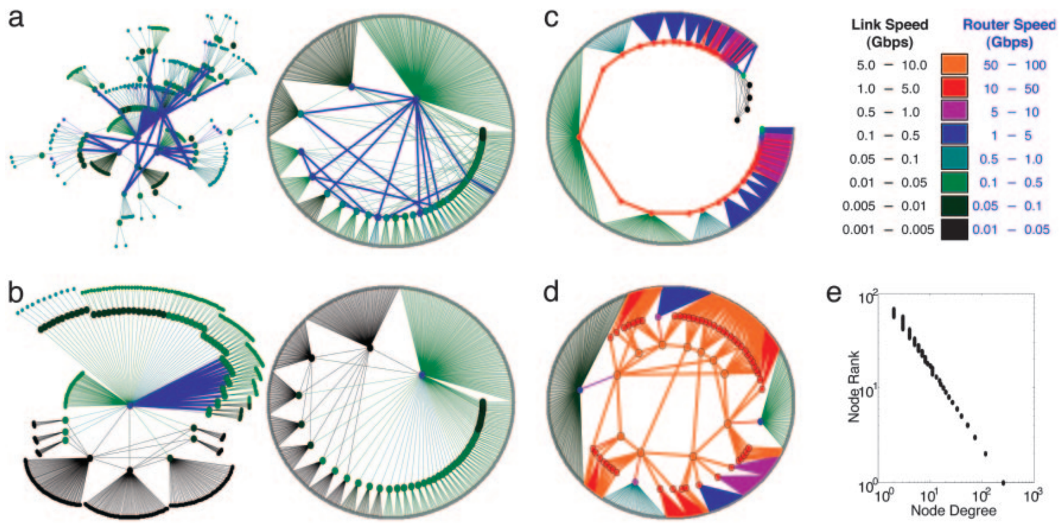
Cautions for use of random graphs

- Ensemble not necessarily representative

- Degree distribution is often not enough:

Doyle, et. al., PNAS **102** (4)2005.

All these have
same deg dist, p_i:



- Graph distance ... complicated to build in Euclidean space

Opportunities for random graphs?

- Enhance/delay onset of percolation
  – Achlioptas, R.D., Spencer, *Science*, 2009.
  – R.D., Mitzenmacher *Phys. Rev. Lett.*, 2010.
  – Chen, R.D. *Phys. Rev. Lett.* 2011.

- Local optimization models:
  tradeoffs between Euclidean and tree metrics
  – R.D., Borgs, Chayes, Berger, Kleinberg, *PNAS* 2007.

