

Energy, Growth, Sustainability and the Underlying Threat of Urbanization

SIZE MATTERS

GEOFFREY WEST

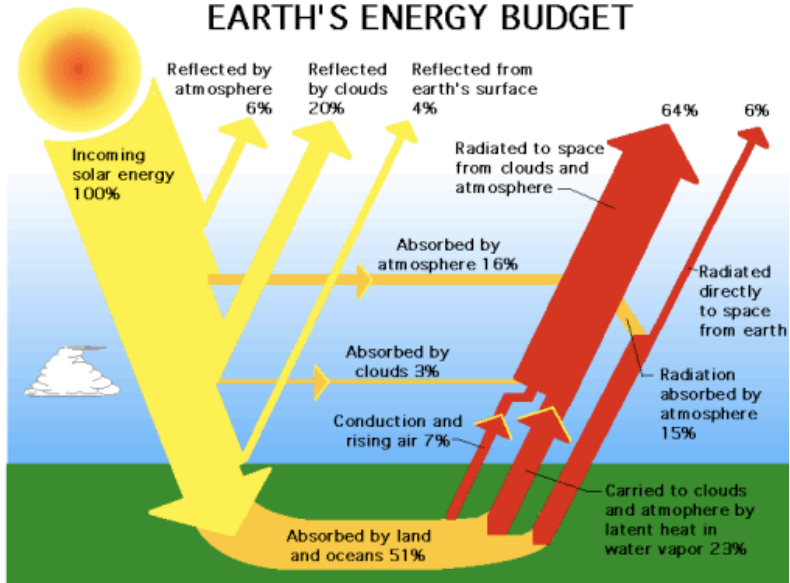
SOME CHARACTERISTICS OF COMPLEX SYSTEMS

- *MANY COMPONENTS*
- *MANY INDIVIDUAL ACTORS / AGENTS*
- *MULTI SPATIAL AND TEMPORAL SCALES*
- *STRONGLY COUPLED / INTERACTING*
- *NON-LINEAR*
- *SENSITIVITY TO BOUNDARY CONDITIONS (CHAOS)*
- *EMERGENT PHENOMENA / MULTIPLE PHASES*
- *UNINTENDED CONSEQUENCES*
- *ADAPTIVE / EVOLVING*
- *HISTORICALLY CONTINGENT / PATH DEPENDENT*
- *ROBUST / RESILIENT*
- *NON-EQUILIBRIUM*
- *UNDERLYING SIMPLICITY*
- *COMPLICATED vs COMPLEX*

- ***COARSE- GRAINED DESCRIPTION***
- ***KINETIC THEORY***
- ***QUARK MODEL***
- ***LONGEVITY***



EARTH'S ENERGY BUDGET





Scaling of economics with energy use









- LIVING/MAINTENANCE

- GROWTH

- REPRODUCTION

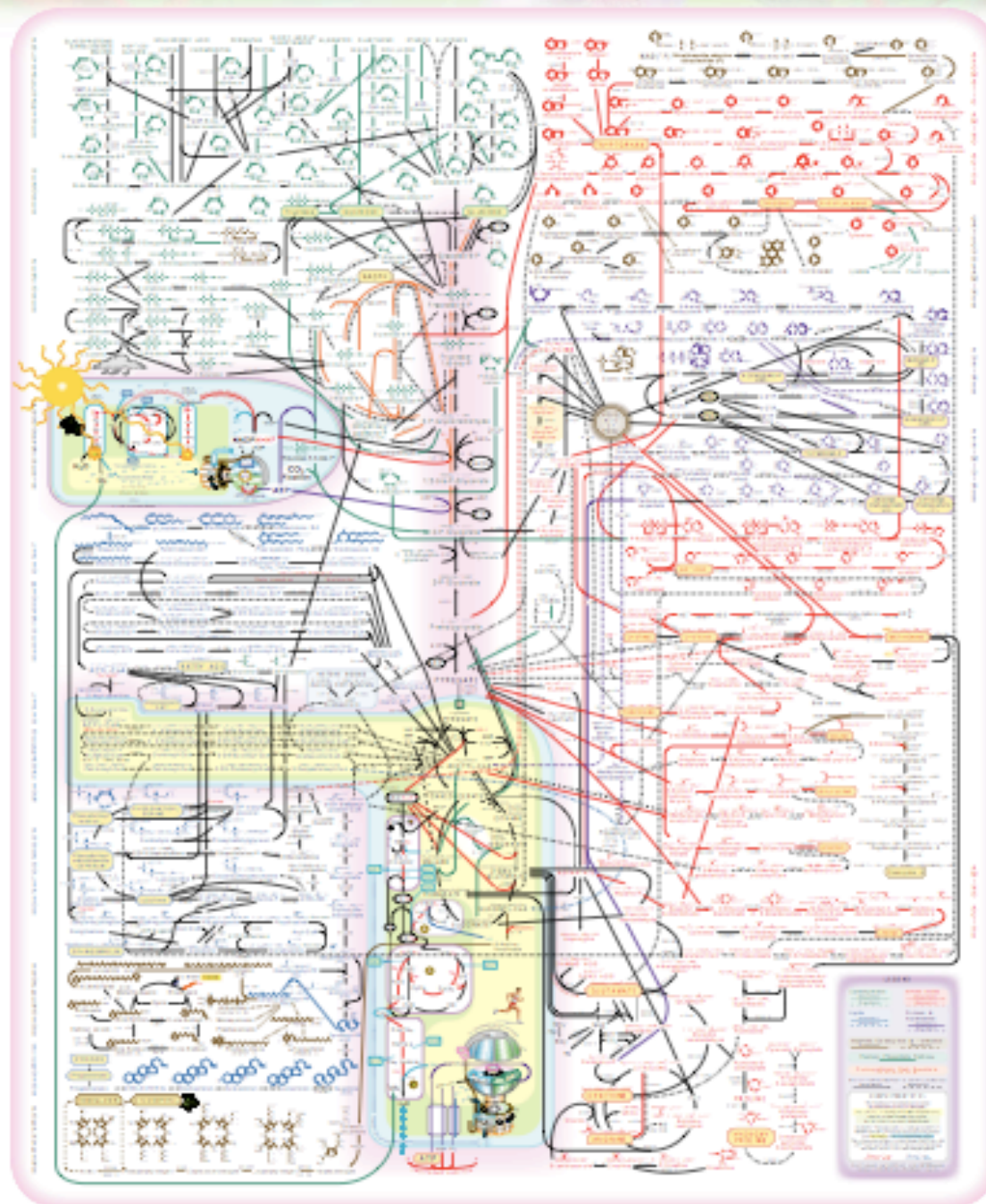
- AGING/DEATH

- EVOLUTION

- SLEEP/REPAIR
- DISEASE/CANCER
- ENERGY & RESOURCES
vs. INFORMATION
- ***THE SEARCH FOR UNDERLYING LAWS
AND PRINCIPLES LEADING TO A
QUANTITATIVE PREDICTIVE
CONCEPTUAL FRAMEWORK***

***ARE BUSINESSES,
CORPORATIONS AND
CITIES JUST VERY
LARGE ORGANISMS
SATISFYING THE LAWS
OF BIOLOGY?***

Metabolic Pathways



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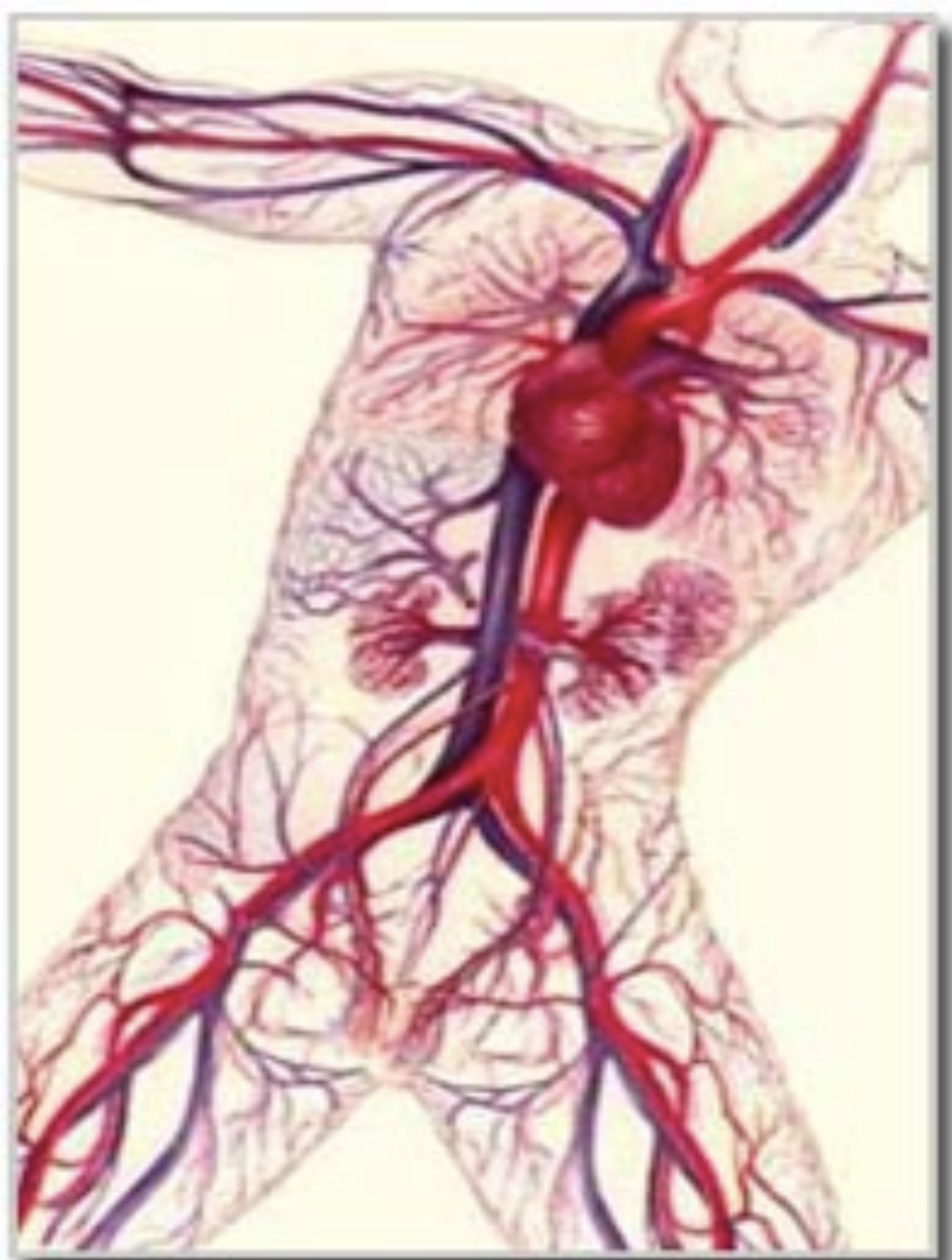
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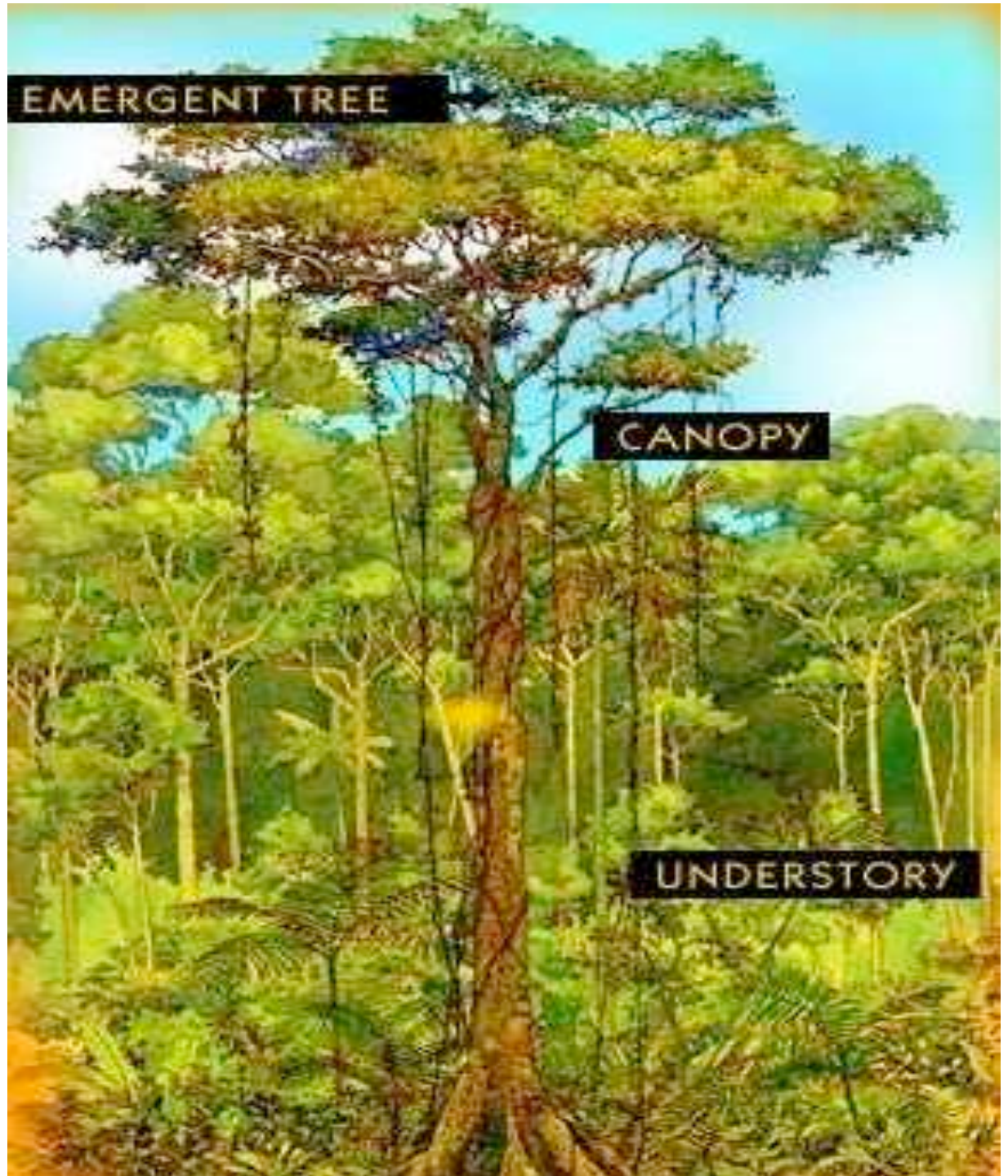






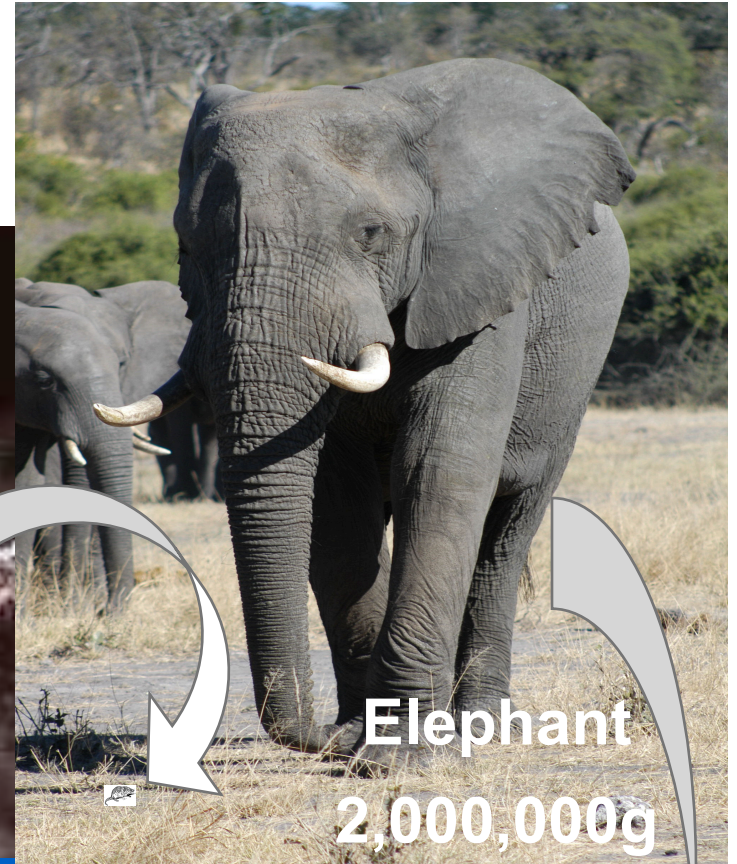
Relation between
number and
diameter of trees
in a forest
recapitulates the
branches of the
largest trees

$$N \propto D^{-2} \propto M^{-3/4}$$

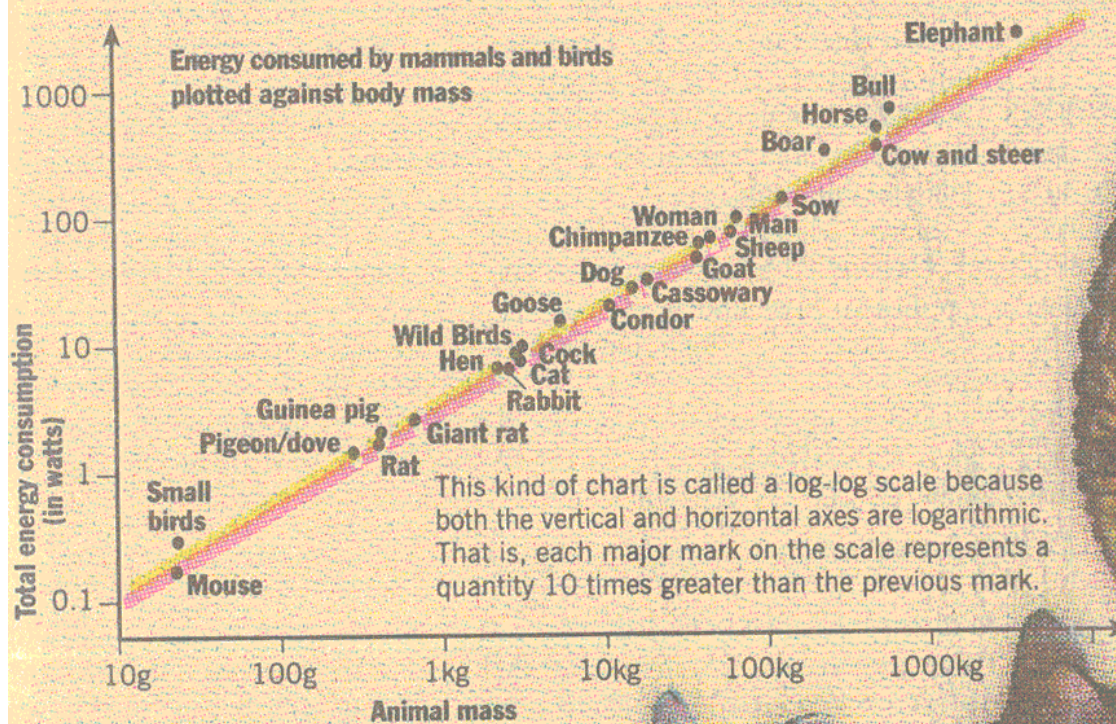




Mammals vary in size by 8 orders of magnitude

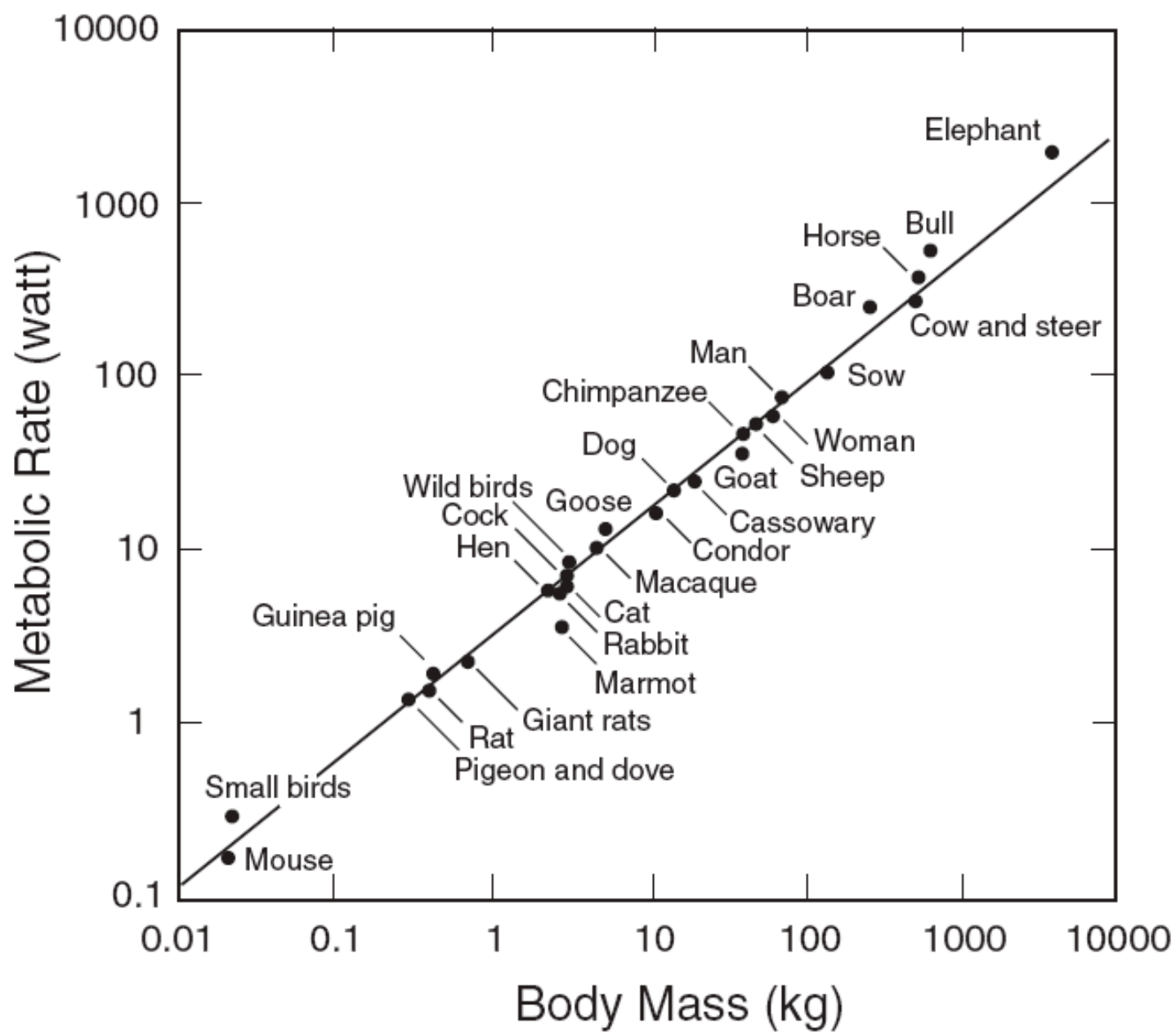


This graph illustrates one of the many scaling laws that are nearly universal among living things. The vertical axis is the amount of energy consumed by an organism per unit time expressed in watts. The horizontal axis is animal mass in grams. There are 28 grams in an ounce, and 1,000 grams (a kilogram) is equal to 2.2 pounds.

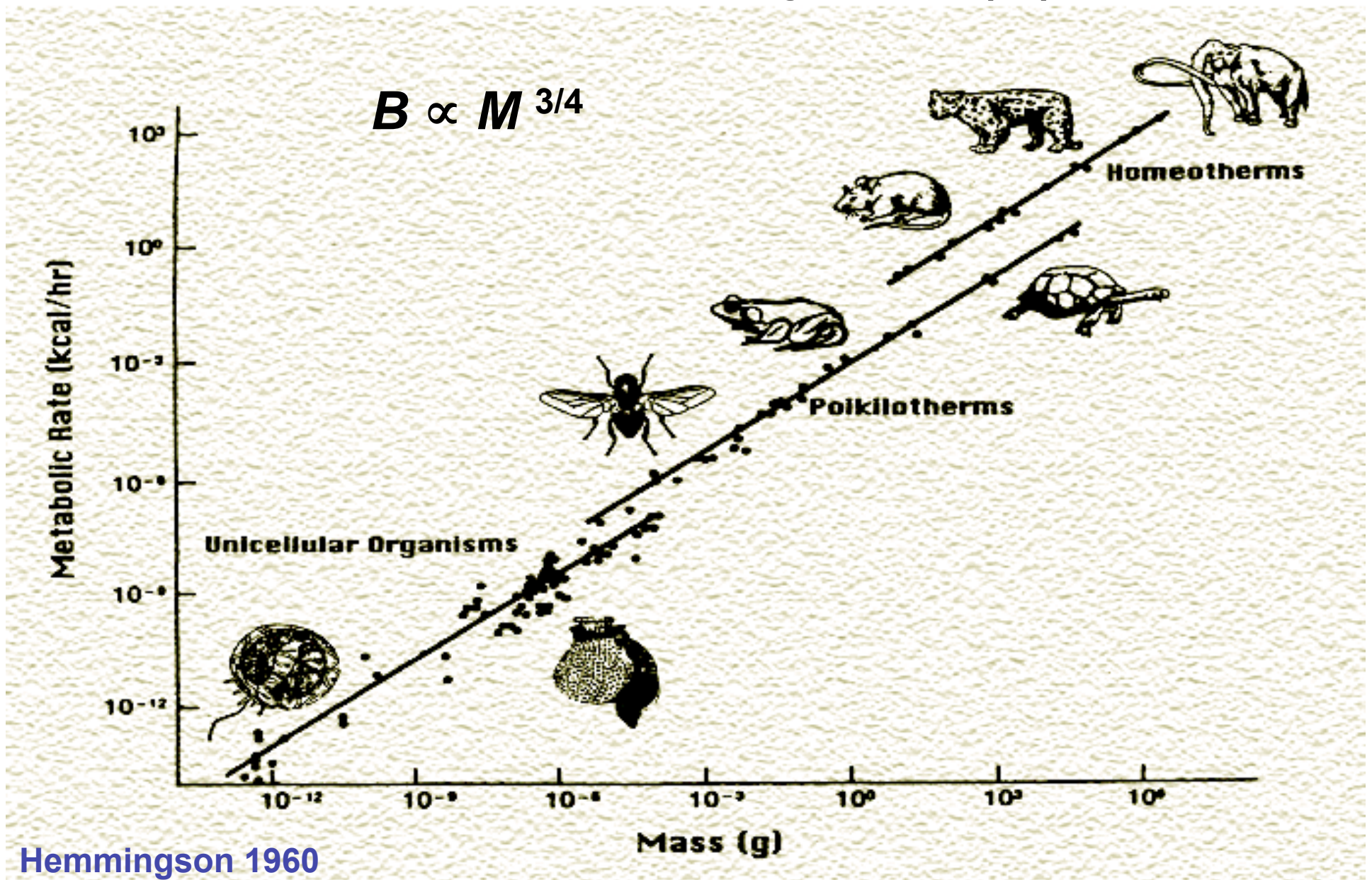


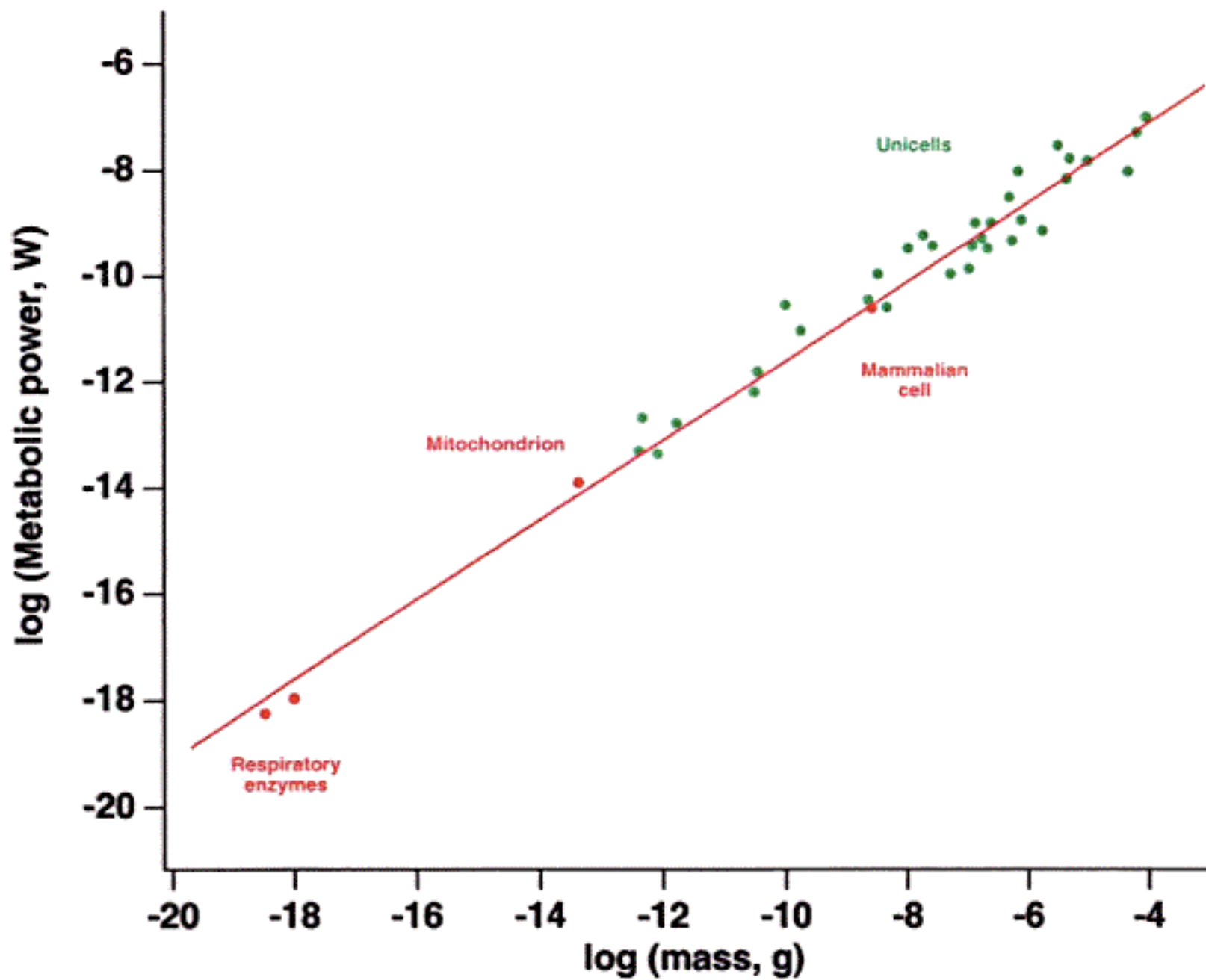
SOURCE: Los Alamos National Laboratory

Kleiber's law: metabolic rate scales as $3/4$ power of body mass



Whole-organism metabolic rate (B) scales as the $3/4$ power of body mass (M)





METABOLIC RATE INCREASES NON-LINEARLY WITH SIZE

EXAMPLE

***NAIVELY EXPECT THAT IF MASS (SIZE)
INCREASES BY A FACTOR OF 10,000 (10^4)***

THEN

***METABOLIC RATE WOULD INCREASE BY A
FACTOR OF 10,000 (10^4)***

BUT

IN FACT,

***METABOLIC RATE INCREASES BY A
FACTOR OF ONLY 1,000 (10^3)***

$$***B \sim M^{3/4}***$$

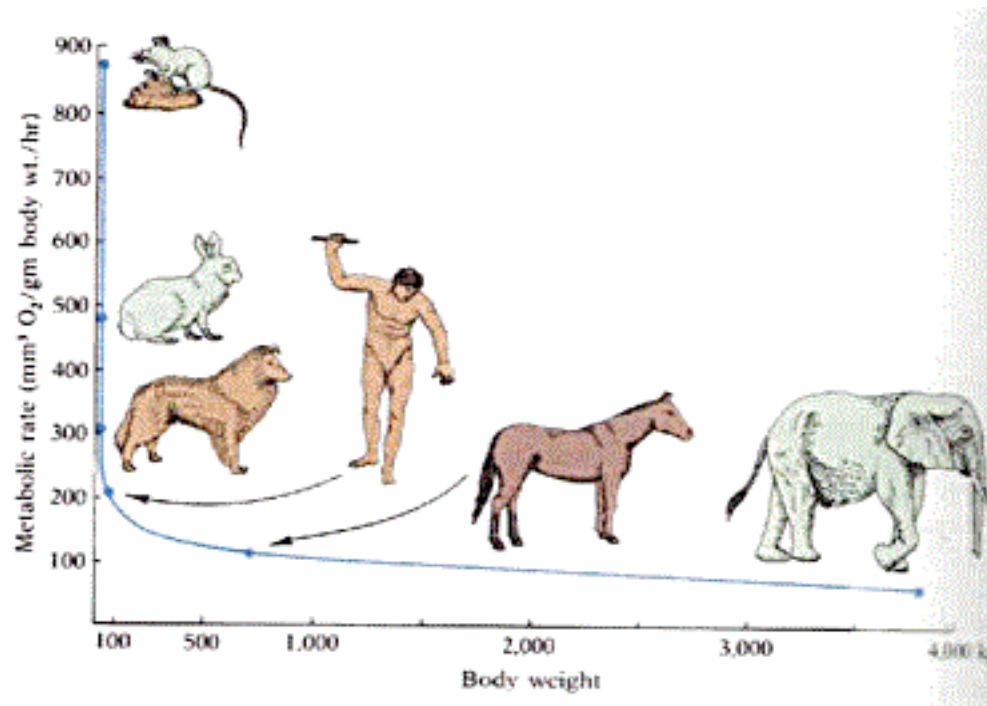
***SPECIFIC METABOLIC RATE (PER
UNIT MASS)***

$$\frac{B}{M} \propto M^{-1/4}$$

***SO METABOLIC RATE OF
AVERAGE CELL***

$$B_{cell} \propto M^{-1/4}$$

LIFE EXHIBITS A SYSTEMATIC ECONOMY OF SCALE



**TO SUSTAIN 1 gm MOUSE REQUIRES 3
TIMES THE POWER FOR 1 gm of DOG
AND 9 TIMES THE POWER FOR 1 gm of
ELEPHANT!!**

**SMALL MAY BE BEAUTIFUL BUT LARGE
IS MORE EFFICIENT**

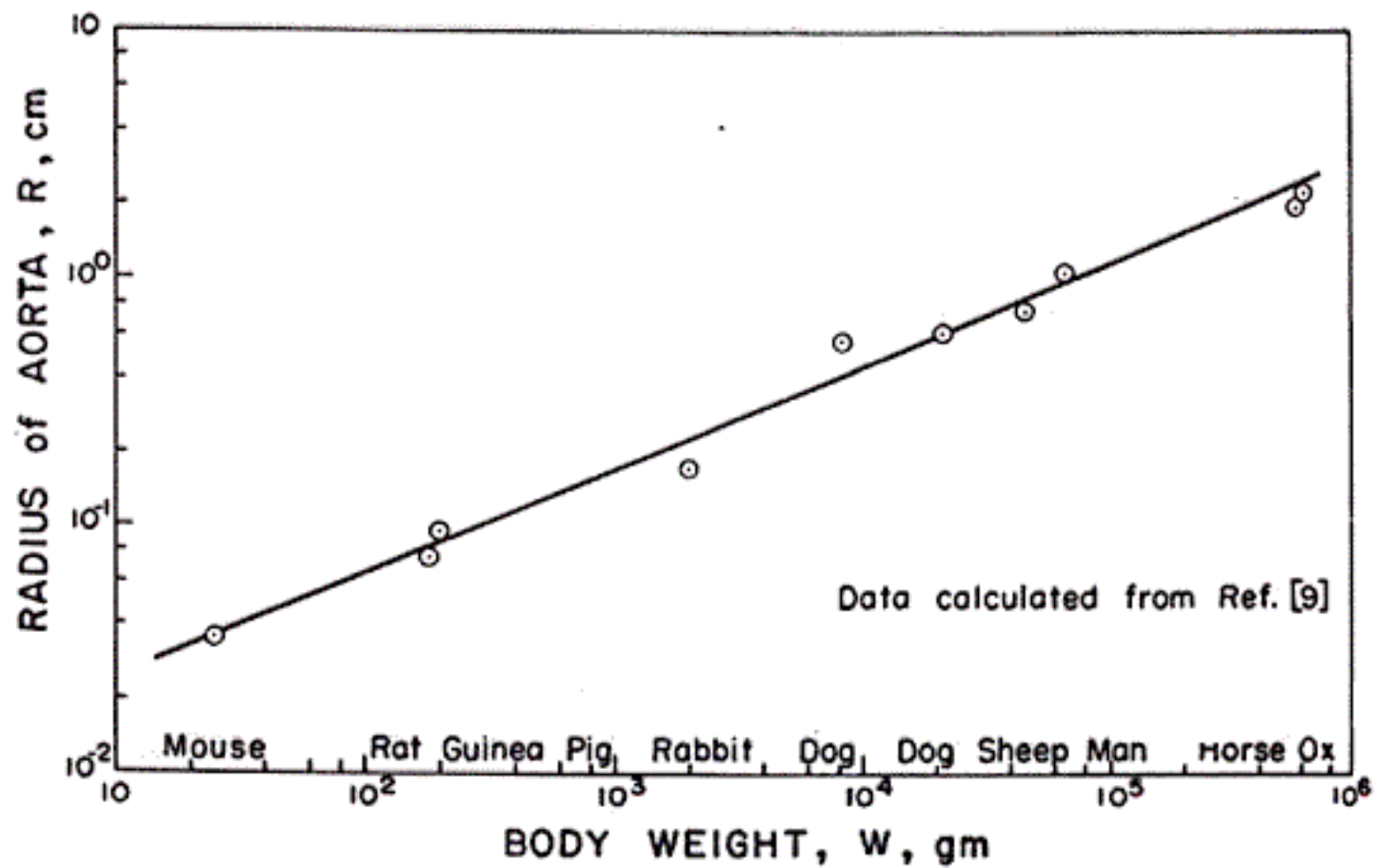
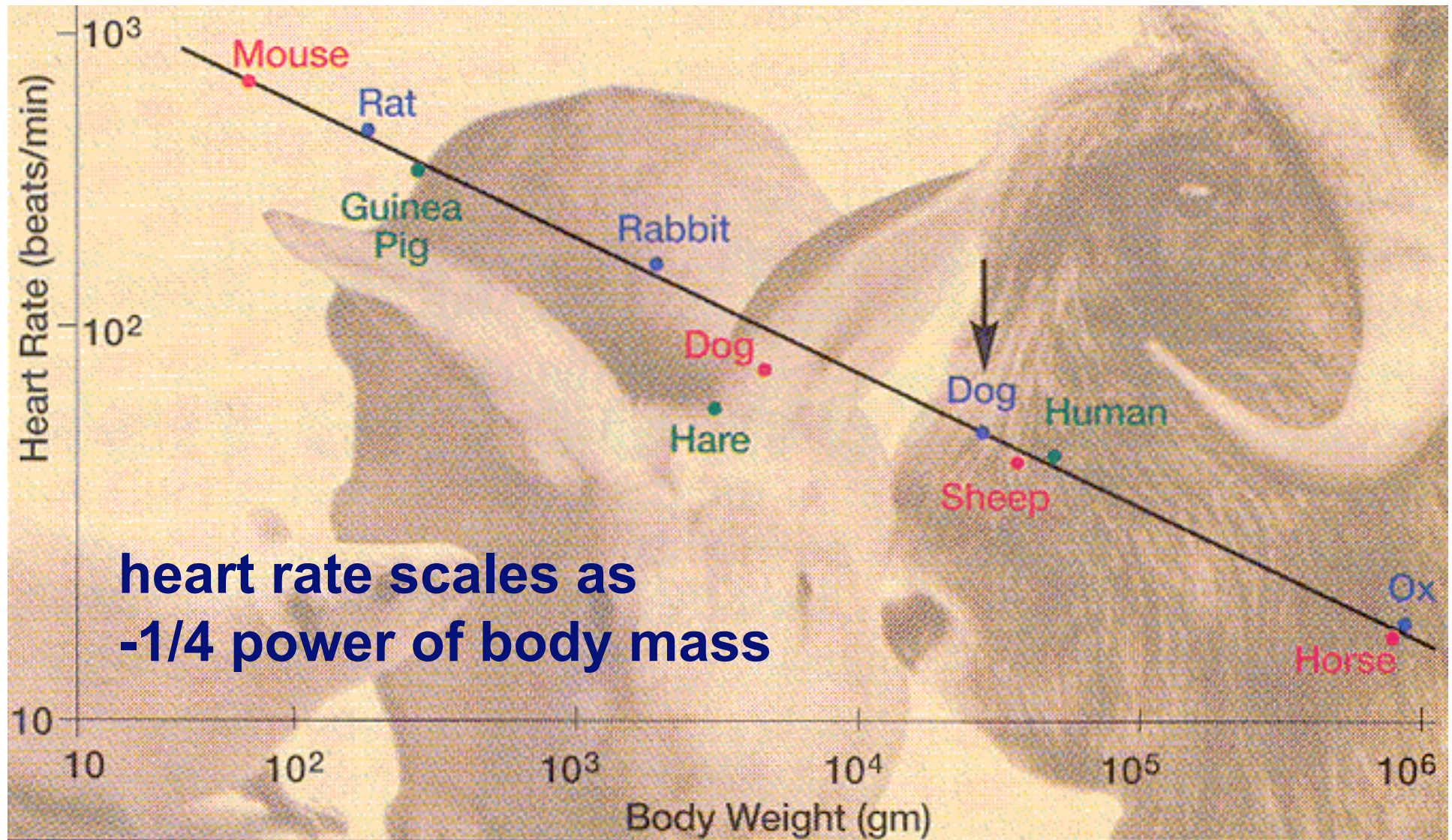


FIG. 4 - VARIATION IN RADIUS OF AORTA WITH BODY WEIGHT

$$r \sim M^{3/8}$$

SAME SCALING FOR TREE TRUNKS

Metabolic rate sets the pace of life small animals live fast and die young



LIFESPAN

$$T \sim M^{1/4}$$

IF HEART-RATE (NUMBER OF BEATS PER SEC.)

$$\sim M^{-1/4}$$

⇒ TOTAL NUMBER OF HEART-BEATS IN A

TYPICAL LIFE-TIME IS INDEPENDENT OF SIZE!

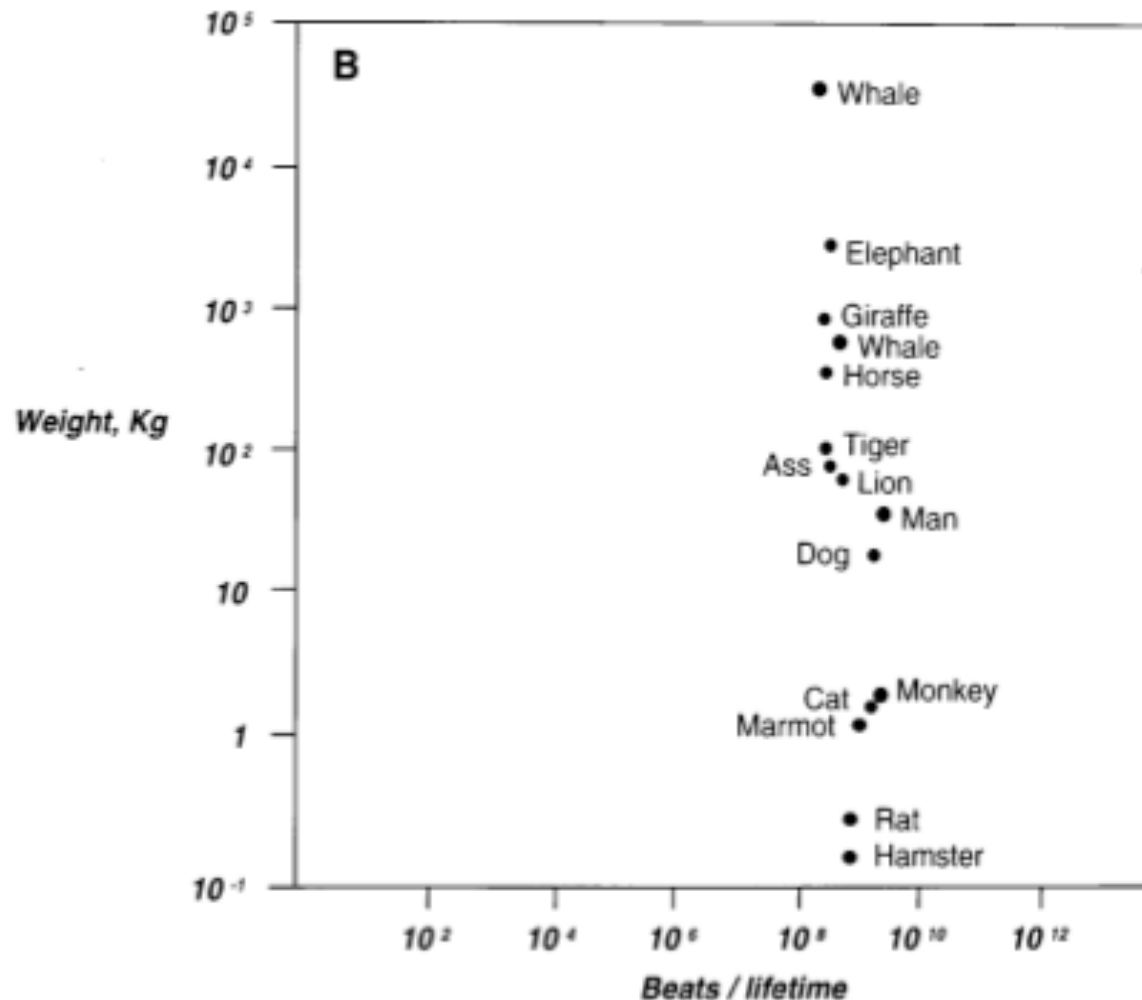
$$\approx 1.5 \times 10^9$$

EACH ANIMAL SPECIES REGARDLESS OF SIZE

HAS APPROXIMATELY THE SAME NUMBER OF HEART-

BEATS IN ITS LIFE-TIME (ROUGHLY 1 BILLION)

**MORE FUNDAMENTALLY, ACROSS AEROBIC METABOLISM:
THE NUMBER OF TURNOVERS IN A LIFETIME OF CytO
ENZYMES (RESPIRATORY COMPLEX) IS AN APPROXIMATE
INVARIANT ($\sim 10^{16}$)**



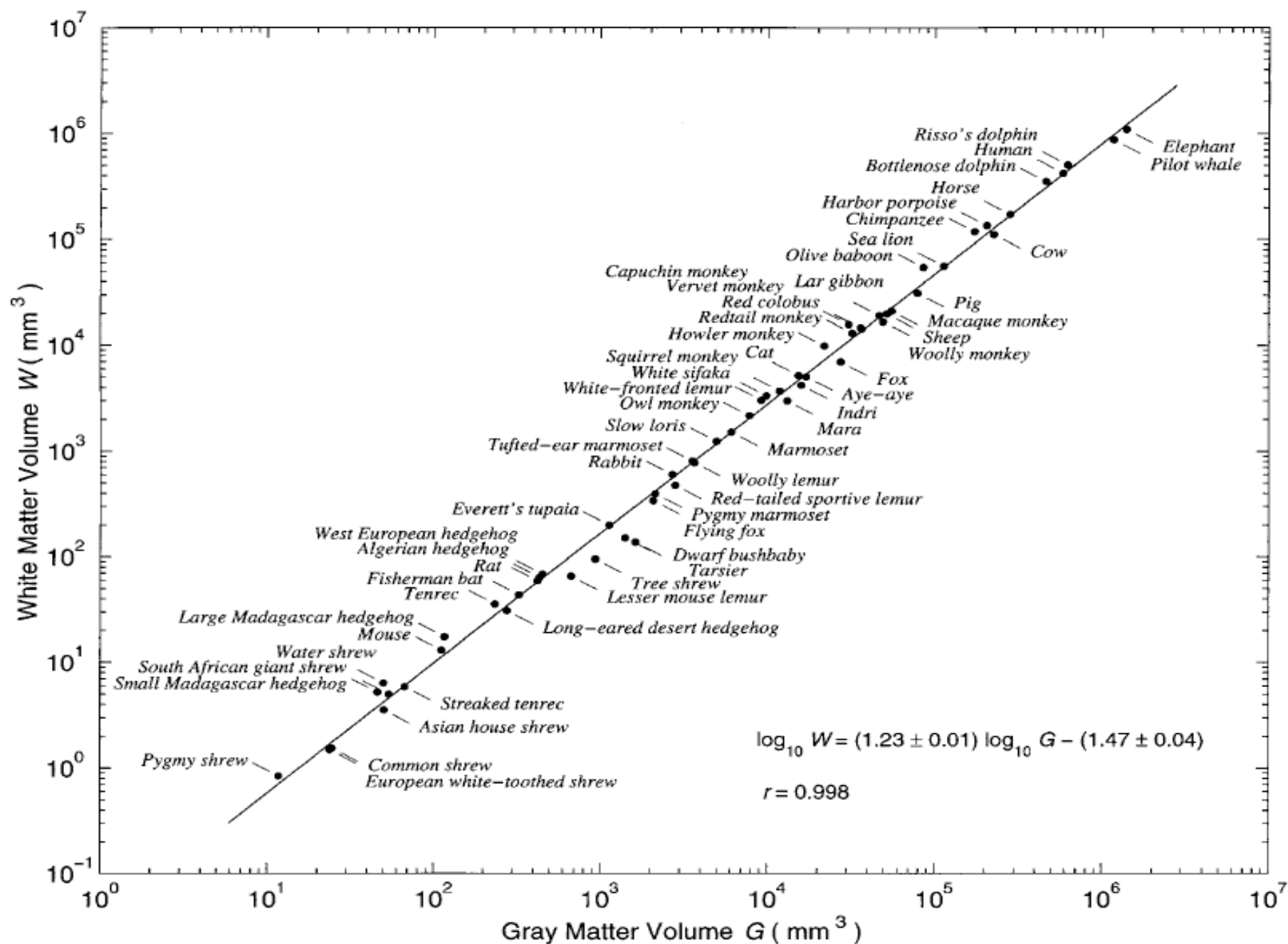
RECALL SPECIFIC METABOLIC RATE

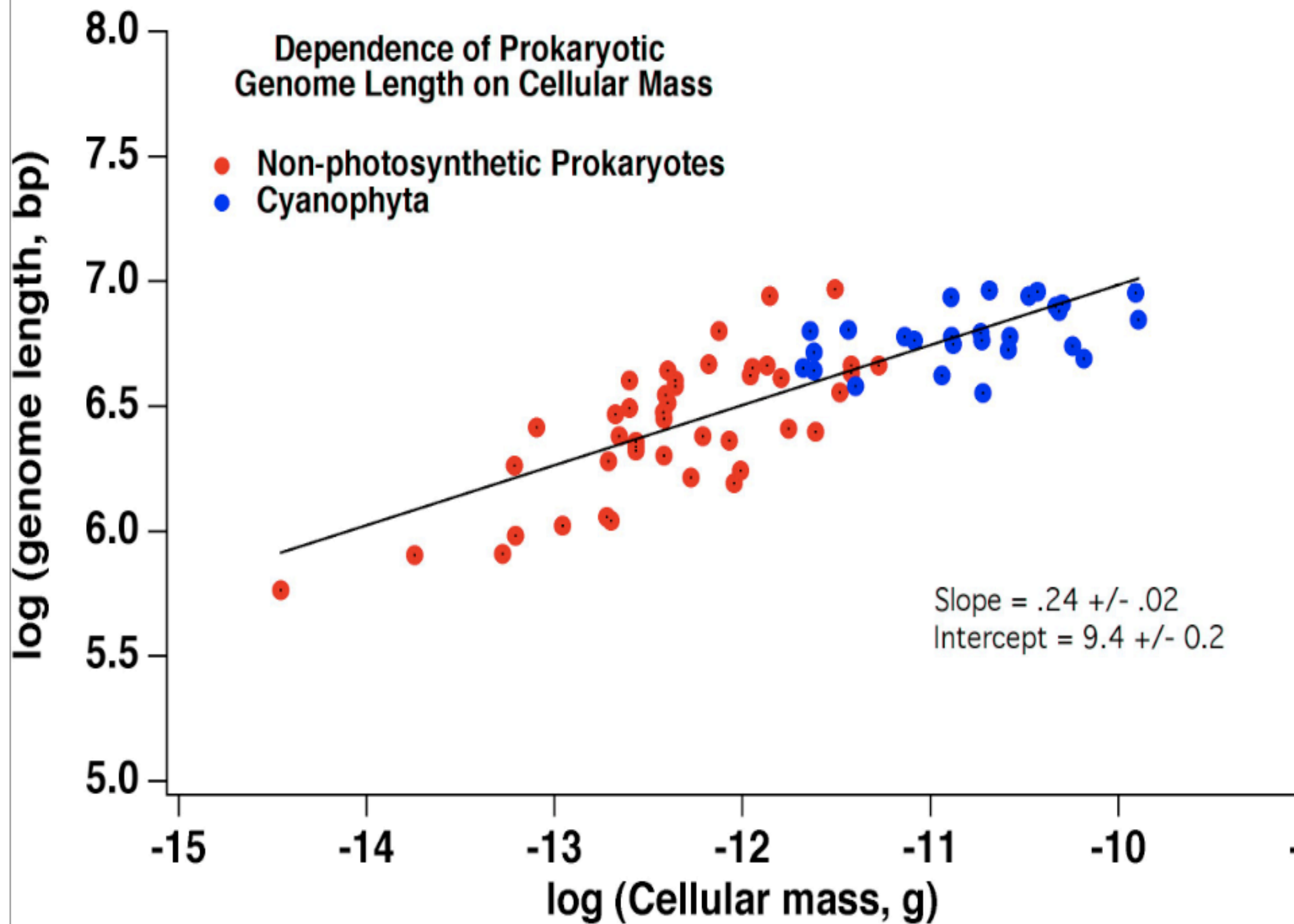
$$\bar{B} = \frac{B}{M} \propto M^{-1/4}$$

⇒ TOTAL ENERGY NEEDED TO SUPPORT UNIT
MASS OF AN ANIMAL DURING A LIFETIME
IS THE SAME FOR ALL ANIMALS REGARDLESS
OF SIZE :

$$\begin{aligned} E_{TOT} &\approx 1.2 \times 10^6 \text{ JOULES / gm} \\ &\approx 300 \text{ kcal / gm} \end{aligned}$$

$$\approx 5.2 \text{ moles O}_2 \text{ / gm}$$





LIFE IS THE MOST COMPLEX SYSTEM

SCALING LAWS ARE REMARKABLE BECAUSE

i) THEY EXIST

ii) THEY ARE VERY SIMPLE

iii) THEY ARE UNIVERSAL

} DOMINANCE OF
 $1/4$ POWER

iv) \Rightarrow BIGGER IS MORE EFFICIENT

v) FEW QUANTITATIVE "LAWS" IN BIOLOGY

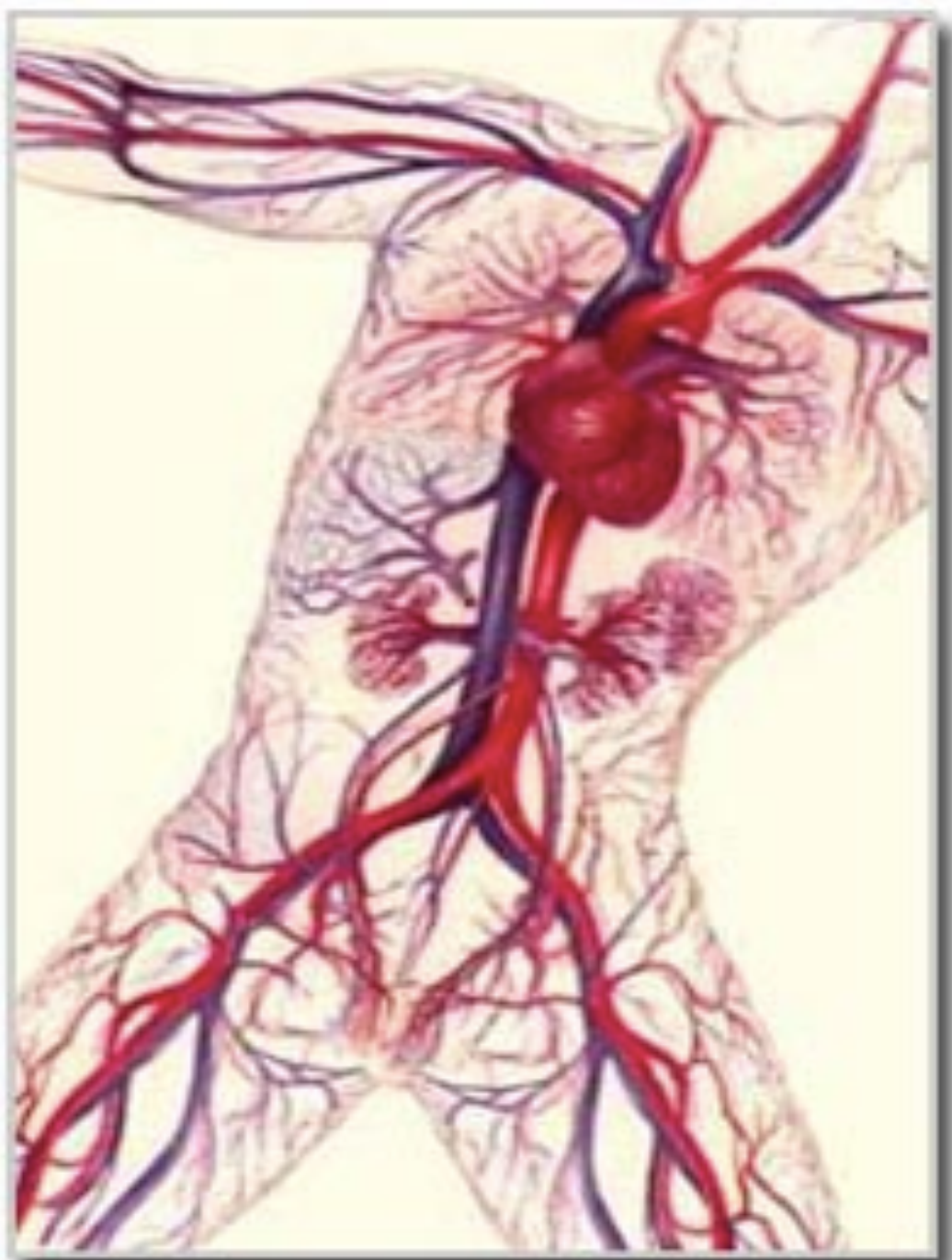
NETWORKS!!!

(FRACTALS!!)

FUNDAMENTAL PRINCIPLES

(NATURAL SELECTION)

- I. AT ALL SCALES ORGANISMS ARE SUSTAINED BY THE TRANSPORT OF ENERGY AND ESSENTIAL MATERIALS THROUGH HIERARCHICAL BRANCHING NETWORK SYSTEMS IN ORDER TO SUPPLY ALL LOCAL PARTS OF THE ORGANISM
- II. THESE NETWORKS ARE SPACE-FILLING
- III. THE TERMINAL BRANCHES OF THE NETWORK ARE INVARIANT UNITS
- IV. ORGANISMS HAVE EVOLVED BY NATURAL SELECTION SO AS TO
 - i) MINIMISE ENERGY DISSIPATED IN THE NETWORKS
 - AND/OR ii) MAXIMISE THE SCALING OF THEIR AREA OF INTERFACE WITH THEIR RESOURCE ENVIRONMENT



**Large vessels
branch into
smaller ones**

Beating heart

**Pulse wave
propagates
through elastic
vessels**

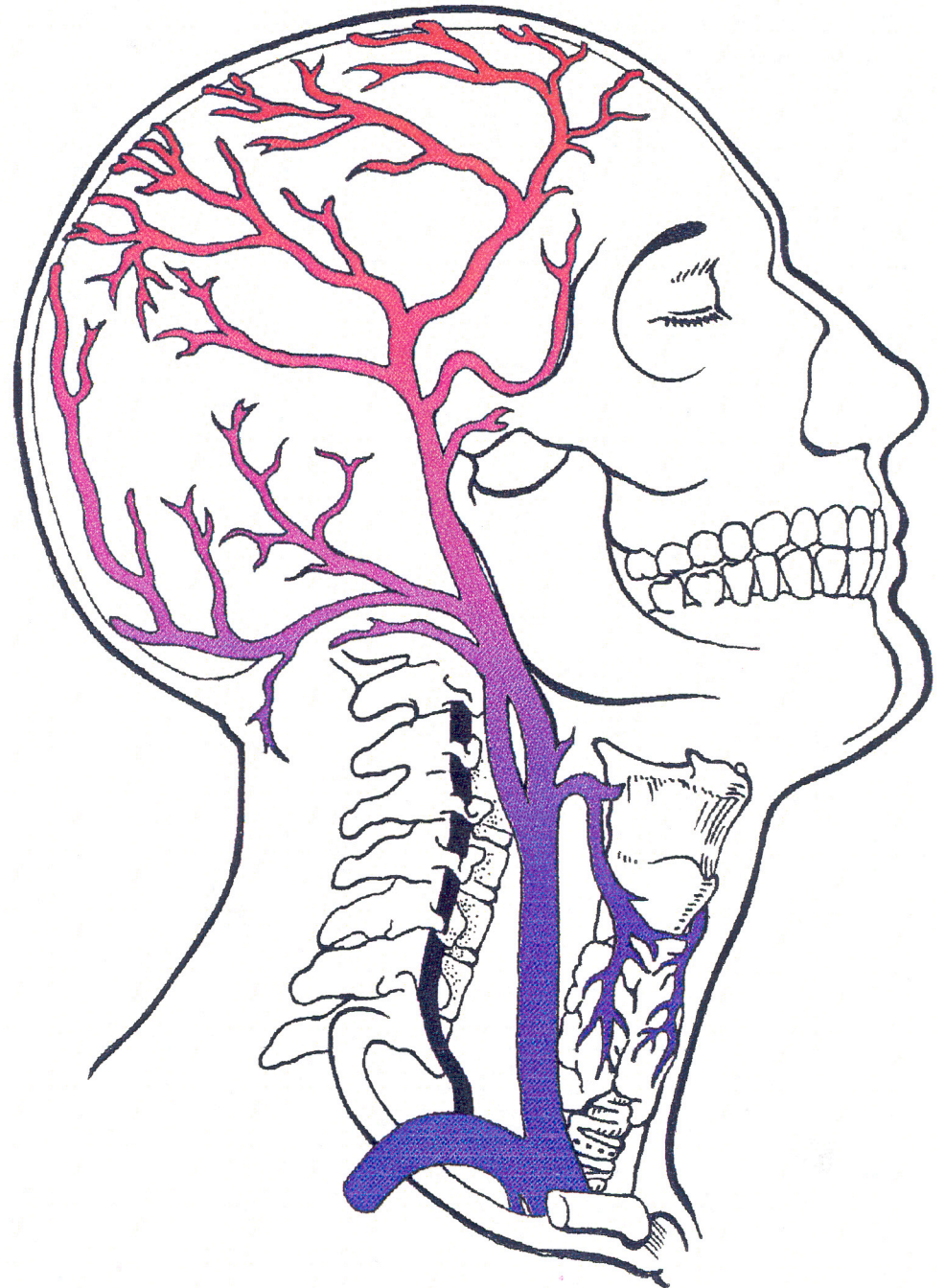
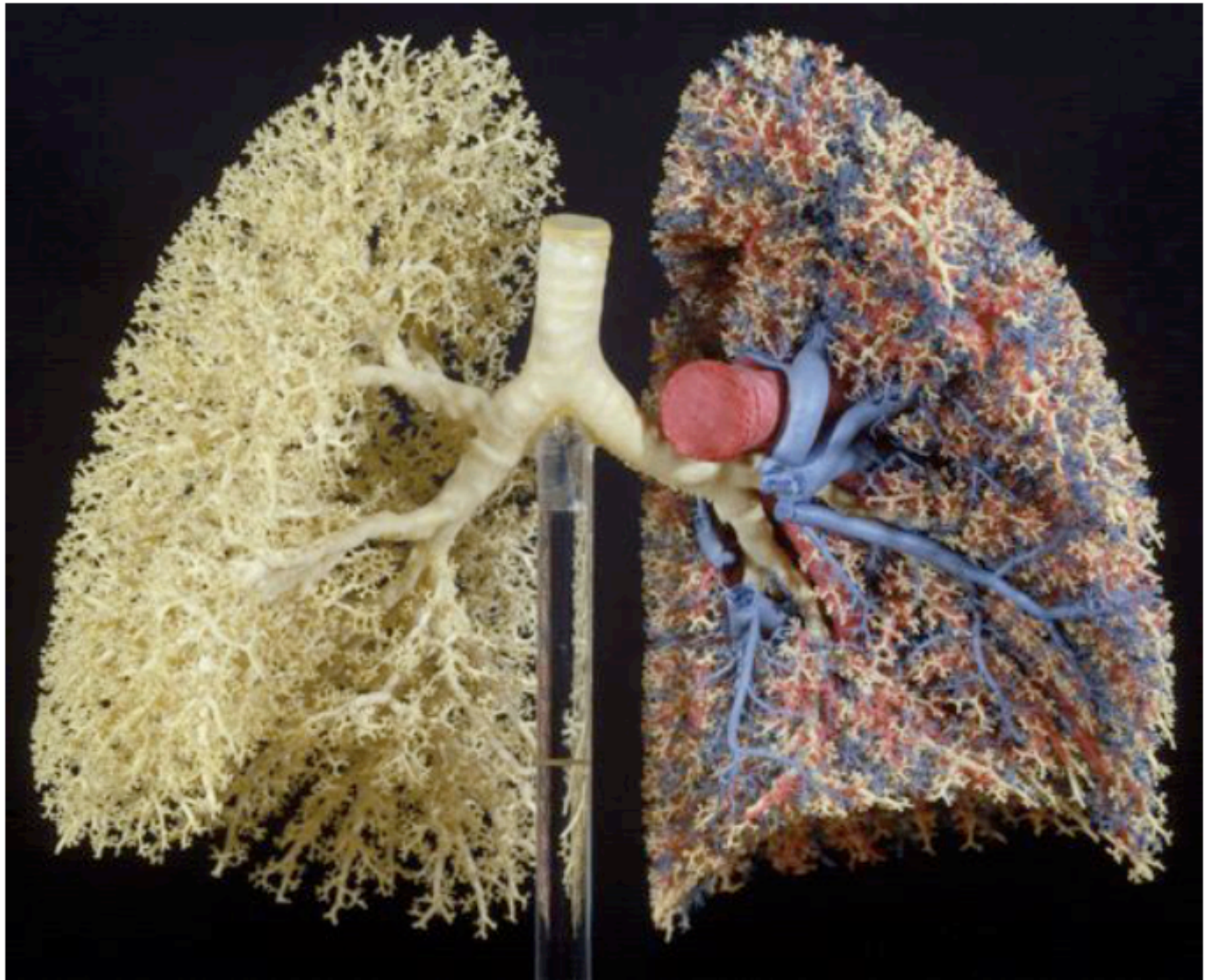


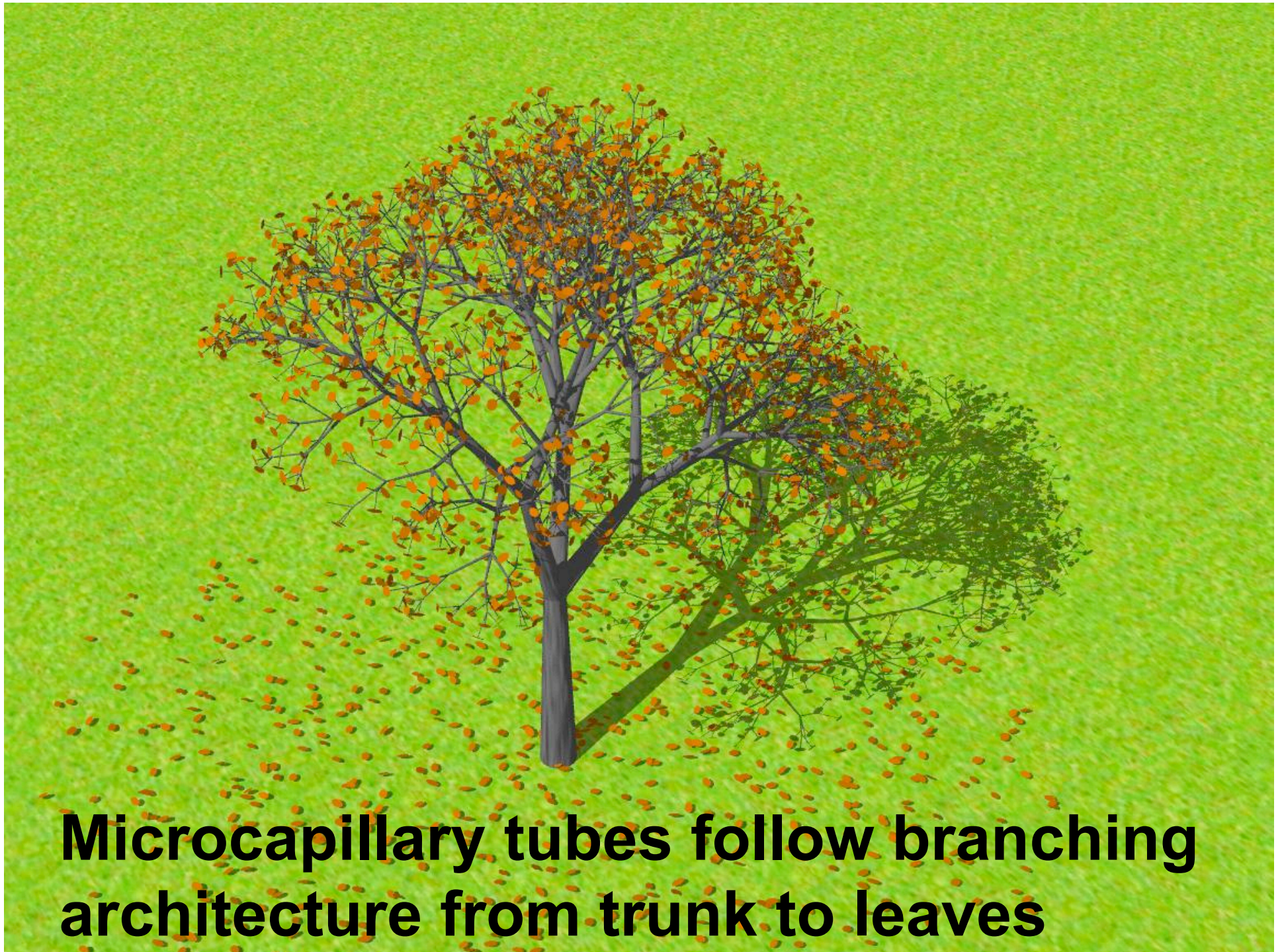


Fig. 5 A slice through the cerebellum showing the progressive branching structure. The white matter is distributed throughout the cerebellar volume. The geometric complexity of these structures provides for rapid dissemination of information (or energy) via a large surface area in a compact space. This feature is a hallmark of structures which maximize the surface area within a finite volume.



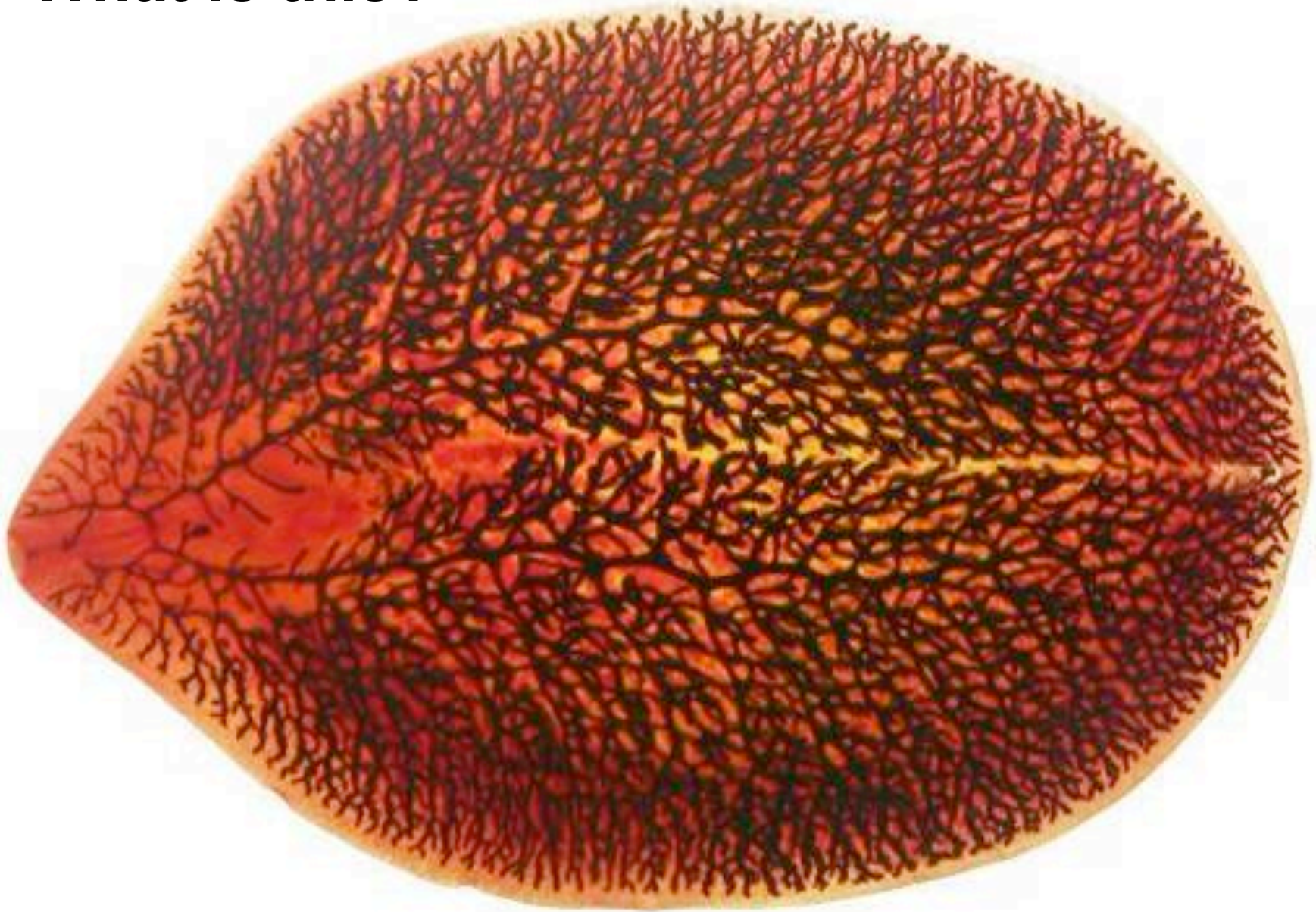
Relation between number and size of branches within a tree

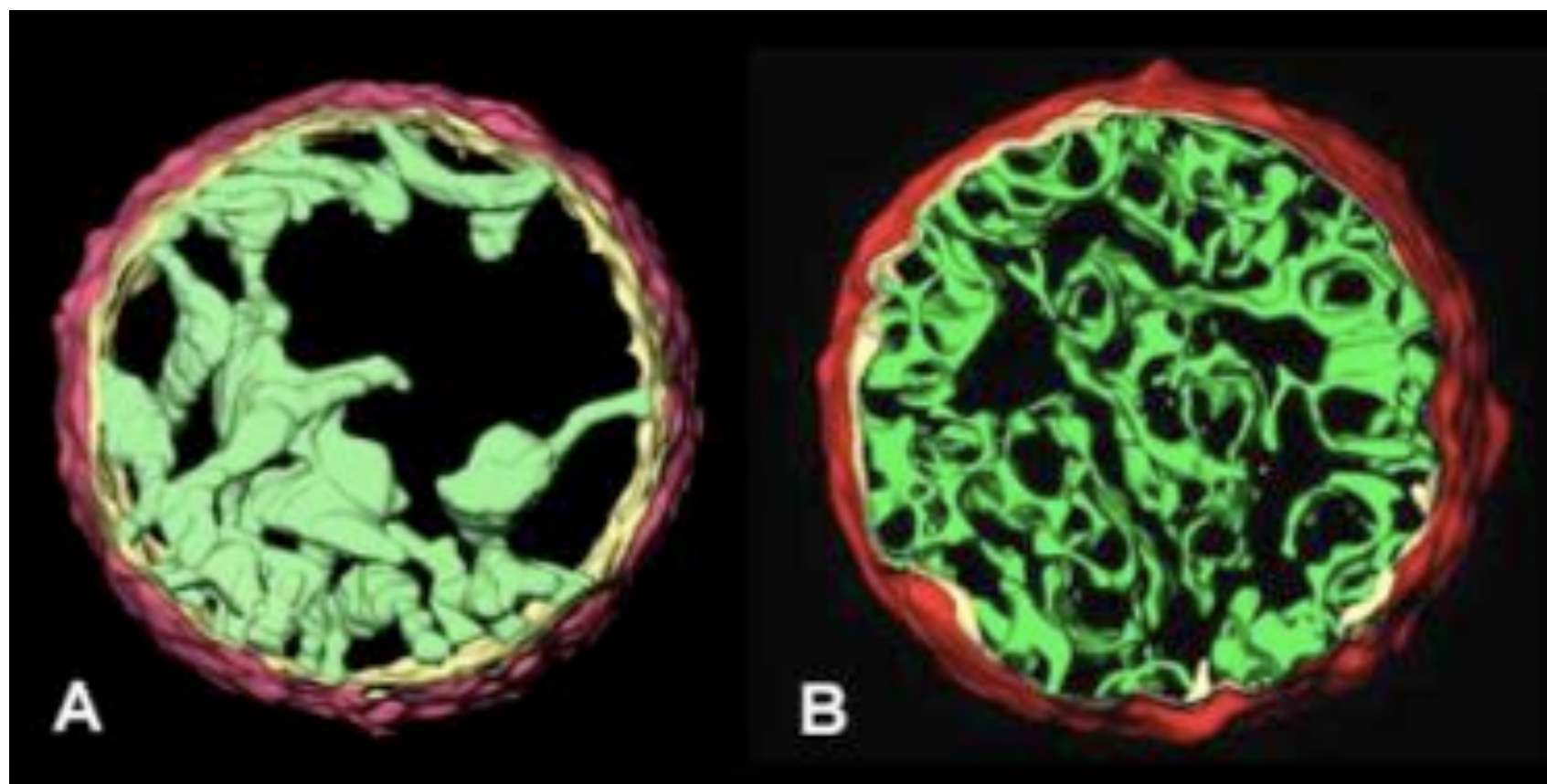




Microcapillary tubes follow branching architecture from trunk to leaves

What is this?





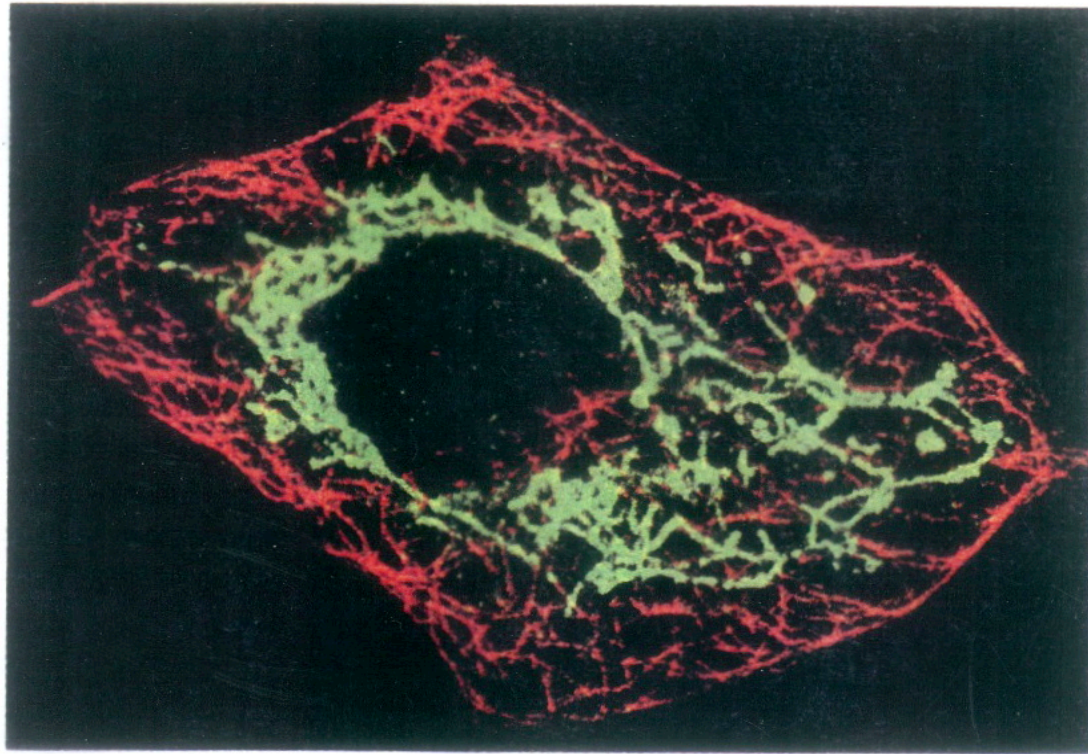
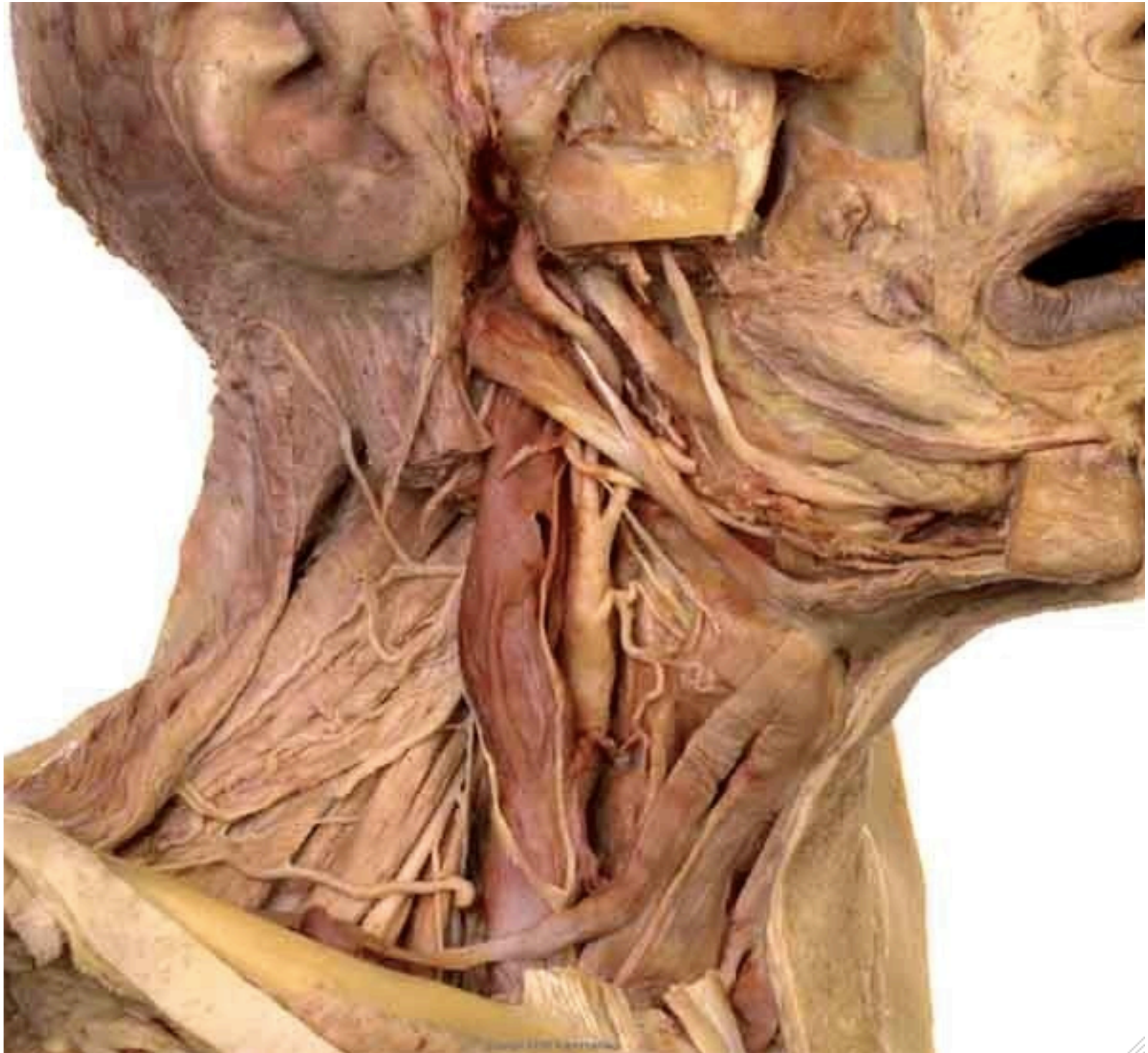
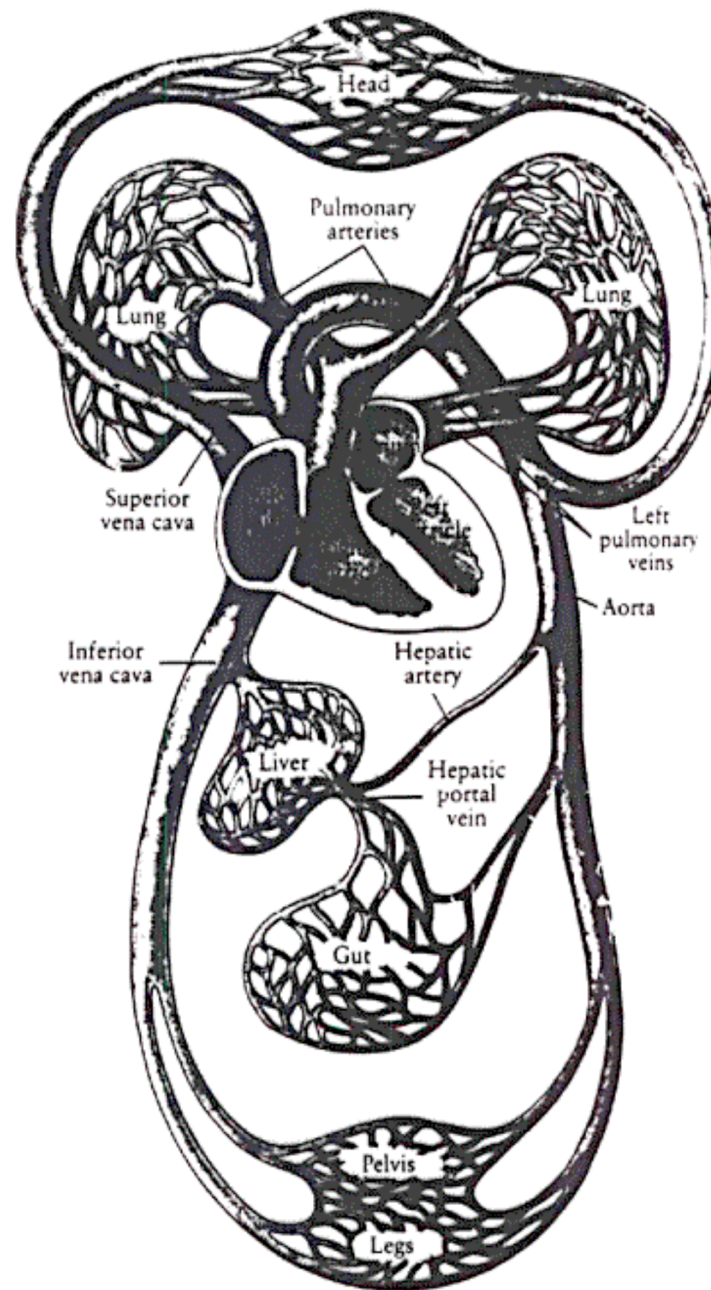


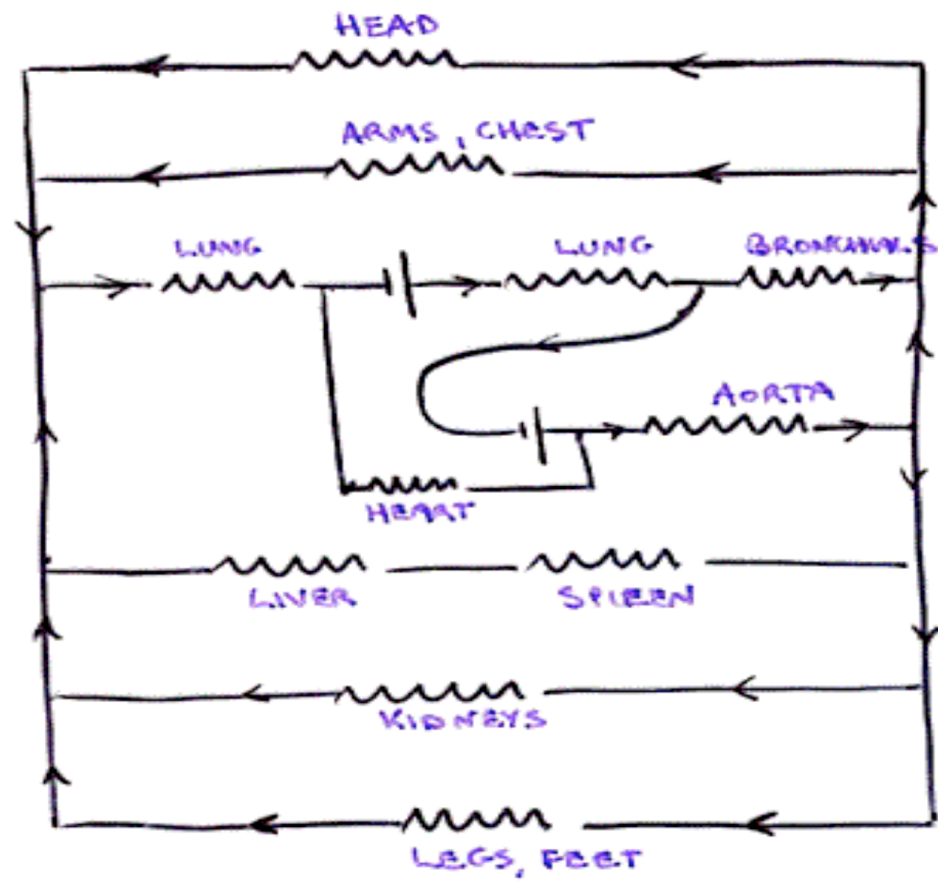
Fig. 1. Mitochondrial network in a mammalian fibroblast. A COS-7 cell labeled to visualize mitochondria (green) and microtubules (red) was analyzed by indirect immunofluorescence confocal microscopy. Mitochondria were labeled with antibodies to the β subunit of the F_1 -ATPase and a rhodamine-conjugated secondary antibody. Microtubules were labeled with antibody to tubulin and a fluorescein-conjugated secondary antibody. Pseudocolor was added to the digitized image. Scale: 1 cm = 10 μ m.

From M. P. Yaffe, *Science*, 283, 1493 (1999).





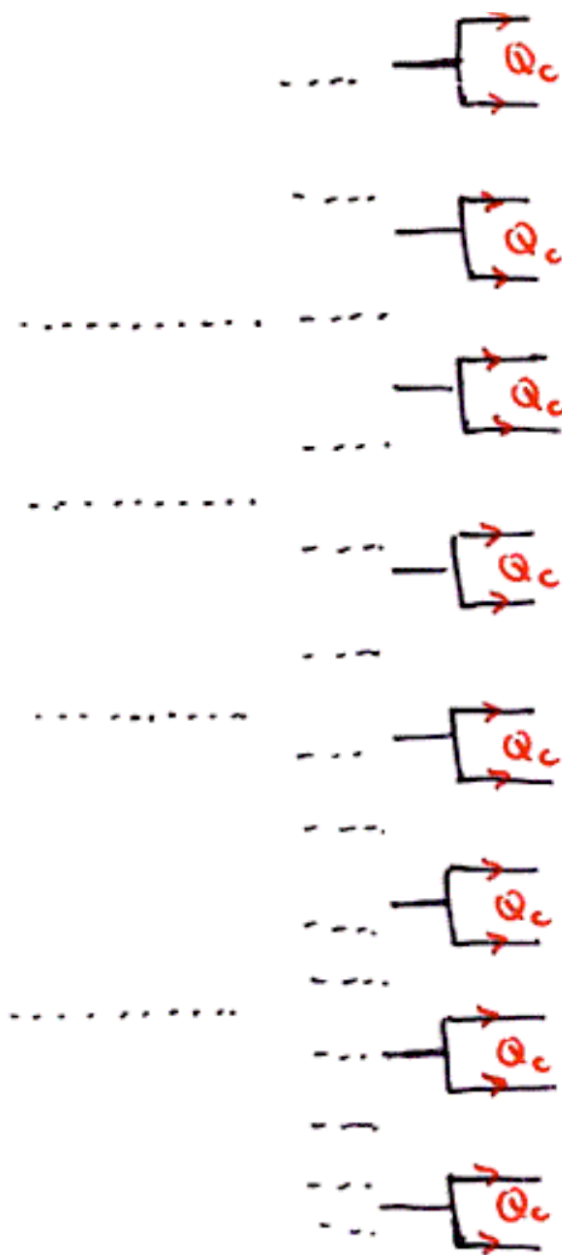
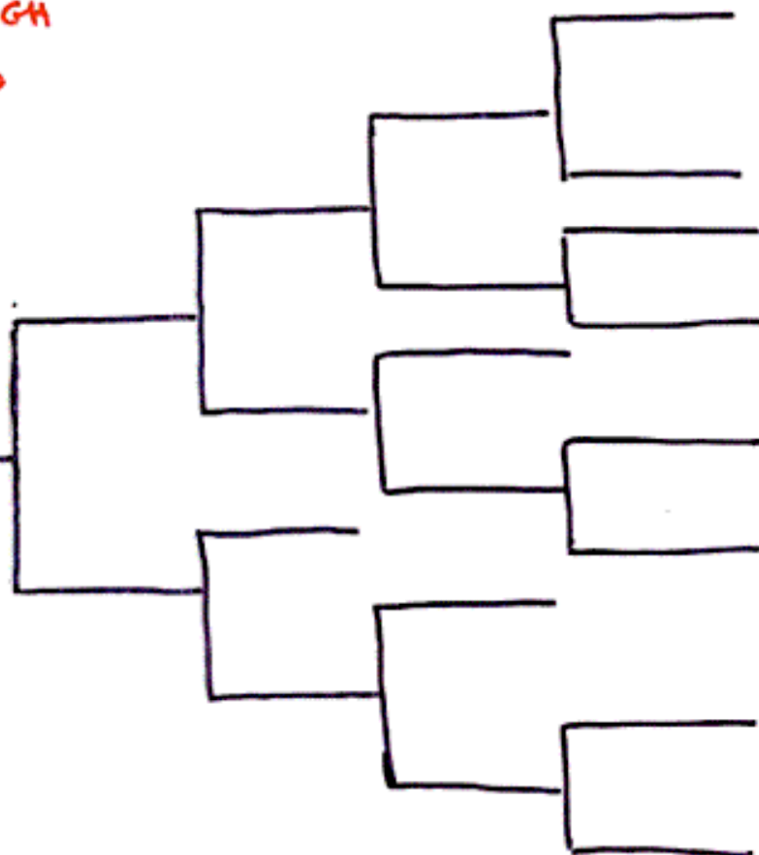
THE VASCULAR CIRCUITRY



FLUID FLOW THROUGH
AORTA $Q_0 \propto B$



AORTA
TRUNK
CEO



CAPILLARIES
PETIOLES
MITOCHONDRIA

SINCE THE FLUID (BLOOD) TRANSPORTS OXYGEN,
NUTRIENTS, ETC FROM THE AORTA TO THE
CAPILLARIES

METABOLIC RATE \propto VOLUME FLOW RATE

$$B \propto Q_o$$

BUT THE CONSERVATION OF FLUID (BLOOD)

$$\Rightarrow Q_o = N_c Q_c$$

TOTAL NUMBER OF CAPILLARIES VOLUME FLOW RATE IN AVERAGE CAPILLARY

CAPILLARY IS AN INVARIANT UNIT

(Q_c IS SAME FOR ALL MAMMALS)

\Rightarrow NUMBER OF CAPILLARIES (N_c) MUST SCALE IN SAME
WAY AS THE METABOLIC RATE ($B \propto Q_o$)

SO, IF $B \sim M^{3/4}$ THEN

$$N_c \sim M^{3/4} \quad (\text{NOT } N_c \sim M)$$

TOTAL NUMBER OF CELLS

$$N_{\text{cell}} \sim M \quad (\text{LINEAR})$$

TOTAL NUMBER OF CAPILLARIES

$$N_c \sim M^{3/4}$$

MISMATCH !

⇒ NUMBER OF CELLS FED BY A SINGLE
CAPILLARY INCREASES AS $M^{1/4}$

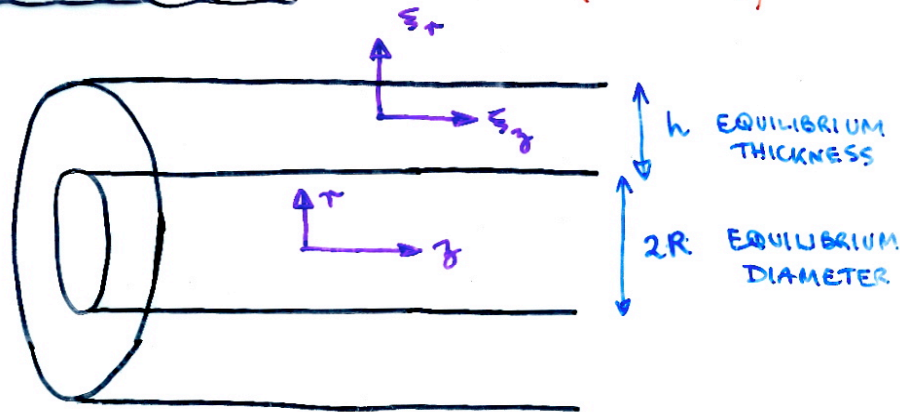
(ANOTHER MANIFESTATION THAT EFFICIENCY
INCREASES WITH SIZE)

IMPORTANT IMPLICATIONS FOR GROWTH AND DEATH !

1) PULSATILE TREATMENT

(WOMERSLEY)

(19)



2) FLUID :

STRESS TENSOR: $\theta_{ij} = \lambda e_{kk} \delta_{ij} + 2\mu e_{ij} - p \delta_{ij}$

(NEWTONIAN)

COEFFICIENTS OF VISCOSITY

PRESSURE

STRAIN TENSOR: $e_{ij} = \frac{1}{2} (\partial_i v_j + \partial_j v_i)$

VELOCITY

EQⁿ. OF MOTION: $\rho \frac{Dv_i}{Dt} = \partial_j \theta_{ij}$

NAVIER-STOKES EQⁿ.

COVARIANT DERIVATIVE $\frac{D}{Dt} = \frac{\partial}{\partial t} + v_i \partial_i$

EQⁿ. OF CONTINUITY: $\frac{\partial \rho}{\partial t} + \partial_i (\rho v_i) = 0$

b) WALLS :

$$\theta_{ij}^w = \lambda e_{kk}^w \delta_{ij} + 2B e_{ij}^w - p \delta_{ij} \quad \text{HOOKE'S LAW}$$

ELASTIC MODULI

$$e_{ij}^w = \frac{1}{2} (\partial_i \xi_j + \partial_j \xi_i)$$

$$\rho_w \frac{Du_i}{Dt} = \partial_j \theta_{ij}^w ; u_i \equiv \frac{\partial \xi_i}{\partial t} \quad \text{NAVIER EQ'NS.}$$

$$\partial_i \xi_i = 0$$

NEGLECT NON-LINEARITIES :

$$\rho_w \frac{\partial^2 \xi_i}{\partial t^2} = B \partial^2 \xi_i - \partial_i p$$

SOLVE USING FOURIER AS WITH FLUID, WALLS AND

FLUID COUPLED VIA BOUNDARY CONDITIONS : CONTINUITY

OF VELOCITY AND FORCE : $u_r = \frac{\partial \xi_r}{\partial t}$

AND $\int_{\text{SURFACE}} \theta_{ij} dS_j$ CONTINUOUS

CAN BE SOLVED : BIG MESS!

SIMPLIFY USING THIN WALL APPROXIMATION

$$\text{i.e. } \frac{h}{R} \ll 1$$

"EVERYTHING" FOLLOWS FROM THE 4 BASIC PRINCIPLES

I. HIERARCHICAL "FRACTAL-LIKE" BRANCHING

NETWORKS

II. SPACE-FILLING

III. INVARIANT TERMINAL UNITS

IV. ENERGY DISSIPATED IS MINIMISED

"AVERAGE
IDEALISED
ORGANISM
OR SYSTEM"

II \Rightarrow i) SPACE-FILLING $\frac{L_{n+1}}{L_n} = \frac{1}{2^{1/3}} \quad \frac{1}{n^{1/3}}$

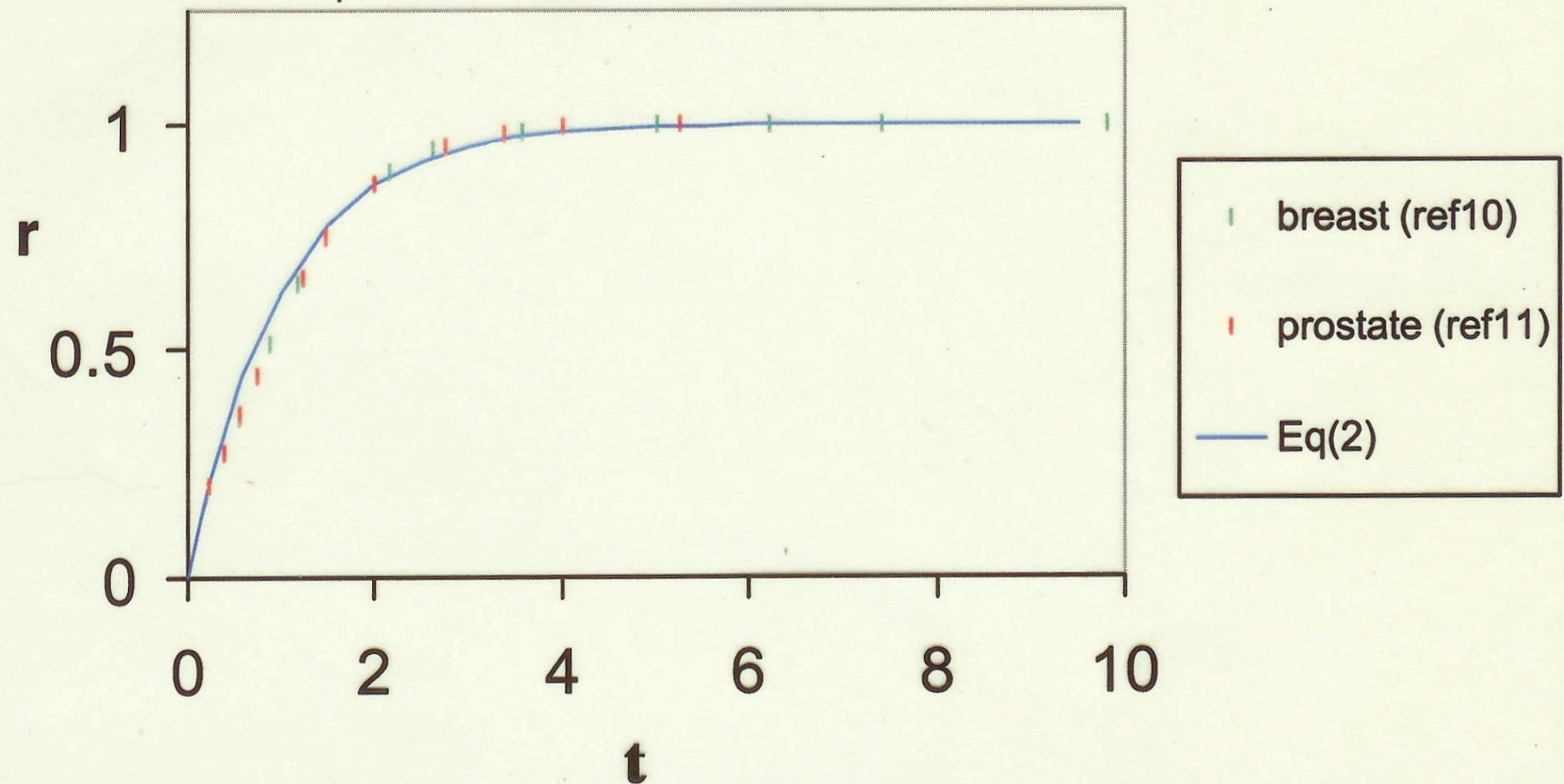
III \Rightarrow ii) INV. TERMINAL UNITS

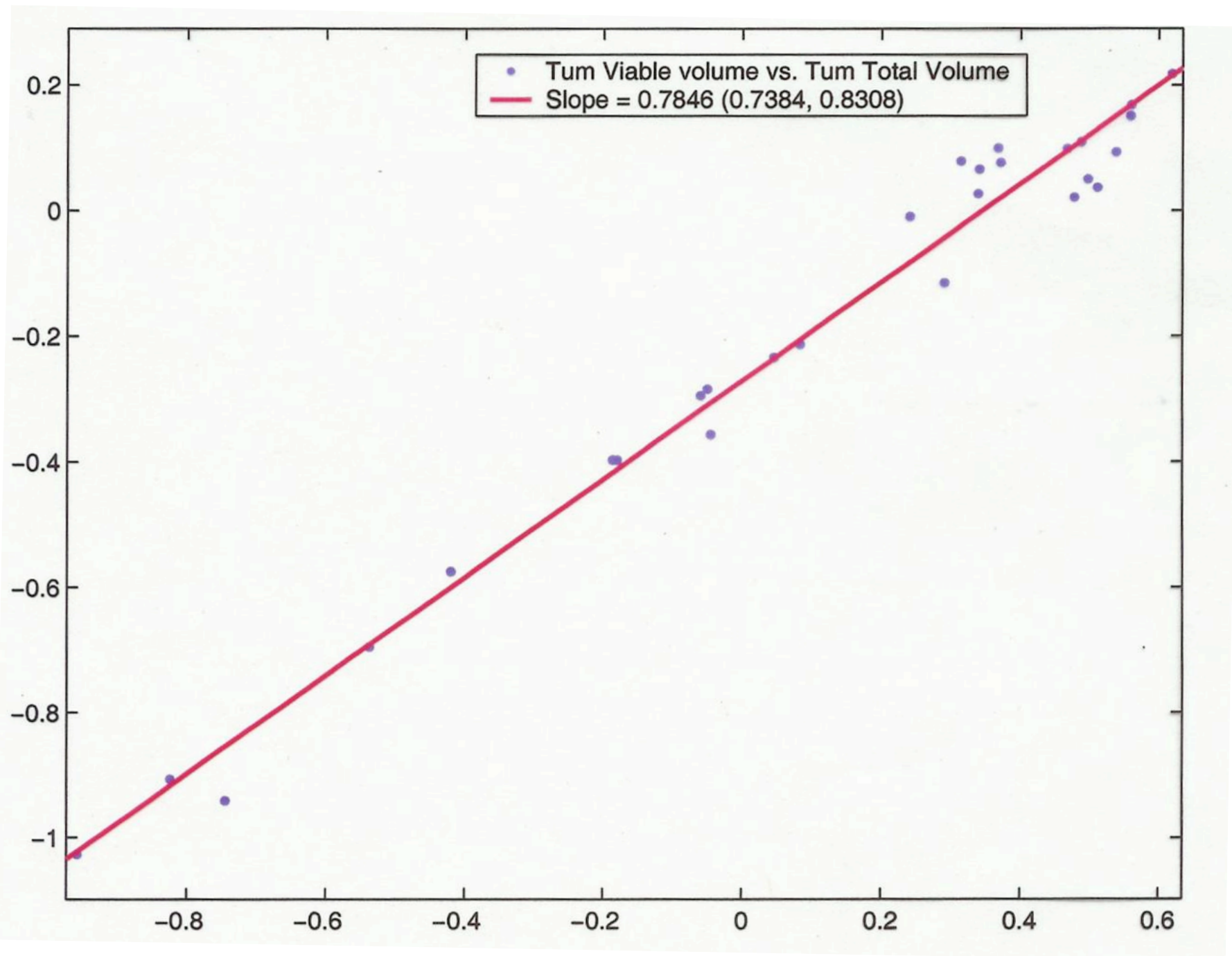
IV \Rightarrow { iii) AREA-PRESERVING BRANCHING $\frac{T_{n+1}}{T_n} = \frac{1}{2^{1/2}}$
iv) VOLUME OF NETWORK (BLOOD)
~ VOLUME OF WHOLE BODY ~ M

$\Rightarrow B \propto M^{3/4}$ (AND MUCH MORE)

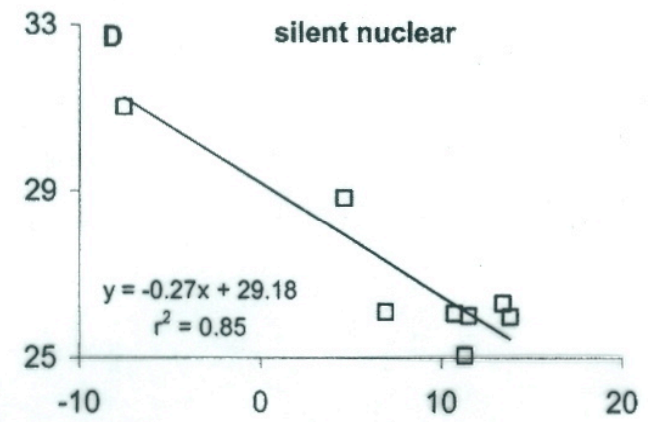
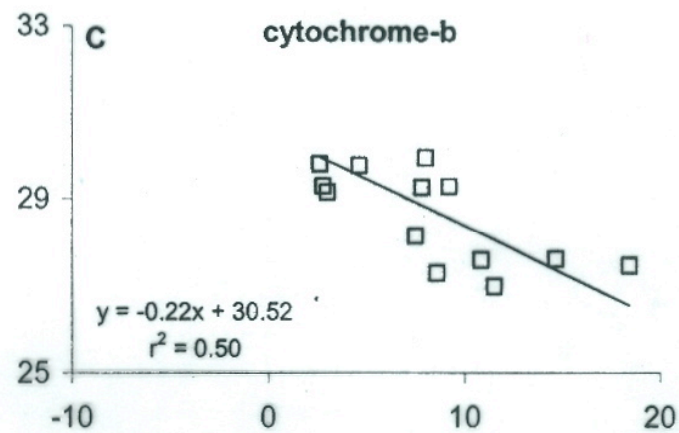
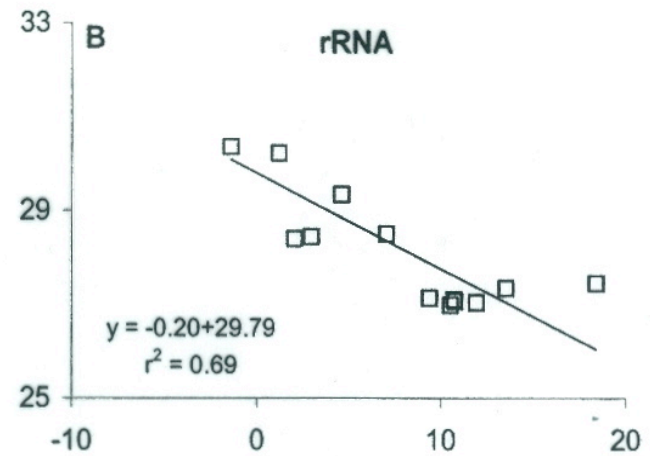
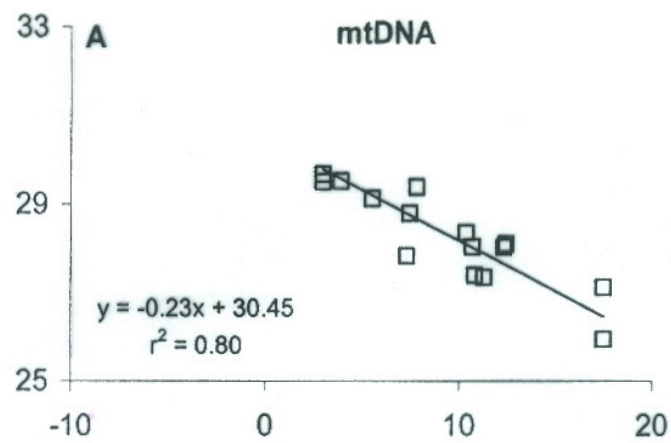
[MORE GENERALLY: $B \propto M^{b(M)}$ WITH $b(M) \approx 3/4$ M LARGE
 $< 3/4$ M SMALL
 ≈ 1 M $< \mu$]
NOT A STRICT POWER LAW!

