Optimal interdependence among power grids

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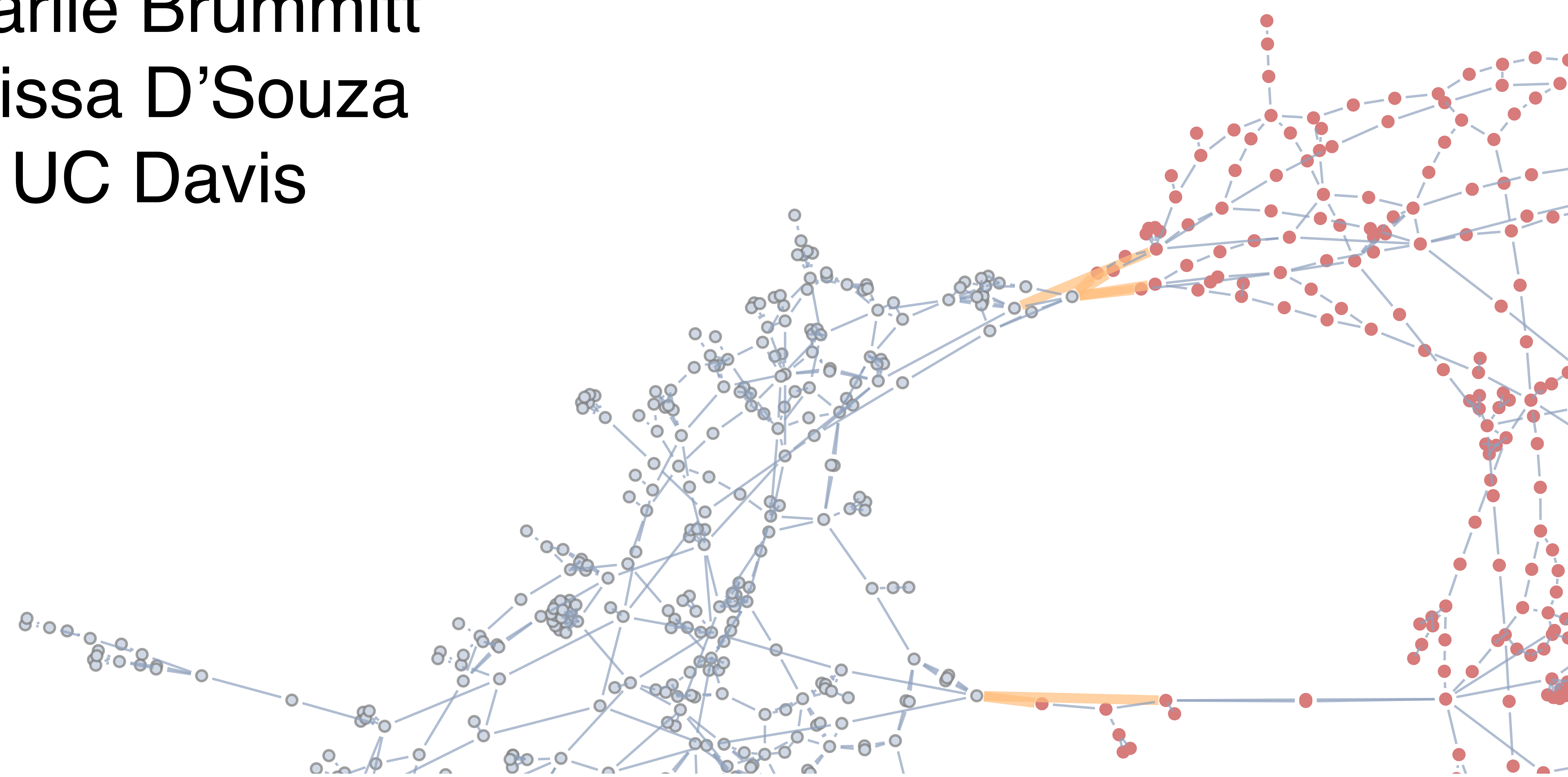
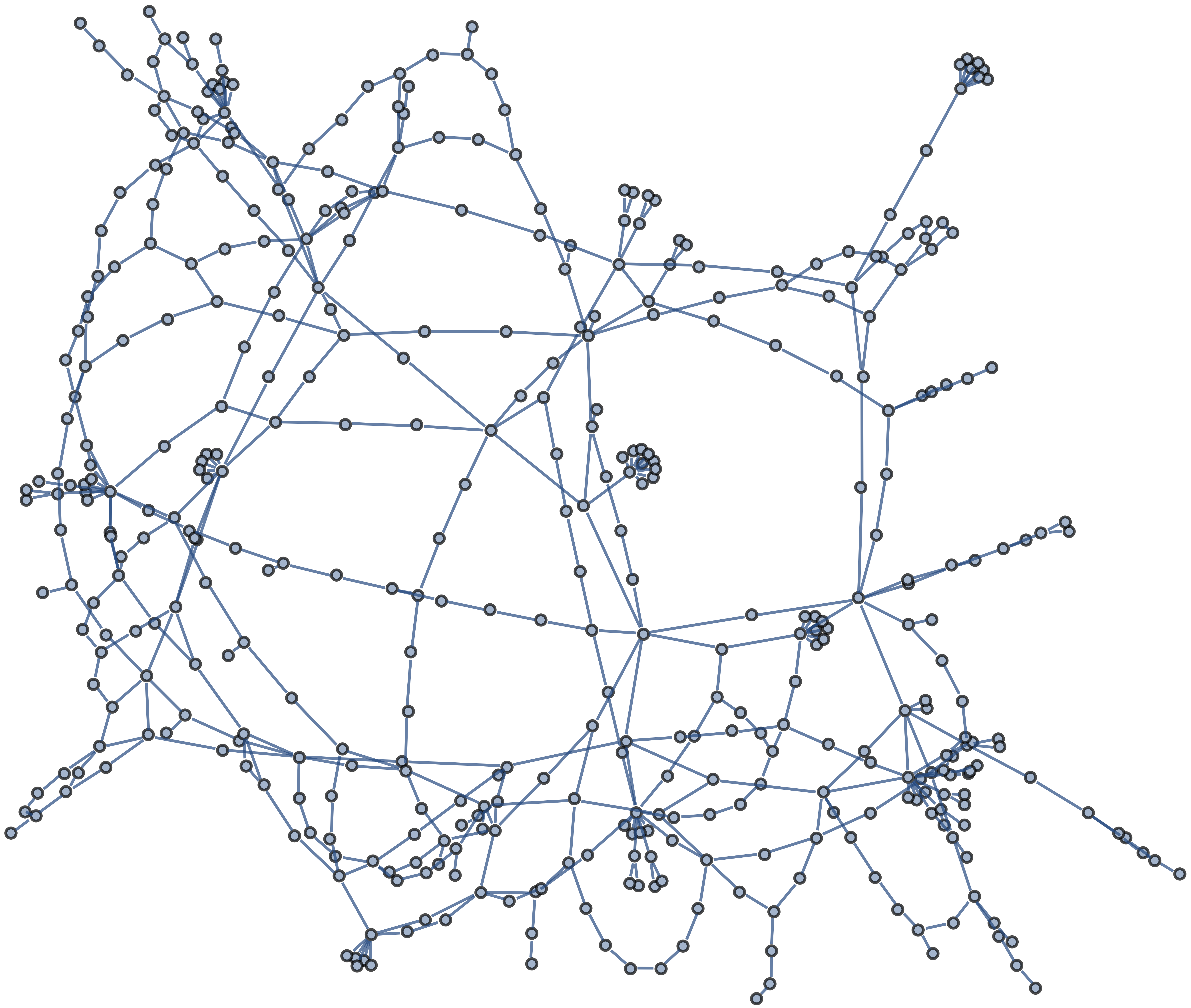


Vulnerability of this power grid?