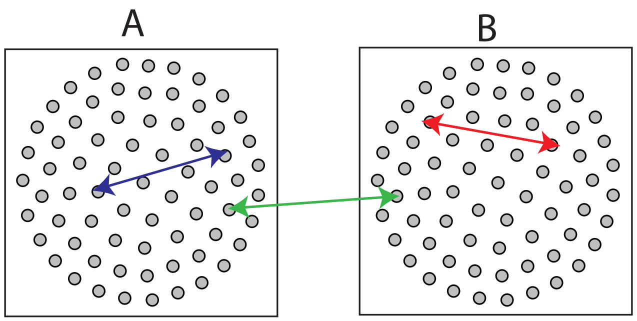
- Epidemic spreading: SIS/SIR

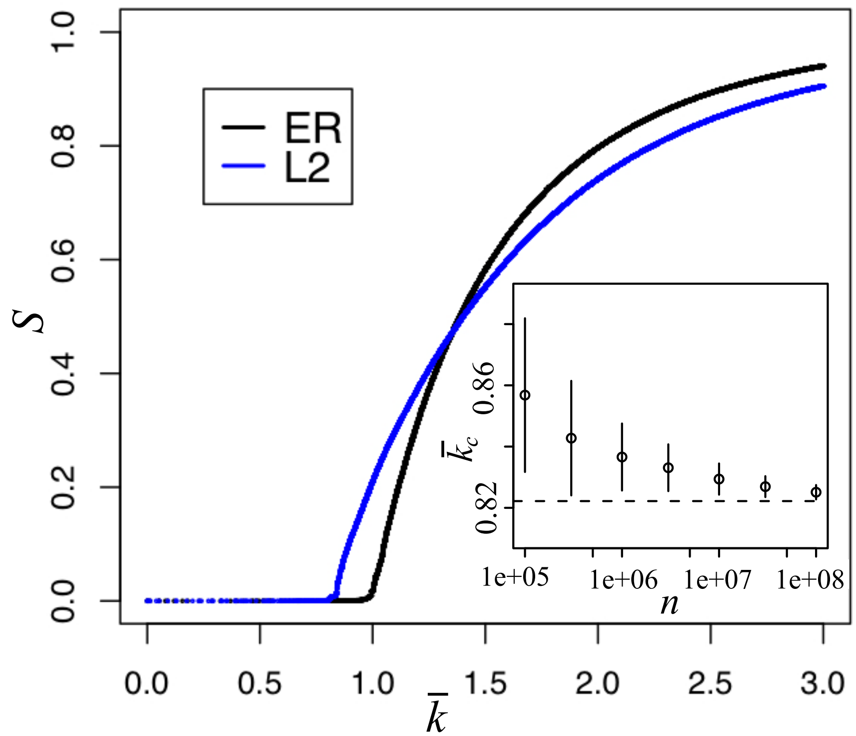
- Socio-technical models ("Task oriented social networks")
  – Wen, R.D, Devanbu, Filkov (under review): OSS systems: shared ownership of tasks good; but need a project lead.

- Signatures for onset of phase transitions

Wiring which respects group structures percolates earlier!

(E. Leicht and R. D'Souza, arXiv:0907.0894)





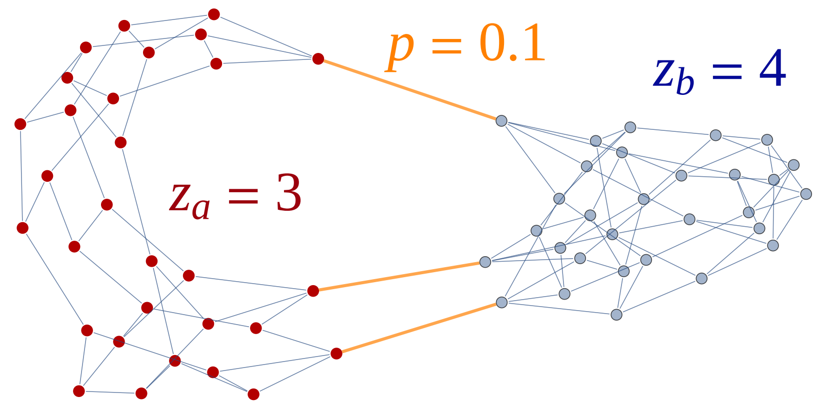
(Also tradeoffs between sparser and denser subnetworks.)

- Probability distribution for node degrees: $\{p^a_{k_a k_b}, p^b_{k_a k_b}\}$
- Generating functions to calculate properties of the ensemble of such networks.

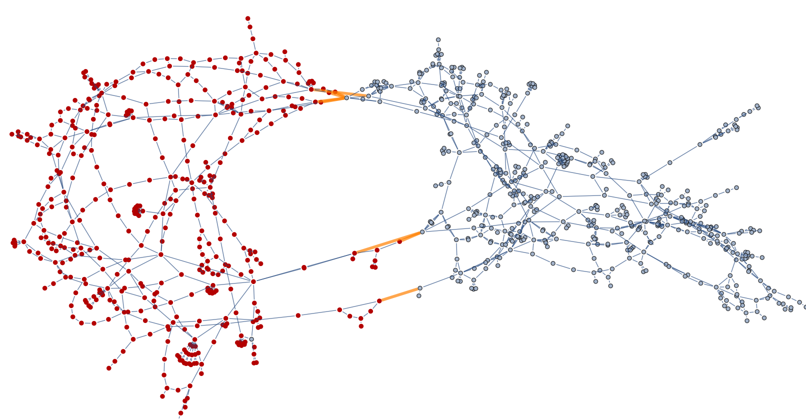
Calculating optimal interconnectivity

[Brummitt, R.D., Leicht, *PNAS* 109 (12) E680-E689, 2012.]
(Author Summary may be of particular interest)