**'in vivo' data
(patients)**





$\ln(\alpha \cdot e^{E/kT})$



$\ln(m)$

IN d DIMENSIONS

$$B \propto M^{\frac{d}{d+1}}$$

WE LIVE IN 3 SPATIAL DIMENSIONS SO $B \propto M^{3/4}$

⇒ "3" REPRESENTS DIMENSIONALITY OF SPACE

"4" INCREASE IN DIMENSIONALITY DUE TO
FRACTAL-LIKE SPACE FILLING

LIFE HAS TAKEN ADVANTAGE OF THE POSSIBILITY OF
USING SPACE-FILLING FRACTAL-LIKE SURFACES
(WHERE ENERGY AND RESOURCES ARE EXCHANGED)

TO MAXIMISE ENERGY TRANSFER FROM THE
ENVIRONMENT

NON-FRACTAL :

AREA

$M^{2/3}$ ← DIMENSIONALITY OF SPACE (VOLUME)

BIOLOGICAL (FRACTAL)

$$M^{3/4}$$

BY ANALOGY : LIFE EFFECTIVELY OPERATES IN
FOUR SPATIAL DIMENSIONS

[FIVE IF TIME IS INCLUDED]

Cardiovascular		
Variable	Exponent	
	Predicted	Observed
Aorta radius r_o	$3/8 = 0.375$	0.36
Aorta pressure Δp_o	$0 = 0.00$	0.032
Aorta blood velocity u_o	$0 = 0.00$	0.07
Blood volume V_b	$1 = 1.00$	1.00
Circulation time	$1/4 = 0.25$	0.25
Circulation distance l	$1/4 = 0.25$	ND
Cardiac stroke volume	$1 = 1.00$	1.03
Cardiac frequency ω	$-1/4 = -0.25$	-0.25
Cardiac output \dot{E}	$3/4 = 0.75$	0.74
Number of capillaries N_c	$3/4 = 0.75$	ND
Service volume radius	$1/12 = 0.083$	ND
Womersley number α	$1/4 = 0.25$	0.25
Density of capillaries	$-1/12 = -0.083$	-0.095
O ₂ affinity of blood P_{50}	$-1/12 = -0.083$	-0.089
Total resistance Z	$-3/4 = -0.75$	-0.76
Metabolic rate B	$3/4 = 0.75$	0.75

Respiratory


Variable	Exponent	
	Predicted	Observed
Tracheal radius	$3/8 = 0.375$	0.39
Interpleural pressure	$0 = 0.00$	0.004
Air velocity in trachea	$0 = 0.00$	0.02
Lung volume	$1 = 1.00$	1.05
Volume flow to lung	$3/4 = 0.75$	0.80
Volume of alveolus V_A	$1/4 = 0.25$	ND
Tidal volume	$1 = 1.00$	1.041
Respiratory frequency	$-1/4 = -0.25$	-0.26
Power dissipated	$3/4 = 0.75$	0.78
Number of alveoli N_A	$3/4 = 0.75$	ND
Radius of alveolus r_A	$1/12 = 0.083$	0.13
Area of alveolus A_A	$1/6 = 0.083$	ND
Area of lung A_L	$11/12 = 0.92$	0.95
O ₂ diffusing capacity	$1 = 1.00$	0.99
Total resistance	$-3/4 = -0.75$	-0.70
O ₂ consumption rate	$3/4 = 0.75$	0.76

Table 1 Predicted values of scaling exponents for physiological and anatomical variables of plant vascular systems.

Variable	Plant mass		Branch radius		
	Exponent predicted	Symbol	Symbol	Exponent	
				Predicted	Observed
Number of leaves	$\frac{3}{4}$ (0.75)	n_0^L	n_k^L	2 (2.00)	2.007 (ref. 12)
Number of branches	$\frac{3}{4}$ (0.75)	N_0	N_k	-2 (-2.00)	-2.00 (ref. 6)
Number of tubes	$\frac{3}{4}$ (0.75)	n_0	n_k	2 (2.00)	n.d.
Branch length	$\frac{1}{4}$ (0.25)	l_0	l_k	$\frac{2}{3}$ (0.67)	0.652 (ref. 6)
Branch radius	$\frac{3}{8}$ (0.375)	r_0			
Area of conductive tissue	$\frac{7}{8}$ (0.875)	A_0^{CT}	A_k^{CT}	$\frac{7}{3}$ (2.33)	2.13 (ref. 8)
Tube radius	$\frac{1}{16}$ (0.0625)	a_0	a_k	$\frac{1}{8}$ (0.167)	n.d.
Conductivity	1 (1.00)	K_0	K_k	$\frac{8}{3}$ (2.67)	2.63 (ref. 12)
Leaf-specific conductivity	$\frac{1}{4}$ (0.25)	L_0	L_k	$\frac{2}{3}$ (0.67)	0.727 (ref. 17)
Fluid flow rate			\dot{Q}_k	2 (2.00)	n.d.
Metabolic rate	$\frac{3}{4}$ (0.75)	\dot{Q}_0			
Pressure gradient	$-\frac{1}{4}$ (-0.25)	$\Delta P_0/l_0$	$\Delta P_k/l_k$	$-\frac{2}{3}$ (-0.67)	n.d.
Fluid velocity	$-\frac{1}{8}$ (-0.125)	u_0	u_k	$-\frac{1}{3}$ (-0.33)	n.d.
Branch resistance	$-\frac{3}{4}$ (-0.75)	Z_0	Z_k	$-\frac{1}{3}$ (-0.33)	n.d.
Tree height	$\frac{1}{4}$ (0.25)	h			
Reproductive biomass	$\frac{3}{4}$ (0.75)				
Total fluid volume	$\frac{26}{24}$ (1.0415)				

Table 1. Similarity of predicted scaling relations for branches within a tree [quantities denoted by uppercase symbols and subscripts i (20)], and for trees within a forest (denoted by lowercase symbols and subscripts k)*

Scaling quantity	Individual tree	Entire forest
Area preserving	$\frac{R_{i+1}}{R_i} = \frac{1}{n^{1/2}}$	$\frac{r_{k+1}}{r_k} = \frac{1}{\lambda^{1/2}}$
Space filling	$\frac{L_{i+1}}{L_i} = \frac{1}{n^{1/3}}$	$\frac{l_{k+1}}{l_k} = \frac{1}{\lambda^{1/3}}$
Biomechanics	$R_i^2 = L_i^3$	$r_k^2 = l_k^3$
Size distribution*	$\Delta N_i \propto R_i^{-2} \propto M_i^{-3/4}$	$\Delta n_k \propto r_k^{-2} \propto m_k^{-3/4}$
Energy and material flux*	$B_i \propto R_i^2 \propto N_i^L \propto M_i^{3/4}$	$B_k \propto r_k^2 \propto n_k^L \propto m_k^{3/4}$

Stand property	Predicted stem radius,  based scaling function
Size class neighbor separation	$d_k \propto r_k$
Canopy scaling	$r_k^{\text{can}} \propto r_k^{2/3}$
Canopy spacing	$d_k^{\text{can}} = c_1 r_k \left[1 - \left(\frac{r_k}{r_{\bar{k}}} \right)^{1/3} \right]$
Energy Equivalence	$\Delta n_k B_k \propto r_k^0$
Total forest resource use	$B_{\text{Tot}} \propto \sum \Delta n_k r_k^2 \leq \dot{R}$
Mortality rate	$\mu_k \approx A r_k^{-2/3}$
Size distribution	$N_k \approx \frac{\dot{R}}{(K+1)b_0} r_k^{-2}$

HYDRODYNAMIC RESISTANCE OF THE NETWORK

$$\sim \frac{1}{M^{3/4}}$$

TOTAL RESISTANCE DECREASES WITH SIZE !!

SMALL MAY BE BEAUTIFUL BUT LARGE IS
MORE EFFICIENT !!

BLOOD PRESSURE $\sim M^0$
AORTA BLOOD VELOCITY $\sim M^0$ } INVARIANT !

RADIUS OF A WHALE'S AORTA ~ 30 cm

RADIUS OF A SHREW'S AORTA $\sim \frac{1}{10}$ mm

YET THEY HAVE THE SAME BLOOD PRESSURE

THIS DECREASE OF B_c WITH SIZE IS DRIVEN

BY THE HEGEMONY OF THE NETWORK

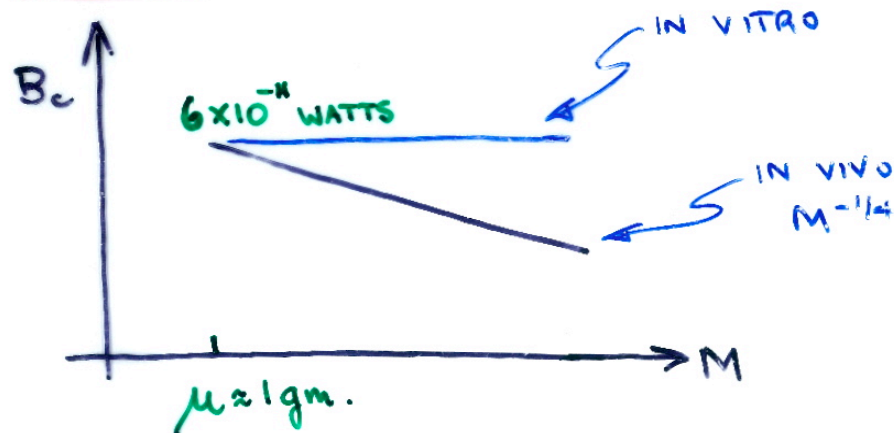
(CONTROLS FUNDAMENTAL BIOCHEMICAL RATES)

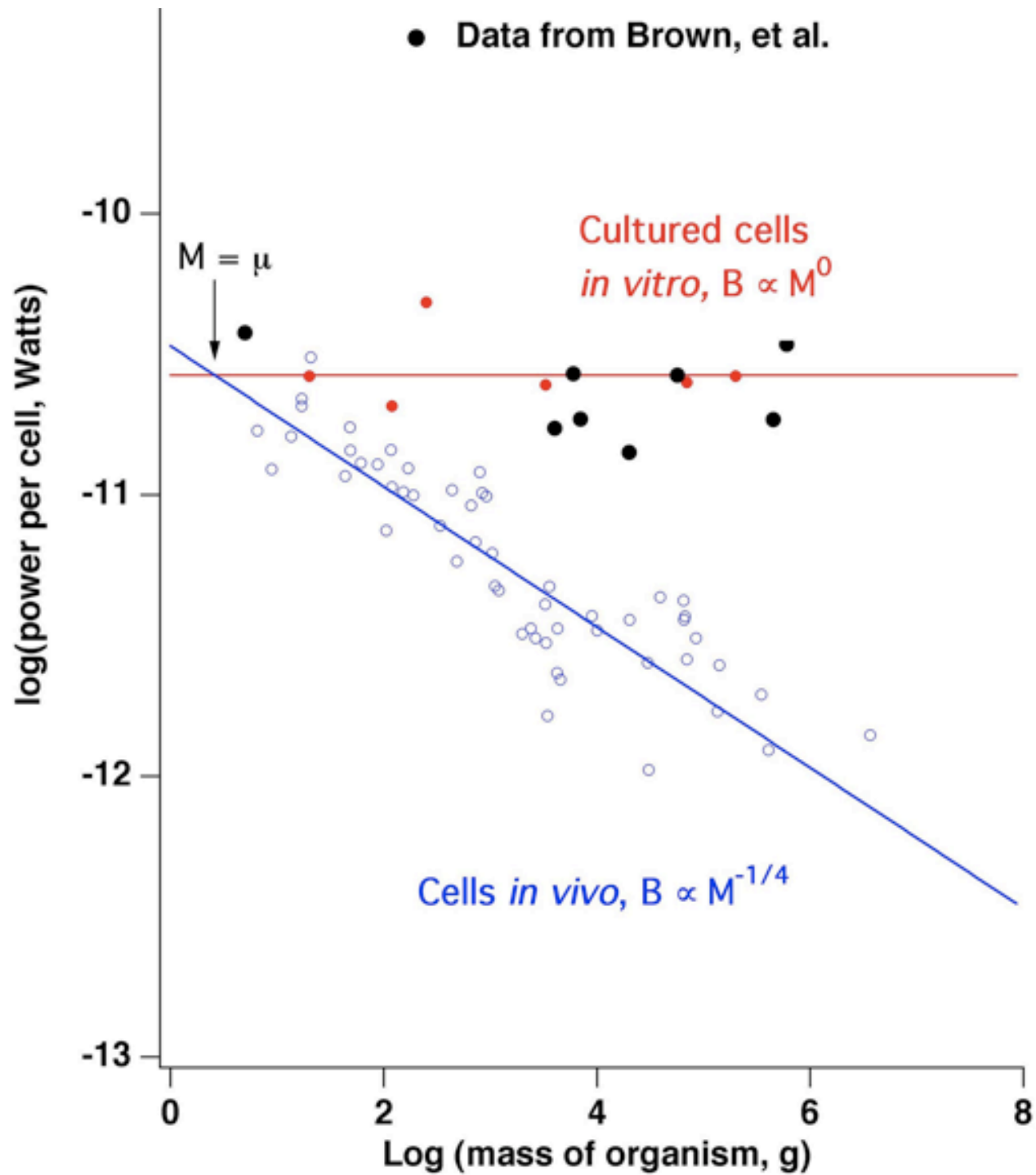
⇒ IF THE NETWORK WERE REMOVED SO CELLS

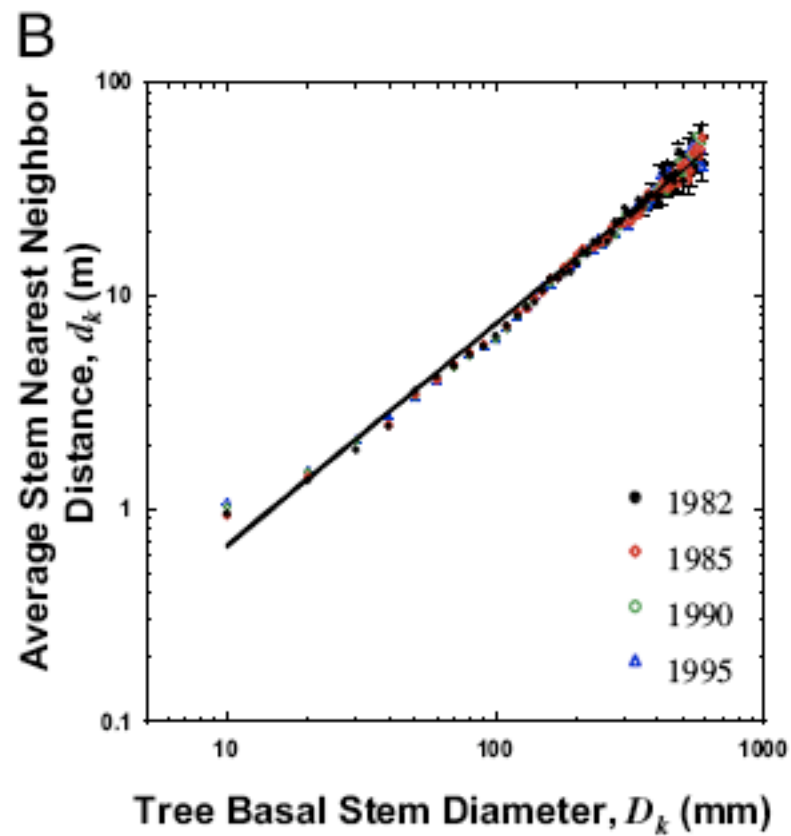
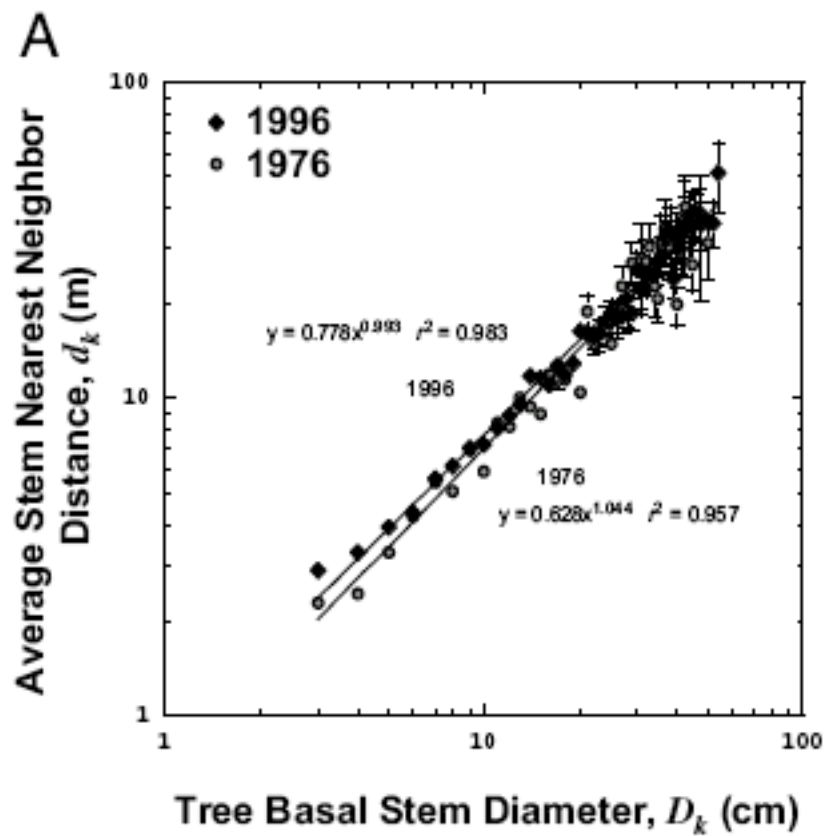
BECOME FREE (IN VITRO) B_c SHOULD BECOME

INDEPENDENT OF WHAT MAMMAL THEY ORIGINATED

IN: PREDICT

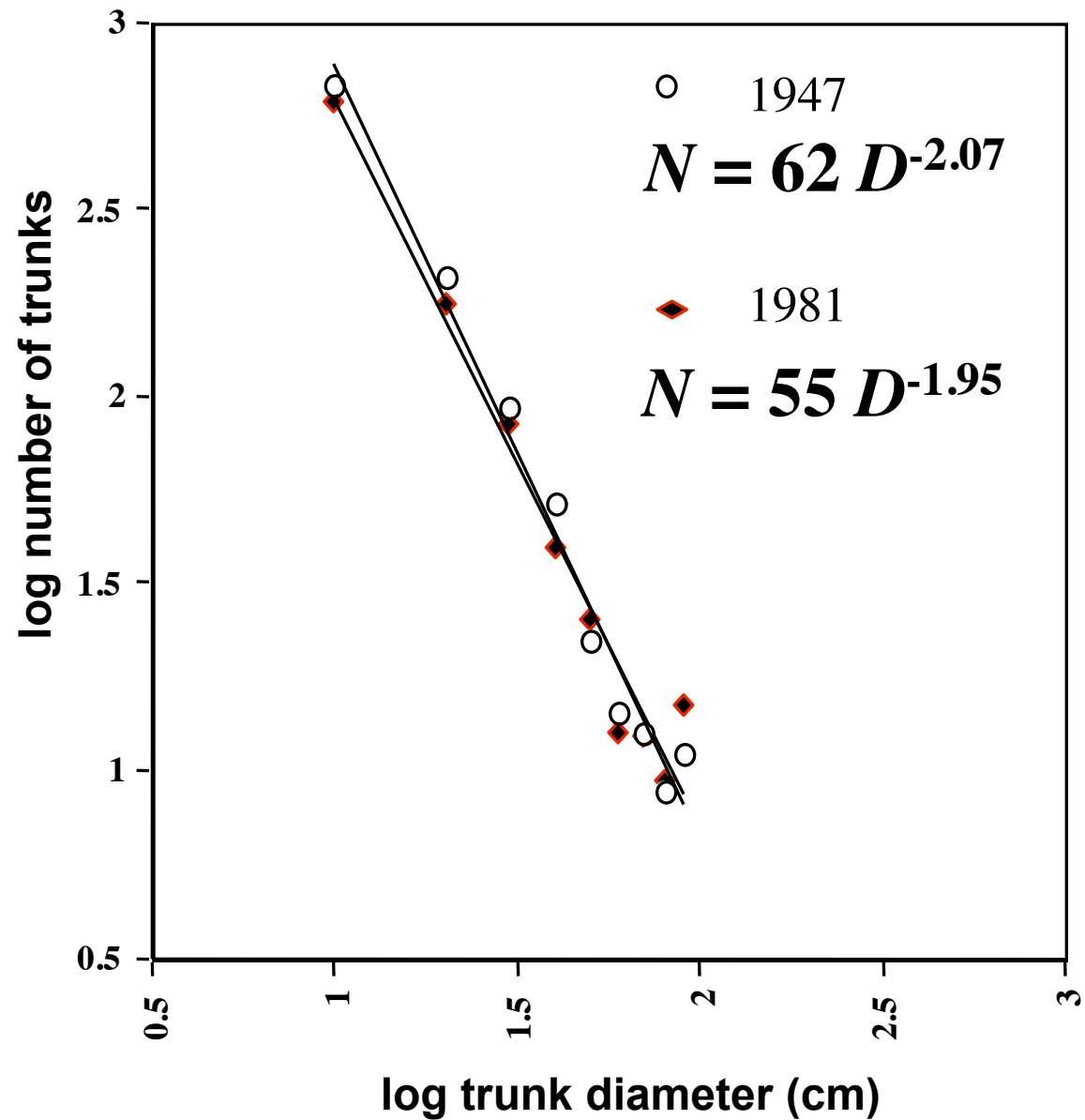




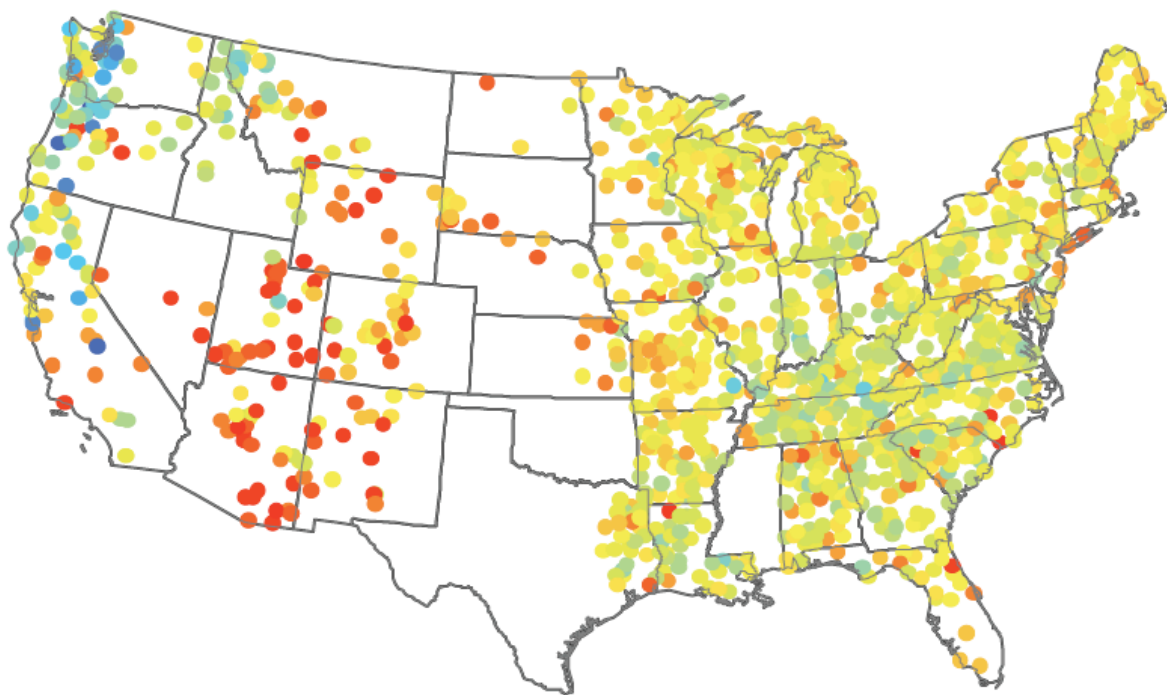
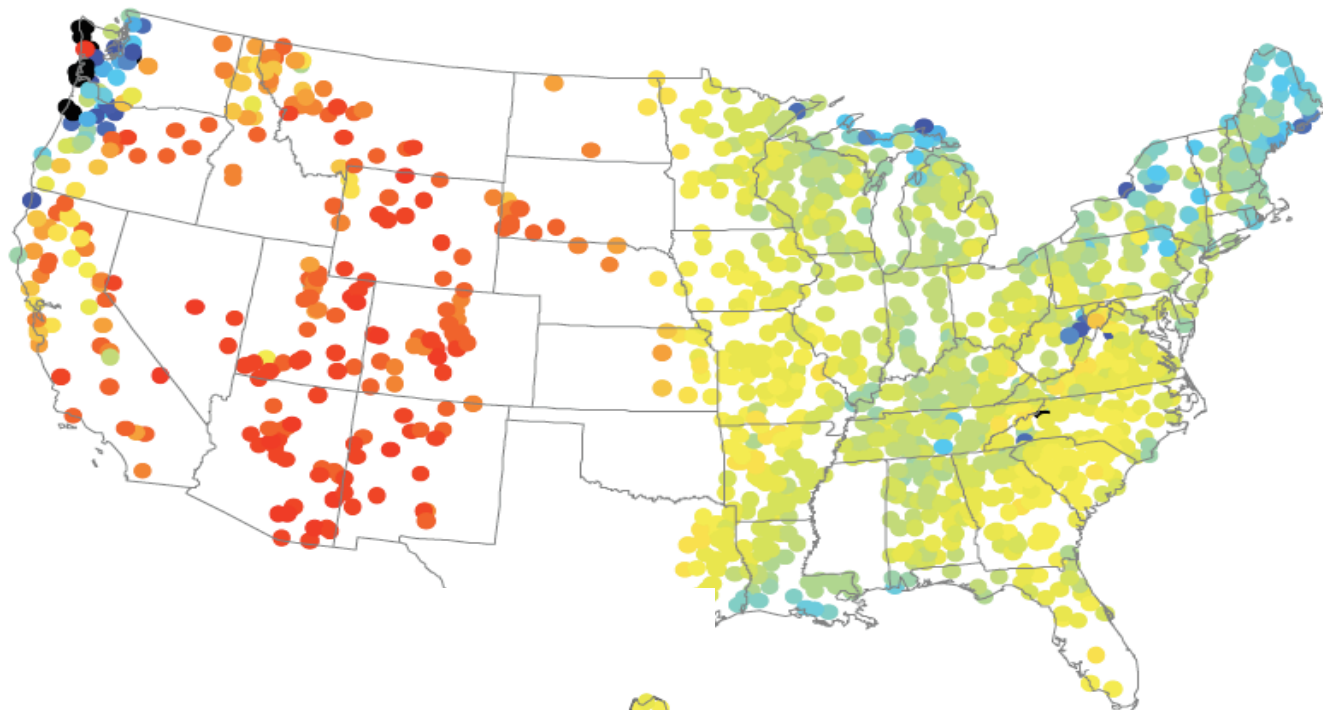


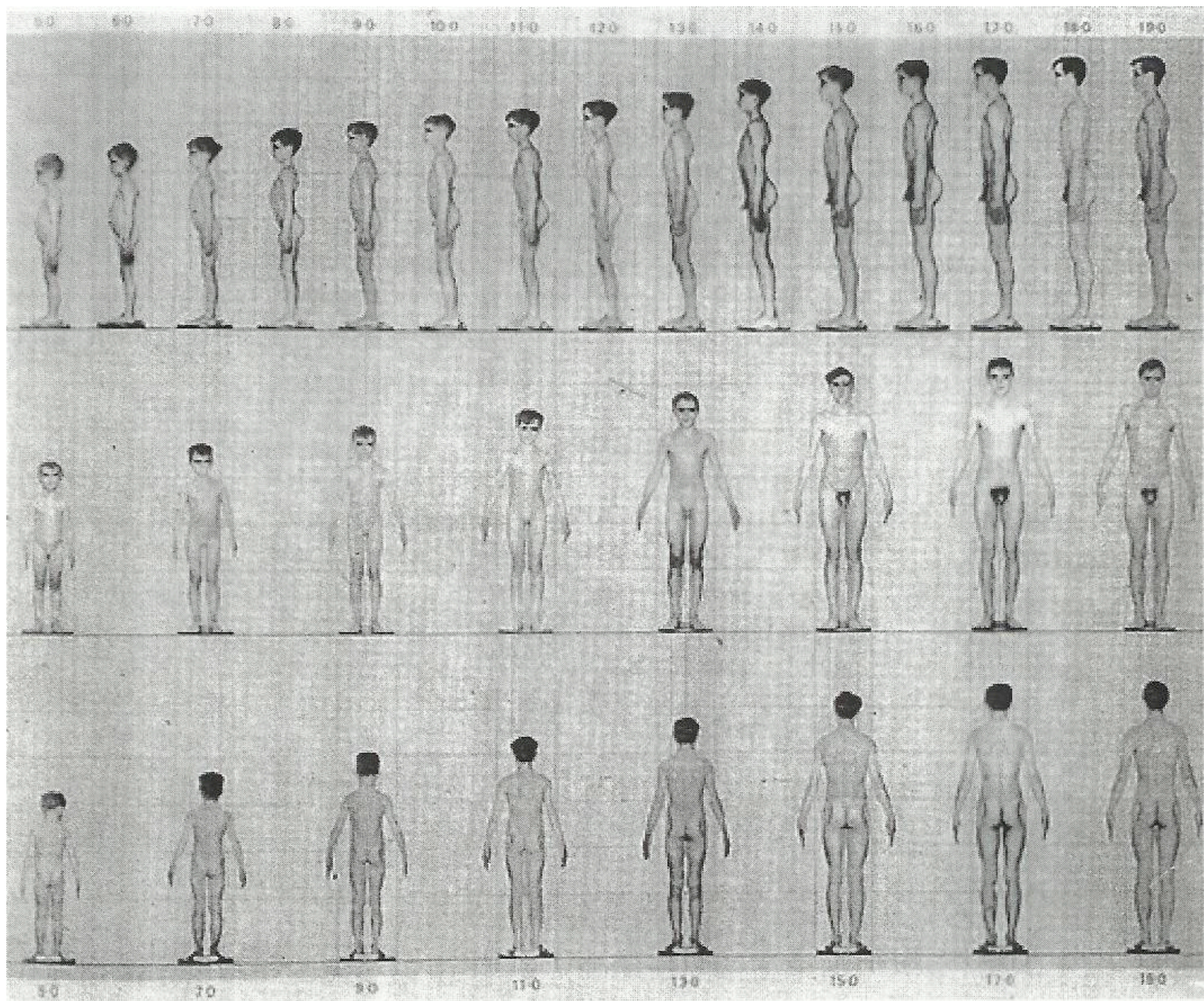
INTERSPECIFIC SIZE DISTRIBUTION

All species in a Malaysian Rainforest



Manokaran and
Kochummen (1987)

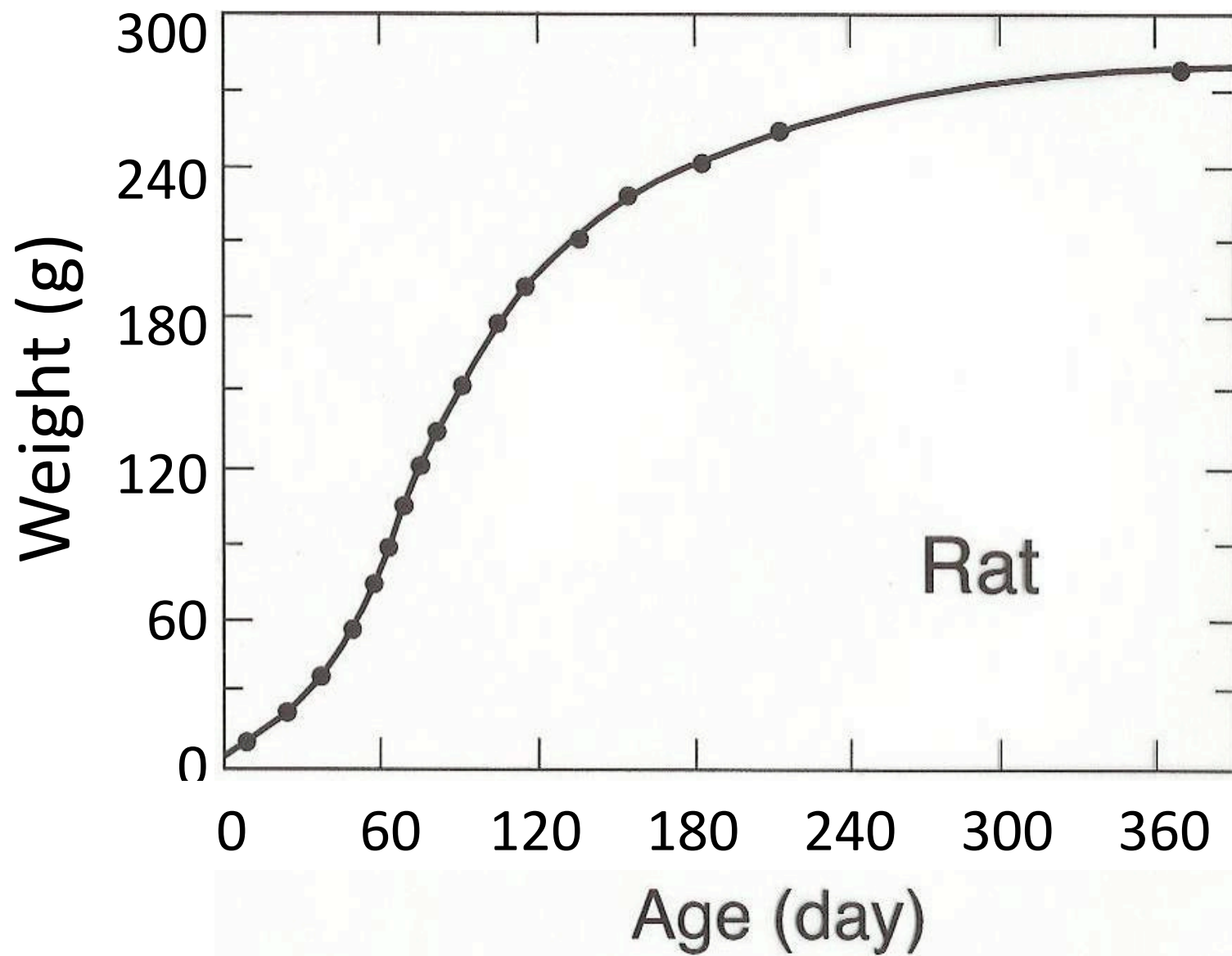






**Food
"fire of life"
Energy**





INCOMING METABOLISED ENERGY



MAINTENANCE
(of existencing cells)

+

GROWTH
(of new cells)

$$B = N_{cells} B_{cell} + E_{cell} \frac{dN_{cell}}{dt}$$

IN TERMS OF MASS AT AGE t

$$\Rightarrow \frac{dm}{dt} = am^{3/4} - bm$$

where

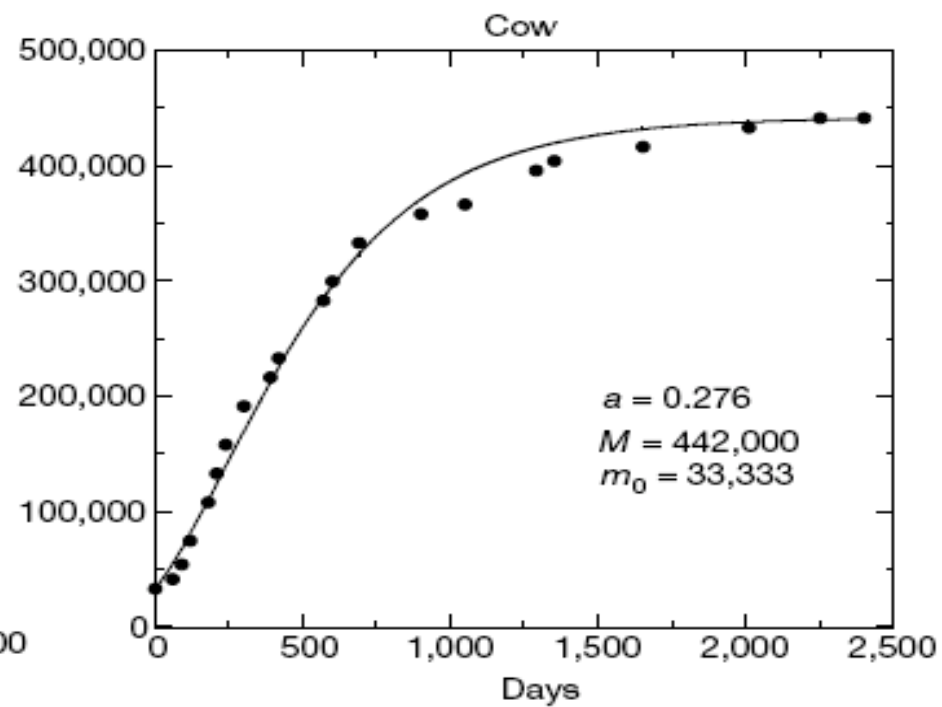
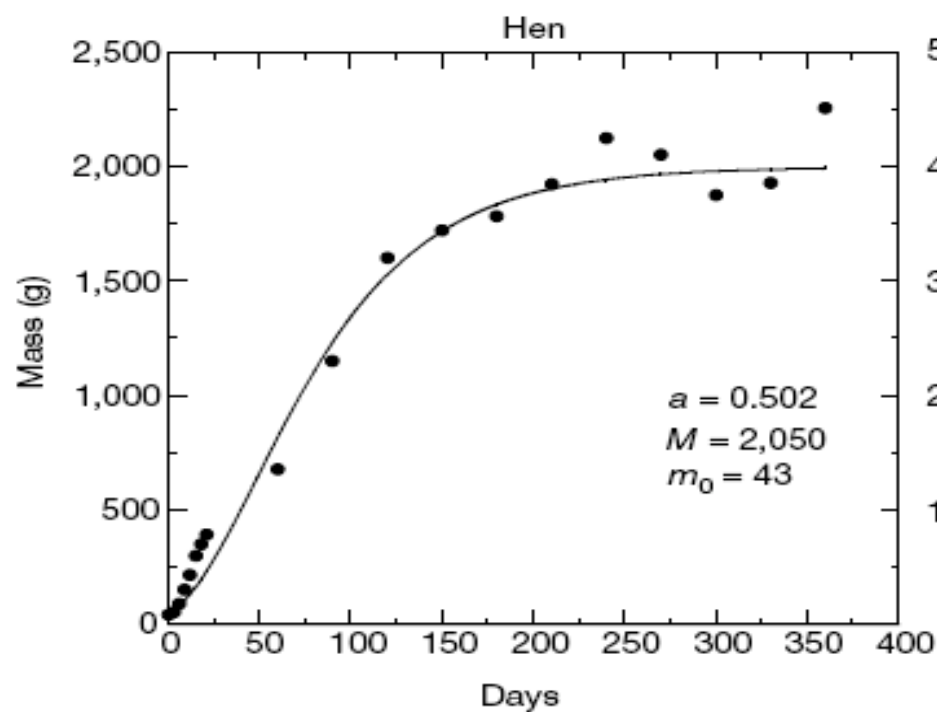
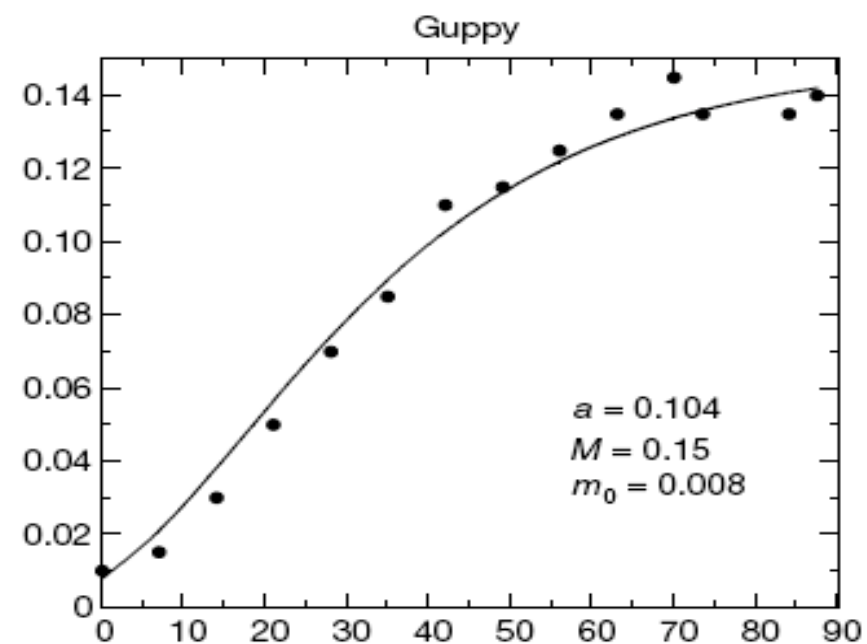
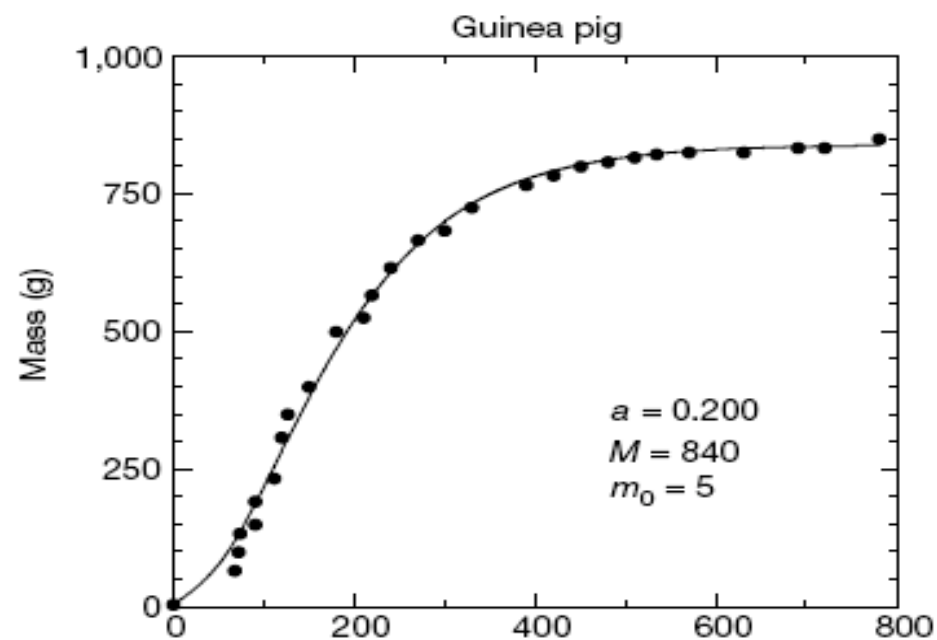
$$a \equiv \frac{B_0 m_c}{E_c}$$

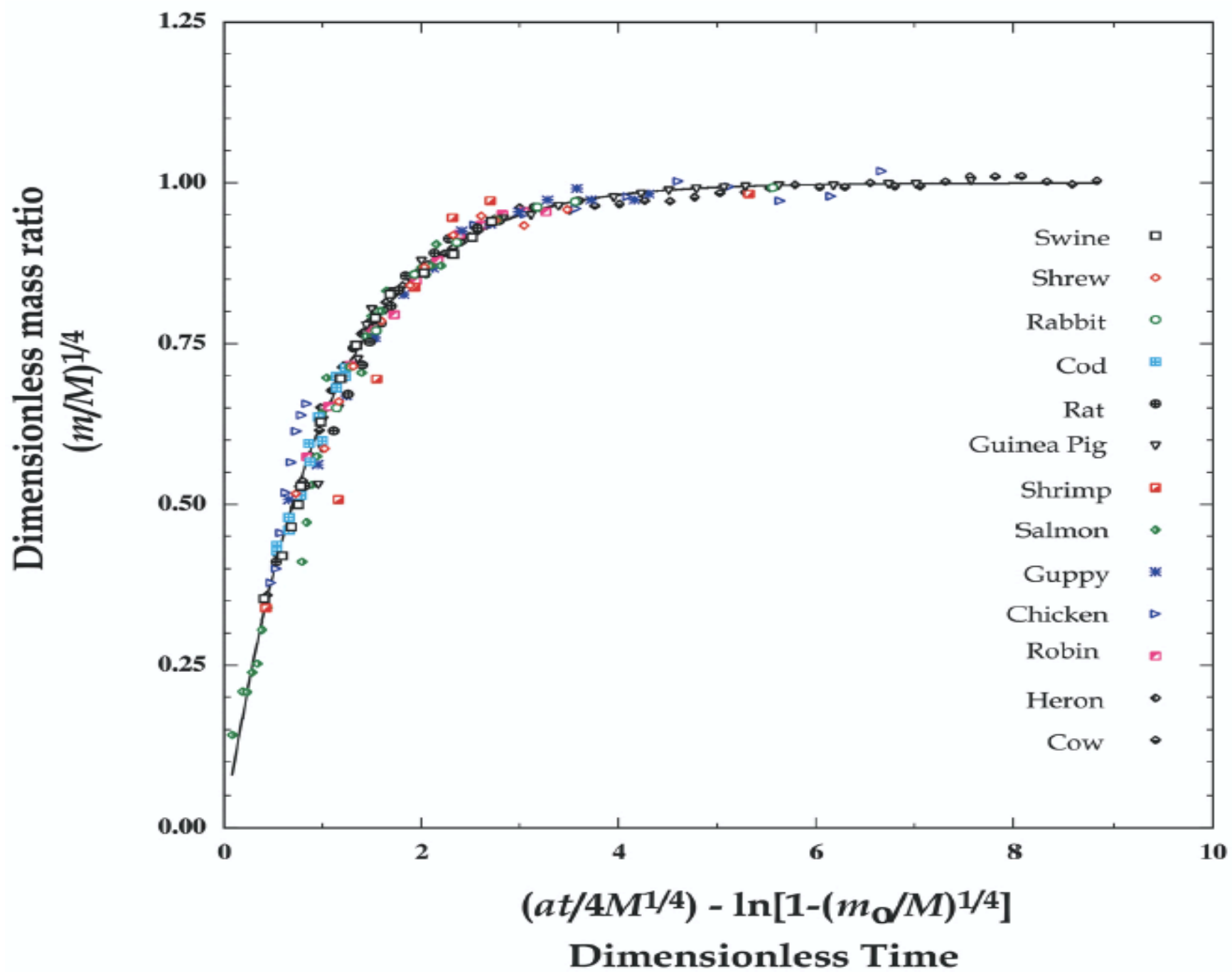
$$b \equiv \frac{B_c}{E_c}$$

SOLUTION:

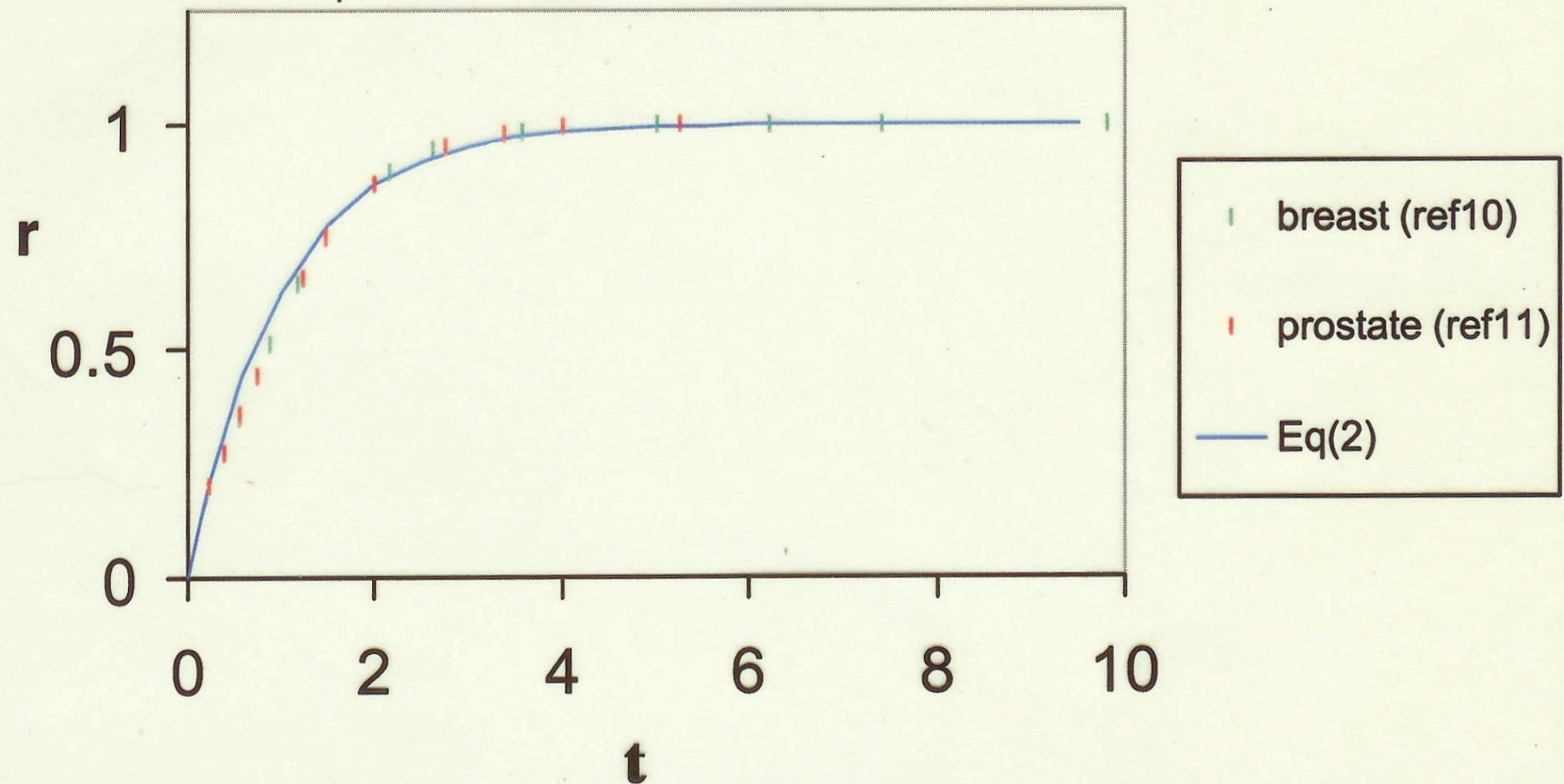
$$\left(\frac{m}{M}\right)^{1/4} = 1 - \left[1 - \left(\frac{M_0}{M}\right)^{1/4}\right] e^{-at/4M^{1/4}}$$

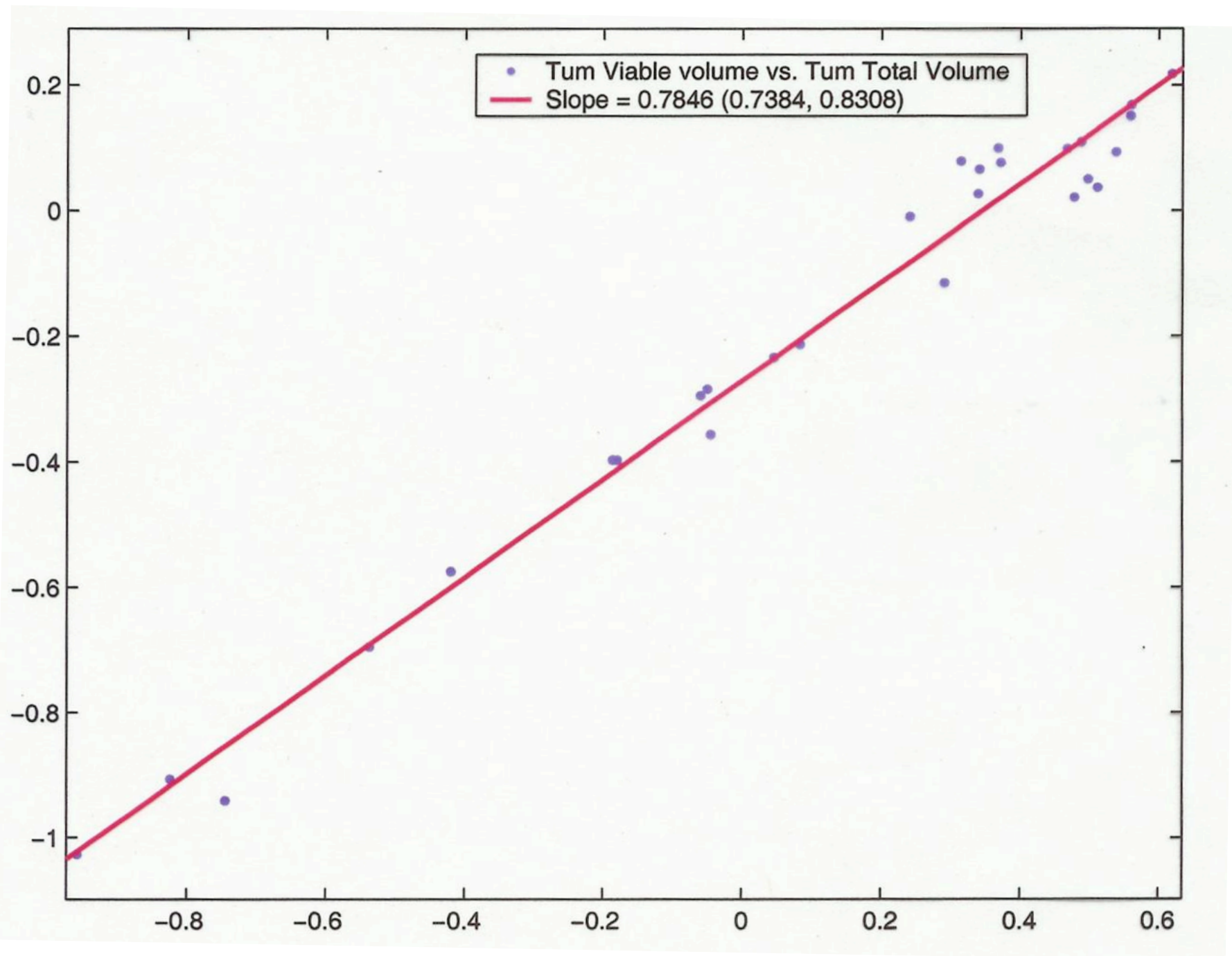
WHERE M_0 = MASS AT BIRTH ($m = M_0$ WHEN $t = 0$)



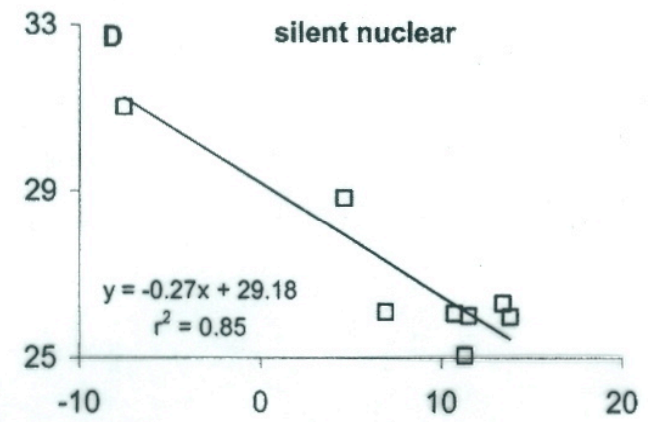
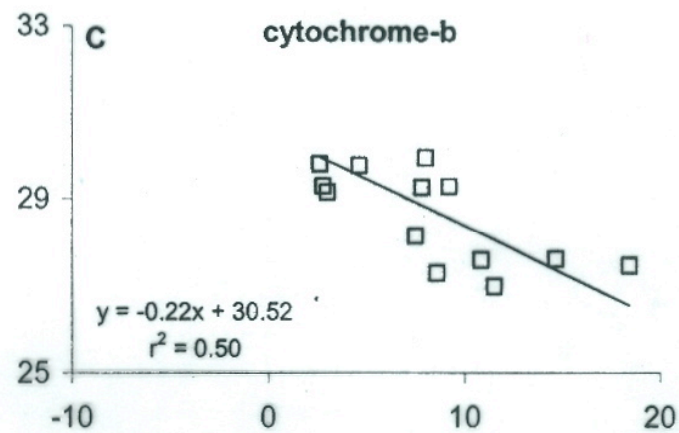
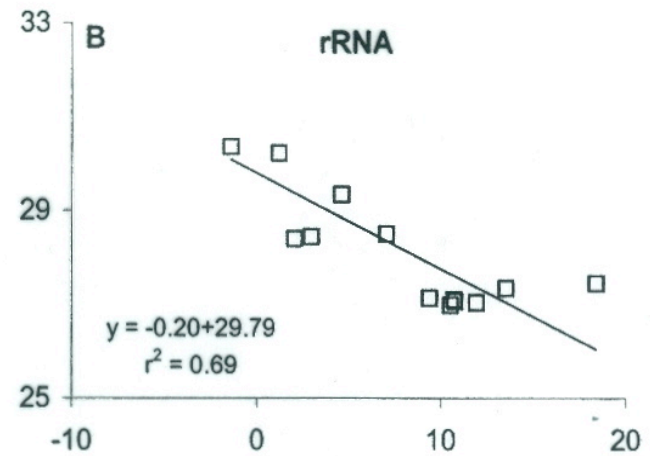
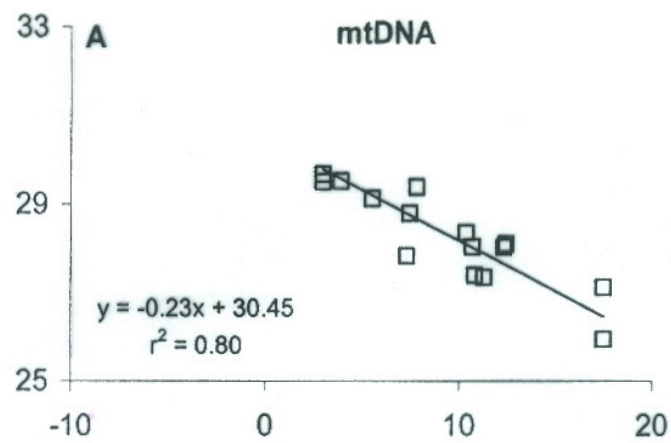


**'in vivo' data
(patients)**





$\ln(\alpha \cdot e^{E/kT})$



$\ln(m)$

Biology Life

- ***NON-LINEAR SCALING LAWS***
- ***UNIVERSAL QUARTER POWERS***
- ***SUB-LINEAR EXPONENTS (< 1)***
- ***ECONOMIES OF SCALE ($\sim M^{-1/4}$)***
- ***PACE OF LIFE DECREASES WITH SIZE:***
 - TIMES $\sim M^{1/4}$***
 - RATES $\sim M^{-1/4}$***
- ***SIGMOIDAL GROWTH CURVES***
- ***STABLE ASYMPTOTE***
- ***SUSTAINABLE***
- ***GOVERNED BY NETWORKS (\sim FRACTAL)***

Social Organizations

(Urban/Corporate Structures)

- Can one construct a **general theory of social organizations** that is quantitative and predictive?
- Are there “**universal**” **scaling laws** that reveal underlying principles?
- Are there **average idealized social organizations**?
- Did they evolve under “**natural selection**” in a “**free market**” environment via competition?
- What is the nature of their **hierarchies and generic network structure**?

Social Organizations

(Urban/Corporate Structures)

- Are there **universality classes of networks**?
- Is there an **optimal** maximum (or minimum) **size**?
- What drives **mergers**?
- Growth, mortality, aging, evolution, ...
- **Energy** (resources) **vs. information**: which dominates?

**Are Cities Approximate Scaled
Versions of Each Other?**

**Do They Obey Power Law
Scaling?** β

**Do Exponents Manifest
“Universality” (analogous
to quarter powers in Biology)?**

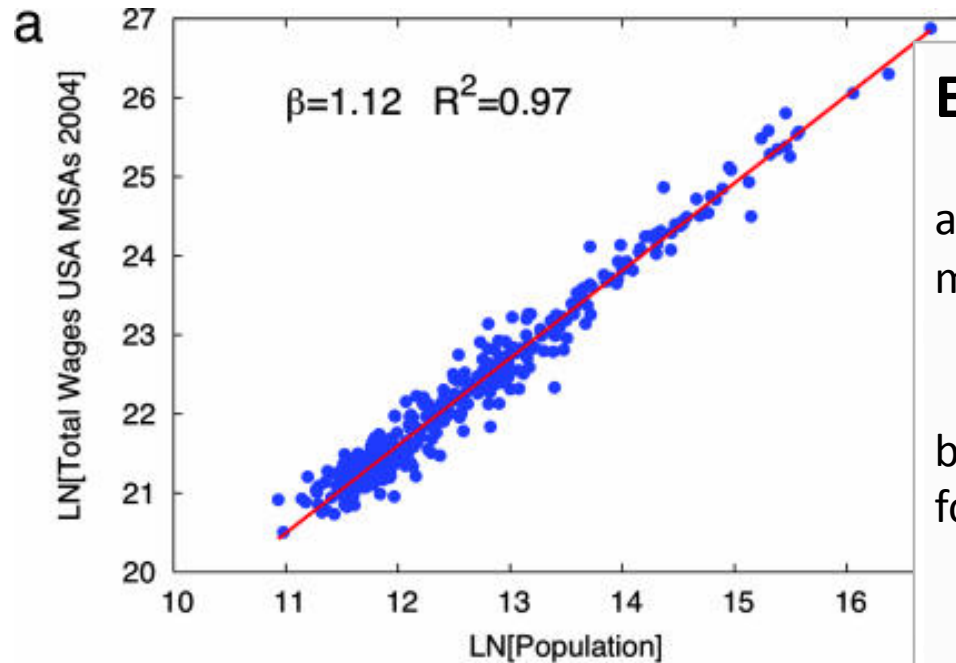
$$R \sim N^b$$

NETWORK DYNAMICS IMPLIES THAT THE
PACE OF LIFE IS DETERMINED BY

$$\text{RATES} \sim N^{b-1}$$

$b < 1$ *PACE OF LIFE SLOWS DOWN*

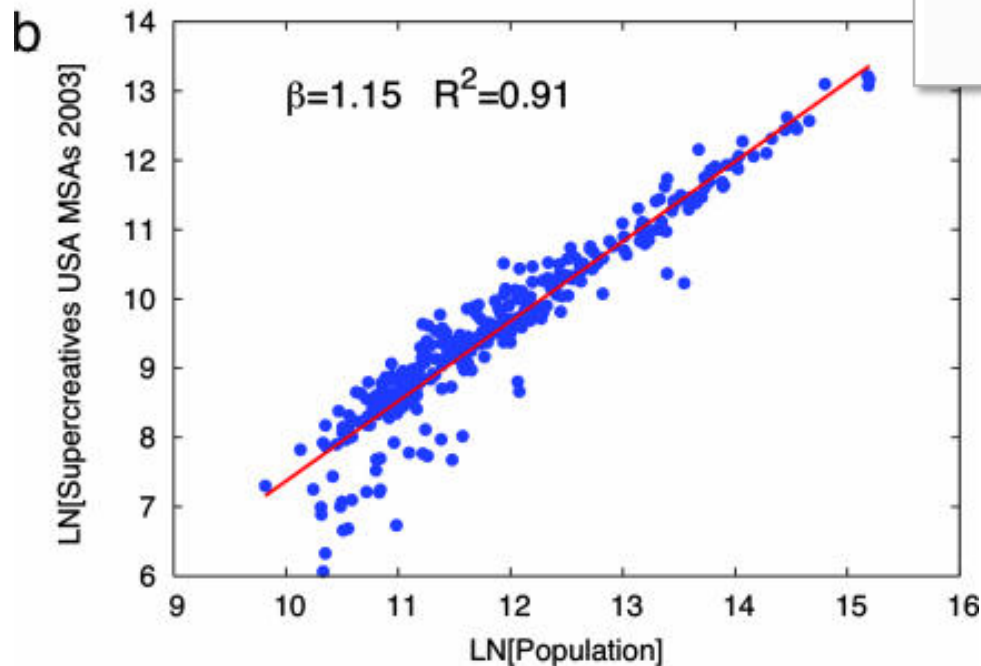
$b > 1$ *PACE OF LIFE SPEEDS UP*



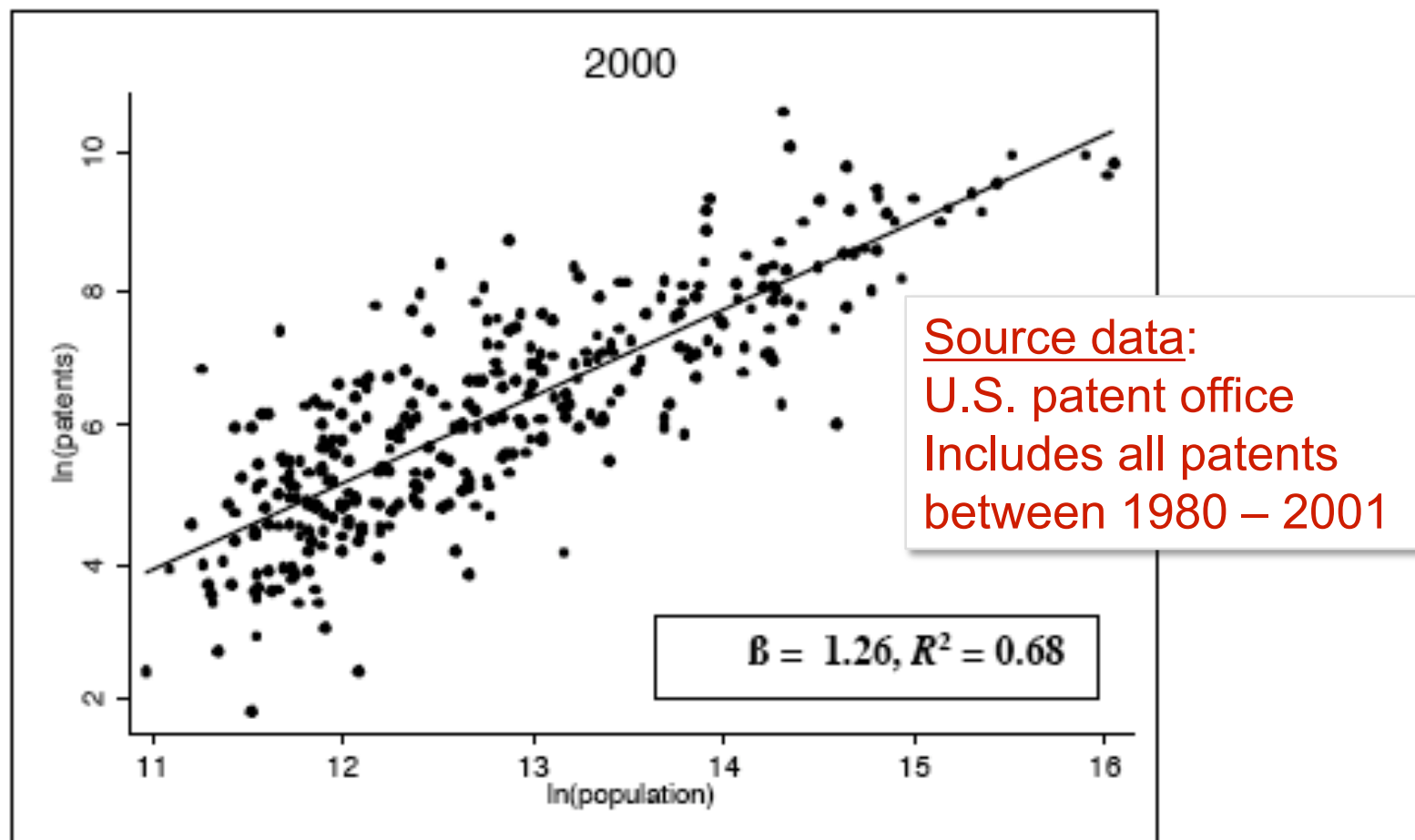
Example of scaling relationships

a) Total **wages** per MSA in 2004 for the USA vs. metropolitan population.

b) **Supercreative employment** per MSA in 2003, for the USA vs. metropolitan population.



Innovation measured by Patents

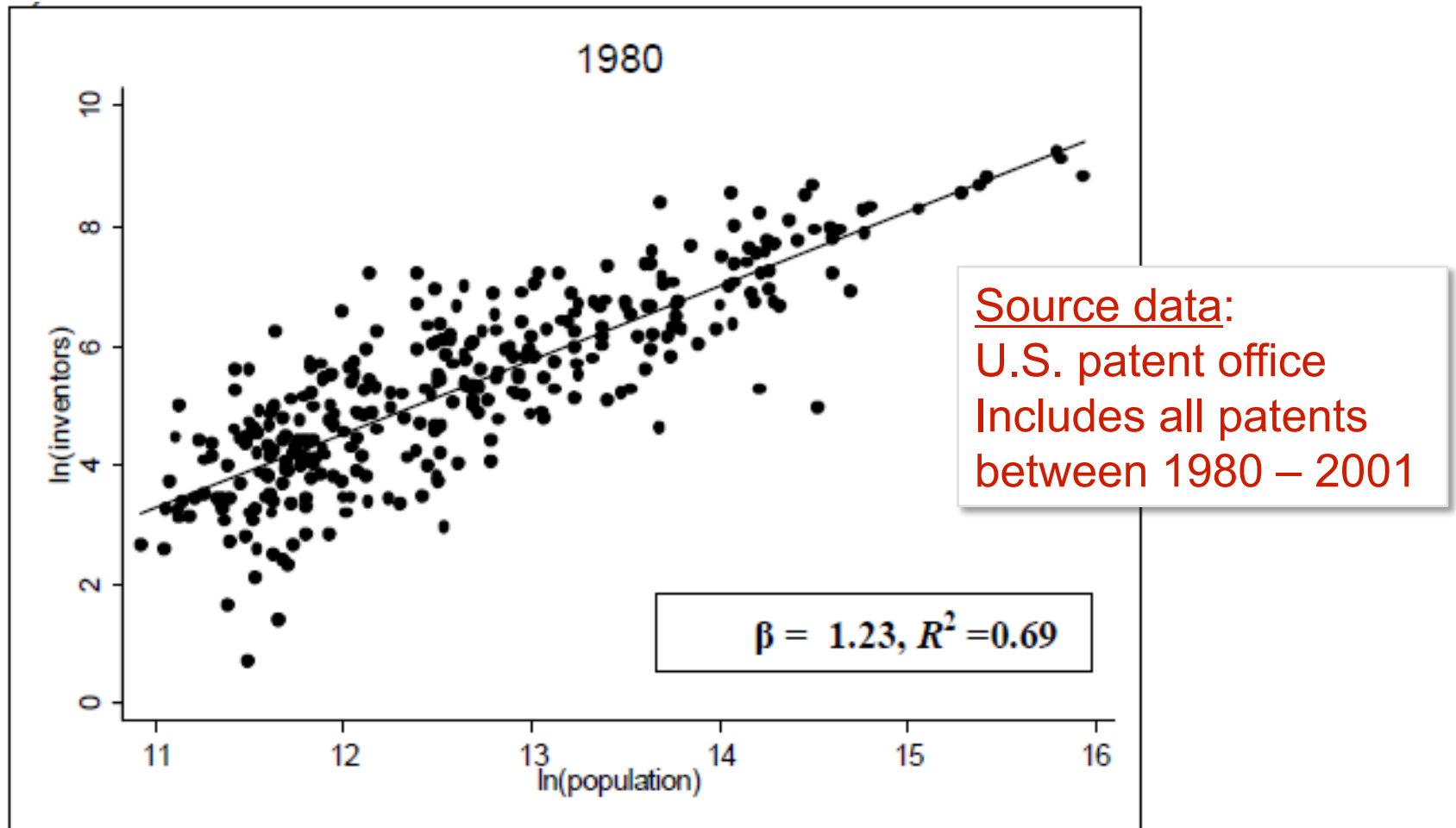


From “**Innovation in the city: Increasing returns to scale in urban patenting**”

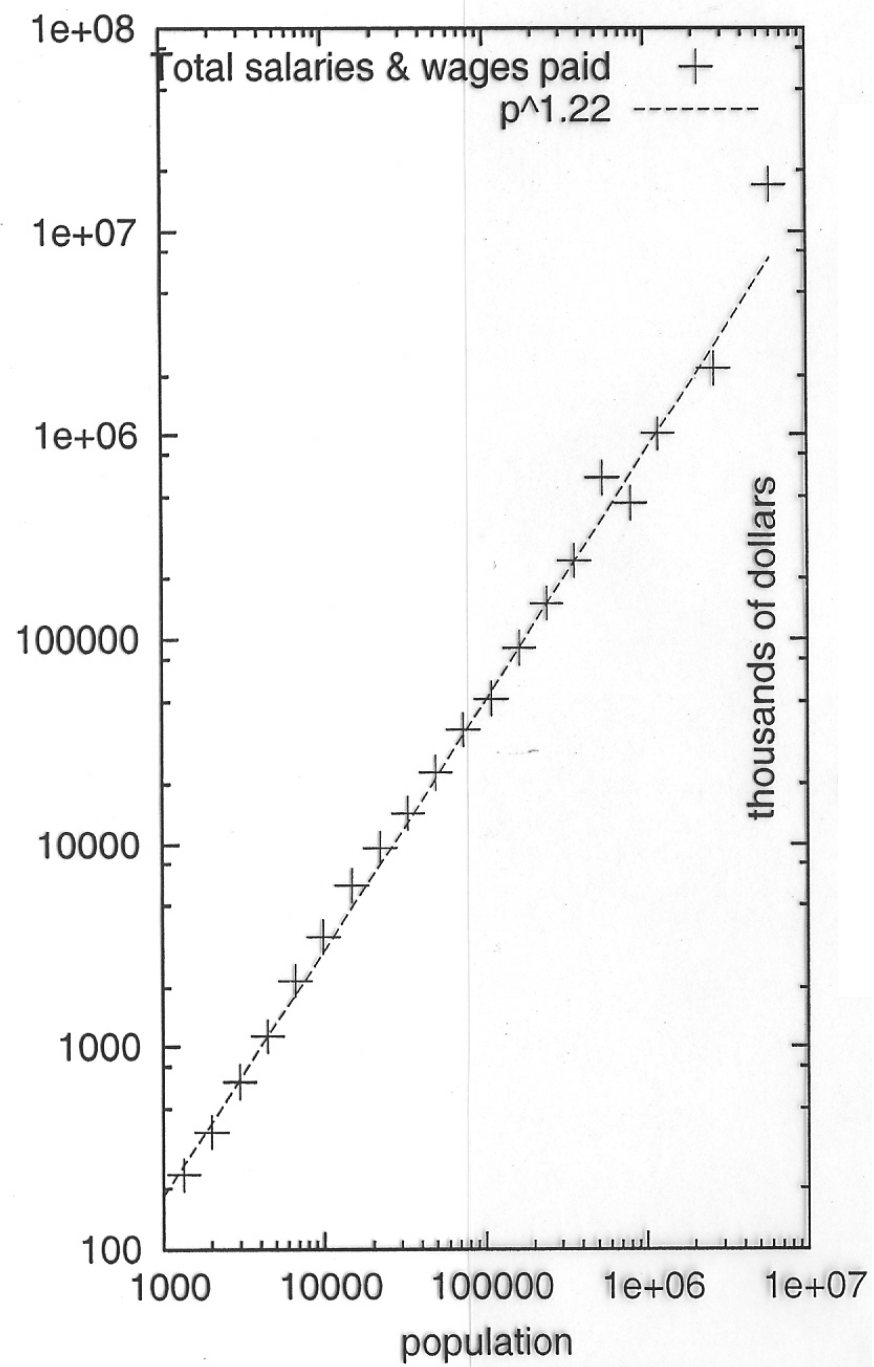
Bettencourt, Lobo and Strumsky

Data courtesy of Lee Fleming, Deborah Strumsky

Or to a **disproportionate agglomeration** of inventors with urban size?



Data courtesy of Lee Fleming, Deborah Strumsky



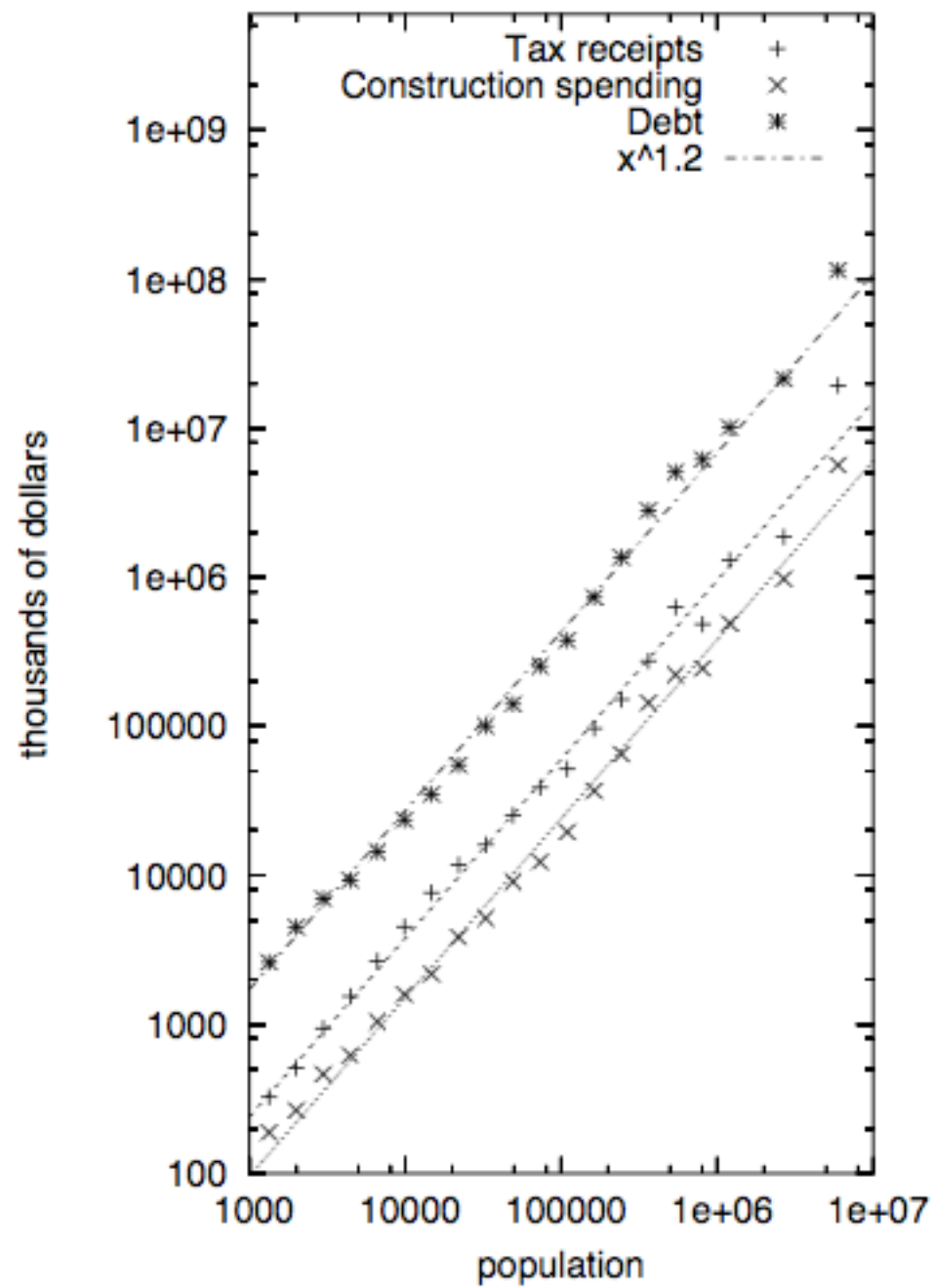
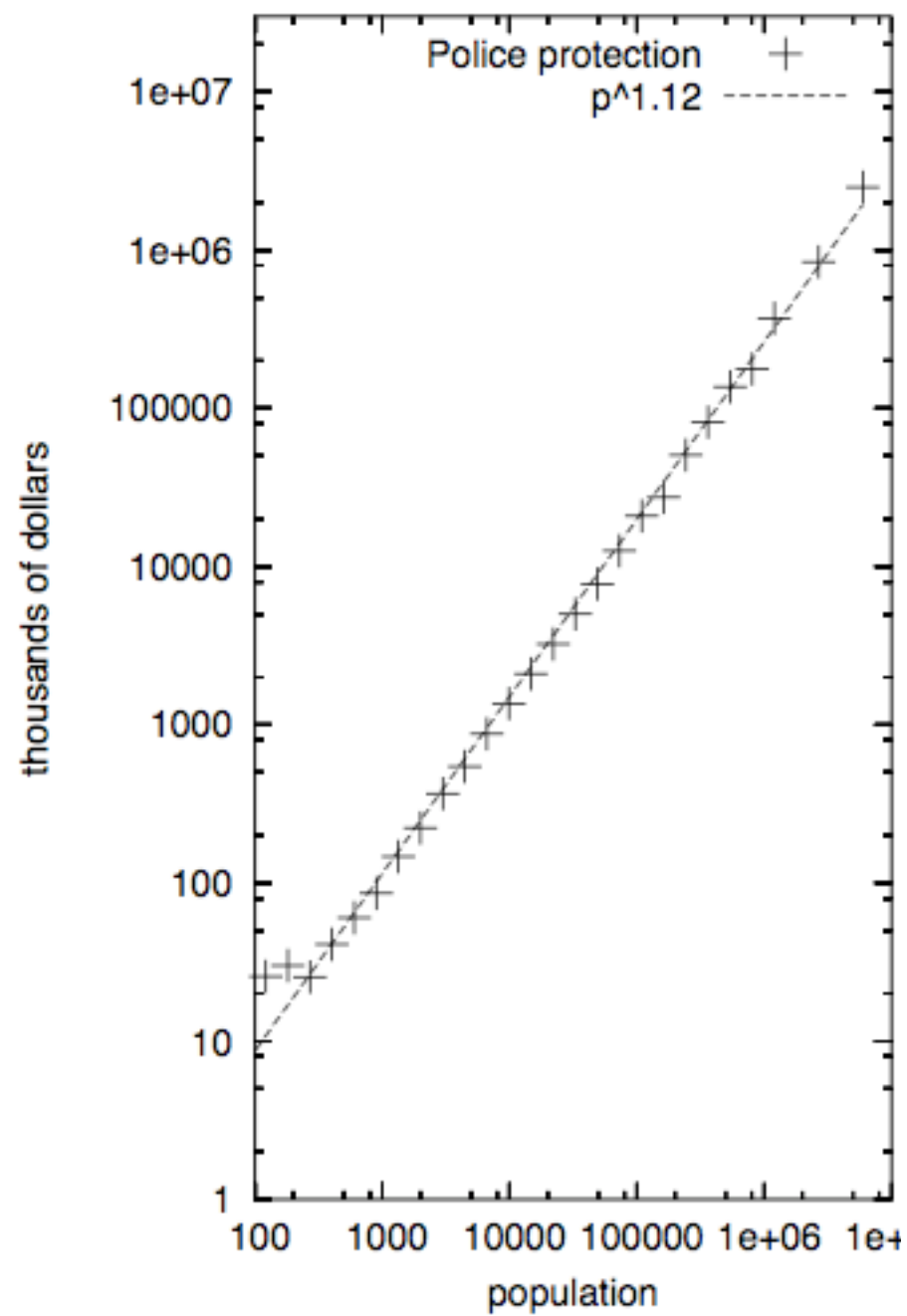


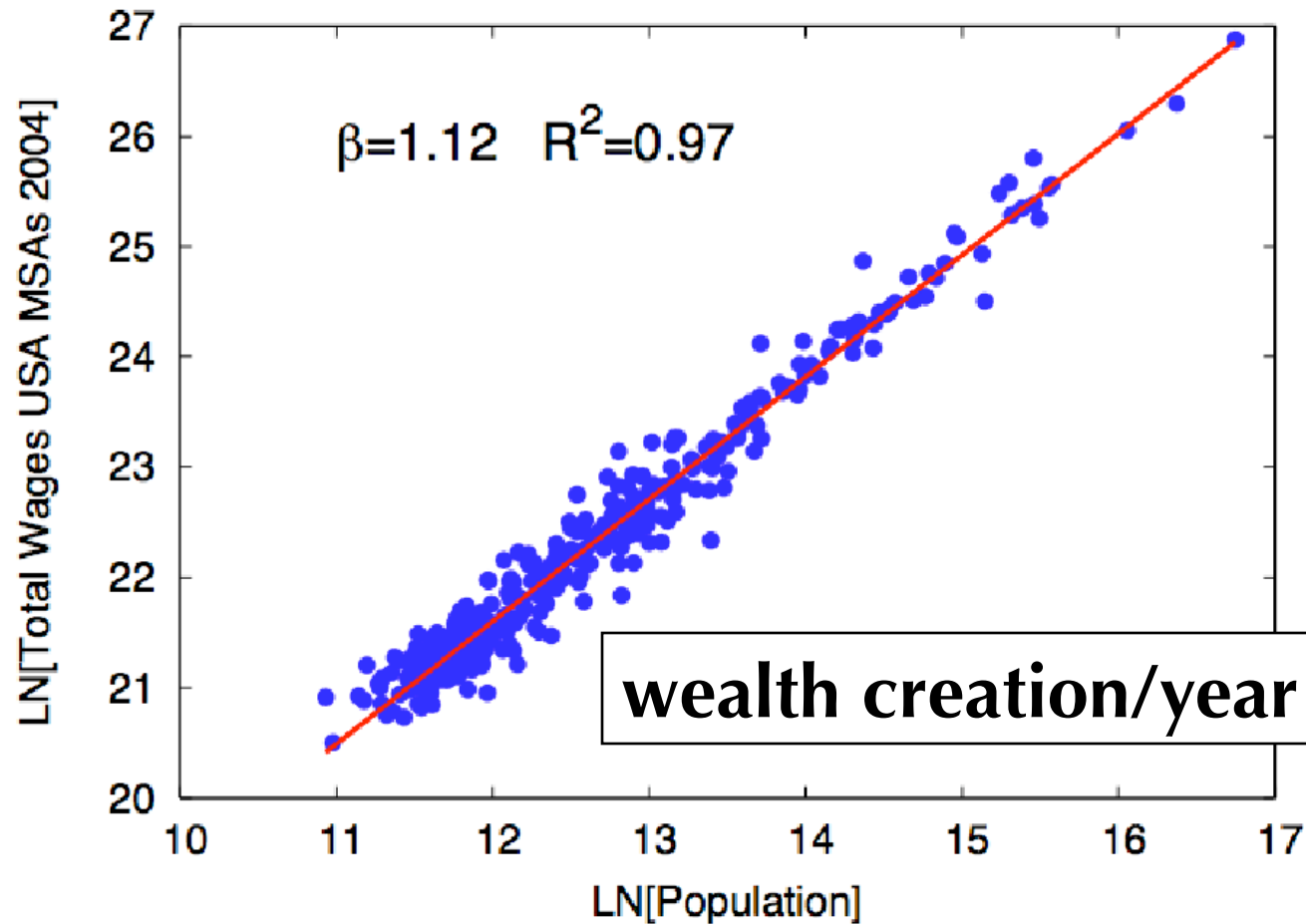
Table 1. Scaling exponents for urban indicators vs. city size

Y	β	95% CI	Adj- R^2	Observations	Country-year
New patents	1.27	[1.25,1.29]	0.72	331	U.S. 2001
Inventors	1.25	[1.22,1.27]	0.76	331	U.S. 2001
Private R&D employment	1.34	[1.29,1.39]	0.92	266	U.S. 2002
"Supercreative" employment	1.15	[1.11,1.18]	0.89	287	U.S. 2003
R&D establishments	1.19	[1.14,1.22]	0.77	287	U.S. 1997
R&D employment	1.26	[1.18,1.43]	0.93	295	China 2002
Total wages	1.12	[1.09,1.13]	0.96	361	U.S. 2002
Total bank deposits	1.08	[1.03,1.11]	0.91	267	U.S. 1996
GDP	1.15	[1.06,1.23]	0.96	295	China 2002
GDP	1.26	[1.09,1.46]	0.64	196	EU 1999–2003
GDP	1.13	[1.03,1.23]	0.94	37	Germany 2003
Total electrical consumption	1.07	[1.03,1.11]	0.88	392	Germany 2002
New AIDS cases	1.23	[1.18,1.29]	0.76	93	U.S. 2002–2003
Serious crimes	1.16	[1.11, 1.18]	0.89	287	U.S. 2003
Total housing	1.00	[0.99,1.01]	0.99	316	U.S. 1990
Total employment	1.01	[0.99,1.02]	0.98	331	U.S. 2001
Household electrical consumption	1.00	[0.94,1.06]	0.88	377	Germany 2002
Household electrical consumption	1.05	[0.89,1.22]	0.91	295	China 2002
Household water consumption	1.01	[0.89,1.11]	0.96	295	China 2002
Gasoline stations	0.77	[0.74,0.81]	0.93	318	U.S. 2001
Gasoline sales	0.79	[0.73,0.80]	0.94	318	U.S. 2001
Length of electrical cables	0.87	[0.82,0.92]	0.75	380	Germany 2002
Road surface	0.83	[0.74,0.92]	0.87	29	Germany 2002

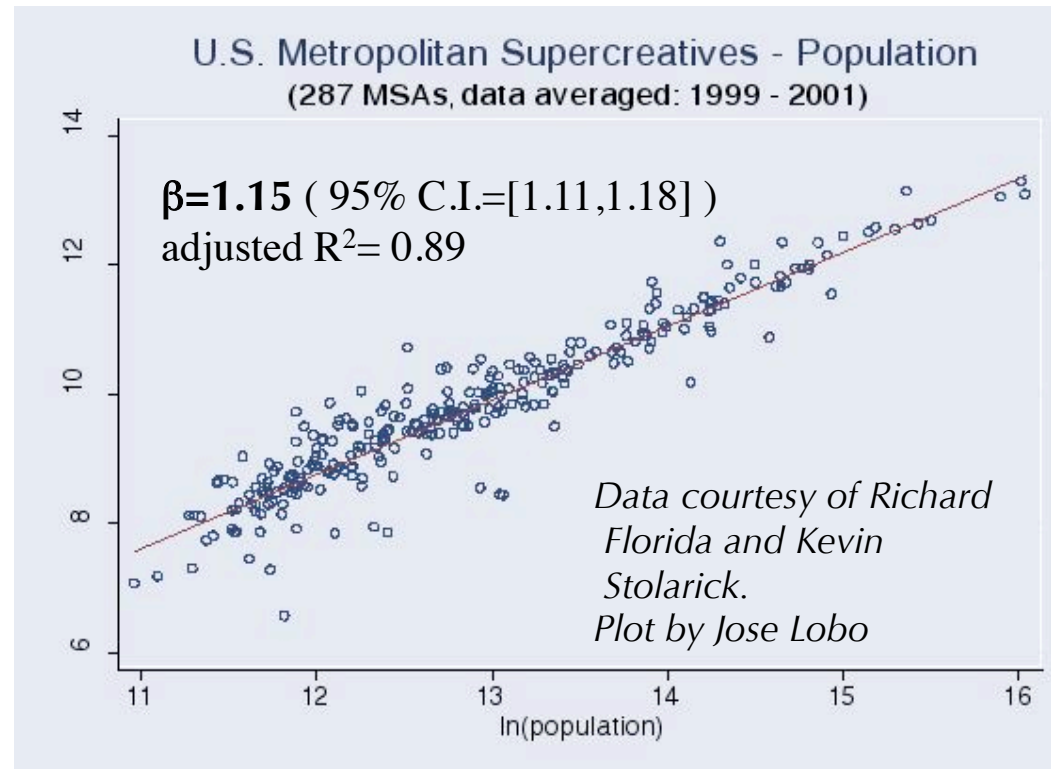
Data sources are shown in [SI Text](#). CI, confidence interval; Adj- R^2 , adjusted R^2 ; GDP, gross domestic product.

See supplementary online materials for further details and data sources.

Increasing returns in cities



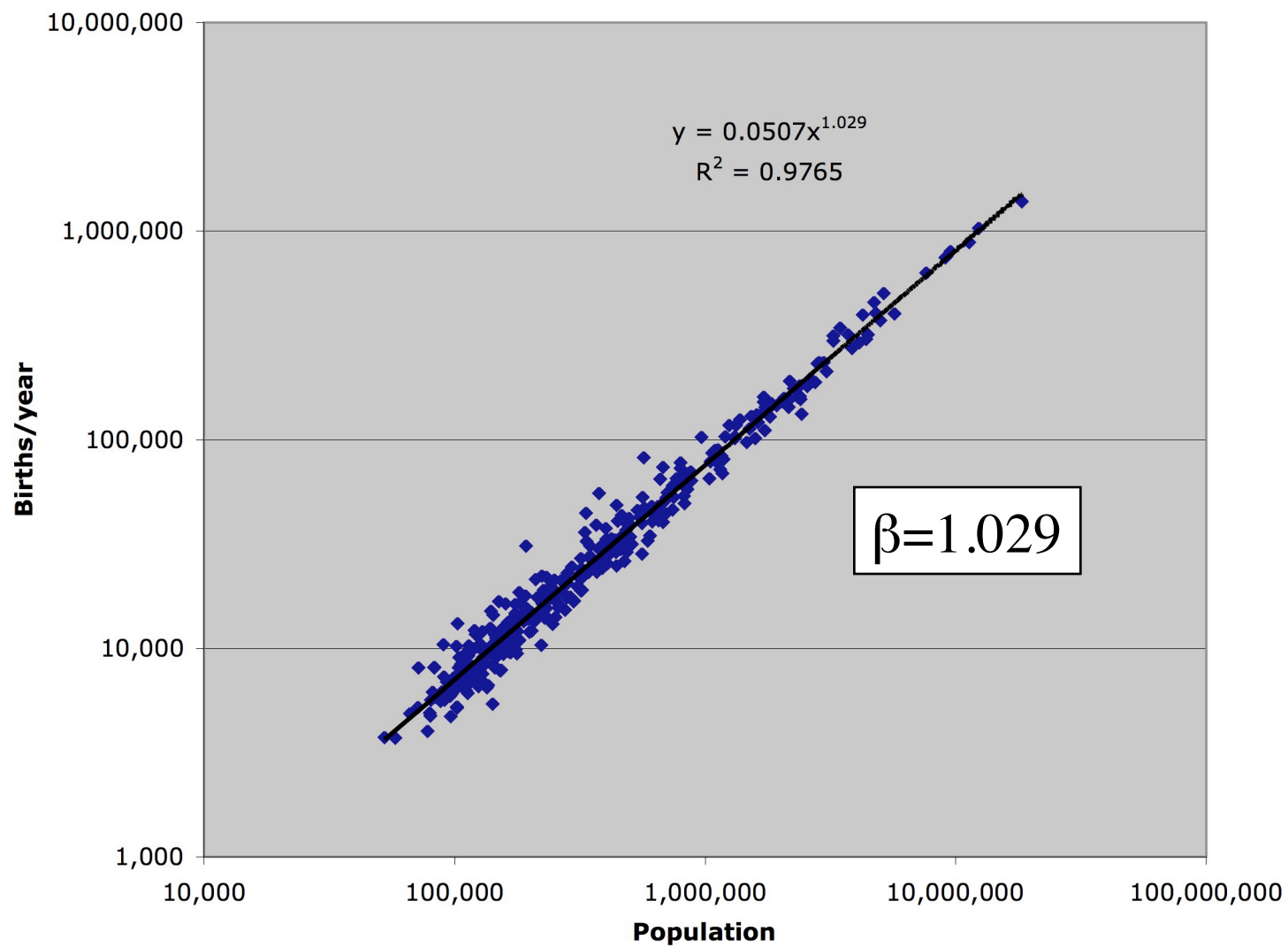
Employment patterns



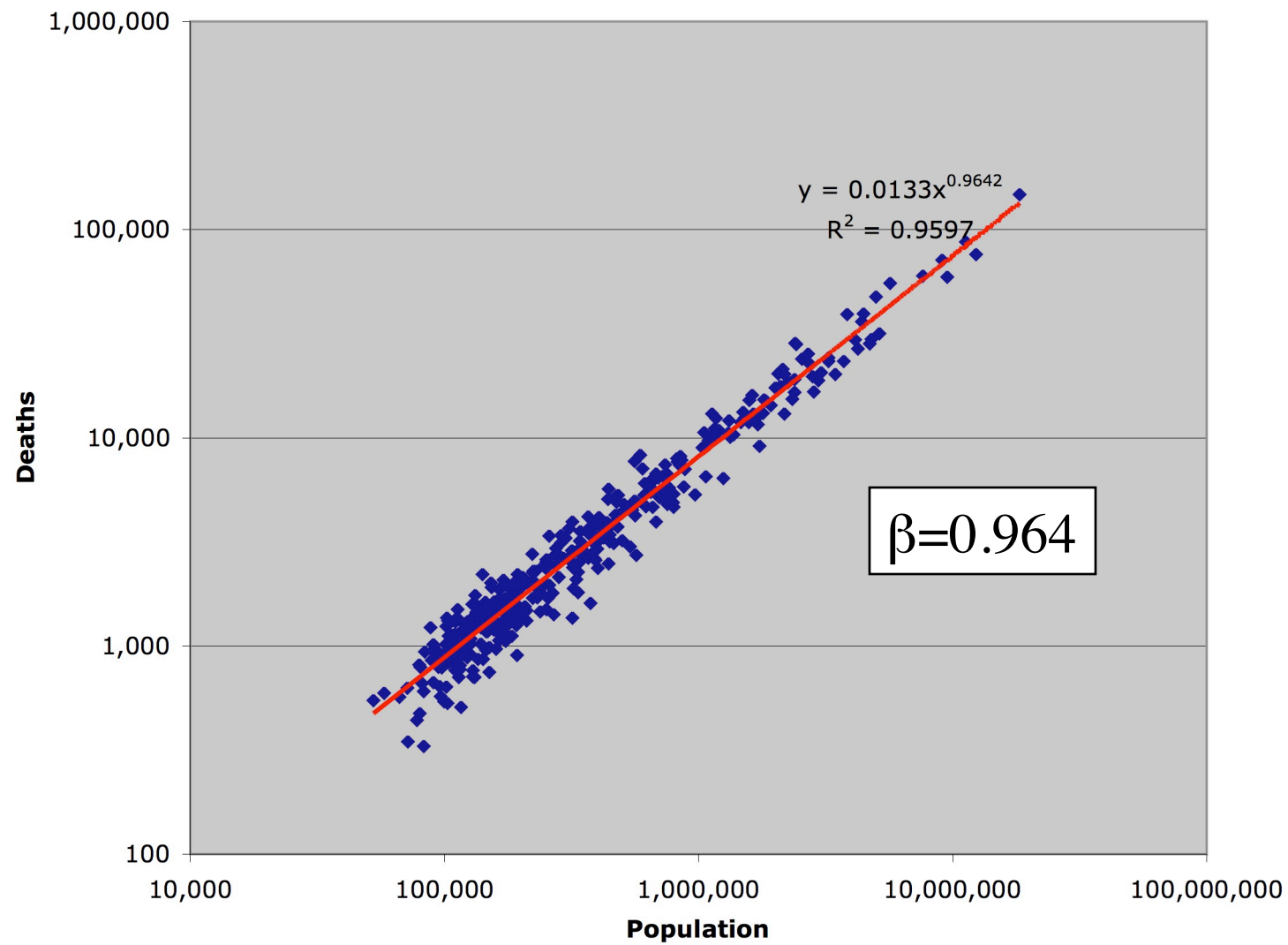
Supercreative professionals [Florida 2002, pag. 327-329] are “Computer and Mathematical, Architecture and Engineering, Life Physical and Social Sciences Occupations, Education training and Library, Arts, Design, Entertainment, Sports and Media Occupations”.

Derived from Standard Occupation Classification System of the U.S. Bureau of Labor Statistics

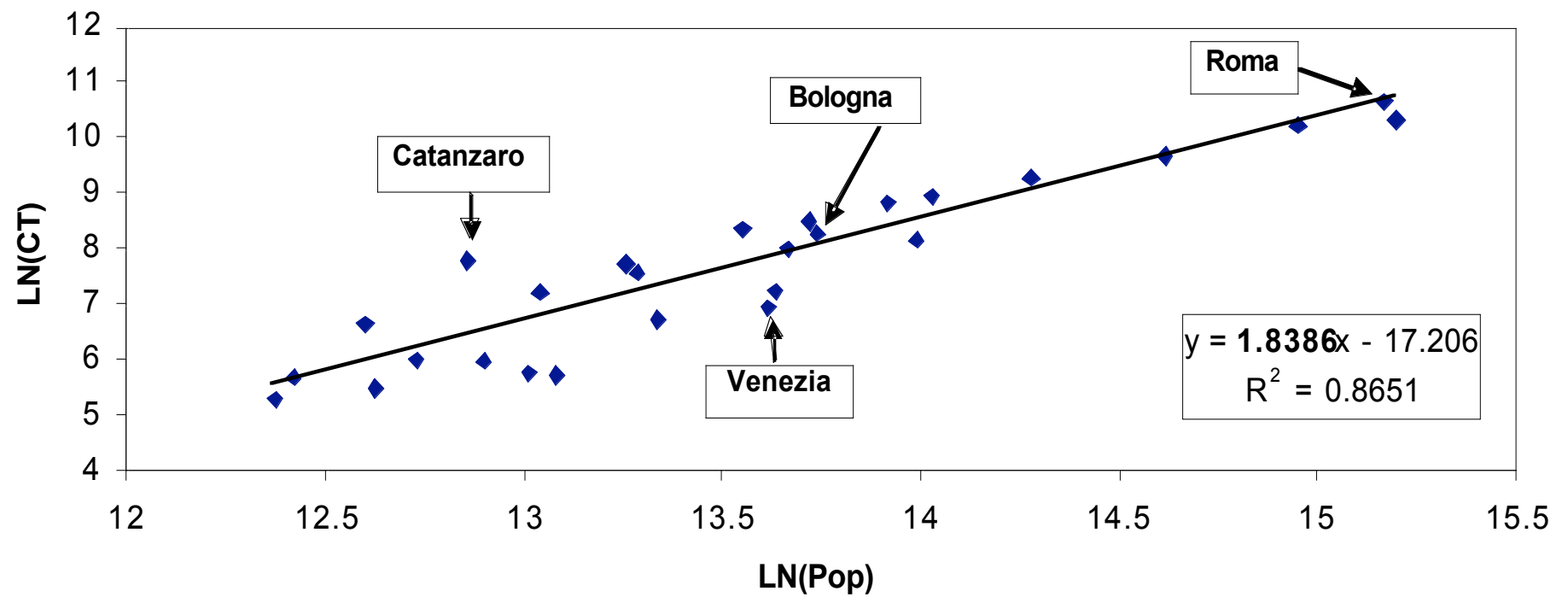
Births vs. Metropolitan Population



Deaths vs. Population



Car Thefts and Urban Population Italy 2001



Material Infrastructure

optimized global design for economies of scale

Y	β	95% CI	adj.- R ²	observations	Country/ year
Gasoline Stations	0.77	[0.74,0.81]	0.93	318	USA/2001
Gasoline Sales	0.79	[0.73,0.80]	0.94	318	USA/2002
Length of electrical cables	0.88	[0.82,0.94]	0.82	387	Germany/2001
Road surface	0.83	[0.74,0.92]	0.87	29	Germany/2002

Note that although there are economies of scale in cables the network is still delivering energy at a superlinear rate:

Social rates drive energy consumption rates, not the opposite

Basic Individual needs

proportionality to population

Y	β	95% CI	adj.- R ²	observations	Country/ year
Total establishments	0.98	[0.95,1.02]	0.95	331	USA/2001
Total employment	1.01	[0.99,1.02]	0.98	331	USA/2001
Total Household electrical consumption	1.00	[0.94,1.06]	0.70	387	Germany/2001
Total Household electrical consumption	1.05	[0.89,1.22]	0.91	295	China/2002
Total Household water consumption	1.01	[0.89,1.11]	0.96	295	China/2002

Also true for the scaling of number of housing units

The urban economic miracle

across time, space, level of development or economic
system

Y	β	95% CI	adj.- R ²	observation s	Country/ year
Total Wages/yr	1.12	[1.09,1.13]	0.96	361	USA/2002
GDP/yr	1.15	[1.06,1.23]	0.96	295	China/2002
GDP/yr	1.13	[1.03,1.23]	0.94	37	Germany/2003
GDP/yr	1.26	[1.03,1.46]	0.64	196	EU/2003

Innovation as the engine

Y	β	95% CI	adj.- R ²	observations	Country/year
New Patents/yr	1.27	[1.25,1.29]	0.72	331	USA/2001
Inventors/yr	1.25	[1.22,1.27]	0.76	331	USA/2001
Private R&D employment	1.34	[1.29,1.39]	0.92	266	USA/2002
“Supercreative” Professionals	1.15	[1.11,1.18]	0.89	287	USA/2003
R&D employment	1.26	[1.18,1.43]	0.93	295	China/2002

Social Side Effects

Y	β	95% CI	adj.- R ²	observations	Country/year
Total elect. consumption	1.09	[1.03,1.15]	0.72	387	Germany/2001
New AIDS cases	1.23	[1.18,1.29]	0.76	93	USA/2002
Serious Crime	1.16	[1.11,1.18]	0.89	287	USA/2003
Walking Speed	0.09	[0.07,0.11]	0.79	21	Several/1979

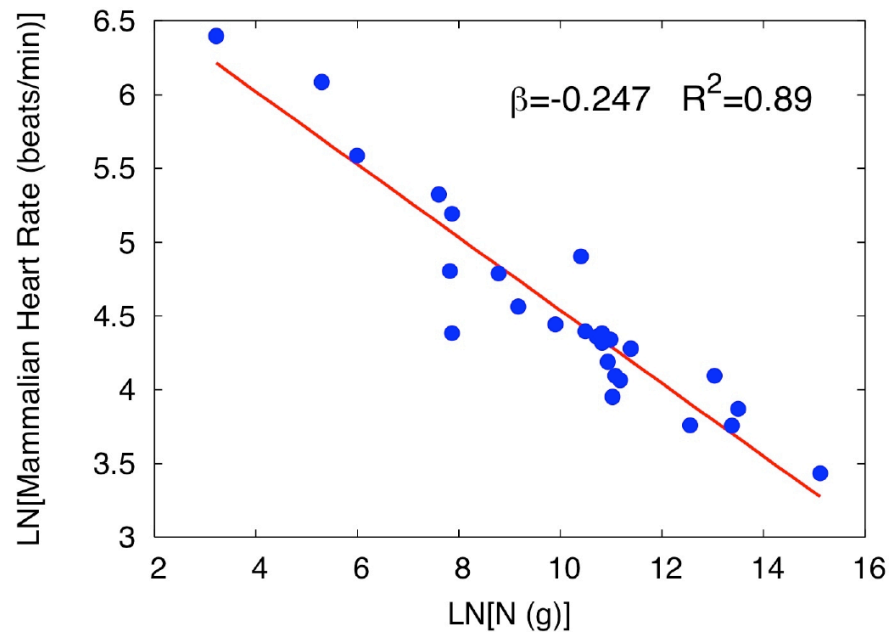
Disease transmission is a social contact process:

$$\frac{dT}{dt} = \beta_c SI \quad \text{Standard Incidence}$$

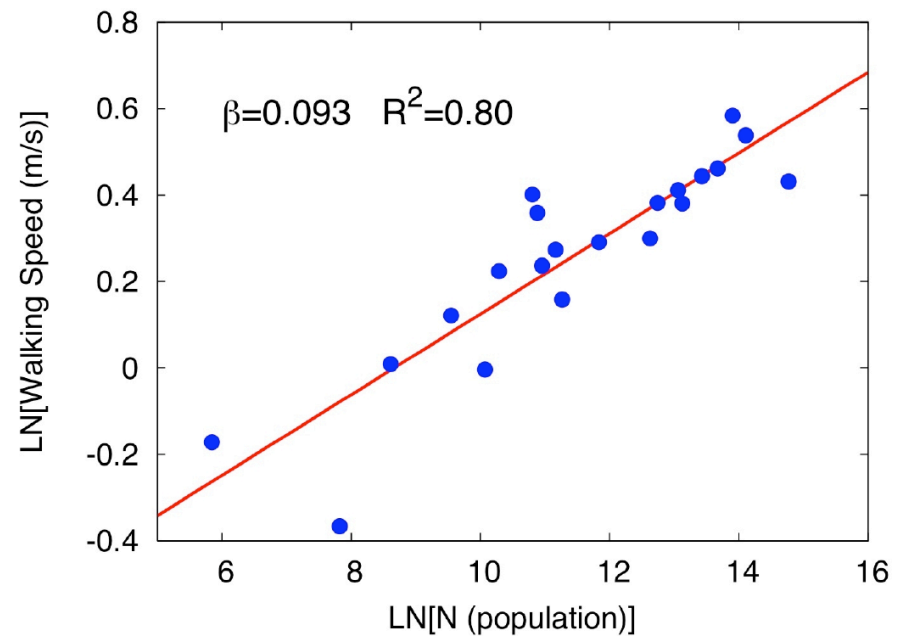
TAXONOMY OF EXPONENTS FALL INTO THREE “UNIVERSAL” CLASSES

- i) $b \sim 0.8 < 1$
INFRASTRUCTURE
(BIOLOGICAL)
SUB-LINEAR → ECONOMIES OF SCALE
DRIVEN BY EFFICIENCY
- ii) $b = 1$
LINEAR → NON-INNOVATIVE
- iii) $b \sim 1.15 > 1$
SUPER-LINEAR → **SOCIO-ECONOMIC**
INNOVATIVE DRIVEN BY
WEALTH CREATION

Pace of biological life vs. Pace of social life



Heart Rate vs. Body Size



Walking Speed vs. Population Size

Urban exponents and the dynamics of growth

Scaling Exponent	Driving Force	Organization	Growth
$\beta < 1$	Optimization, Efficiency	Biological	Sigmoidal long-term population limit
$\beta > 1$	Creation of Information, Wealth and Resources	Sociological	Boom / Collapse finite-time singularity/unbounded growth accelerating growth rates / discontinuities
$\beta = 1$	Individual Maintenance	Individual	Exponential

2003 Patenting Rankings

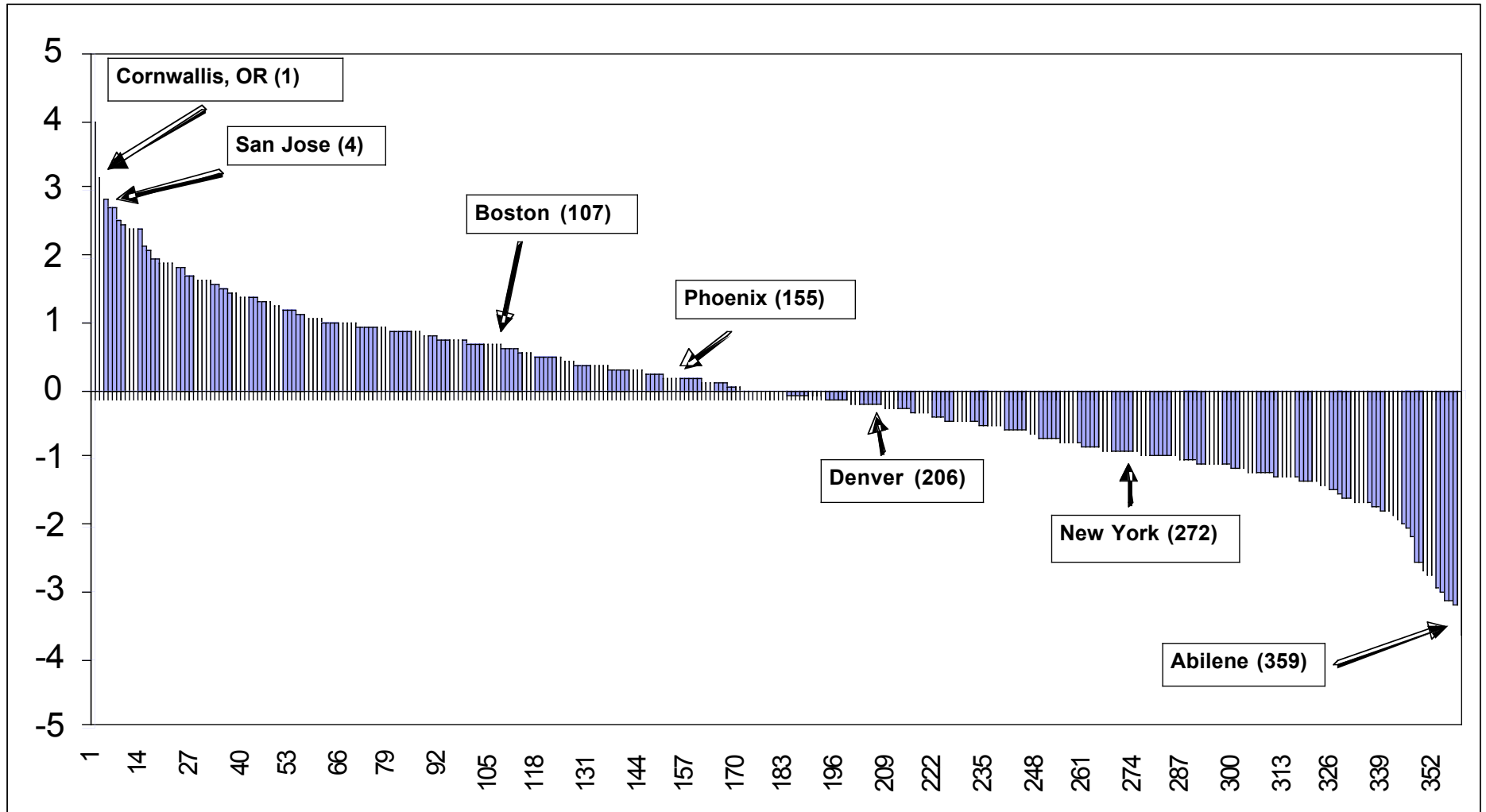
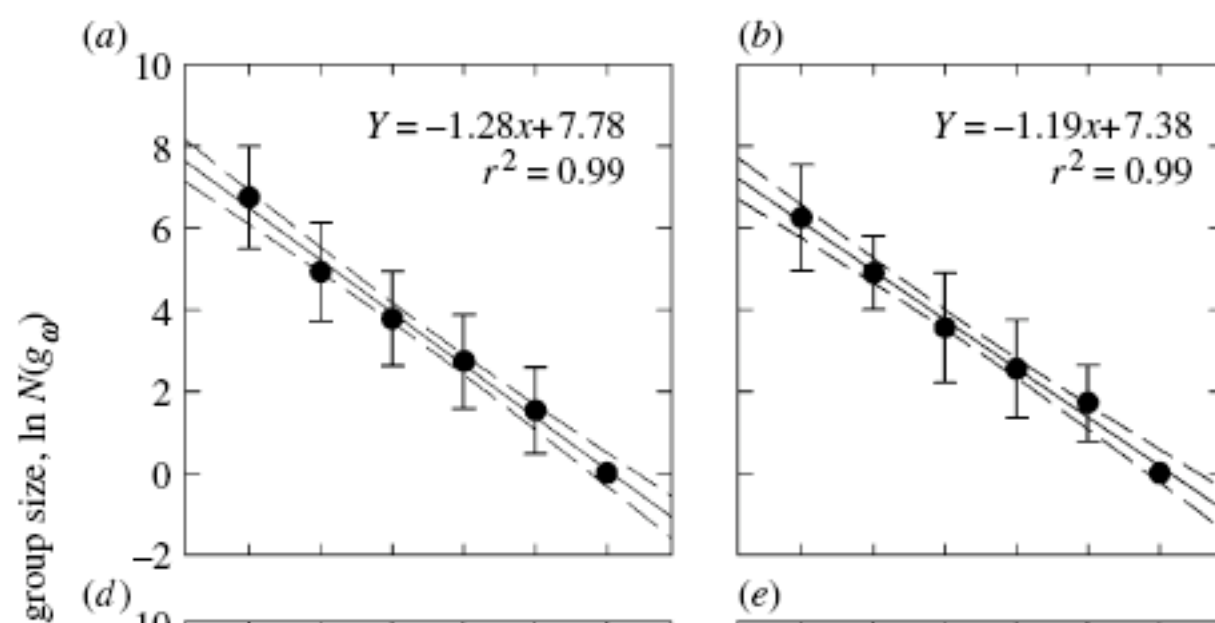


Table 1. Hunter-gatherer group sizes and frequencies.

organizational level	Horton order	sample size	ln mean	s.d.	geometric mean
group size (g)	ω	n	$\langle \ln g \rangle$	s_g	\bar{g}
individual	1	—	0	—	1
family	2	114	1.50	0.23	4.48
dispersed group	3	227	2.75	0.46	15.60
aggregated group	4	297	3.98	0.71	53.66
periodic aggregation	5	213	5.11	0.66	165.32
population size	6	339	6.73	1.25	839.19
frequency, $N(g)$	ω	n	$\langle \ln N(g) \rangle$	$s_{N(g)}$	$\bar{N}(g)$



Growth Equation

Total incoming rate (Resources, Products, ... “Energy” or “Dollar” equivalent)
 \approx **Maintenance** (Repair, Replacement, Sustenance, ...) + **Growth**

$$R \approx NR_0 + E_0 \frac{dN}{dt}$$

Resources, etc.
needed to maintain
individual

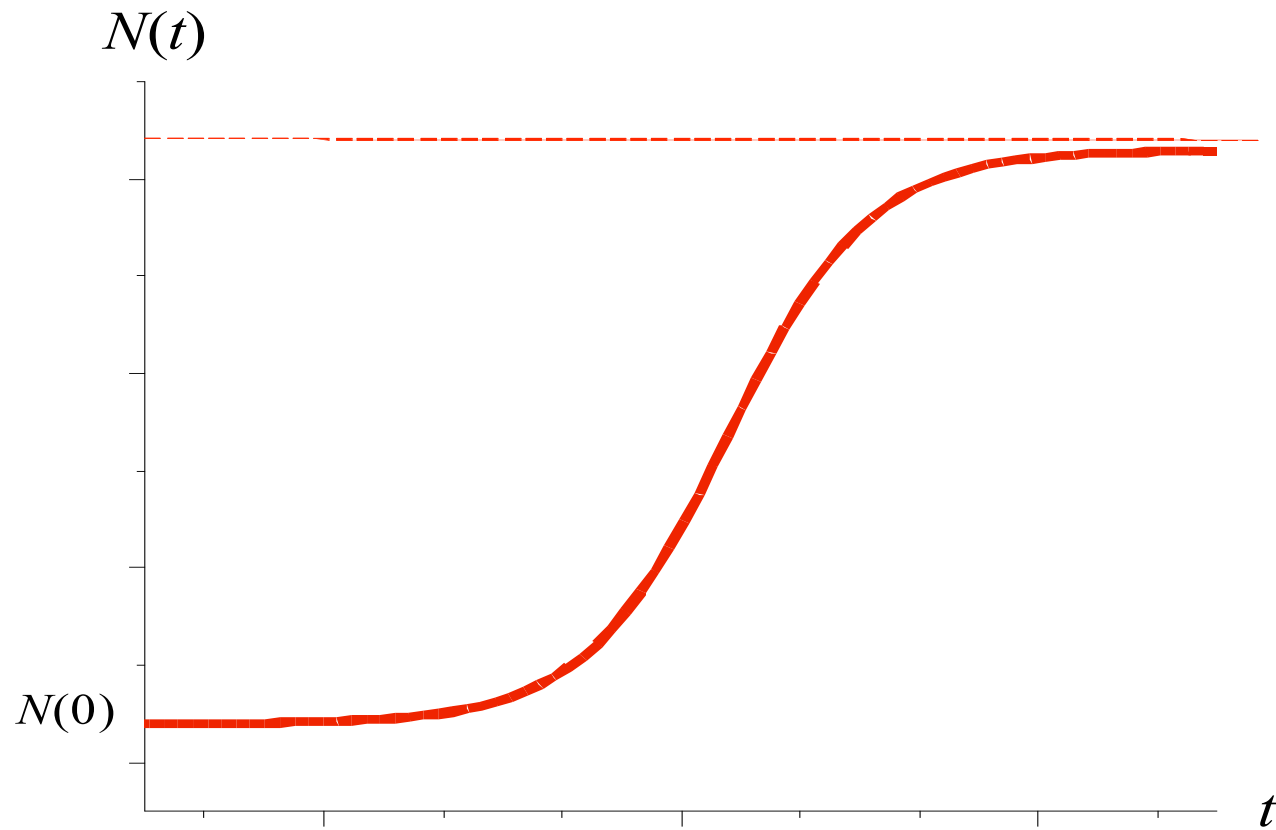
Energy/resources, etc.
needed to create new
individual

$$\frac{dN}{dt} = \left(\frac{R_1}{E_0} \right) \left[N^\beta - \left(\frac{R_0}{R_1} \right) N \right]$$

SOLUTION:

$$N^{1-\beta} = \frac{R_1}{R_0} + \left[N^{1-\beta}(0) - \frac{R_1}{R_0} \right] e^{-\frac{R_0}{E_0}(1-\beta)t}$$

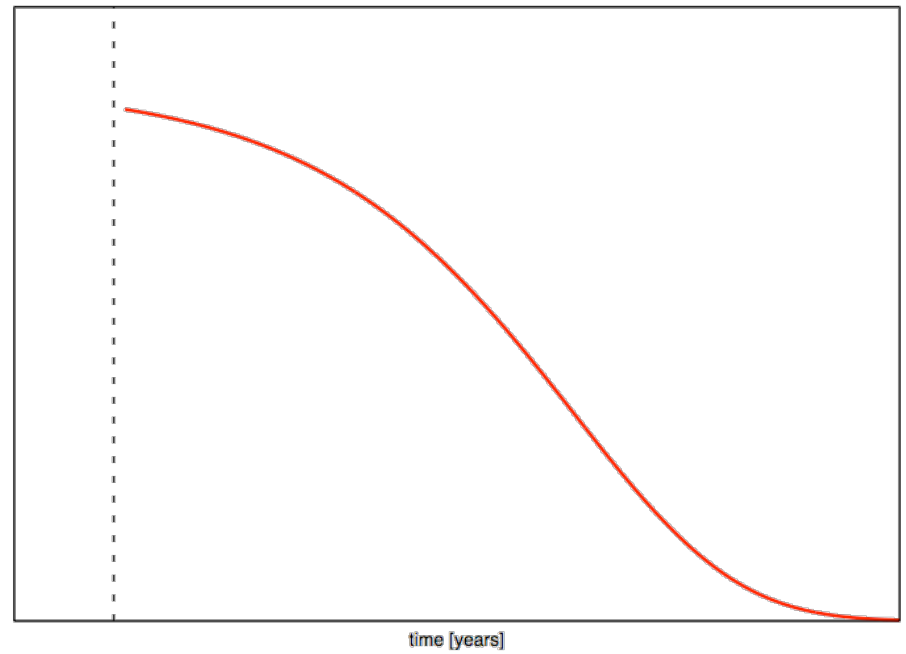
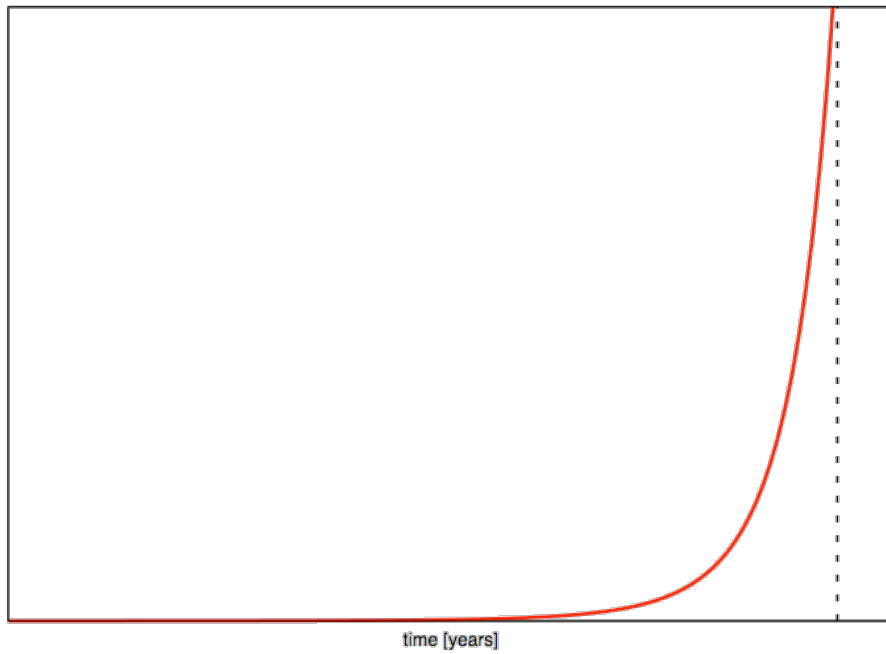
CHARACTER OF SOLUTION SENSITIVE TO $\beta >, =, < 1$



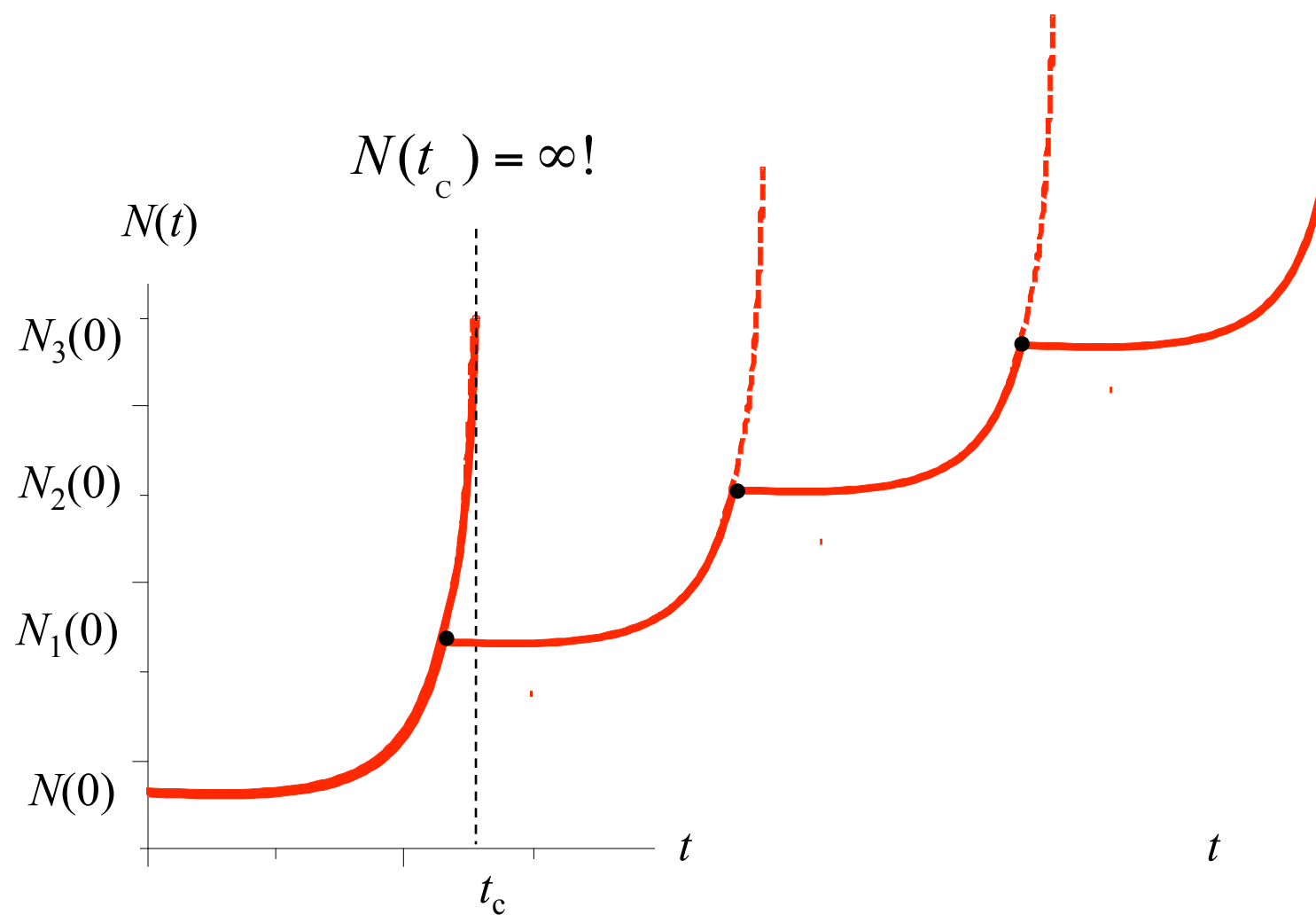
$$b < 1$$

SIGMOIDAL BOUNDED GROWTH

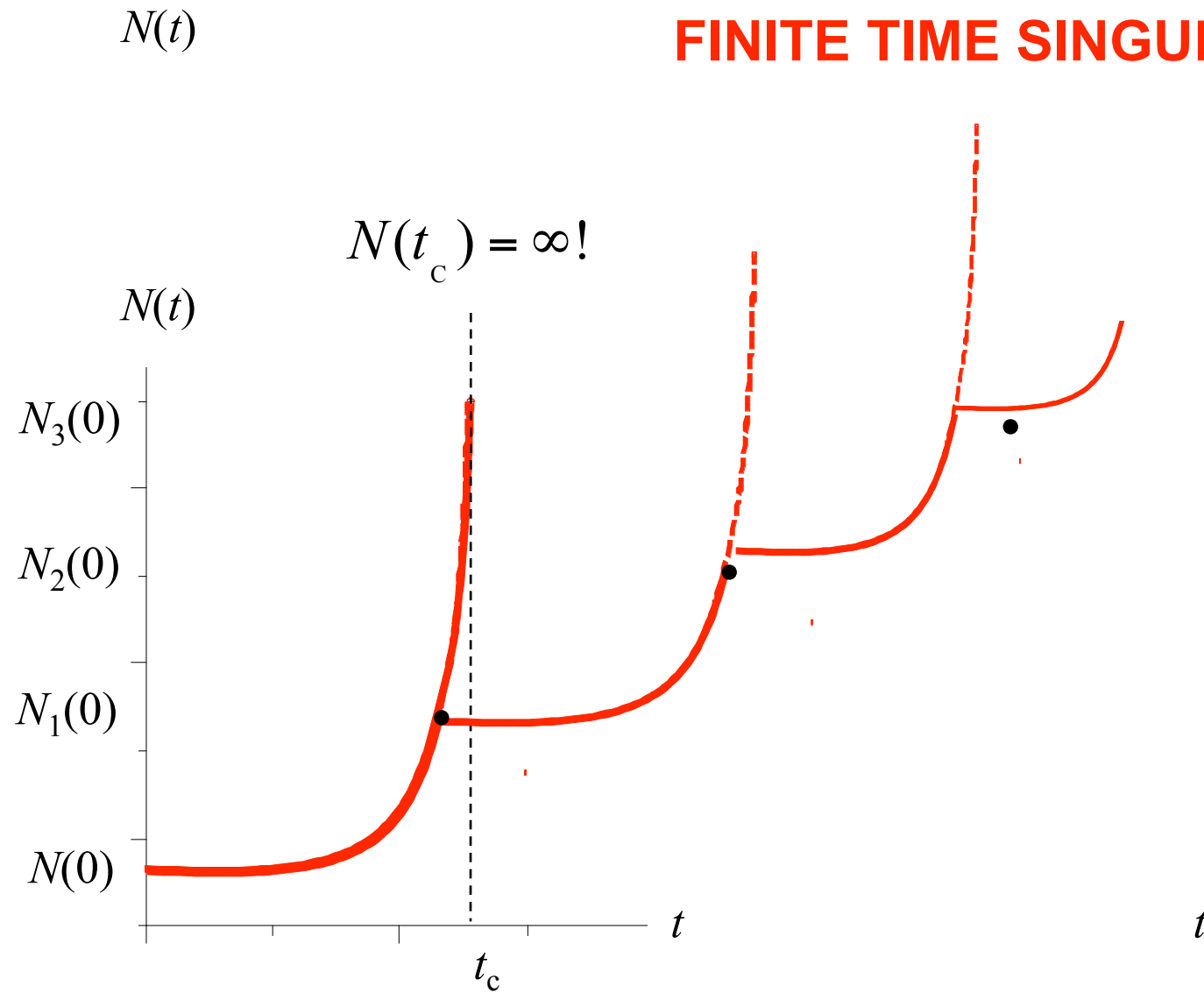
$b > 1$: Finite time Boom and Collapse



**$b > 1$ UNBOUNDED GROWTH UP TO
FINITE TIME SINGULARITY**



**$b > 1$ UNBOUNDED GROWTH UP TO
FINITE TIME SINGULARITY**



TO MAINTAIN CONTINUOUS GROWTH, MUST HAVE:

i) $b > 1$ AND

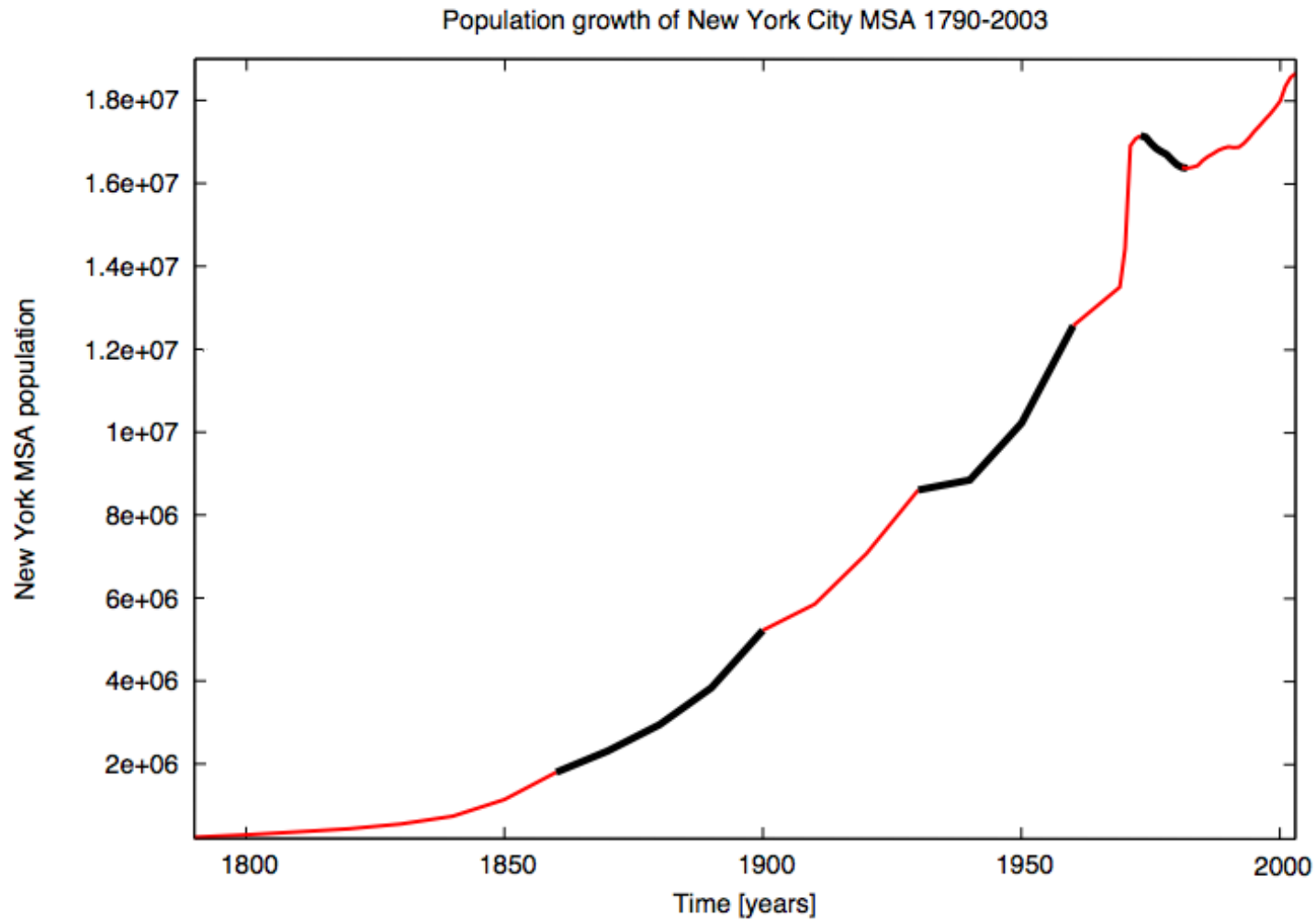
ii) CONTINUOUS MAJOR INNOVATIONS OR PARADIGM SHIFTS AT AN ACCELERATING RATE

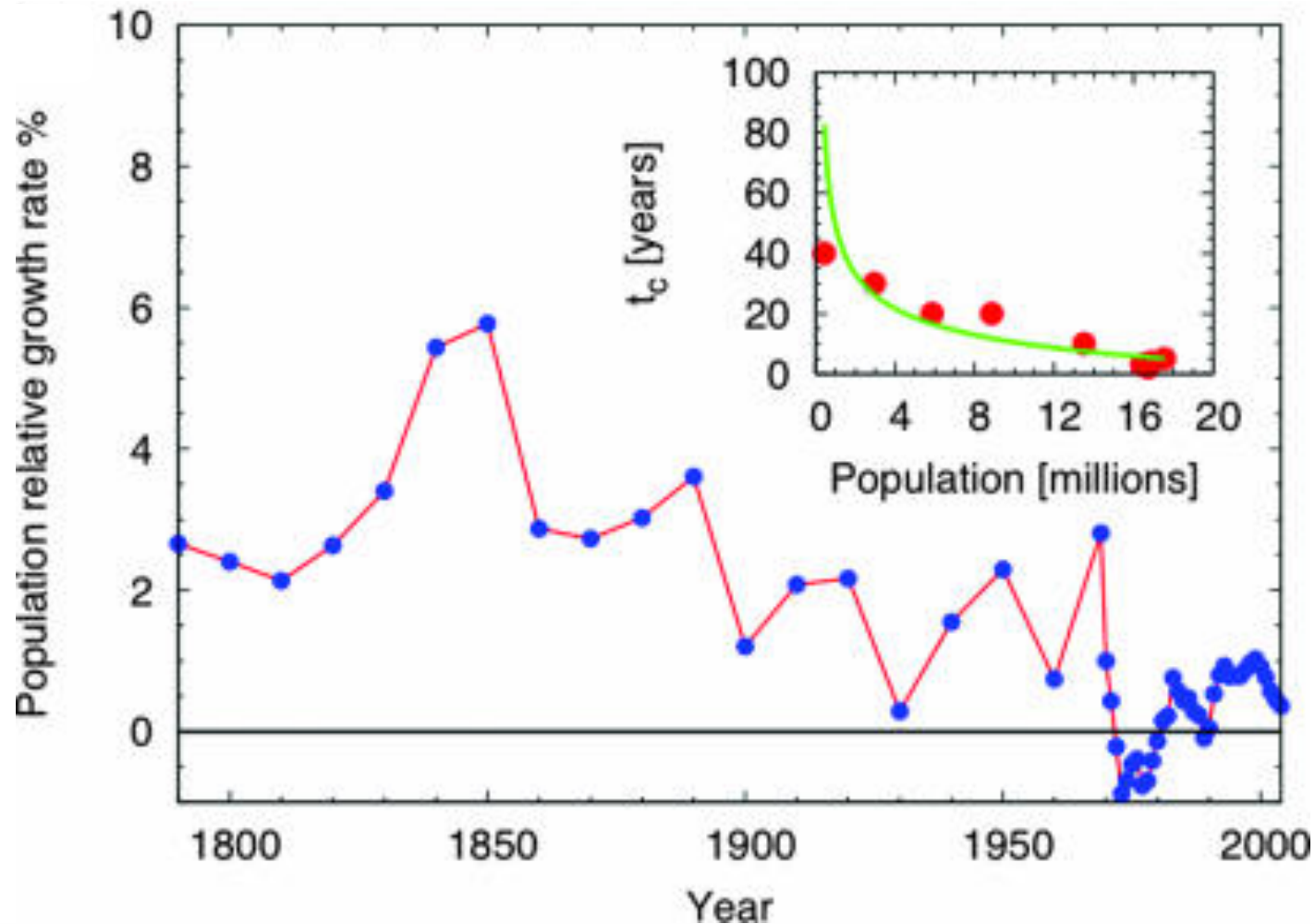
iii) TIME BETWEEN INNOVATIONS DECREASES SYSTEMATICALLY WITH GROWTH:

$$t_c \sim N^{1-b} \sim N^{-0.15} \sim 1/t$$

Population growth for New York City

1790 - 2003





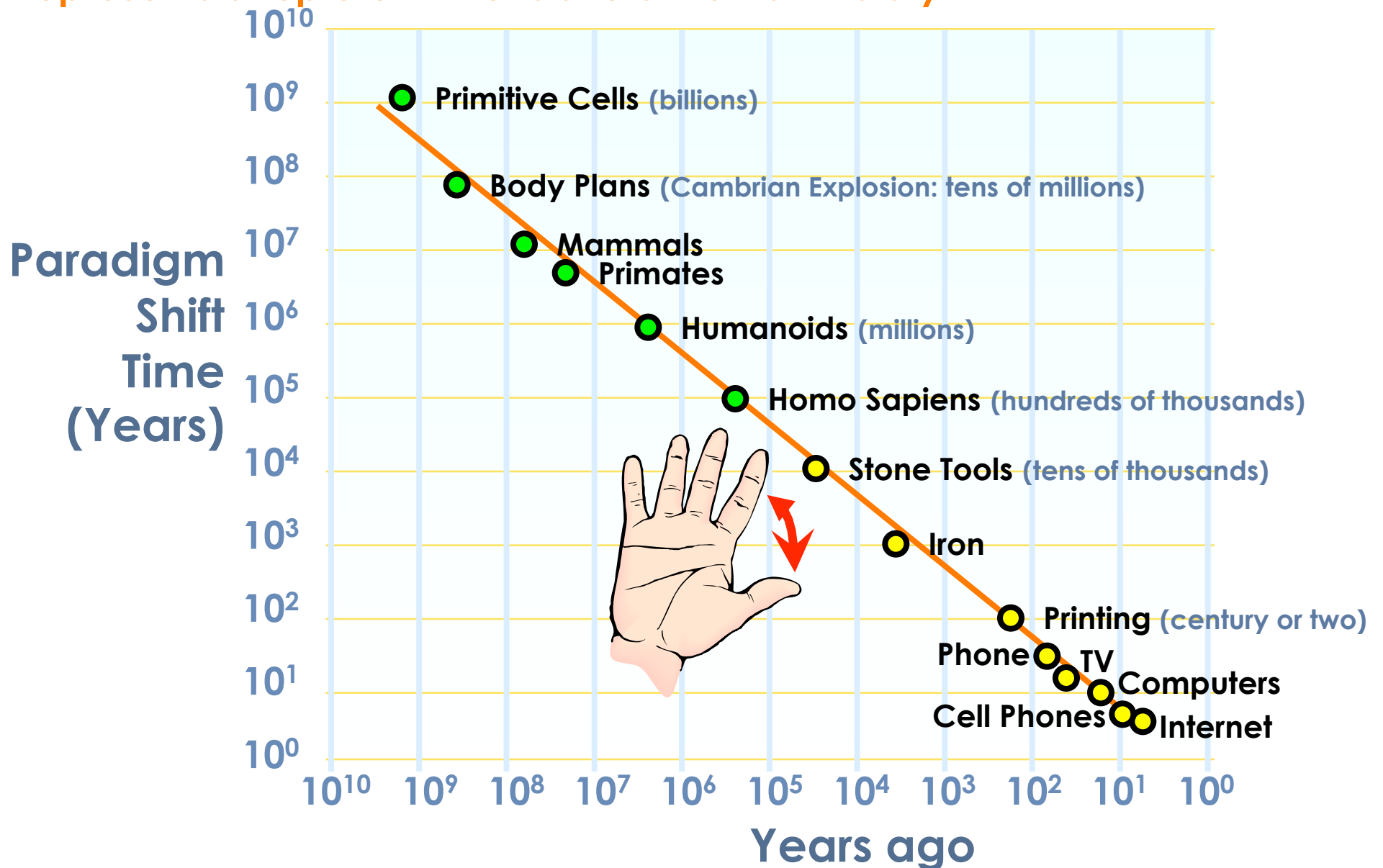
Successive cycles of superlinear innovation reset the singularity and postpone instability and subsequent collapse. The relative population growth rate of New York City over time reveals periods of accelerated (super-exponential) growth. Successive shorter periods of super exponential growth appear, separated by brief periods of deceleration. (Inset) t_c for each of these periods vs. population at the onset of the cycle. Observations are well fit with $\beta = 1.09$ (green line).

Social Corporate Urban

- ***NON-LINEAR SCALING LAWS***
- ***THREE UNIVERSAL CLASSES***
- ***SUPER-LINEAR EXPONENTS (> 1)***
- ***WEALTH CREATION INNOVATION ($\sim N^{0.15}$)***
- ***PACE OF LIFE INCREASES WITH SIZE:***
 - TIMES $\sim N^{-0.15}$***
 - RATES $\sim N^{0.15}$***
- ***UNBOUNDED SUPER-EXPONENTIAL GROWTH***
- ***FINITE TIME SINGULARITY***
- ***ACCELERATING CYCLES OF INNOVATION***
- ***SUSTAINABLE?***
- ***GOVERNED BY NETWORKS (FRACTALS?)***

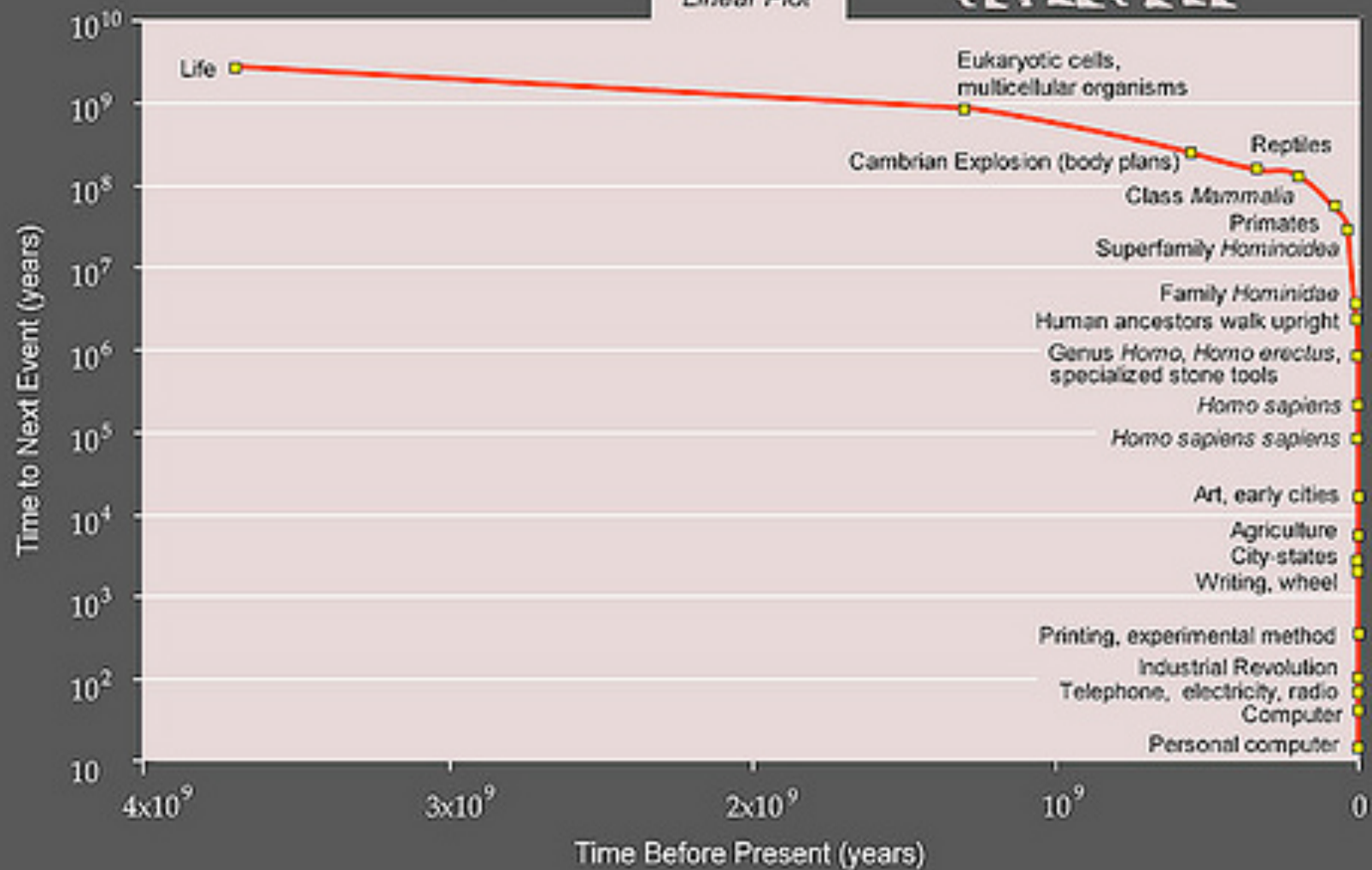
Countdown to singularity

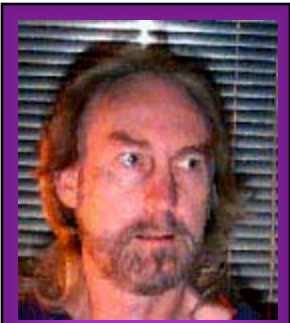
Singularity is technological change so rapid and so profound that it represents a rupture in the fabric of human history



Countdown to Singularity

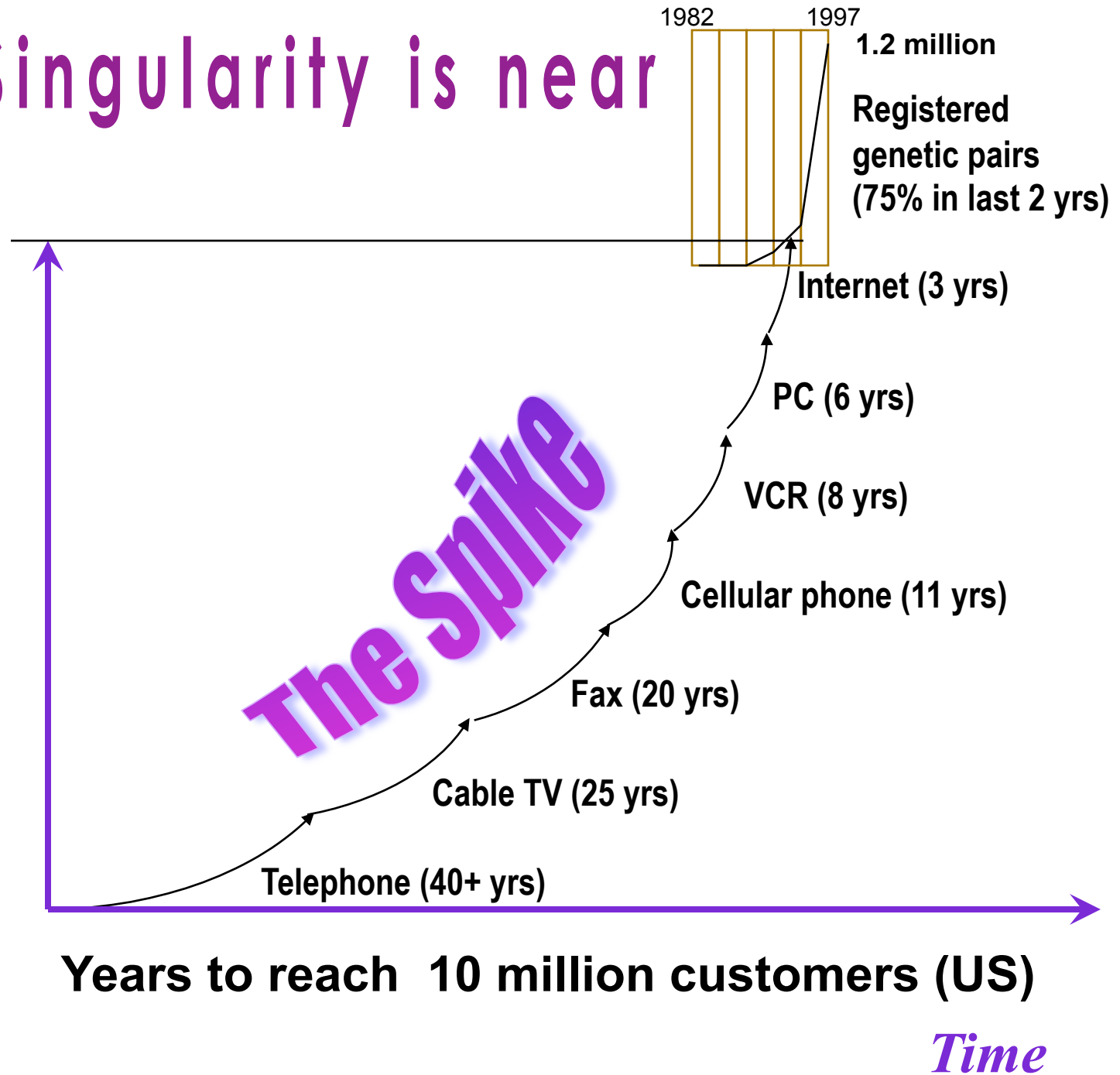
Linear Plot





Damien
Broderick

Singularity is near



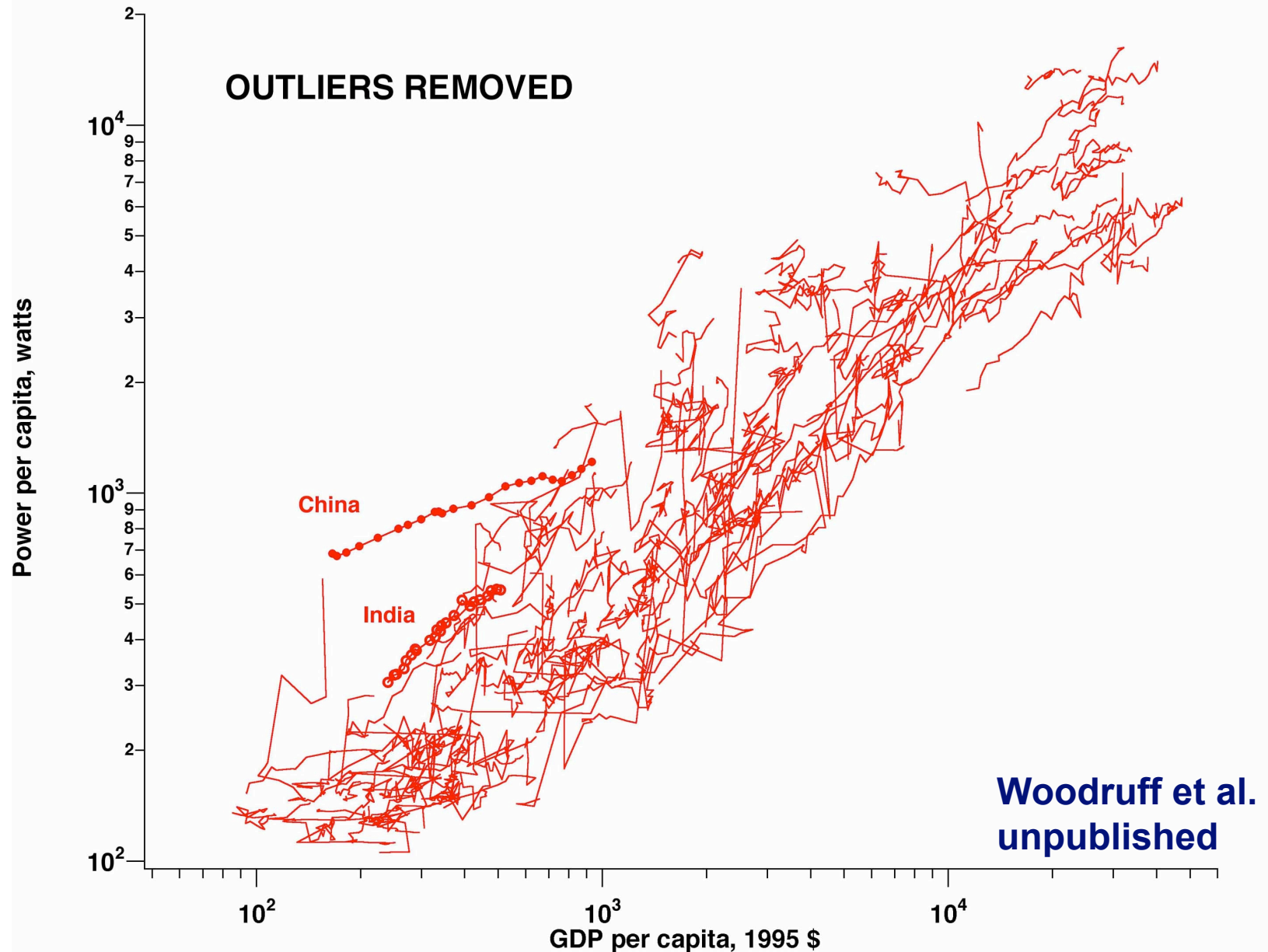
Singularity is near

The ever accelerating progress of technology....gives the appearance of approaching some essential singularity in the history of the race beyond which human affairs, as we know them, could not continue.



John von Neumann
(1903 - 1957)

PER CAPITA POWER CONSUMPTION AS A FUNCTION OF PER CAPITA GDP 1980-2000



Human ecology: reproductive rate in modern nations



- Biological metabolic rate (B) is 100 watts
- Per capita rate of total energy use, including fossil fuels, varies

from 300 watts in developing nations
to 11,000 watts in developed nations

- Predicted fecundity rate (F)

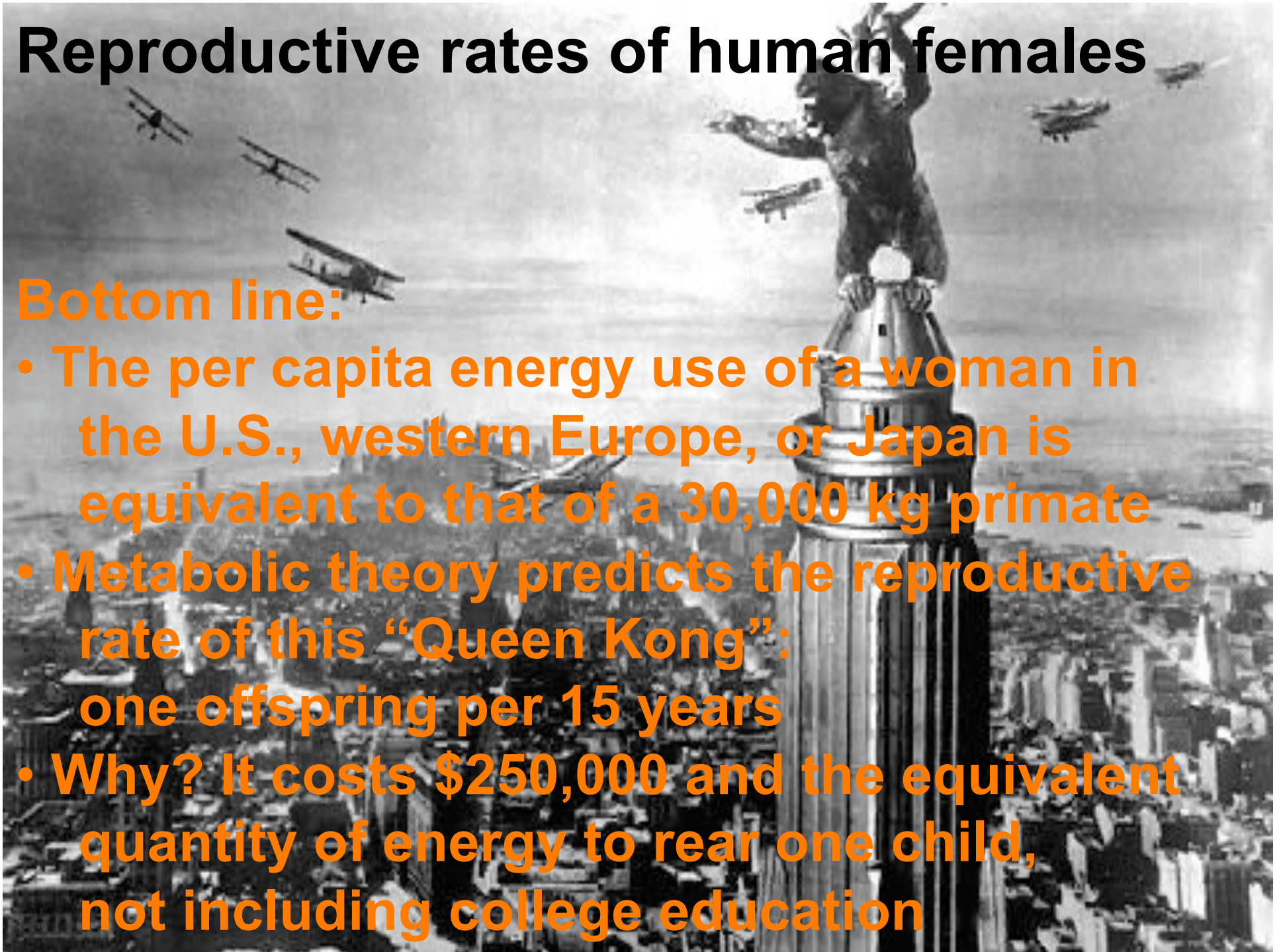
$$F \propto M^{-1/4} \text{ and } B \propto M^{3/4},$$

$$\text{so } F \propto B^{-1/3}$$

Reproductive rates of human females

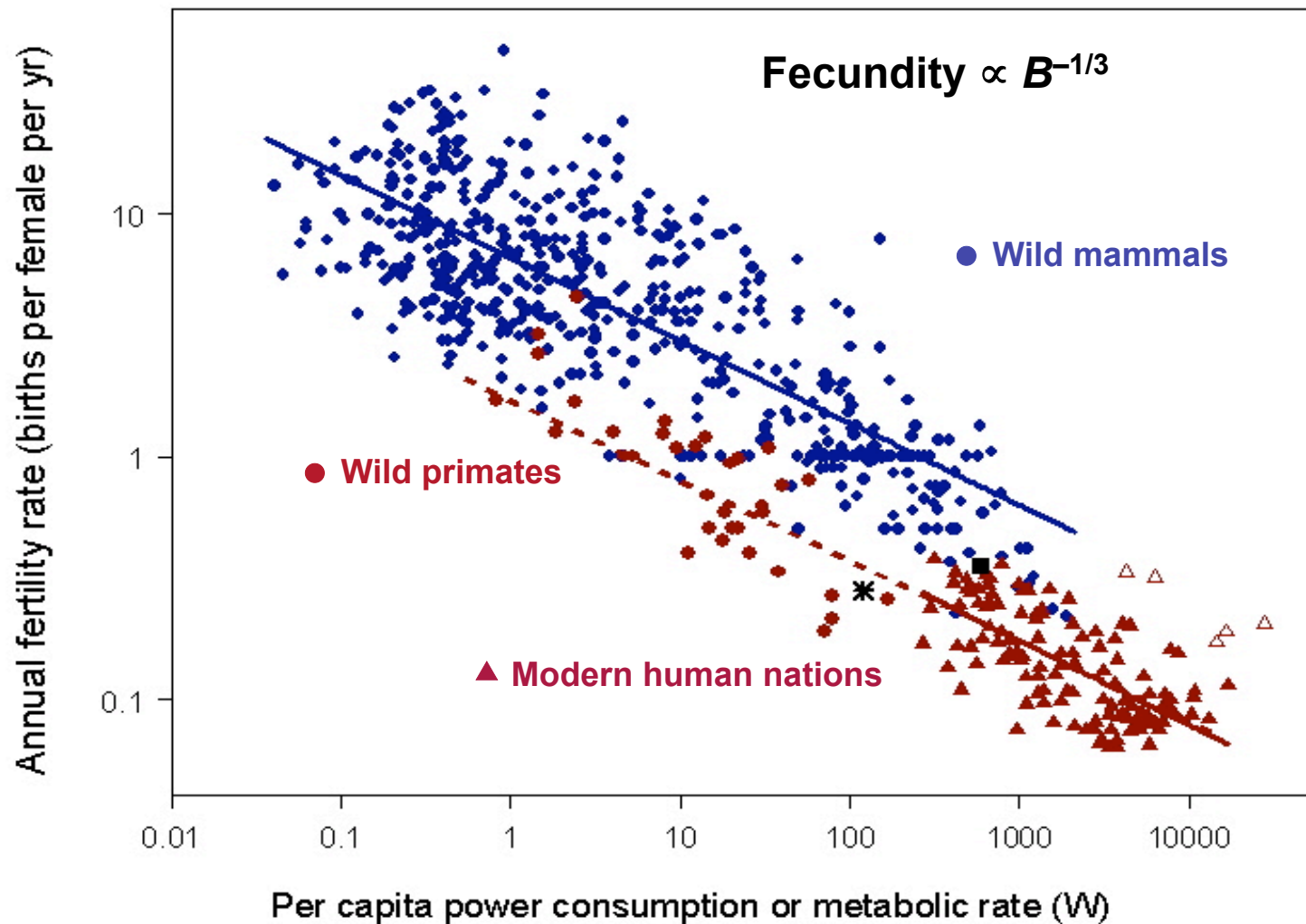
Bottom line:

- The per capita energy use of a woman in the U.S., western Europe, or Japan is equivalent to that of a 30,000 kg primate
- Metabolic theory predicts the reproductive rate of this “Queen Kong”: one offspring per 15 years
- Why? It costs \$250,000 and the equivalent quantity of energy to rear one child, not including college education

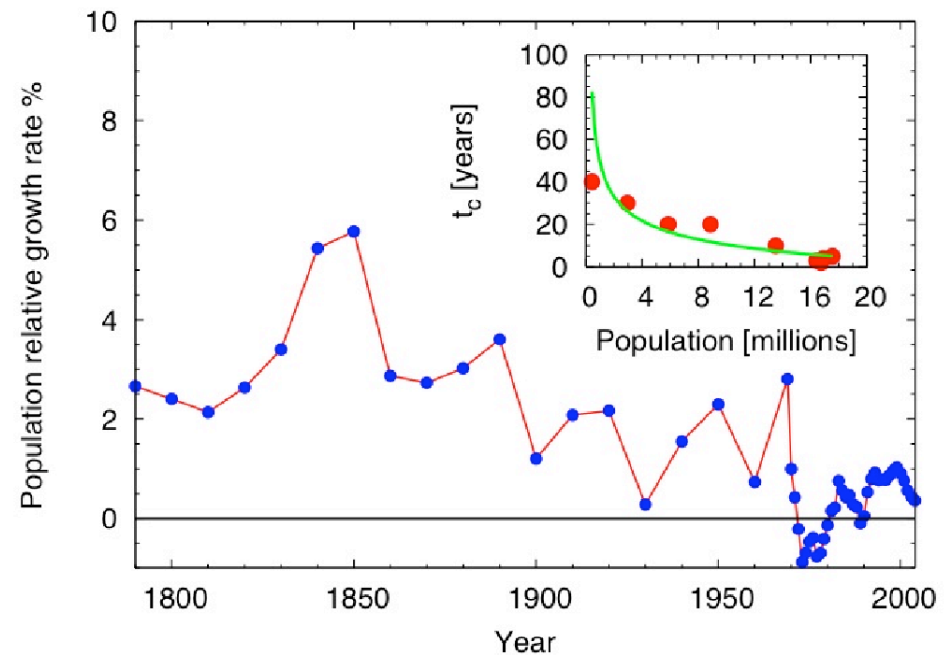
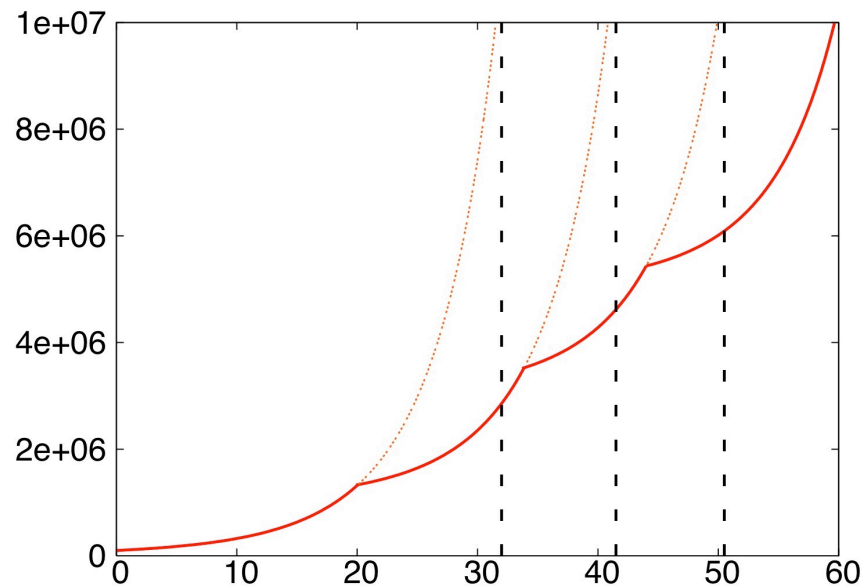


Reproductive rates of mammals, primates, and humans

Moses and Brown 2002



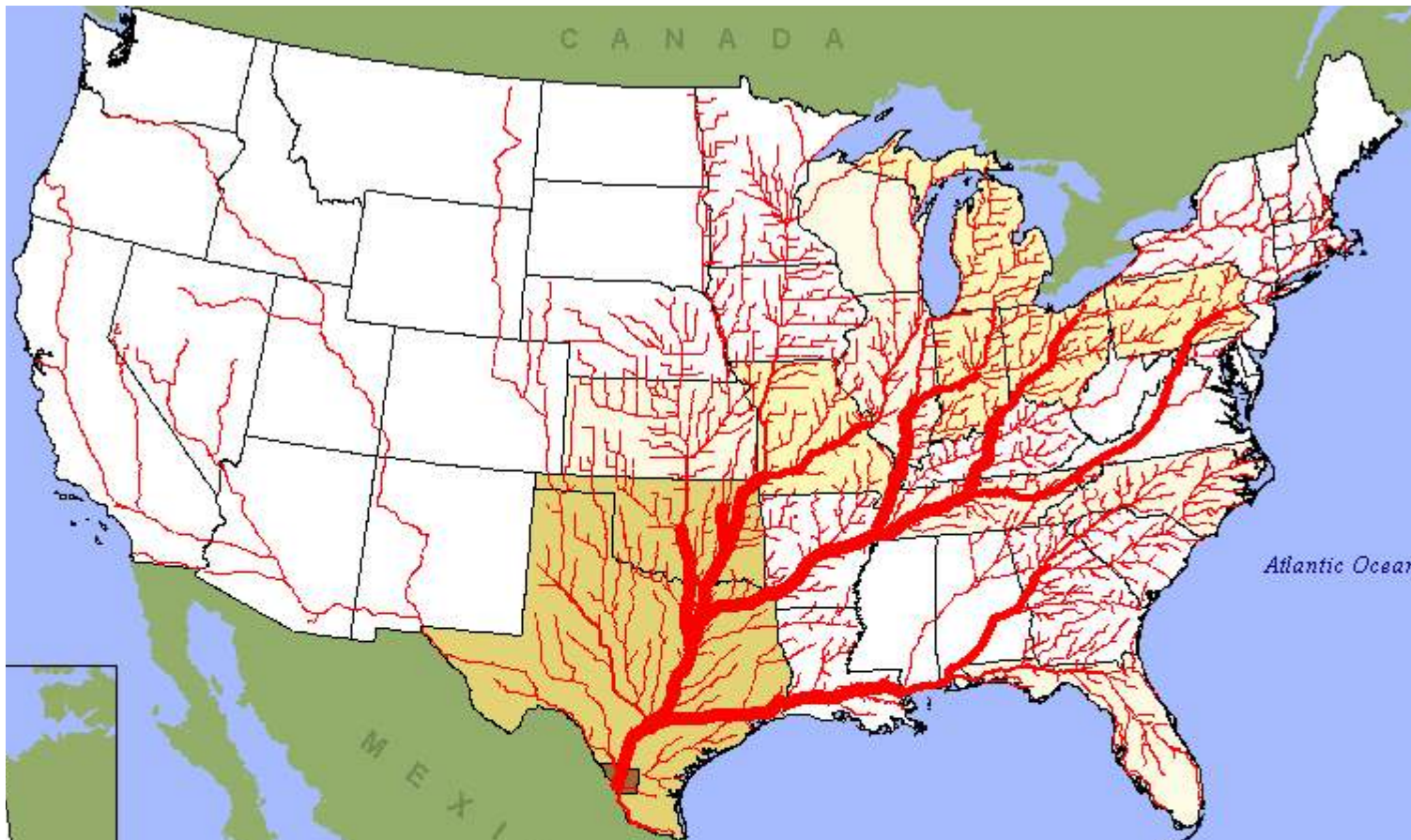
Escaping the singularity with $\beta > 1$: cycles of successive growth & innovation



$$t_{crit} \approx \frac{E_0}{(\beta - 1)R_a} N^{1-\beta}(0) \cong 50 \frac{T}{n^{\beta-1}} \text{ years.}$$

→ t_{crit} shortens with population size N

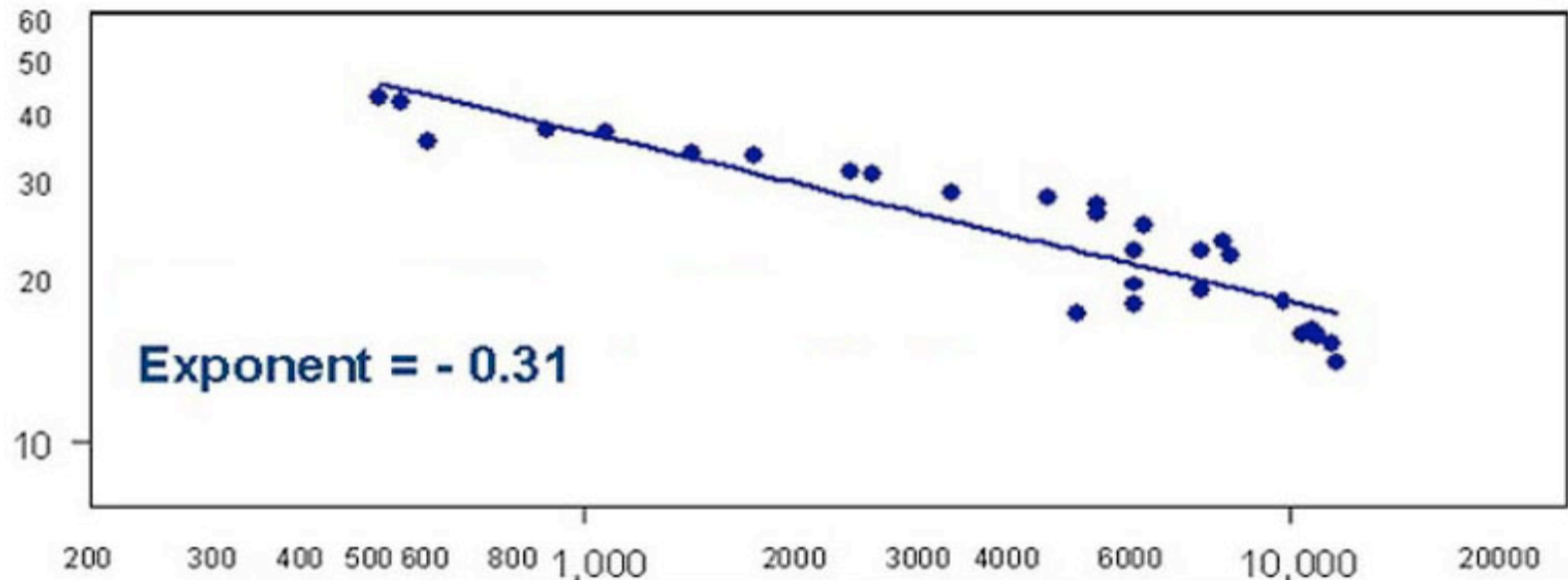
Combined truck traffic through the Laredo area



Federal Highway Administration
Office of Freight Management and Operations

Reproductive rates of U.S. females: Temporal change 1870-2000

reproductive rate
(births per thousand per year)



per capita power consumption
(watts)

Moses and Brown 2002

Energy, Urbanization and Sustainability; Lessons from Biology

GEOFFREY WEST

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LOS ALAMOS NATIONAL LABORATORY

Energy, Sustainability and the Underlying Threat of Urbanization

SIZE MATTERS

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Time List of 100
Harvard Business Review