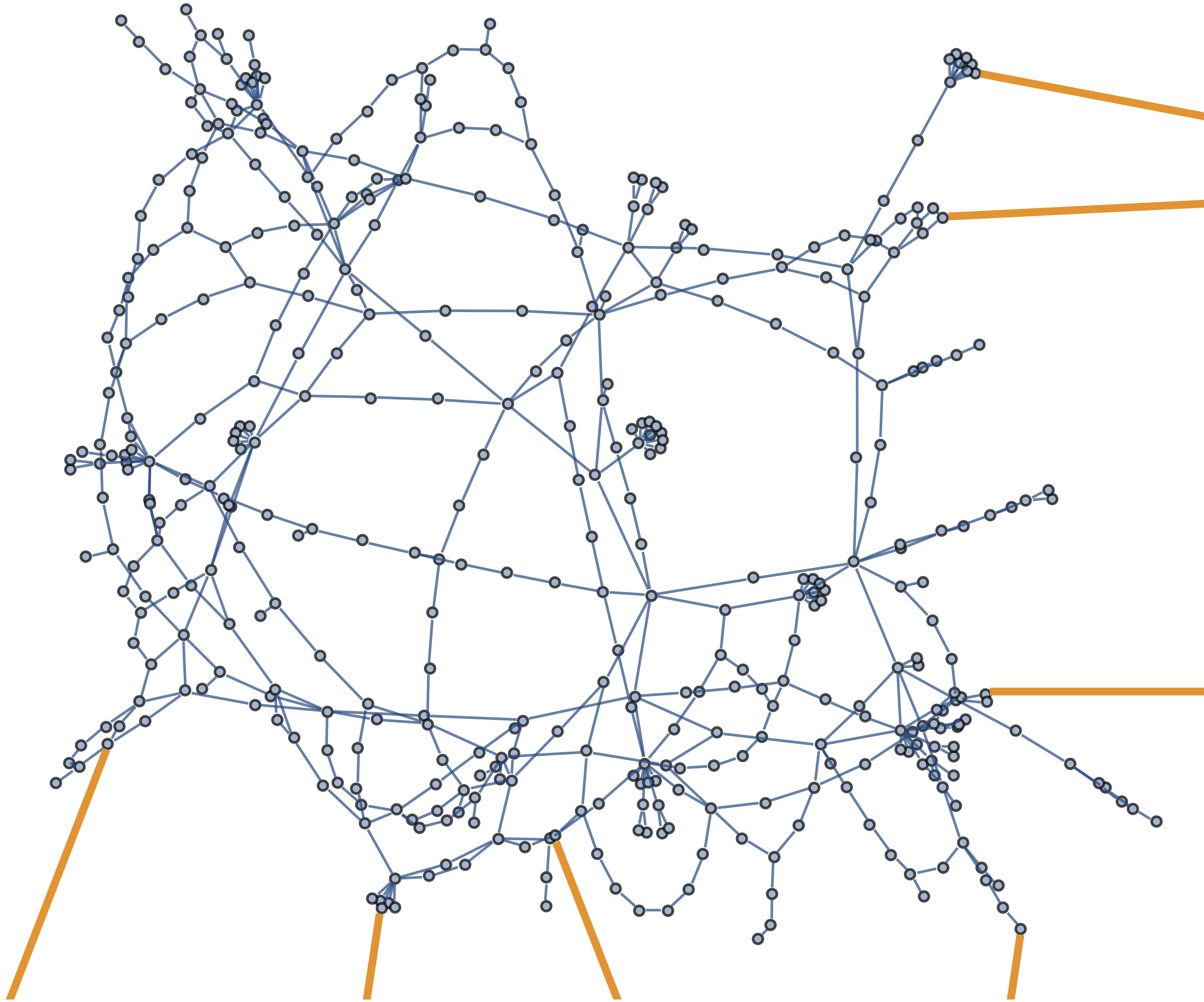


Vulnerability of this power grid?

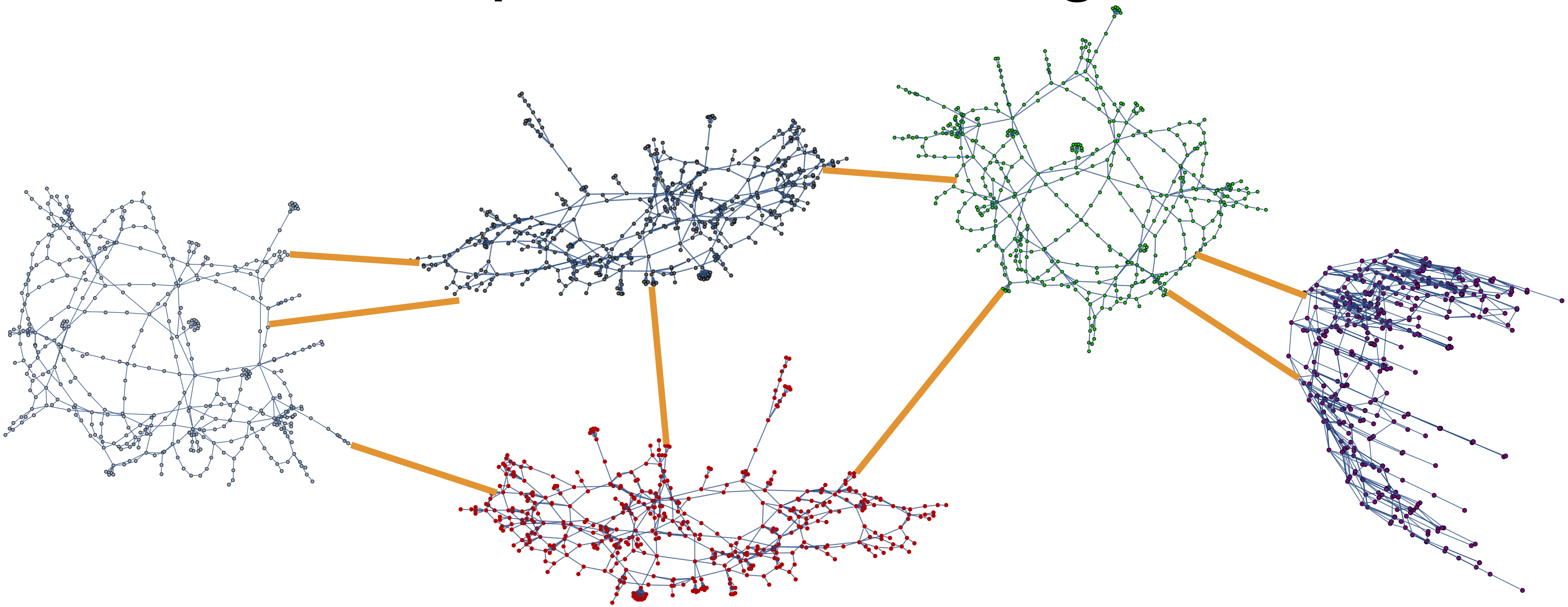
- Study robustness to
  cascading line outages
  hidden failures, etc.

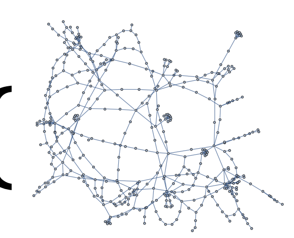
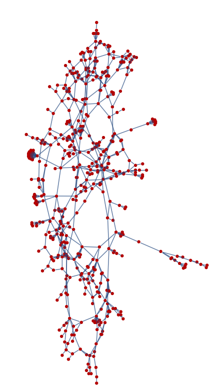
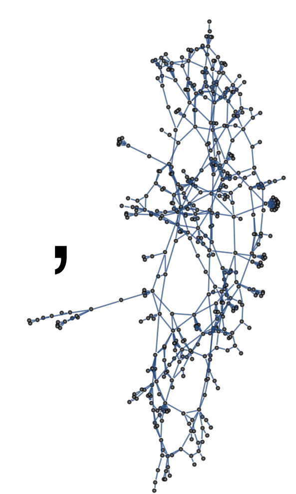
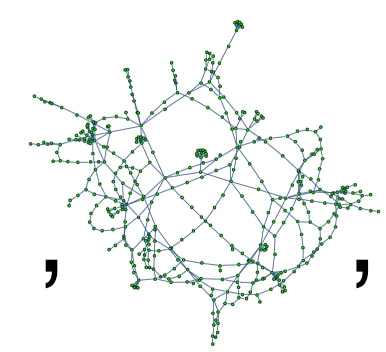
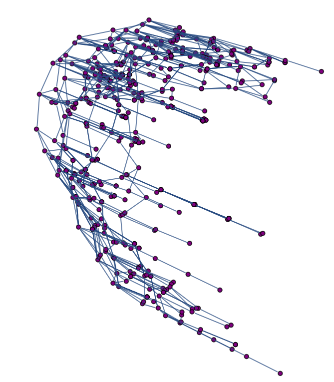
  $\mathbb{P}$(blackout of size *x*)

- But what about those transmission lines to other grids?

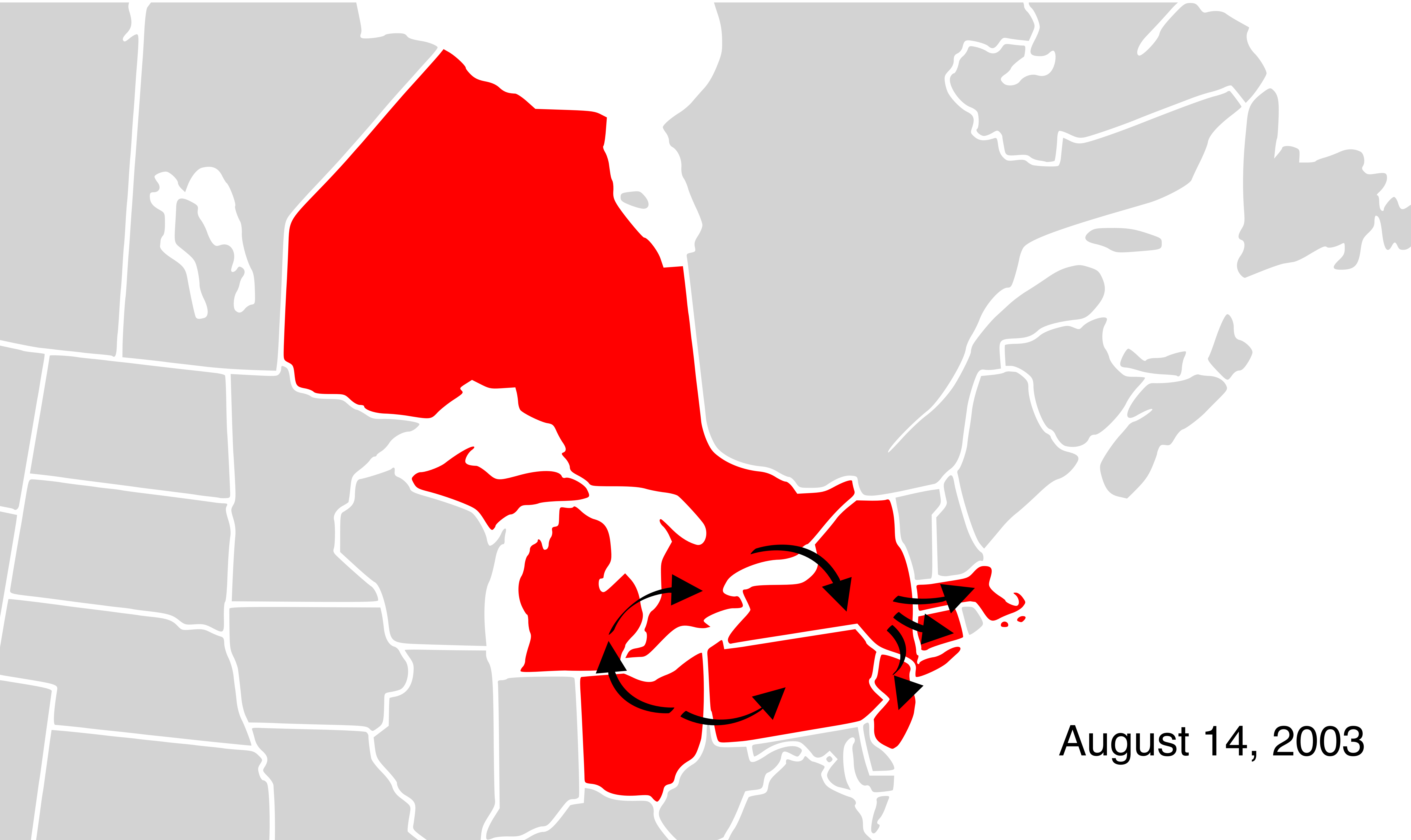


How interdependent should grids be?



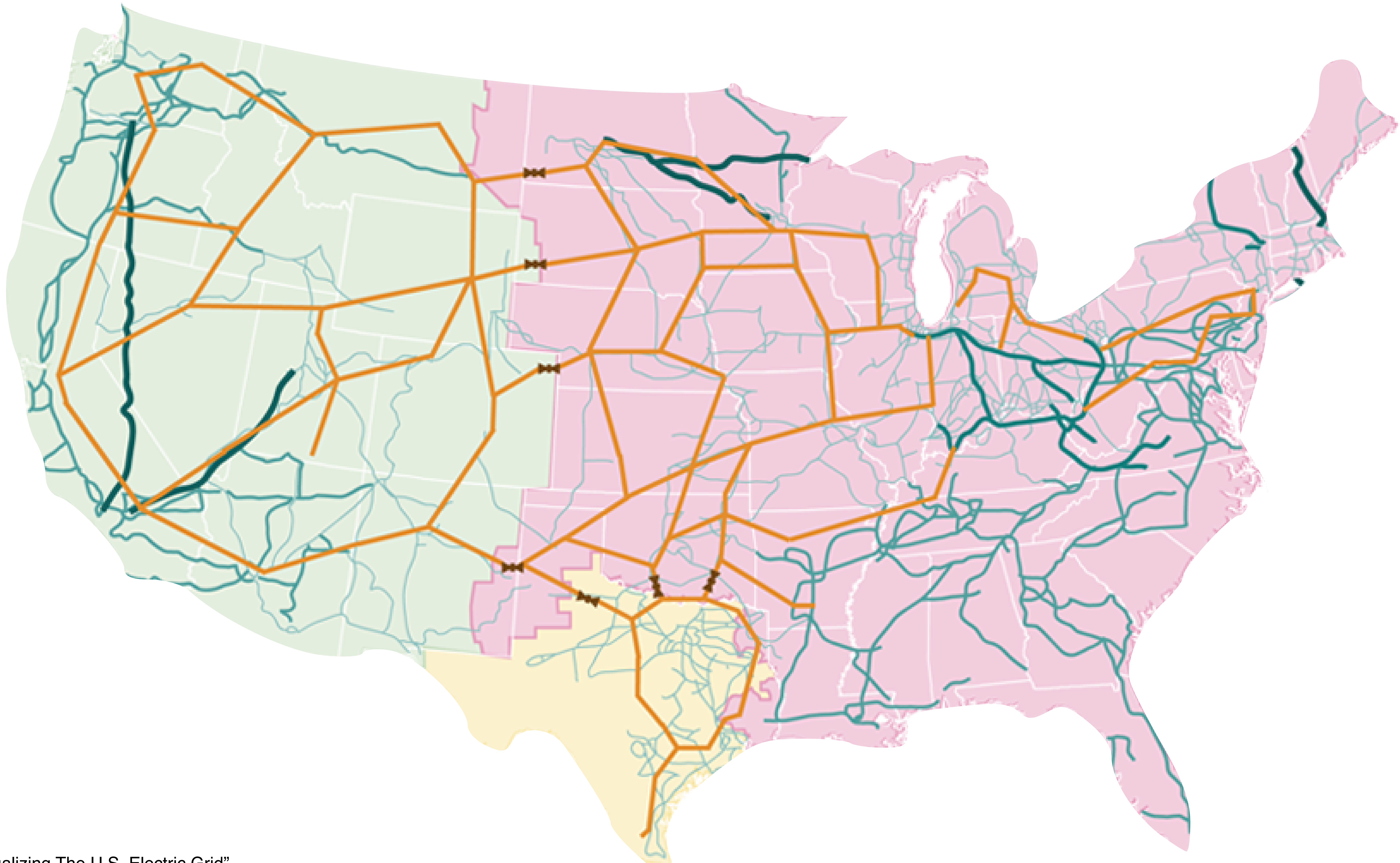
If regional grids were isolated: largest blackout ≤ **max**{  ,  ,  ,  ,  }

Connections among grids enable blackouts



August 14, 2003

We’re building lines among grids



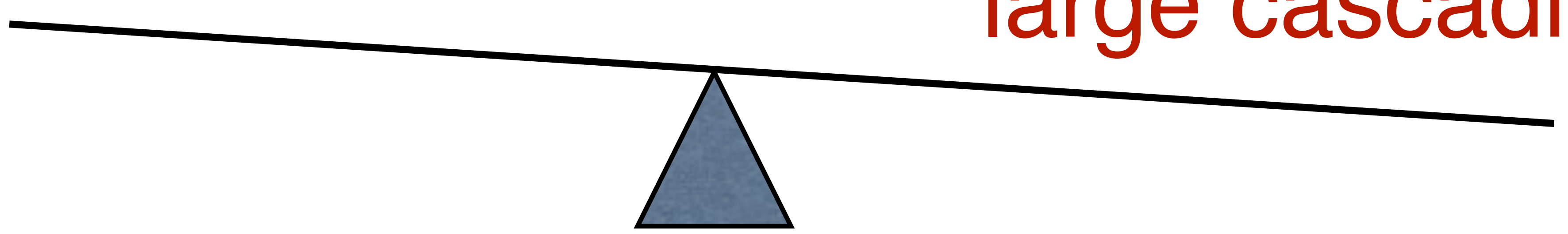
National Public Radio, “Visualizing The U.S. Electric Grid”

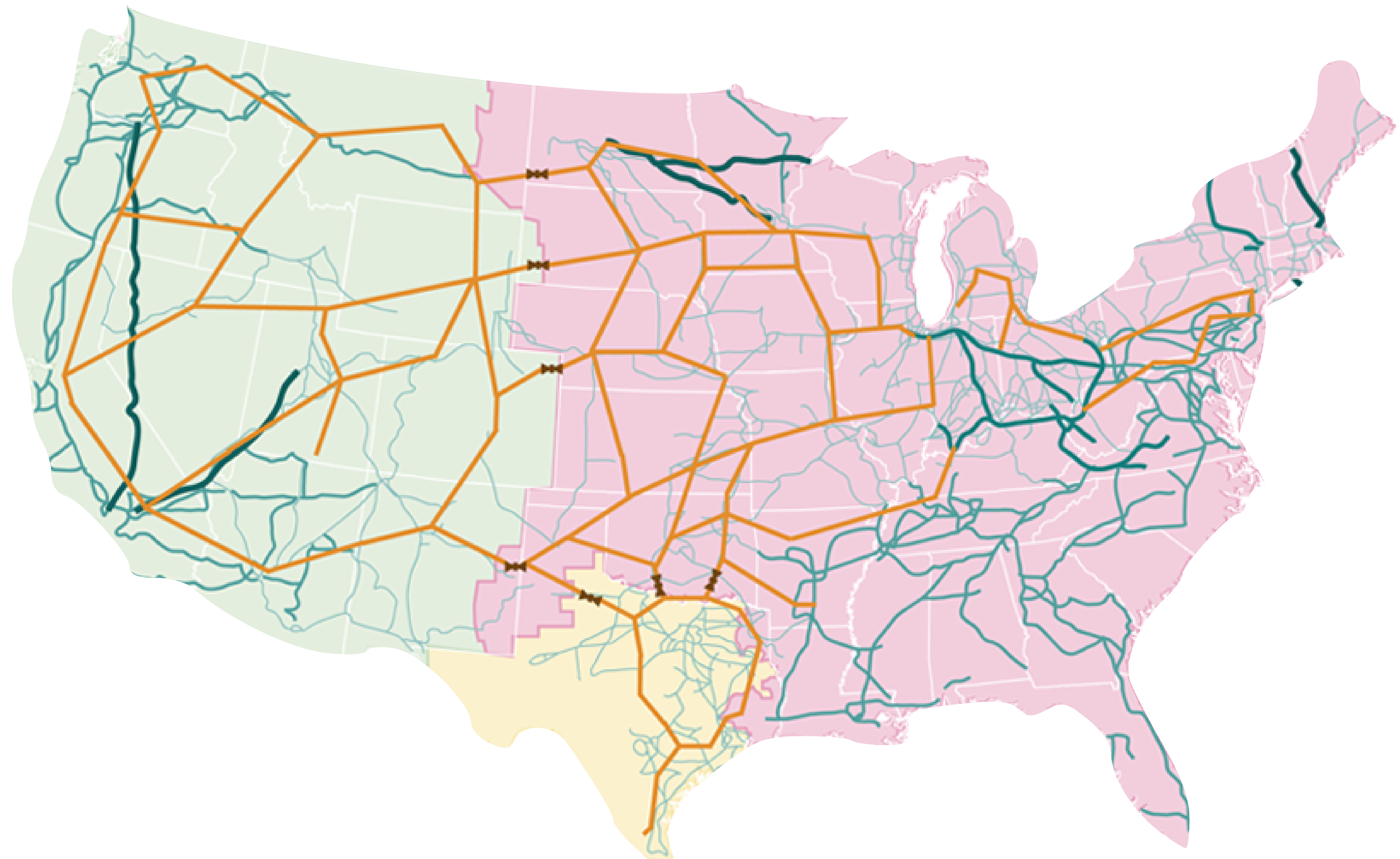
We’re building lines among grids

Tradeoffs of interdependence:

long distance trade,
shares risk

large cascading failures

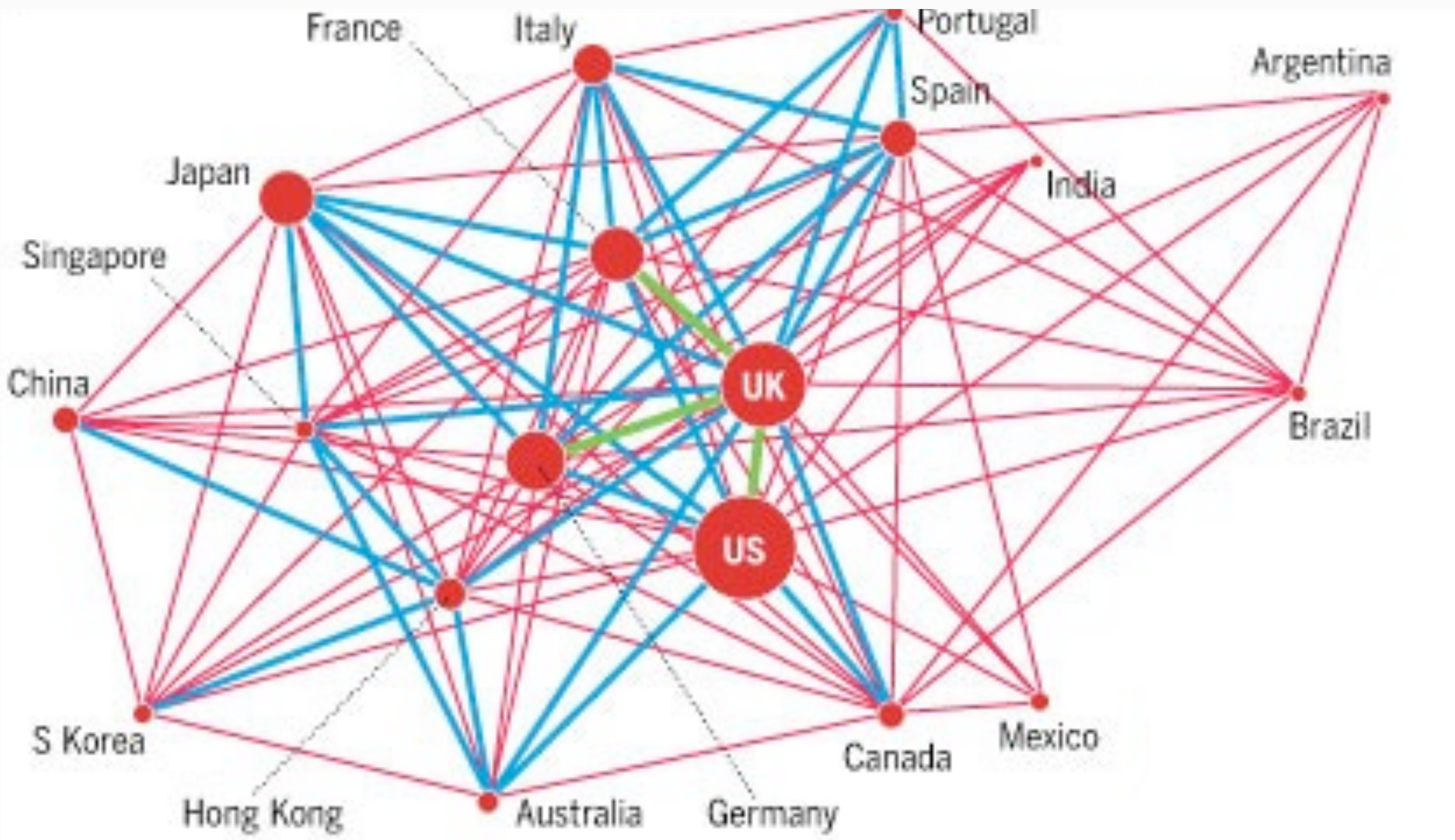




What interdependence balances these tradeoffs?

National Public Radio, “Visualizing The U.S. Electric Grid”

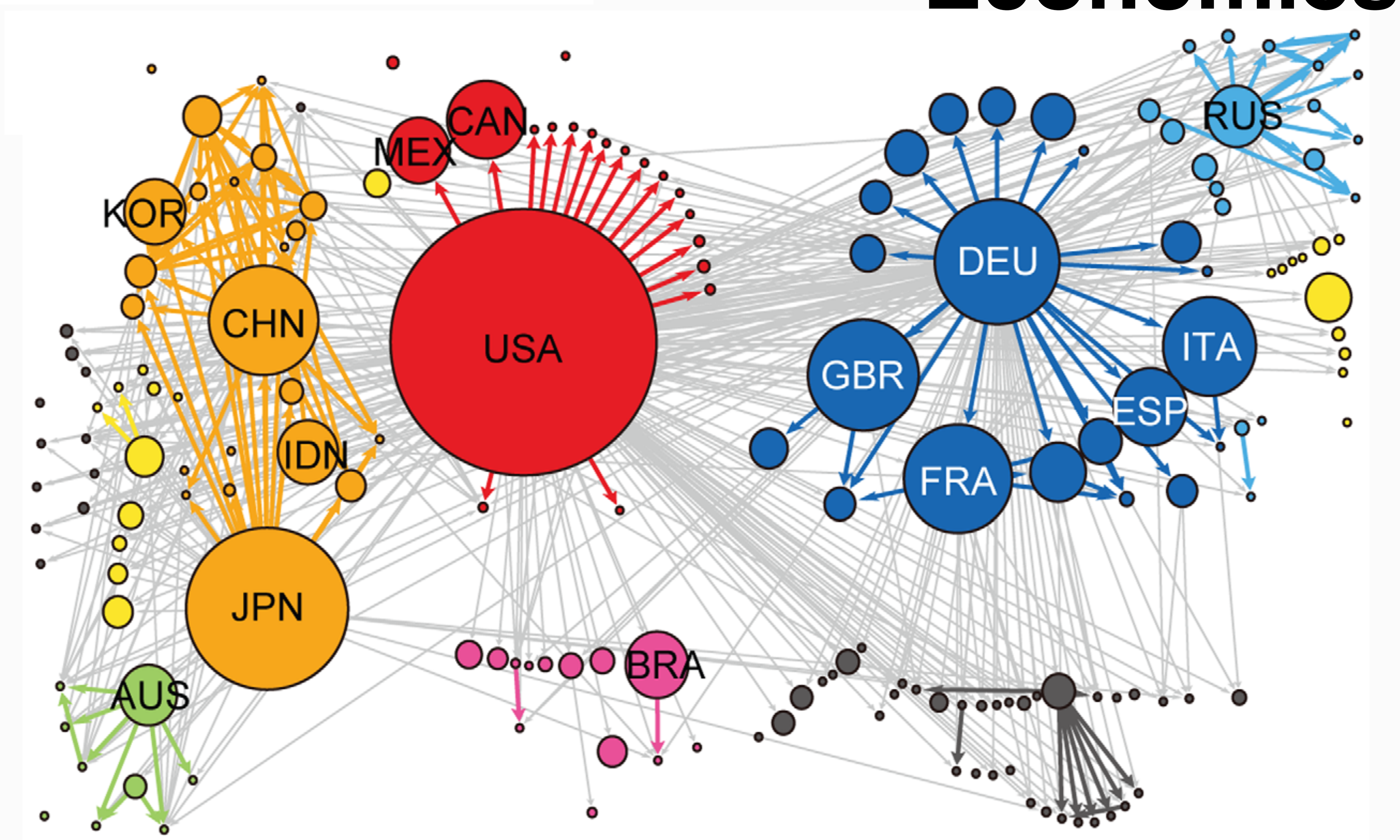
How interdependent should *X* be?



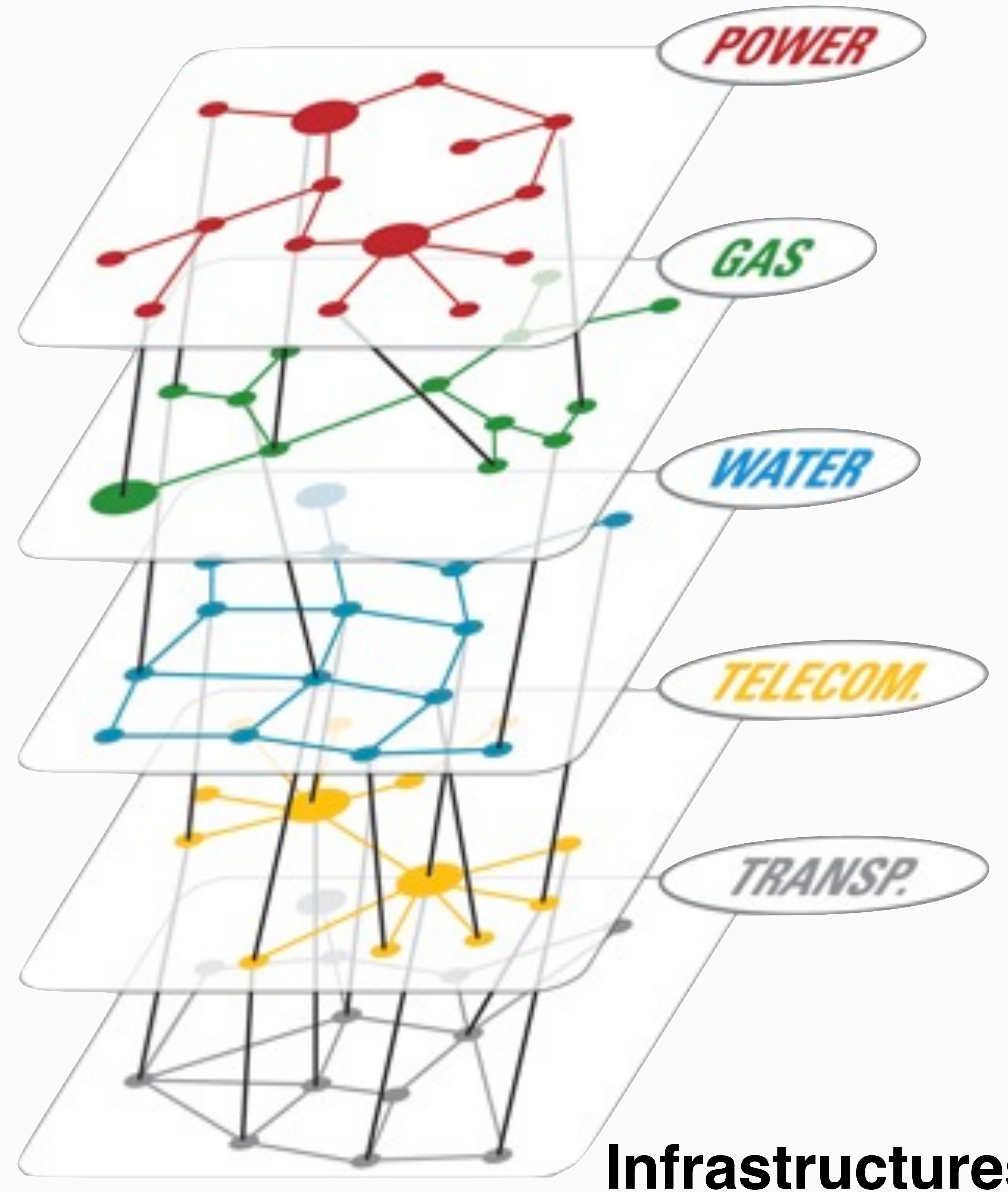
Andrew Haldane/Bank of England

Financial networks

Economies



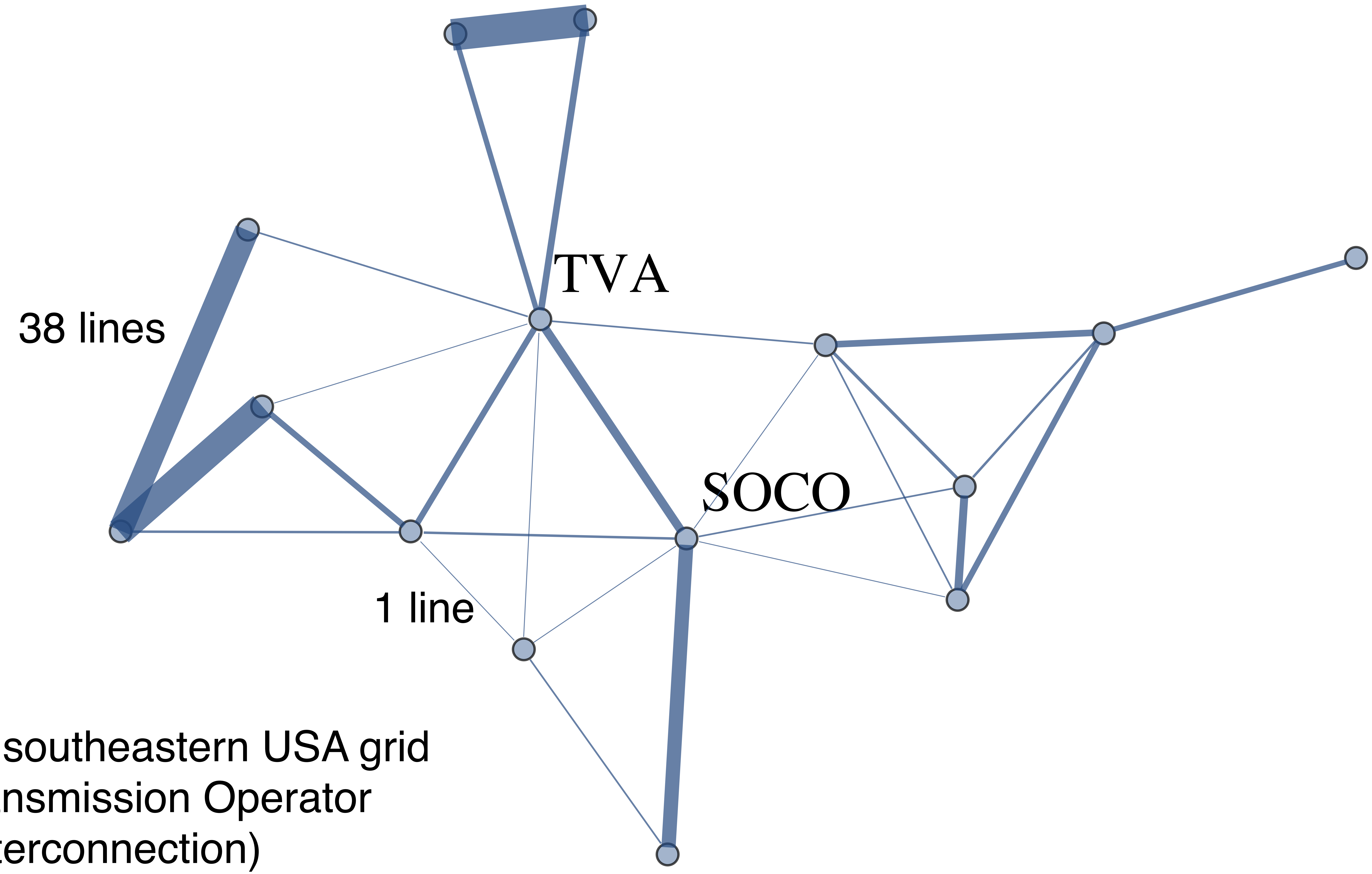
Lee et al., PLoS ONE 2010



Infrastructures

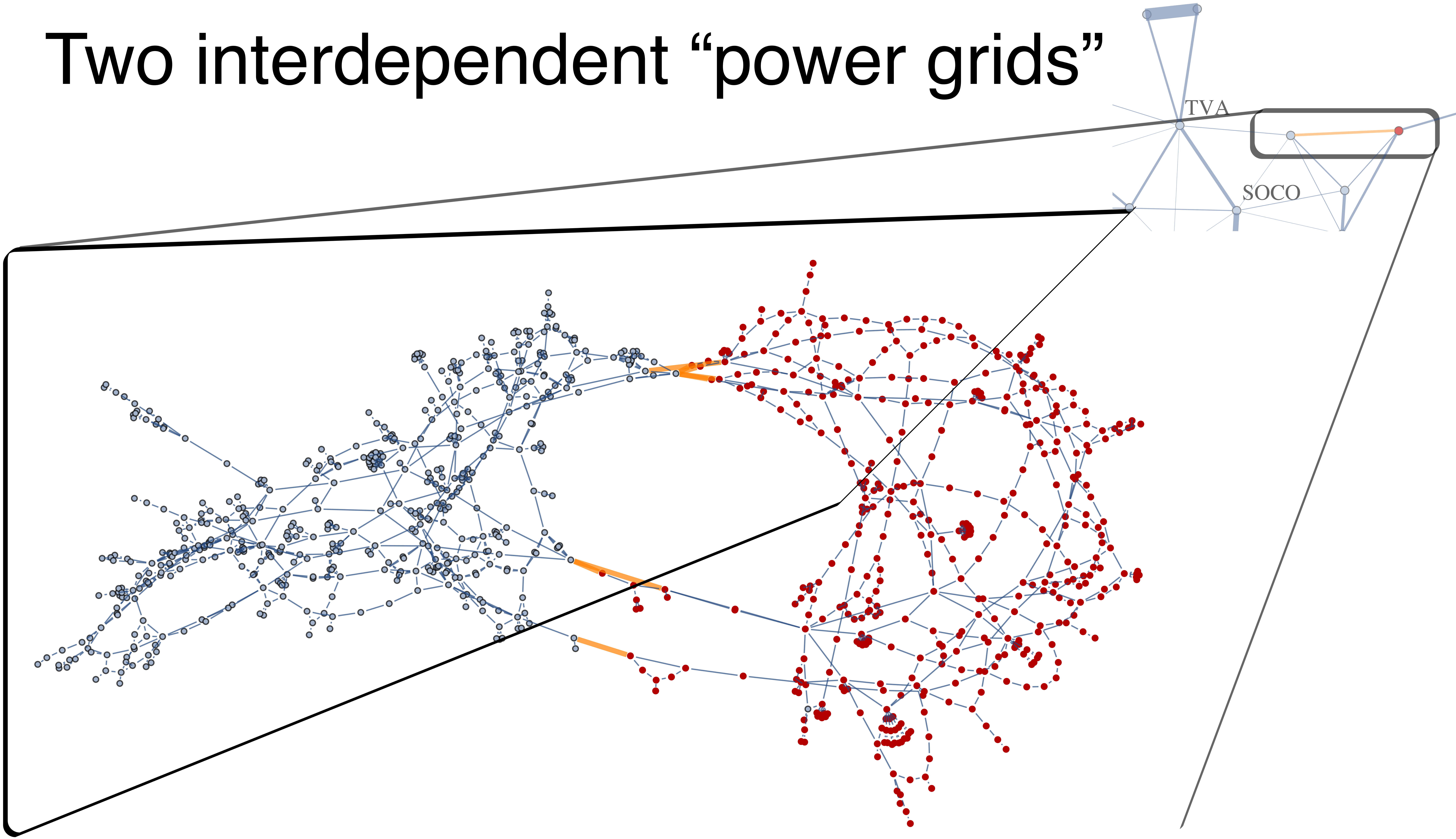
L. Dueñas-Osorio

Power grids: interdependent networks



Nodes: “areas” of the southeastern USA grid
(PJM Regional Transmission Operator
$\subset$ Eastern Interconnection)

Two interdependent “power grids”

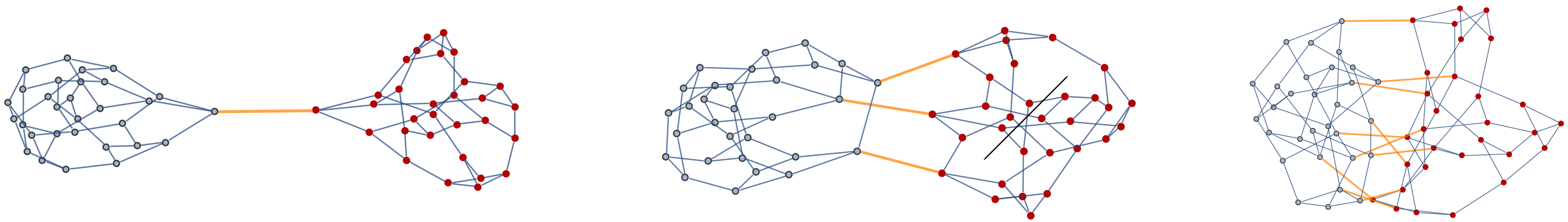


Our approach

today: sandpiles,
DC power flow

Study ≥ 2 “power grids” subject to certain **dynamics**.

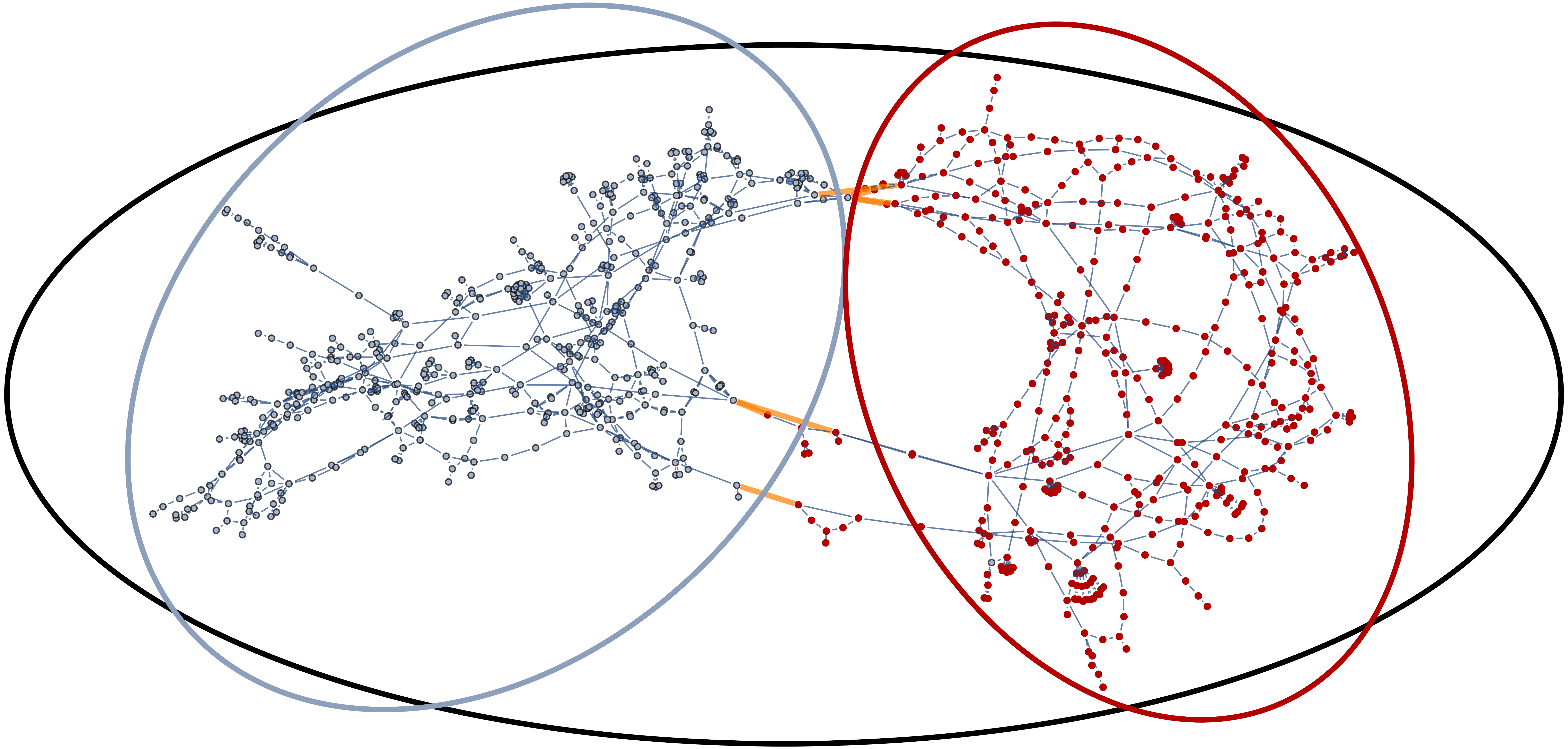
What amount of interdependence



interdependence

most reduces **risk** (e.g., to a large blackout)?

Our approach



Goal 1: reduce risk of the whole system.

Goal 2: each “grid” reduces its own risk.

Dynamics #1: sandpile model

systems possibly driven to "critical points"?

hypotheses

"All models are wrong; some models are useful." – George E. P. Box

sandpile

power grid outage models:

topological networks — flight simulators

infrastructure cascades:

topological networks — flight simulators

...

Dynamics #1: sandpile model

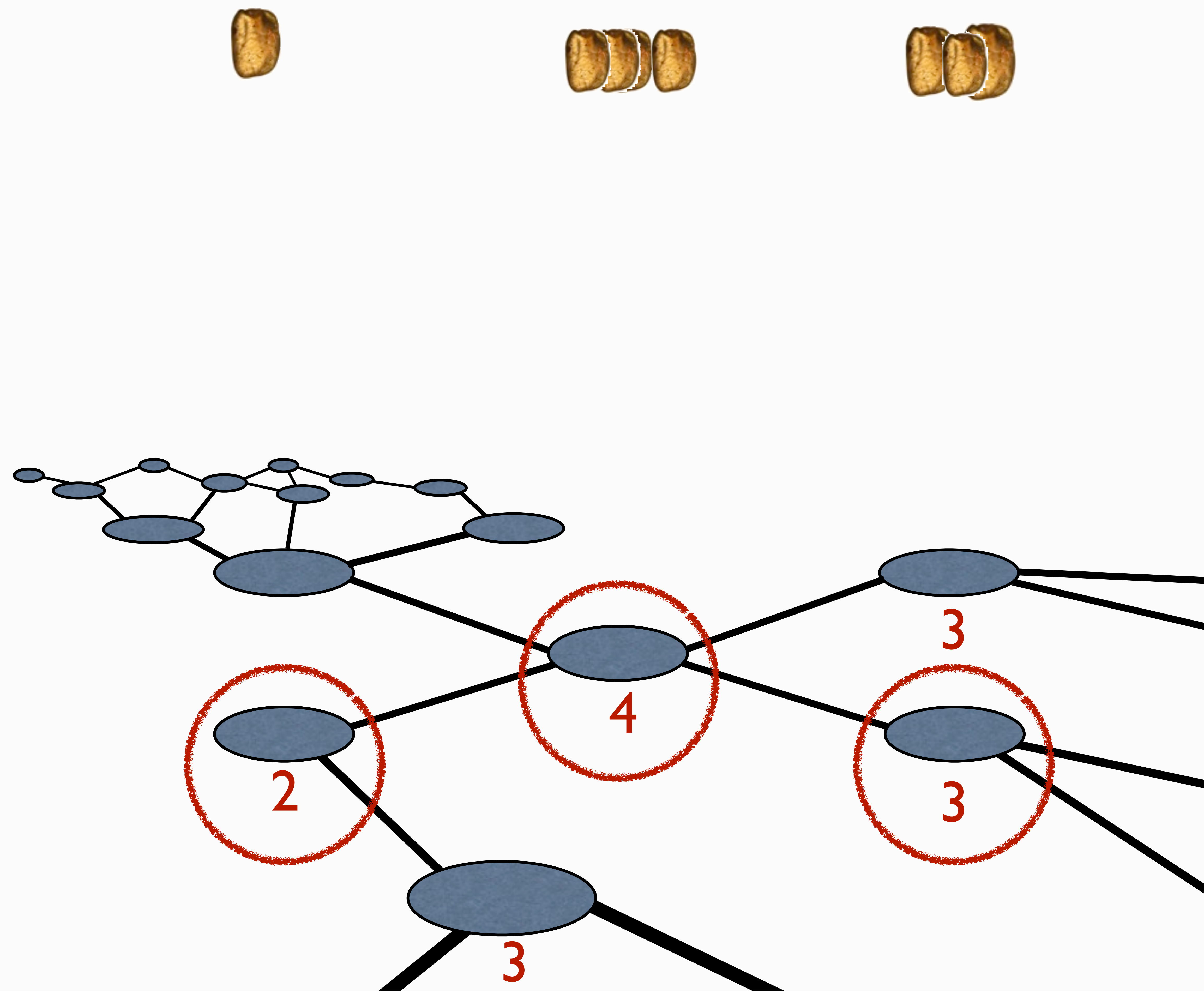
"All models are wrong; some models are useful." – George E. P. Box

- Stylized model of overwhelmed nodes moving load to neighbors
- Result: an intermediate equilibrium in interdependence.

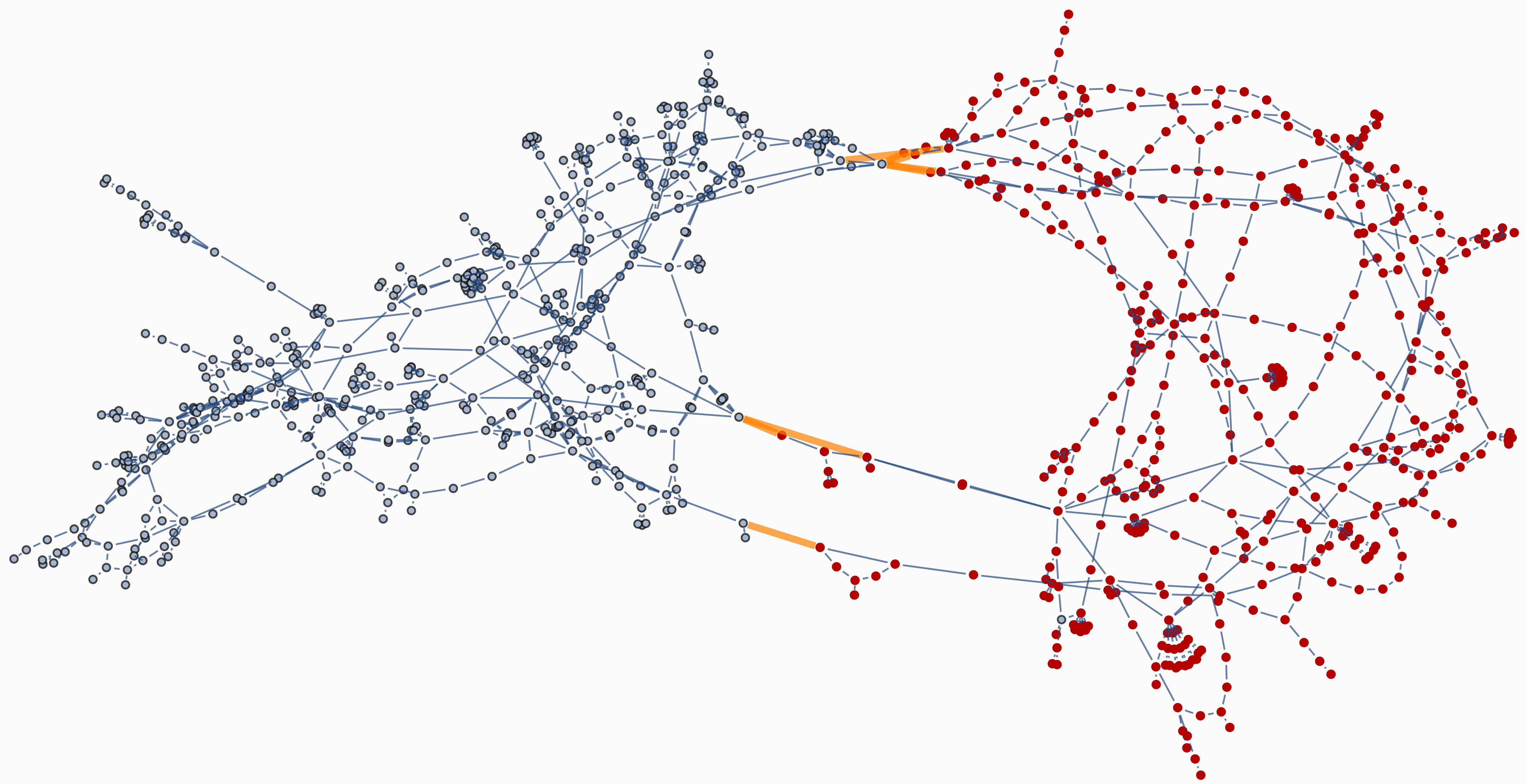
CB, R. D'Souza, E. Leicht. Suppressing cascades of load in interdependent networks. *PNAS* **109** (12), E680–689 (2012).

Sandpile model on networks

- Start with a network
- Drop units of load  randomly on nodes
- Each node has a threshold.
  Here: degree.
- Load on a node ≥ threshold
  ⇒ node topples, moves sand to neighbors
- Neighbors may topple. Etc.
  Cascade (or avalanche) of topplings.
- Delete sand with probability $\varepsilon \ll 1$.



Interdependent power grids



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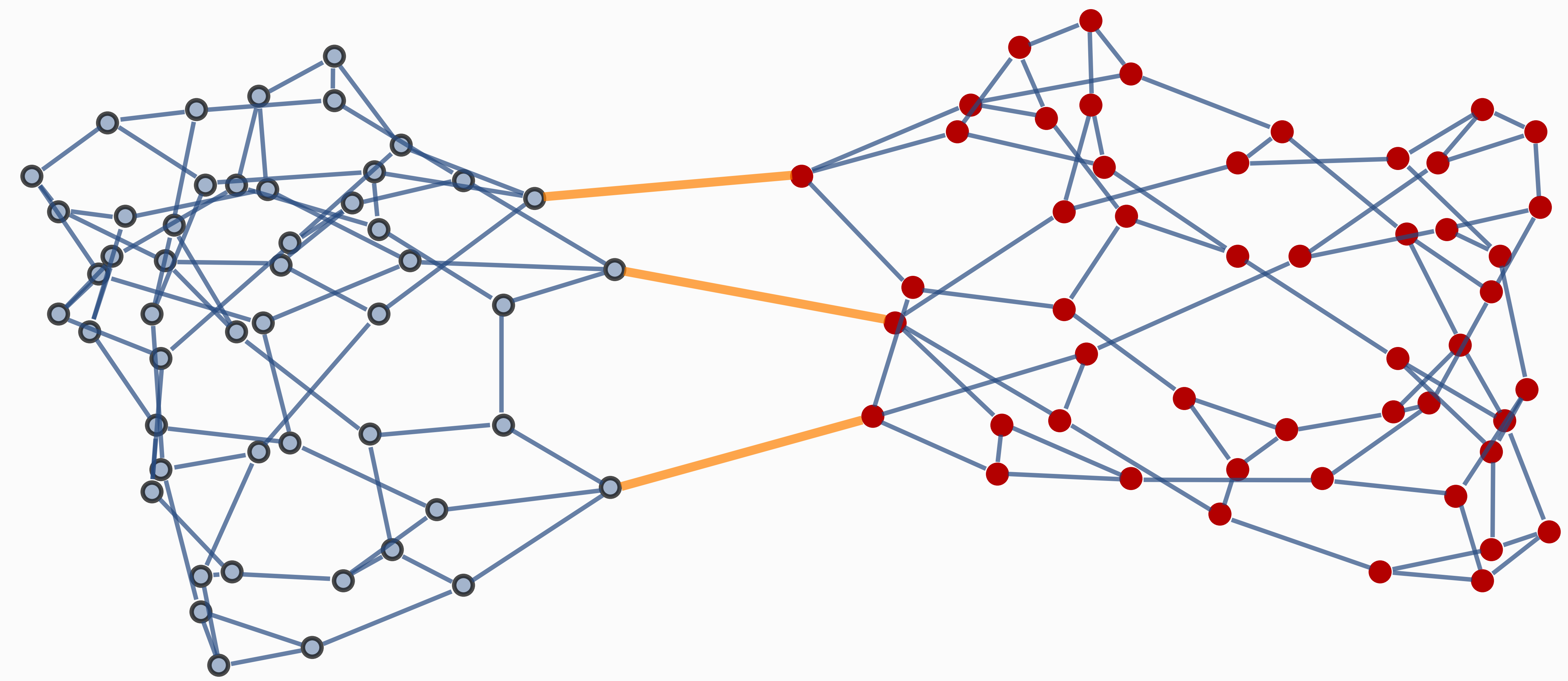


Power grids
(2 areas in SE USA)