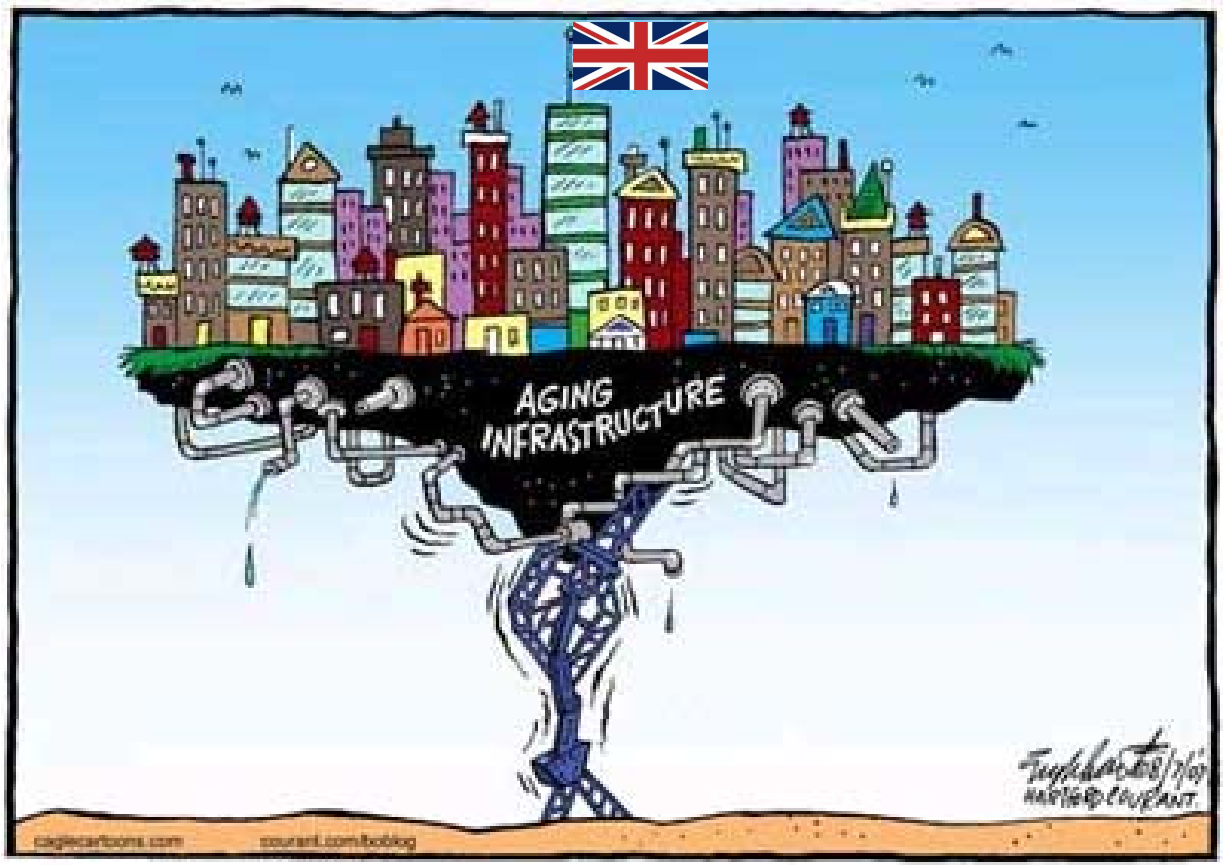
- Branching process on multi-type random-regular graphs



- Simulations of sandpiles on real power grid topologies



A view from the UK's Chief Science Advisor



(Source: Prof. Brian Collins, Chief Science Advisor, UK Dept of Transport)

Thanks to: