The roundtable

An agent-based model of conversation dynamics

Santa Fe Institute working paper

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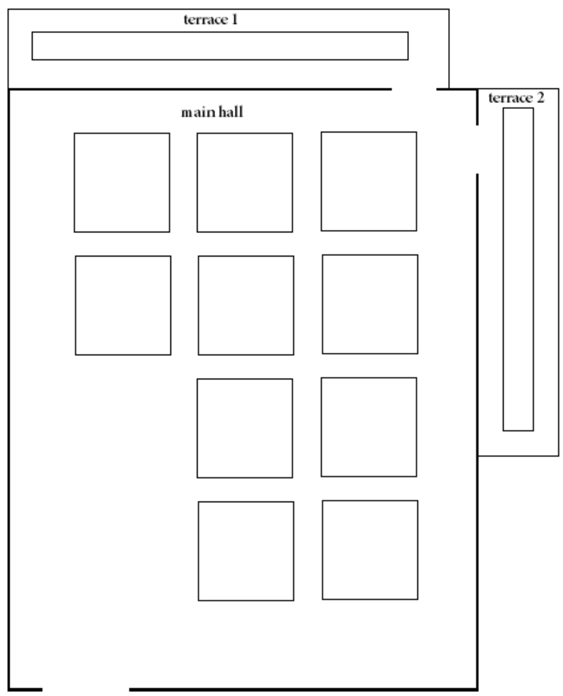
Abstract. We present and analyze an agent-based model of conversation dynamics. The model develops from intuitive assumptions derived from experimental evidence, it abstracts from conversation content and semantics while including topological and psychological information, and is driven by stochastic dynamics. The model exhibits rich behavior and can capture many aspects of real-life conversations. Its potential generalizations - including individual traits and preferences, memory effects and more complex conversational topologies - may find useful applications in other fields of research where dynamically-interacting and networked agents play a fundamental role.

1. Introduction

The glorious Santa Fe Institute's Complex Systems Summer School 2009 took place mainly in St. John's College - in the high end of beautiful Santa Fe. Breakfast, lunch and dinner were taken during fixed time slots in a fixed location, the cafeteria (sketched in Figure 1). Nearly all tables, both inside the cafeteria's main hall and on the external terraces, were arranged so that groups of up to 20 people could comfortably sit in square-like ensembles, consume their meals together and enter into inspiring conversation.

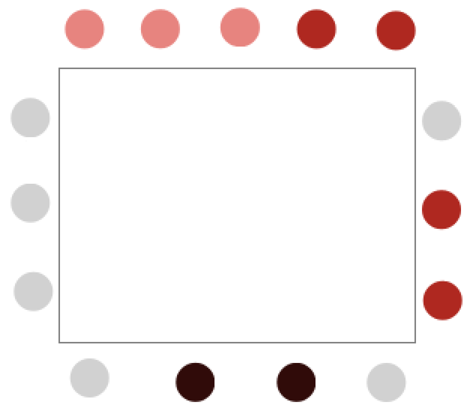
Figure 1: scheme of Saint John's cafeteria



Consequently, participants ate together many times during the four week-long summer school. Participants arrived in the cafeteria and picked available seats in an essentially random sequence. Therefore, the chance of assembling the same ensemble of chatting people more than few times over time was pretty low.

An observer could even incidentally discern that table conversations were not stable. Not only did table topics change in space and time; interestingly, not all participants seated around the same, specific table took part at all times in a table-wide conversation: usually, they took part in conversations that involved only a subset of the people seated around that table. As a result, each table had multiple, separate sub-conversations going on at the same time. Moreover, people taking part in one of these parallel chats usually did not remain involved in the same sub-conversation indefinitely, but tended to leave their original sub-conversation and join another, possibly neighboring one going on at their same table. A typical table situation is depicted in Figure 2, where people taking part in the same sub-conversation are represented by dots of the same color.

Figure 2: a typical table conversation situation.



We thought this phenomenon was no coincidence nor accident, but evidence of a probably general, underlying conversation dynamics. This can evidently match the experience of anyone. Thus, we got inspired by these systematic yet striking in-the-field observations to model the dynamics of conversational groups through interacting software agents. Our model is a pure act of creation and – hopefully - insight. We were not aware of nor consulted any model or reference eventually already existing in related literature - we kept ourselves deliberately ignorant.

2. The baseline model

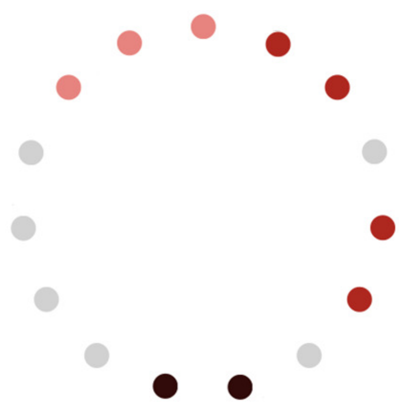
We defined our *baseline agent-based conversational model* by instantiating a set of simplifying yet, in our opinion, reasonable assumptions:

0. *Homogeneous initial condition.* At the beginning, all people participate in a unique conversation and are in the same state. The conversation starts with a random participant entitled to speak – she will be called *the speaker* - while all other participants are *listeners* (Figure 3). Other initial configurations can be imposed.

Figure 3: initial state of a conversation after the first speaker has been chosen

1. *Roundtable*. The participants are arranged in an ideal roundtable (*i.e.* a one-dimensional torus with periodic boundary conditions; see Figure 3 and following): each participant can in principle speak with any other participant, but she is in direct (*i.e.* spatial) contact only with her two nearest neighbors - which define her own topological neighborhood. Neighborhoods can be subsets of conversations, though this is not necessary nor always the case. While simplifying an actual conversational spatial topology[1] – wherein nodes (*i.e.* agents) may have arbitrary and time-varying neighborhoods - the roundtable assumption encodes a non-trivial spatial topology (a network with node degree equal to 2); particularly, it allows embedding what we below define as the "conversational principle of least effort".

Figure 4: the ideal roundtable

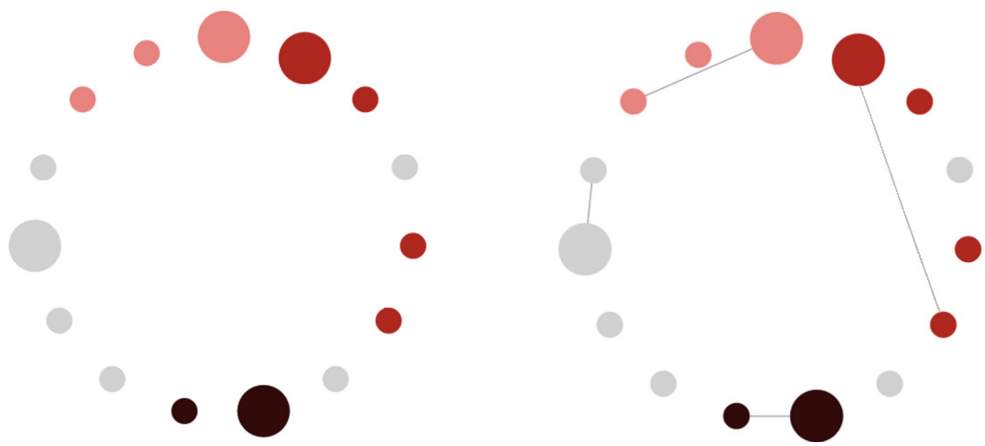


2. *Politeness*. In every conversational group, only one person (the speaker) speaks at any given time (see Figures 5a; speakers are depicted as larger circles) before another participant (a listener) of the same subgroup is entitled to speak. The speakers of all groups are appointed concurrently and simultaneously[2] (*i.e.* at every single conversation turn, all speakers assume their role only once, and do not change their status – *i.e.* stop speaking, appoint a new speaker and become listeners - before all groups have appointed their own speakers). This update rule introduces a turn-taking dynamics: at every turn each conversational group has a speaker different from that of the previous turn (see Figures 5b; turn taking is depicted by a link between the old and the new speaker agents).

[1] The conversational topology can be easily generalized adopting more connected or clustered networks. Also, one can introduce geometry-dependent probability distributions for the choice of speakers and/or other conversational phenomena.

[2] This synchronous and concurrent update rule can be relaxed by allowing the present speaker to be a possible outcome of a probability distribution, or allowing more than one speaker at the same time.

Figures 5: conversational groups with only one speaker (a), and turn taking (b).



3. *Abstraction from conversational content*. We model the succession of speakers within any given conversation group as a stochastic process. In principle, it is possible to use any kind of speaker-dependent[3] or history-dependent[4] probability distribution to establish the choice of the new speaker. Note that this approach is coherent with content abstraction, while still capable of generating formally complex conversation patterns, as documented below. The probability distribution adopted in the baseline model is uniform, *i.e.* speaker- and history-independent.

4. *Joining/leaving force balance*. Participants in a specific conversation want to remain in the conversation as long as they have the opportunity to speak regularly, while they wish to leave the conversation as soon as they feel excluded from or unexcited by the conversation, *e.g.* because they do not have the opportunity to speak or to be somehow actively involved in it up to their preferred degree. We model this lively behavior by assigning a happiness status to each participant of the conversation. The baseline scenario has all participants initially involved in the same table-wide conversation and assigned with the maximum level of the happiness scale, which is equal to that of anyone else[5] – *i.e.* we optimistically assume a person is happy to take part in a conversation that is about to start. The happiness level is then subjected to dynamic change. It is decreased by one unit for every conversation turn during which the participant

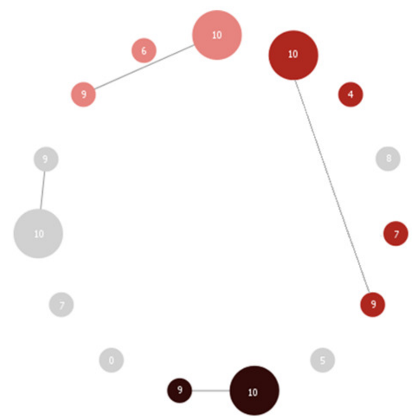
[3] A speaker-dependent probability distribution would reflect the speaker's own preferences or her own different probabilities to evoke responses from the listeners due to *e.g.* geometric closeness, common interests, social hierarchies and more.

[4] History-dependent probability distributions would introduce memory effects as well as reflect the ability of speakers to evoke responses from the listeners due to previous discourse patterns, *e.g.* some speakers might want to encourage interaction of people who have not spoken for a long time or, alternatively, they might have a tendency to keep on addressing participants who have spoken recently or better suit their moods.

[5] Such scale can be unique to each participant in the general case, to reflect her personality and individuality.

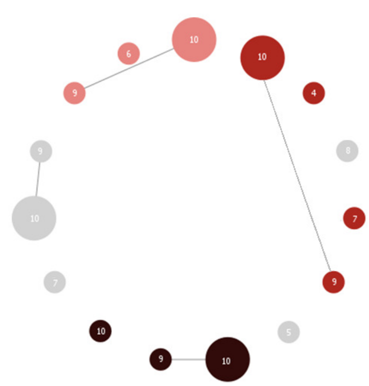
cannot speak, while it is reset back to the initial level when the person gets a new opportunity to speak[6] (see Figure 7). As soon as the happiness level drops to the minimum tolerated level, the participant becomes latent, *i.e.* she feels excluded enough to watch out around her for opportunities to enter another or new conversation[7]. Corollaries: a) a speaker is always fully happy; b) a latent is necessarily a listener.

Figure 6: happiness levels with maximum level of 10



5. *Neighborhood-based schism dynamics*. When a participant is in the latent state, she will look to her topological neighbors to be eventually engaged in a new conversation – schism then takes place. She will first check whether at least one of the neighbors is in turn latent: if this is the case, she will start a new conversation with her/them. Otherwise, she will anyway join the ongoing conversation of either of her neighbors[8]. In any case, her happiness level will recover and will be reset to the maximum level (see Figure 7 for an example of how a latent joins another conversation). The use of only local resources to escape from a stagnant conversation is what we define as the conversational principle of least effort.

Figure 7: participant in latent state joining another conversation



[6] More generally, the new speaking appointment can increase the happiness level by a fixed or even time-dependent number of units.
[7] Our agents can thus be considered as finite states automata with a set (ideally, a continuum) of states between the fully conversation-integrated state (*i.e.* the speaker - state of maximal happiness) and the fully excluded state (*i.e.* the latent - state of minimal happiness).
[8] In this case the choice of the conversation to be entered may be driven by a fixed priority (*e.g.* left neighbor first at all times), or stochastic (again, eventually history- or participant-dependent).

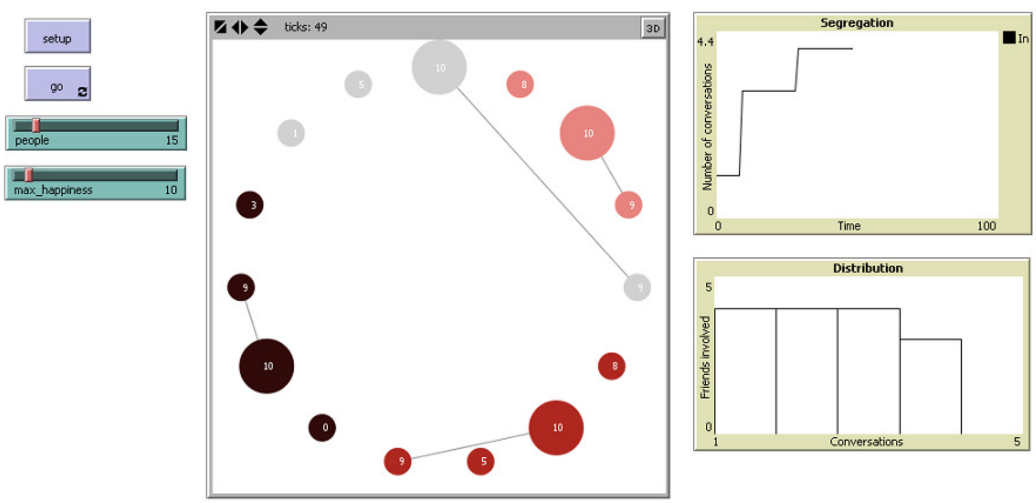
To verify the extent to which our naive assumptions capture realistic features of real-life conversations, we implemented them and inspected the emergent behavior they generate in an agent-based model. The simulative investigations were complemented with analytical methods and insights where possible.

3. Agent-based implementation of the baseline model[9]

As described in the previous section, in the baseline model every participant in the original conversation has the same initial happiness level (set to the maximum value), and makes use of a uniform probability distribution for the choice of a new speaker among the listeners. One can think of this configuration as a situation where a homogenous group of people is engaged in leisurely chat without selection biases due to accidental geometry, common interests, hierarchies or previous discourse patterns. We implemented the baseline model in NetLogo[10]; Figure 8 shows the customized graphical user interface.

If people would not switch to latent status and eventually leave the initial table-wide conversation, the whole process could be characterized by a standard Markov chain process defined by a transition matrix with 0's on the main diagonal (since each speaker cannot entitle herself to speak in the following turn) and all other entries equal to 1/(N-1). This process would evolve toward a unique stationary distribution over the participants as characterized by a transition matrix with all entries equal to 1/N.

Figure 8: customized NetLogo graphical user interface for the baseline model



The possible disappearance of people from a conversation group could also be handled by analytic methods in a straightforward manner. As long as there are more than 2 people in a conversation, there exists always a non-null probability that one participant will not speak before her happiness level decreases to the minimum value, driving her to leave the conversation. This is true independent of the number of participants in

[9] The NetLogo implementation of the basic model (file name basic.nlogo) can be found on the CSSS 2009 Wiki page.

[10] NetLogo is freely available at: http://ccl.northwestern.edu/netlogo/

the conversation and of their maximum happiness level. Number of participants in the conversation and maximum happiness levels influence anyway the expected waiting time until the stationary distribution is reached.[11] In short, the stationary state corresponds is achieved in the presence of conversation groups including only 2 people that speak alternatingly, plus a single group of 3 people (whose identity changes in time) in the case of odd total number of participants in the starting conversation.[12]

Possible addiction of newcomers into an ongoing conversation renders a direct analytic approach, even in this very basic scenario, more difficult or at least not obvious. Its implementation through an agent-based model, on the other hand, is quite straightforward. In the following sections we outline the main results of the software investigations.

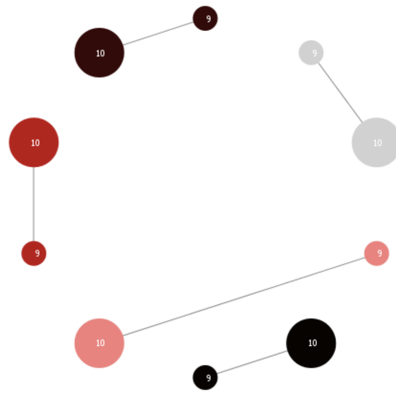
3.1 Stationary states

Running the agent-based model with the homogeneous initial condition, it is found that the initial table-wide conversation group splits over time into smaller and smaller conversation groups. This is akin to a spatial symmetry-breaking phenomenon: the initial, spatially-homogeneous system (*i.e.* lacking boundaries) evolves into one with spatially-defined boundaries. This splitting process continues - despite temporary increases of the sizes of conversation groups – until the conversation groups cannot split any further. When simulating an even number of people in the initial conversation, later conversation groups cannot split any further when all of them consist of 2 people. Then a stationary distribution over the size of conversation groups is reached. In this even case, the final, degenerate state is an absorbing state of the system. Moreover, the pairings within the 2-people conversation groups do not change over time. The final pairings within the 2-people groups are determined randomly, but are usually confined to the direct topological neighbors. A possible stationary state for the even case is shown in Figure 9.

[11] The more the participants in a conversation group, the higher the individual probability of leaving the conversation group and the shorter the expected waiting time until exit for any given person in the conversation group. By the same token, the higher the maximum happiness level, the smaller the individual probability of leaving and the longer the expected waiting time until exit.

[12] We are assuming that the speakers change with certainty, and that the maximum happiness levels are bigger than 1.

Figure 9: typical end state for even initial number of participants

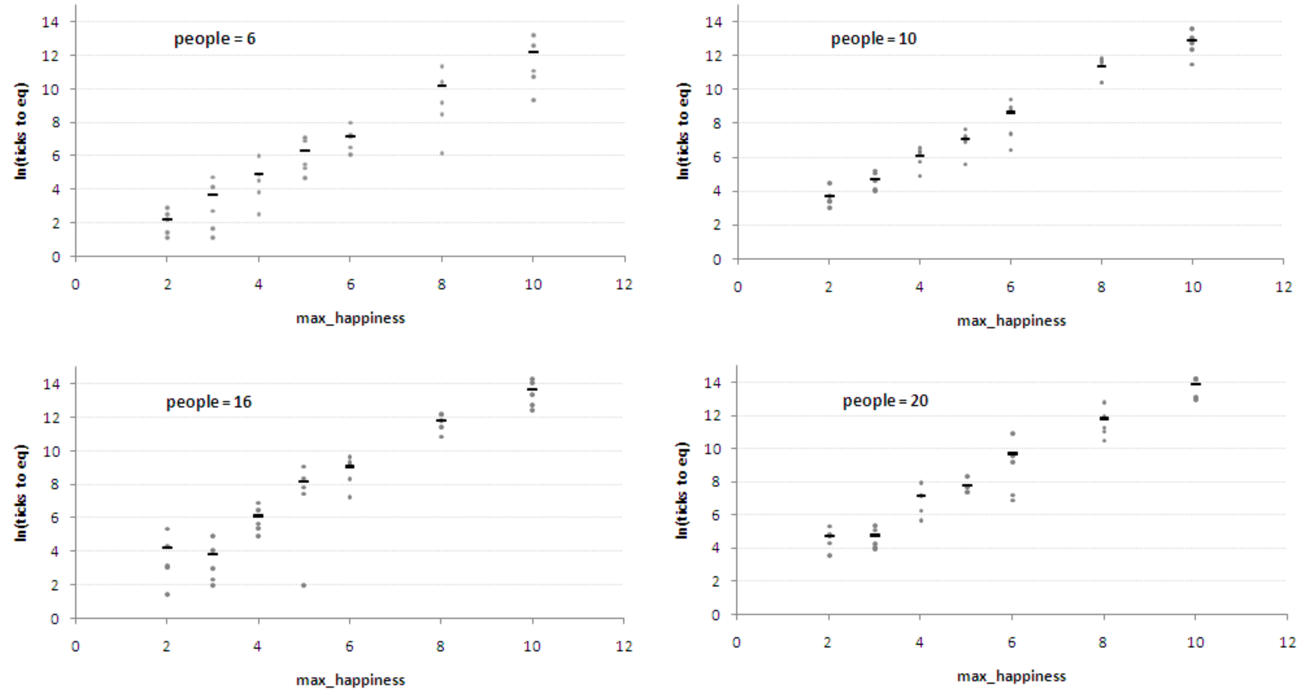


The number of turns needed to reach the stationary distribution depends on the initial number of participants and on the maximum happiness levels. Table 1 shows such statistical dependence for 5 different runs of the agent-based model. The average number of turns to equilibrium of the 5 different runs is the best estimate of the expected waiting time until equilibrium. The same data can be visualized by plotting the logarithm of the number of turns needed until equilibrium is reached versus the given maximum level of happiness (see Figure 10).

Table 1: turns to equilibrium for 5 conversation runs, given maximum happiness and even participant number

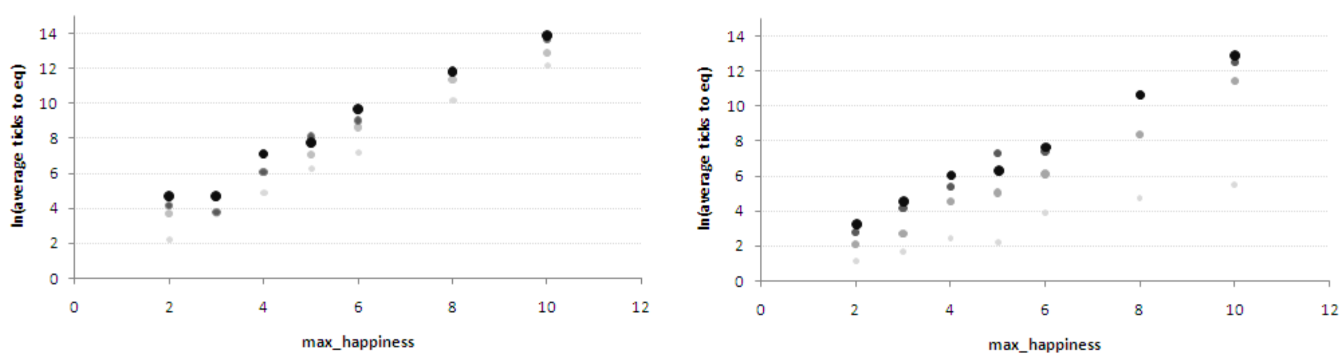
maximum happiness	ticks to equilibrium					average	ticks to equilibrium					average
	people = 6						people = 10					
2	3	18	9	4	12	**9.2**	30	20	42	29	88	**41.8**
3	15	111	63	5	3	**39.4**	183	60	58	151	101	**110.6**
4	46	395	93	130	12	**135.2**	136	541	323	674	560	**446.8**
5	189	108	1170	980	240	**537.4**	2102	1371	1277	260	1041	**1210.2**
6	1350	660	2880	440	1200	**1306**	6670	1620	640	11960	8010	**5780**
8	83100	4600	32400	9700	470	**26054**	33400	130500	107400	136400	33800	**88300**
10	62800	44700	286000	556000	11000	**192100**	346000	816000	470000	95900	243000	**394180**
	people = 16						people = 20					
2	20	4	23	212	71	**66**	35	126	203	76	116	**111.2**
3	10	7	60	20	134	**46.2**	164	55	51	72	223	**113**
4	284	212	138	977	619	**446**	2750	1310	525	296	1318	**1239.8**
5	7	4270	2560	8570	1680	**3417.4**	4190	2230	1640	2280	1605	**2389**
6	11450	4130	10860	14940	1400	**8556**	53900	1340	14030	9980	1010	**16052**
8	191000	51200	140200	88900	192600	**132780**	77900	147300	362000	36300	62700	**137240**
10	346000	1325000	1660000	246000	634000	**842200**	422000	1478000	1485000	1499000	502000	**1077200**

Figure 10: plots for the data of Table 1 (logarithmic scale for turns to equilibrium; the average of the 5 runs is displayed as a straight line, the outcomes of the 5 individual runs as dots)



As seen, the expected waiting time to equilibrium increases exponentially with the maximum happiness level. Moreover, the expected waiting time increases with the initial number of participants (see Figure 11).

Figure 11: dependence of average number of turns to equilibrium on maximum level of happiness and initial number of participants (the higher the initial number of people – 6, 10, 16, 20 or 5, 9, 15, 19 - the darker and bigger the dots)



Similar results are found running the agent-based model starting with an odd number of participants in the conversation (odd case). Now the initial table-wide conversation group splits over time into smaller conversation groups until all conversation groups, except one, contain only 2 people, the singular conversation group containing instead 3 people (see Figure 12). Then a stationary distribution is reached, and as in the even case the final state is degenerate. Anyway, in this case the pairings within the 2-people conversation groups change over time. Indeed, sometime the happiness level of a participant in the 3-people conversation group drops to the minimum value, and that person consequently joins another conversation group, thereby decreasing the size of her old conversation group to 2 and increasing the size of her new conversation group from 2 to 3. This mechanism can never stop and results over time in a rearrangement of pairings in the conversation groups.

Figure 12: typical end state for odd initial number of participants

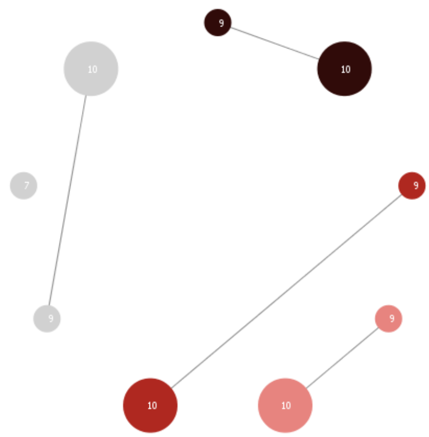
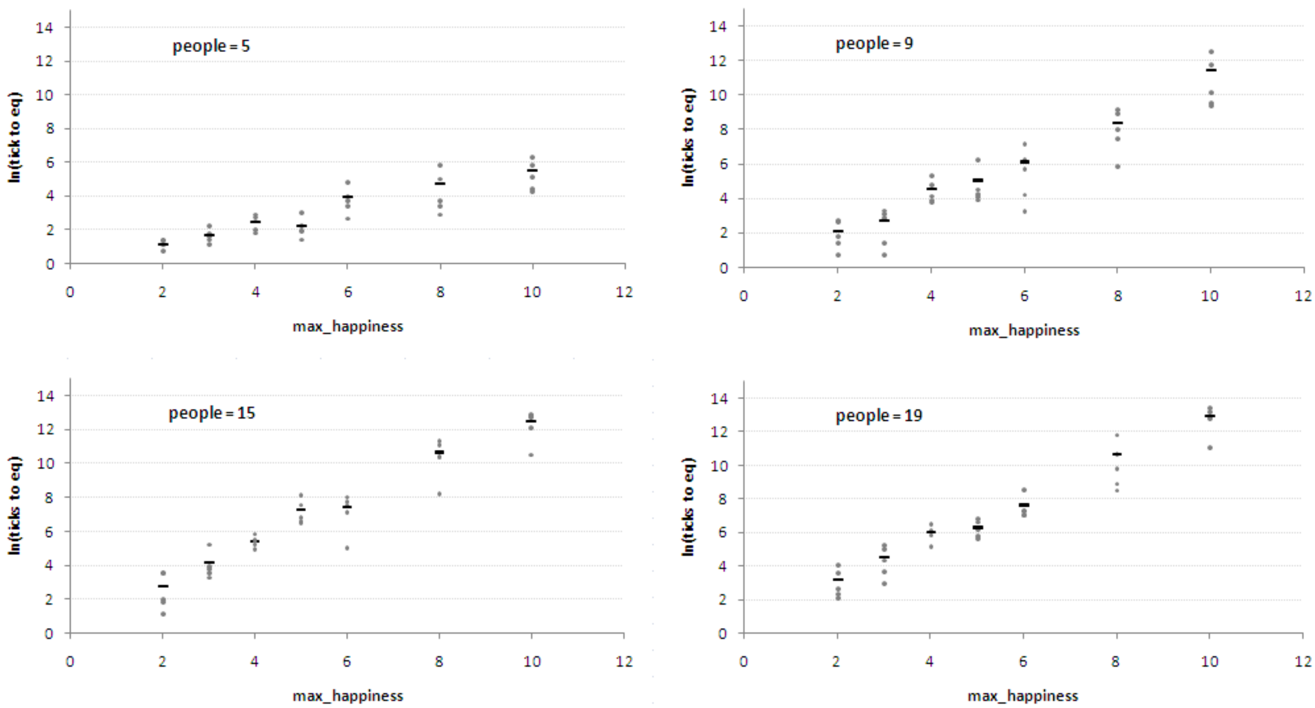


Table 2: turns to equilibrium for 5 conversation runs, given maximum happiness and odd participants

maximum happiness	ticks to equilibrium					average	ticks to equilibrium					average
	people = 5						people = 15					
2	3	4	2	3	4	**3.2**	3	6	33	34	7	**16.6**
3	5	3	9	4	6	**5.4**	180	26	50	33	43	**66.4**
4	12	7	15	6	18	**11.6**	135	183	242	339	220	**223.8**
5	7	4	20	9	7	**9.4**	3260	898	1870	653	711	**1478.4**
6	40	121	50	14	29	**50.8**	1190	150	2990	1820	2290	**1688**
8	150	30	337	18	40	**115**	31000	3700	63700	85600	32100	**43220**
10	349	80	540	170	70	**241.8**	176800	345000	392000	36800	375000	**265120**
	people = 9						people = 19					
2	4	14	15	2	6	**8.2**	14	10	57	34	8	**24.6**
3	27	2	18	4	23	**14.8**	76	19	151	180	40	**93.2**
4	49	115	63	201	45	**94.6**	463	332	424	642	174	**407**
5	91	58	50	510	72	**156.2**	329	880	742	270	476	**539.4**
6	540	66	26	300	1350	**456.4**	1150	1412	5120	1084	1470	**2047.2**
8	7600	9040	3030	1730	360	**4352**	42000	4780	134000	18200	7370	**41270**
10	133300	13600	287000	12100	27000	**94600**	392000	520000	368000	65000	686000	**406200**

Figure 13: plots of the data in Table 2 (logarithmic scale for turns to equilibrium, the average of the 5 runs is displayed as a straight line, the outcomes of the 5 individual runs as dots)

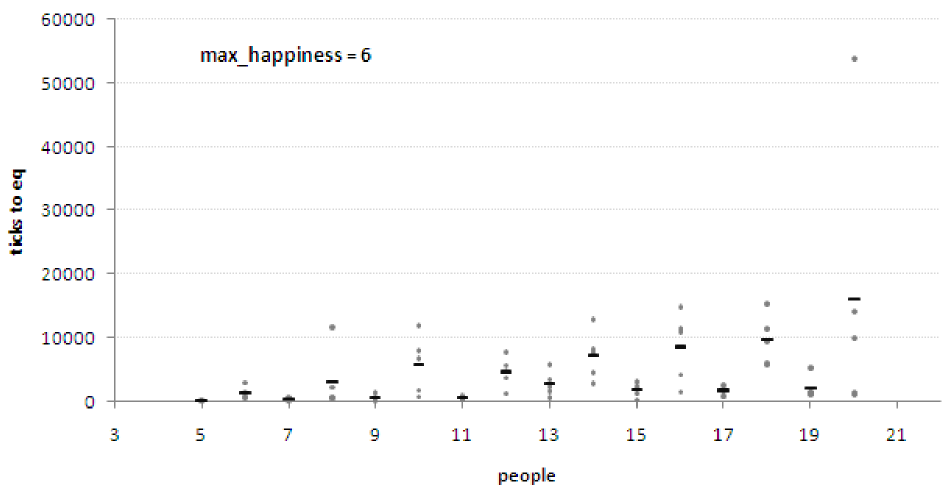


As for the previous case, the dependency of the number of turns needed to reach the stationary distribution on the initial number of people and the maximum levels of happiness was simulated in 5 different runs of the agent-based model (see Table 2). The same data are visualized in a semi-logarithmic plot (see Figure 13). The expected waiting time to equilibrium increases exponentially with the maximum happiness level also for odd numbers of people. Again the expected waiting time increases with the initial number of people. Interestingly, simulated data also show that the expected time to equilibrium is substantially lower for odd numbers of people than for even numbers of people with roughly the same size (*e.g.* 19 and 20; see Figure 14). This finding becomes more evident when one studies the expected time to equilibrium for a given maximum level of happiness in dependence on the initial number of people. Odd initial numbers of people generally result in more rapid convergence to the stationary distribution than even initial numbers of people of roughly the same size. Moreover, the variance of the waiting time to equilibrium is far lower for odd initial numbers than for even initial numbers (see Table 3 and Figure 14).

Table 3: turns to equilibrium for 5 conversation runs for a specific maximum happiness level and several initial participants

people	ticks to equilibrium					average
	max_happiness = 6					
5	40	121	50	14	29	**50.8**
6	1350	660	2880	440	1200	**1306**
7	665	27	64	244	165	**233**
8	413	539	11500	480	2220	**3030.4**
9	540	66	26	300	1350	**456.4**
10	6670	1620	640	11960	8010	**5780**
11	580	359	160	955	332	**477.2**
12	7730	3630	5580	4700	1180	**4564**
13	1530	2220	3330	5740	512	**2666.4**
14	2780	4440	8180	12850	7770	**7204**
15	1190	150	2990	1820	2290	**1688**
16	11450	4130	10860	14940	1400	**8556**

Figure 14: plot of the data contained in Table 3 (logarithmic scale for turns to equilibrium, the average of the 5 runs is displayed as a straight line, the outcomes of the 5 individual runs as dots)



3.2 Transient dynamics

So far only the stationary states were discussed. In reality, 2-people conversation groups seem to be fairly stable, as opposed to 3-people conversation groups. It seems evident, on the other hand, that large table-wide conversations do not usually converge to a situation where sub-conversations take place mainly within 2-person conversation groups, if only because the duration of an average conversation may not be sufficient to reach that asymptotic state. If one wants to apply the agent-based model to real table conversation dynamics, one can focus on the transition dynamics of the model, or modify the model assumptions in such a way that a different stationary distribution results. Here the focus is on transition dynamics, while major modifications of the model assumptions are left for future research (see Section 6).

The strong model assumption of simultaneous turn taking (#2) roughly defines the characteristic time unit of the model (1 tick = 1 conversation turn) as well as the empirically relevant range of the total number of turns taking place during a reasonable table talk. In the real world, turns occur on average rarely quicker than every 10 seconds. A one hour table talk then would not allow for more than about 360 turns. This information, together with the need to avoid the unrealistic stationary distribution for large table conversations previously discussed, allows putting a lower bound on the range of permissible maximum levels of happiness. For all (even and odd) numbers of participants larger than 5, avoidance of convergence to the stationary distribution within the first 360 turns can be achieved by setting the maximum happiness level larger than about 8 – *i.e.* this is the minimum number of conversation turns which needs to be tolerated without being entitled to speak (and therefore before leaving the conversation) to avoid precocious conversation convergence. Tables of participants with higher maximum levels of happiness would be able to maintain large conversation groups for longer periods of time. Figure 15 shows the stationary distribution and the transition dynamics up to 376 ticks of a model run with 15 participants and a maximum happiness level of 8. The initial table-wide conversation splits right after the beginning into 4 smaller sub-conversations because the happiness levels of some table members necessarily become minimal at the same time, and it is highly improbable that no one of the latents is close to another latent. The 4 smaller group conversations persist for 150 ticks before another conversation group is formed. No other conversation group is formed until 376 ticks, *i.e.* the end of the table conversation (see also table 4). The geometric location of, and the very participants involved in a group conversation, are persistent over time. People join or leave conversations only when they are located next to another conversation group or, in a much rarer case, when they find themselves next to a person whose happiness level has also decreased to her minimal value. Conversation groups rarely include people who are not direct geometric neighbors of other people in the same conversation. Also, latents can be trapped within a conversation group (see *e.g.* at ticks 10 and 53 in the upper left and right quarters of Table 4; latents are colored in dark grey). Finally, it is evident that the number of conversation groups increases monotonously over time (see Figure 16).

Figure 15: transition dynamics for a 15-people conversation with maximum happiness level of 8

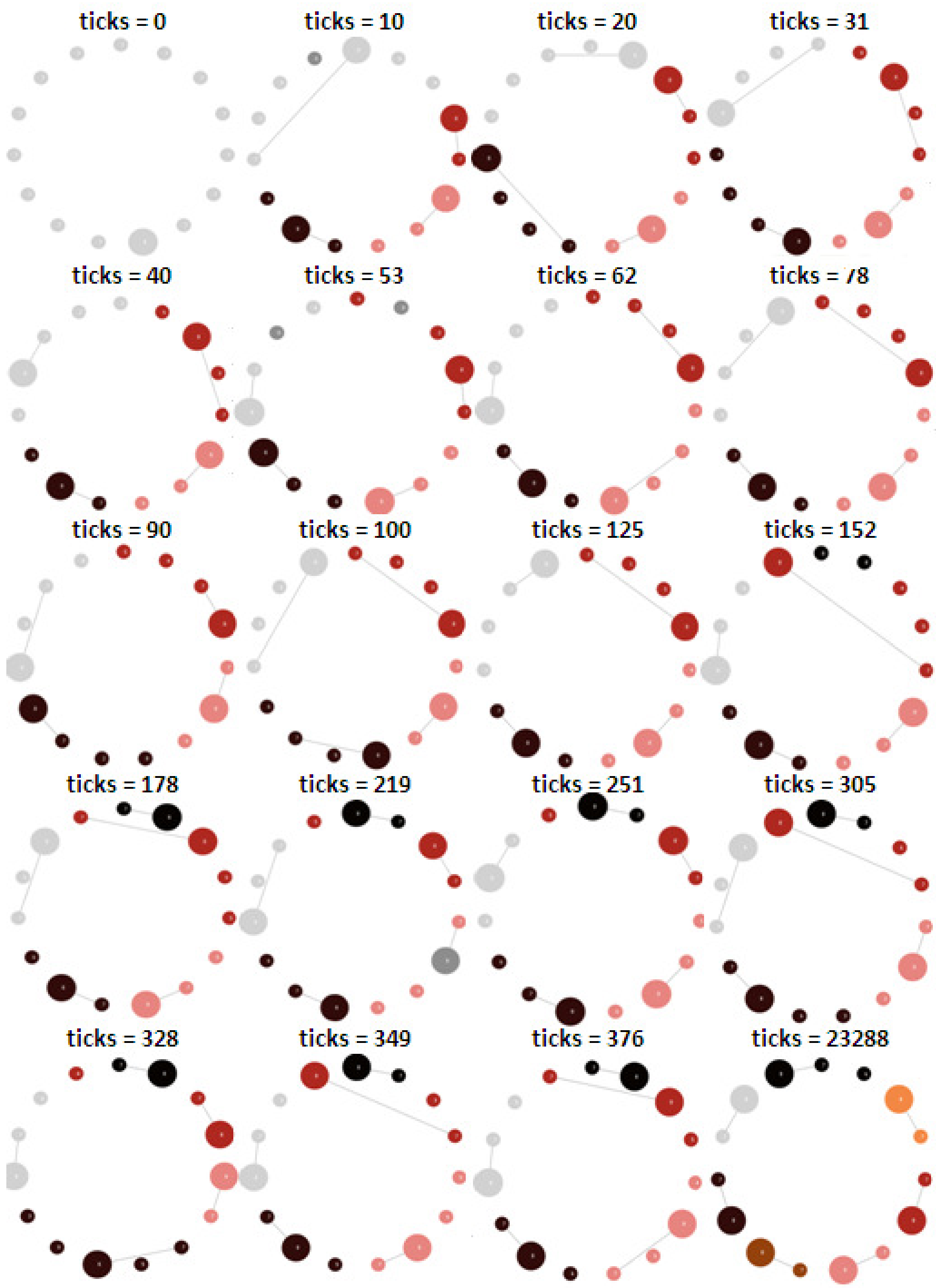
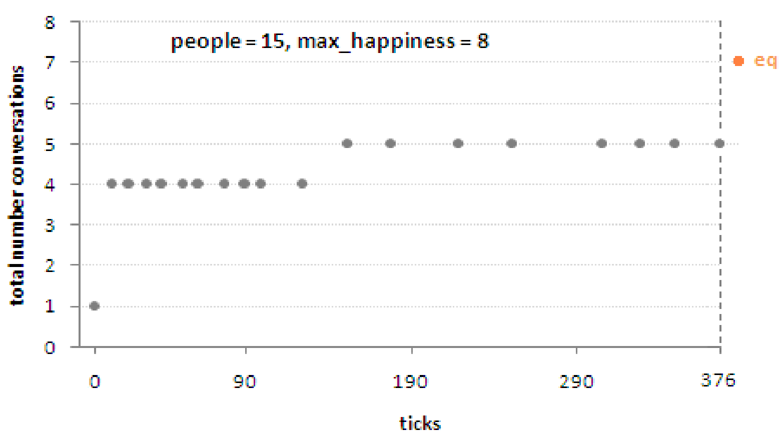


Table 4: time evolution of conversation groups for the conversation of Figure 15

ticks	conversation groups of size						Total
	15	7	5	4	3	2	
0	1						1
10		1			2	1	4
20			1	1	2		4
31				3	1		4
40			1	1	2		4
53			1	1	2		4
62				3	1		4
78				3	1		4
90				3	1		4
100				3	1		4
125				3	1		4
152				1	3	1	5
178				1	3	1	5
219				1	2	2	5
251				1	3	1	5
305				1	3	1	5
328			1		2	2	5
349				1	3	1	5
376				1	3	1	5
23288					1	6	7

Figure 16: history of total number of ongoing conversations for the conversation of Figure 15 (total number of conversations at equilibrium is shown as an orange dot)



All the above findings hold in general and not only in the special case illustrated above. Importantly, in spite of abstracting from conversation contents and contexts, many of these findings seem to correspond at least qualitatively to phenomena that can be observed in real table conversations. For example: table-wide conversations involving a large number of people are unstable, while smaller conversation groups persist over longer periods of time; people sometimes change conversation groups, and when this happens they confine themselves to nearby conversations (the conversational principle of least effort is the reason why party organizers often pay so much attention to the initial table population and configuration, if it is supposed to remain fixed); people within a conversation group change from time to time, but the conversation group has a tendency to remain in a specific geometric location, and only a limited number of people around the table join a specific conversation group; people who have left a conversation group often return to that same conversation later; sometimes people would like to leave a conversation, but nonetheless they may remain in it because they are trapped between two people eagerly taking turns in that very conversation.

To determine the extent to which the model replicates qualitatively or even quantitatively real world table conversation dynamics, one should compare the predicted dynamics to large data sets. While a detailed comparison is left for future investigations, we have performed a small preliminary survey within the CSSS'09 fellows in order to roughly estimate the maximal size of an optimal conversation group. In the survey we asked the fellows to answer to the following question:

In your opinion, what is, on average, the maximum number of people that can be in the same conversation before this conversation gets uncomfortable?

A total of 31 CSSS fellows answered the question. Figure 17 shows the histogram of the answer's frequency. While the sample size is not large enough to be statistically significant, one can see that the distribution has a well-defined average and finite moments. The maximal value for the size of a stable conversation group (N = 4) approximately matches the conversation group sizes that were reached in the metastable state (*i.e.* the transition state up to the conversation characteristic time) in our simulations, something that gives credit to our model assumptions. Again, further empirical data should be obtained in order to confirm these preliminary results.

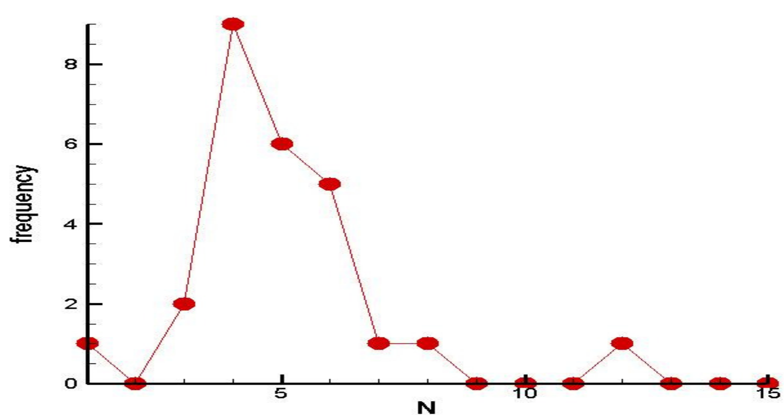


Figure 17: Distribution of the maximal number of people for a comfortable distribution (*i.e.* maximal group size for a stable conversation) according to our internal CSSS'09 survey. The distribution is peaked around N = 4, in fairly good agreement with the numerical prediction of the baseline model.

4. Summary and discussion

In spite of its simplicity, the proposed agent-based model of conversation dynamics predicts a surprisingly familiar behavioral scenario:

I. Large conversation groups are unstable, while smaller conversation groups can persist for a long time.

II. Conversations with an even number of participants asymptotically approach a stationary configuration that consists exclusively of 2-people conversation groups. Once this state (which belongs to a degenerate absorbing state set) is reached, the pairings within the 2-person-conversation groups do not change.

III. Conversations with an odd number of participants asymptotically approach a stationary configuration that consists of exactly one 3-people conversation group and otherwise 2-people conversation groups. Once the stationary configuration is reached, the pairings within the 3-person and 2-person-conversation groups continue to change indefinitely.

IV. Given the number of participants, the expected waiting time to the equilibrium state increases approximately exponentially with the maximum level of happiness of the participants.

V. Given the maximum level of happiness, the expected waiting time to the equilibrium state increases approximately linearly with the number of participants.

VI. The expected waiting time to equilibrium is much lower for odd than for even total numbers of participants when similar initial conversation sizes are compared for both cases.

VII. The variance of the waiting time to equilibrium is much lower for odd than for even numbers of participants when similar initial conversation sizes are compared for both cases.

VIII. The formation of new conversation groups is a relatively rare event after the initial conversation split: the conversation dynamics mainly consists of people joining and leaving already existing conversation groups.

IX. Table conversations rarely involve people who are not geometric nearest neighbors.

X. Participants may remain trapped within their present conversation group, in spite of their dissatisfaction.

XI. The location of and the very people involved in a conversation group are persistent, *i.e.* a conversation remains close to its original geometric location, and the people within a conversation group change rarely.

XII. The number of conversation groups increases monotonically in time, except for transient events; *i.e.* it is impossible that the total number of conversation groups decreases irreversibly over time.

Having established these results, one can use analytic reasoning to start making sense of them. In the following two subsections, first some exploratory, non-formal arguments are given, whose formalization and validation are left for future work; then, an analytical mean-field approximation is proposed.

4.1 Heuristic arguments

The assumed agents' behavioral rules make it impossible for agents in a 2-people conversation group to leave the group. Indeed forced turns taking (as the specification of the probability function does not allow the current speaker to be the new speaker as well) and resetting of happiness levels of the new speakers do not allow escape until a third person eventually enters the group. At the same time, 2-people conversation groups remain open to newcomers, and thereby allow rearrangement of all conversation groups as long as there exist at least one conversation group comprising more than two people - the equilibrium behavior of a table with an odd number of people illustrates this observation: 2-people conversation groups are rearranged continuously due to the presence of a 3-people conversation group. Any rearrangement is possible that does not decrease the total number of conversations (impossible because 2-person conversation groups cannot dissolve) and is generally feasible (*e.g.* a conversation group involving people that are not direct geometric neighbors need to have at least 2 people from another conversation group between them). All possible rearrangements of a (degenerate) table configuration have therefore a non-null probability, and will eventually be realized at some point in time if the total number of conversation groups does not increase. Sooner or later 2 latents will end up next to each other and form a new conversation group. This process will continue until all people belong to 2-people conversation groups or until no additional 2-people conversation groups can be formed (as in the case of odd total numbers of participants). These reasonings can explain findings i, ii, iii and xii.

Finding iv is understood by reminding that the formation of new conversation groups depends on 2 participants having simultaneously minimal happiness levels and being geometric nearest neighbors. This implies that the 2 participants have spoken simultaneously and then have not spoken for a number of turns that is at least equal to their maximum happiness level. Also, the longest fraction of the waiting time to equilibrium is needed to match the last 2-conversation group. The reason for the waiting time needed to produce 2 direct geometric neighbors that have spoken simultaneously and can form the last possible 2-people conversation group can be grasped analyzing the probability for simultaneous minimal happiness levels conditional on such setting. If there are N_1 persons in the first conversation group and N_2 persons in the second conversation group, then the probability for the formation of a new 2-people conversation group is $(1\text{-}1/N_1)^{MAX} \bullet (1\text{-}1/N_2\text{-}1)^{MAX}$, where MAX is the maximum happiness level. An increase in the maximum happiness level exponentially reduces this probability and thereby exponentially increases the expected waiting time until this event is realized. Finding v is also related to the mechanism that creates the last new 2-people conversation group. Now one cannot abstract from the waiting time needed to produce 2 direct geometric neighbors that have spoken simultaneously and can form the final 2-people conversation group. The more people around the table, the lower the probability that two people belonging to the different 3-people conversation groups are direct geometric neighbors. This results in a longer waiting time to equilibrium.

As mentioned above, the formation of the very conversation group that moves the system to the stationary distribution is usually the most time consuming process. There are usually less arrangements of conversation groups conducive to the formation of the crucial conversation group if the total number of participants is even than if it is odd. If the total number is even, much time is spent in states where the crucial conversation group cannot form. If the total number is odd, the arrangement of conversation groups allows for the formation of the crucial conversation group most of the time. This difference results in a difference of average waiting times for the formation of the crucial conversation group (finding vi). Moreover, an even total number of people results in fewer opportunities for the formation of the crucial conversation group and a large dispersion of waiting times. Rapid formation of the crucial conversation group depends on very specific favorable realizations of the stochastic process. Favorable realizations of the stochastic process are less specific when the total number of people is odd. Therefore, the variance of waiting times is lower when the total number of people is odd (finding vii).

Large conversation groups result in higher individual probabilities of not being chosen as a speaker. The probability of arriving at a minimal happiness level after having spoken is $(1-1/N)^{MAX_i}$. The higher the number of people, the higher is this probability and consequently the more probable it is that people leave the conversation (finding viii). The probability that 2 geometric neighbors belonging to 2 different conversation groups form a new conversation group after they have spoken simultaneously is $(1-1/N1)^{MAX_i} \bullet (1-1/N2-1)^{MAX_i}$. To join another conversation group it is enough that one person drops to her minimal happiness level. This probability is much higher: $(1-1/N)^{MAX_i}$. At the beginning, happiness levels decrease simultaneously for all non-speakers and it is probable that non-speakers will be direct geometric neighbors when they reach minimal happiness levels (finding ix). It is as probable for a central person in a conversation group to reach the minimal happiness level as it is for persons in the same conversation group who have direct geometric neighbors in other conversation groups. Therefore, one should expect occurrence of latents being trapped within their conversation groups as often as the occurrence of people joining other conversation groups (finding x). Conversation groups involving people who are not direct geometric neighbors necessitate the previous formation of a new conversation group out of a bigger conversation group. This is a low probability event with a probability of $(1-2/N)^{MAX_i}$ similar to the formation of a new conversation from different conversation groups (finding xi).

The most frequent reason for a change in conversation groups is one person joining or leaving a conversation group. Every person in a conversation group has generally the same probability of reaching minimal happiness levels. Therefore, there exists no leftward or rightward bias for the enlargement or diminution of the conversation group. Moreover, conversation groups never disappear completely. These features relate to the geometric (and equivalently personal) persistence of the conversation groups (finding xii).

4.2 Mean-field approach

In a mean field treatment of the baseline model, we will assume that the probability p_i of a participant in a conversation to be entitled to speak by the present speaker at time t is independent of previous conversation history and constant in time. In general, $p_i = p_i(N,i)$ where N is the number of participants, and the specific dependence of p_i on each participant characterize individuality, both intrinsic (*e.g.* psychological factors) or extrinsic (*e.g.* conversation geometry). Let $F_i(t)$ be the happiness level of participant i at time t; for what said before, $F_i(t)$ is semipositive definite.

Evolution equation

Participant i at time t+1 will have probability p_i of being a speaker - and thus of increasing F_i to its maximum level MAX_i), and probability $1 - p_i$ of being a listener - thus of decreasing her happiness level by one: $F_i(t+1) = F_i(t) - 1$. Hence we have the following N-dimensional map $g(F_i)$:

$$F_i(t+1) = p_i \cdot MAX_i + (1-p_i)(F_i(t)-1)\text{, for all i = 1,…,N}$$

Fixed point and stability analysis

To find the fixed points F_i^* of each of these equations, we drop the time dependence, *i.e.*:

$$F_i^* = p_i \cdot MAX_i + (1-p_i)(F_i^* - 1)$$

from which we get:

$$F_i^* = MAX_i + 1 - 1/p_1 \text{ for all i = 1,…,N,}$$

which are the fixed points of the system.

The fixed point F_i^* is stable when $-1 < dg(F_i^*)/dF_i^* < 1$. We have:

$$dg(F_i^*)/dF_i^* = -p_i$$

Accordingly, F_i^* is stable when $-1 < p_i < 1$, which is always fulfilled being $0 < p_i < 1$. We conclude that $F_i^* = MAX_i + 1 - 1/p_i$ is the stable fixed point of each participant.

Now, a participant becomes latent when $F_i = 0$. In order for a participant to be active in the steady state, we must have $F_i^* > 0$. This translates into $MAX_i > 1/p_i - 1$ which is a restriction in the waiting time (*i.e.* patience) of agent i. Note that depending on p_i, each agent will have a different critical patience.

As an example, in our baseline model we suppose that every agent has the same probability of being a speaker. Imposing probability normalization ($\sum_{i=1}^{N} p_i = 1$), we have $p_i = 1/N$ for all i=1,…,N. In this condition an active steady state is achieved for $MAX_i > N-1$ for all i=1,…,N. That is, in order for every participant to be active in the same conversation, their maximal waiting time cannot be less than the number of participants minus one (the participant herself). If this requirement is fulfilled, the initial conversation will, on average[13], be stable - the agents will remain active as time evolves.

Extinction cascade and sociological interpretation

The same analysis as before can be performed iteratively. Suppose that we start at time t = 0 with N agents such that

I. $p_i = 1/N$ for all agents

II. $MAX_i > N - 1$ for i=1,…, N-1

III. $MAX_i < N - 1$ for the last agent

Then the last agent is - statistically speaking - doomed to reach latency (and eventually leave the conversation). In order to find the critical values of patience of the other agents, a similar analysis can be performed, for $N' = N-1$, and we can conclude that the conversation will be stable if all the rest of speakers have a patience level such that $MAX_i = N'-1 = N-2$. This can be applied iteratively (the limit is N = 2, that leads to $MAX_i > 1$, that is, a standard turn taking conversation between two agents, as resulting from ABM simulations). A straightforward conclusion is the following: the number of speakers within a conversation will decrease until everybody feels comfortable (*i.e.* until the patience thresholds of everybody are above the critical values), and from there, it will remain as a stable conversation that every speaker will profit of.

5. Extended Model[14]

The baseline model has one free parameter (the happiness level) that can be used to fit empirical data. This also means that all agents are seen as homogenous and follow the same time-independent behavioral rules. On the other hand, it seems obvious that the large heterogeneity and variety of human behavior manifests itself also in table conversations. For example, people in a conversation group could evoke more responses from people that are geometrically close to them. Alternatively, some people in a conversation group could follow the turn taking

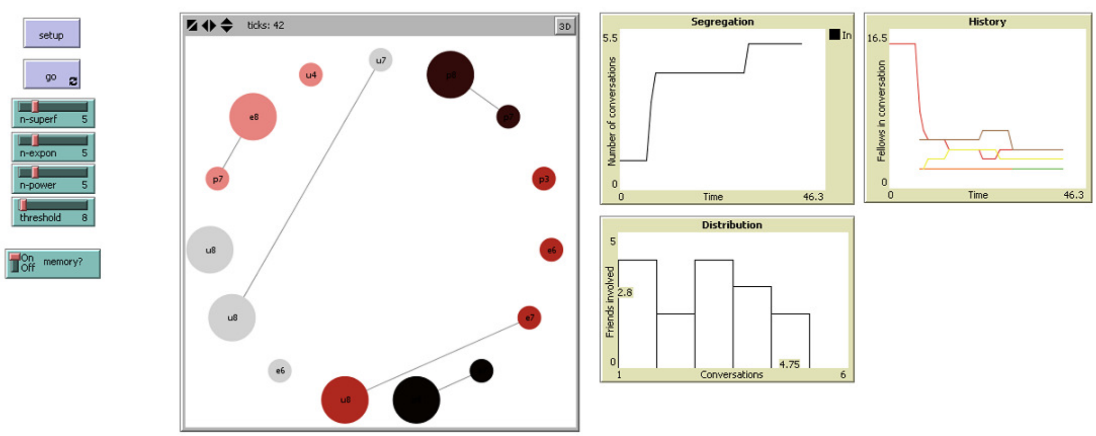
[13] This is a mean field approximation and addresses the typical (statistical) behavior of the system. In this sense, while in a concrete experiment we may find that an agent becomes latent even if $MAX_i > N - 1$, the average of many experiments will behave as in the solution.

[14] A Netlogo implementation of the extended model (file name extended.nlogo) can be found on the CSSS 2009 Wiki webpage.

within a conversation and actively try to let people speak who have not spoken for a long time and/or seem unhappy. The opposite behavior is also possible: people might tend to address people in their conversation group who have contributed recently. As this kind of behavior in the agent-based model is largely determined by the probability distribution that determines the next speaker, it is natural to allow for speaker-dependent and time-dependent probability distributions.

Our extended version of the agent-based model does implement these remarks. It allows for a definable number of 3 types of agents - which for lack of better terms were called "superficial" (short form "u", for uniform distribution), "exponentials" (short form "e", for exponential distribution) and "power laws" (short form "p", for power law distribution) to be involved in the original conversation. They differ in the probability distributions they use to choose the next speaker. Moreover, a "memory" feature was implemented that results in speakers memorizing the history of the conversation and assigning probabilities of choice based on the historical sequence of the last speakers. One would expect that agents who preferably talk to people who are closer to them or have spoken recently will accelerate the splitting of conversation groups while agents who preferably talk to people who have not spoken for a while will tend to slow down the same process. The graphical user interface of the extended model is shown in Figure 18.

Figure 18: graphical user interface of Netlogo implementation (extended model)



The focus of the present investigation was on the more parsimonious baseline model of conversation dynamics. The investigation of the extended model is left to future research. It is expected anyway that the extended model, with the possibility of agent heterogeneity and memory effects, will be able to better fit empirical data. How much better the fit will be is an open question.

6. Future directions

The model presented above originated as an attempt to model and understand the dynamics of conversations. Further progress in this direction depends on the matching of simulated and experimental data, which might well entail the refinement of the model assumptions. Further reflections may originate from the content independency of the model.

On a more abstract level, the whole system can be described as a set of Markov chains (of high order, in presence of memory effects) or, equivalently, as a set of random walks on networks. If isolated from each other, these Markov chains are ergodic. The peculiarity of this model consists in dynamically reconfiguring these Markov chains based on geometry and threshold levels. It describes the dynamics of interacting sub-networks where the network interaction derives from random walks taking place on these sub-networks. A superficial literature review suggests that these kinds of models do not exist yet.

The actual table conversation setting and the above abstract view suggest various model modifications:

1. Inclusion of the current speaker in the probability function that determines the speaker of the next turn. This eliminates the table-wide simultaneity of turn taking, and allows a different interpretation of the characteristic time of the system. It also removes the stability of 2-people conversations, and makes the stationary states potentially more interesting - if one further assumes that 1-person conversation group cannot socially exist, and lonely people have to join other conversation groups instead.
2. Modify the conversation geometry so that nodes can form conversation groups with more than only two neighbors; any number of neighbors becomes possible (*e.g.* connectivity of brain networks). A dynamic topology might eventually reproduce cocktail party dynamics.
3. Define fixed sub-networks and allow linkage of two different sub-networks (*i.e.* let the random walk take place on the linked sub-networks) whenever one node in a sub-network reaches the lower threshold and joins another sub-network; a sub-network within the linked sub-networks reverts back to its isolated existence when one of its nodes reaches the lower threshold.
4. Leave the random walk framework and allow multiple interactions/links within a sub-network at a given time.

These generalizations might prove useful to model phenomena like volatility surges during financial crises, background noise of brain activity, or split of existing communities and reformation of new ones if regular interaction/communication is absent.

Acknowledgments

The principal merit - among many - of the Santa Fe Institute's Complex System Summer School is to posit the minds of the collective (or the collective mind, if preferred) of the attending pupils in a perpetually excited and intellectually fertile state. This is a no small accomplishment. We thank SFI and their wonderful people to have honored us to be enrolled in such great collective phenomenon. Special thanks goes to Dan Rockmore for helping us stay on track and on Earth, Tom Carter for lending us his wise eyes to revise our NetLogo code, and all attendees of CSSS'09 – a wonderful crowd to say the least, if only for its inspiring way of attending meals.