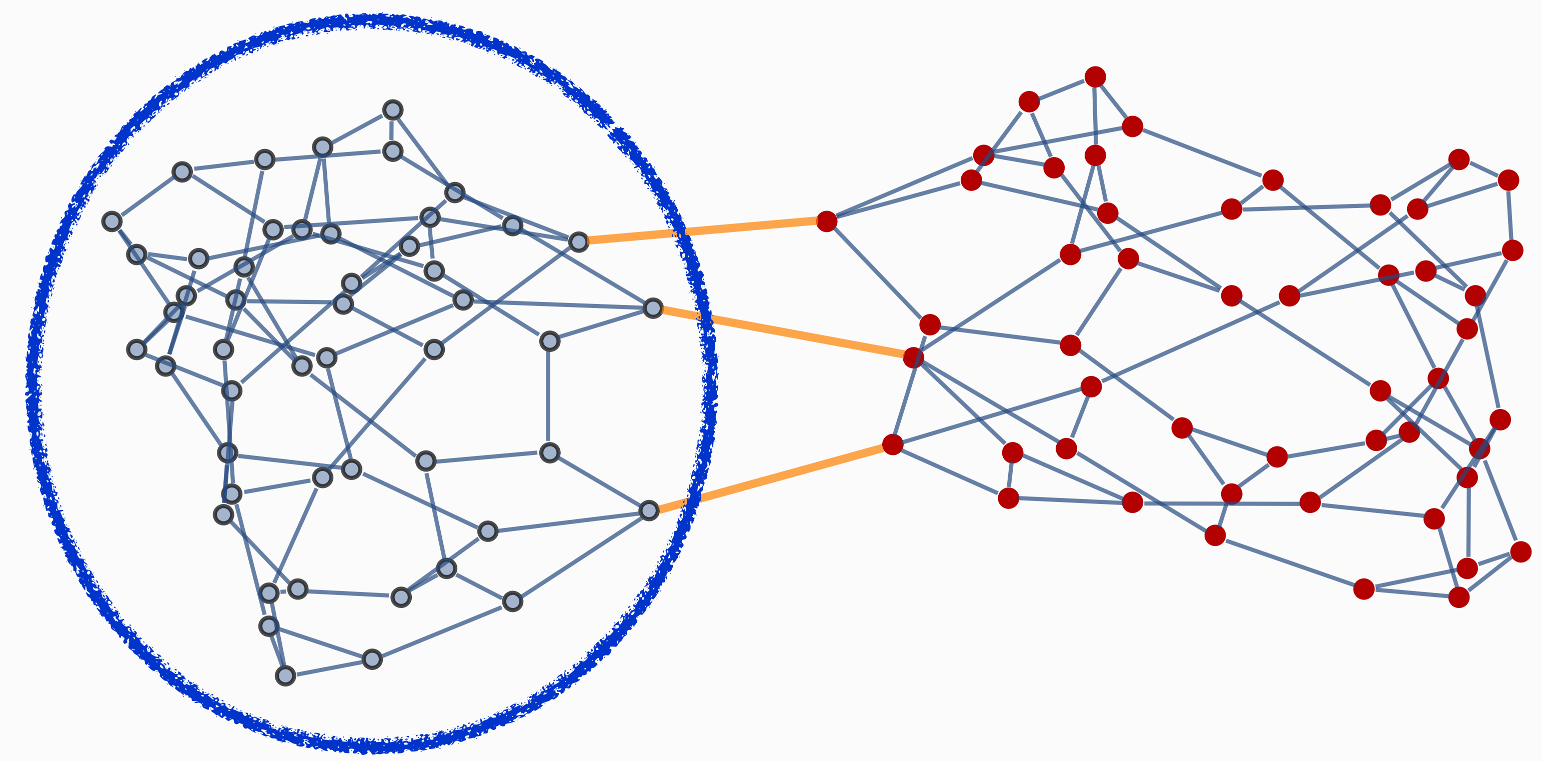
Random 3-regular graphs

Equilibrium in interdependence

Suppose we own the power grid network *a*.

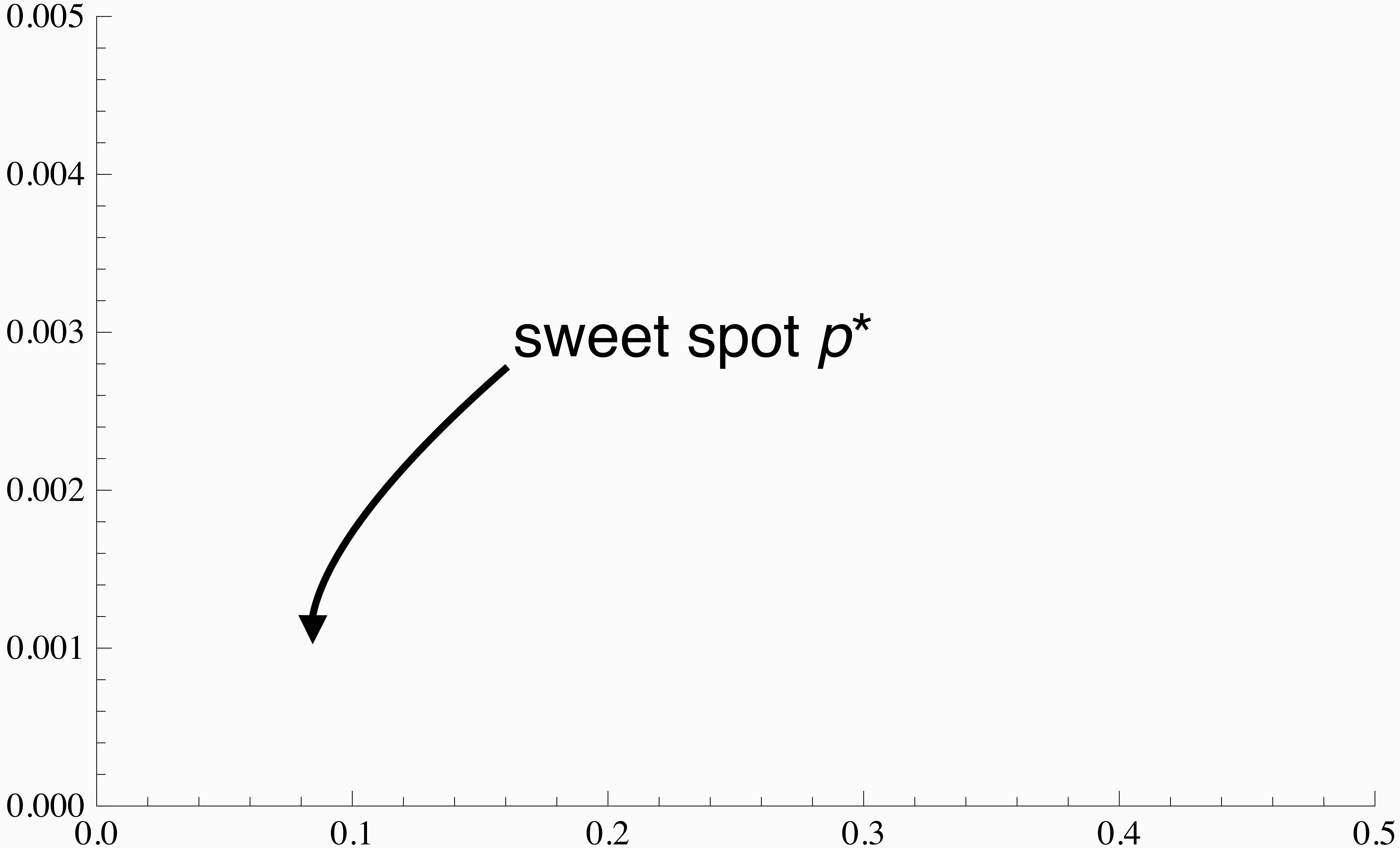
Want to mitigate large cascades in our grid.

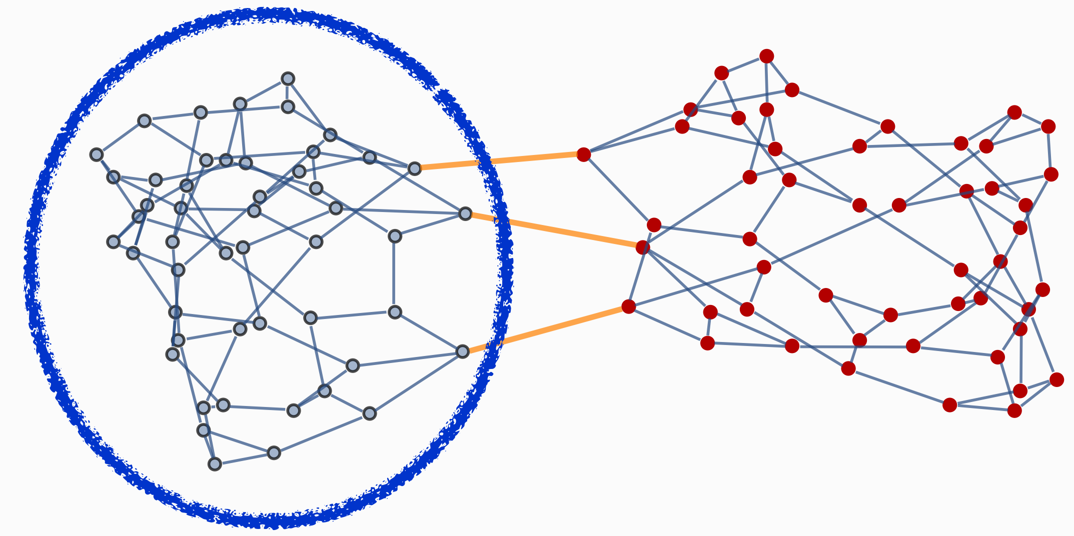
topple ≥ half network *a*



Equilibrium in interdependence

Risk to the blue network
=
ℙ(cascade topples ≥ 1/2 of blue network)

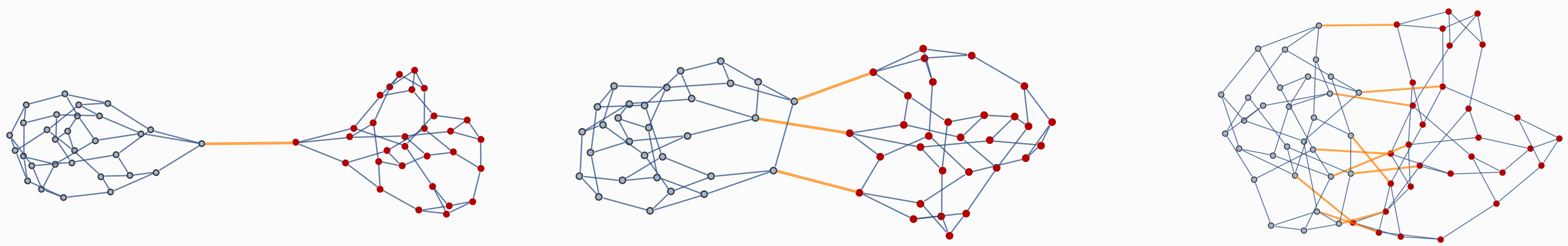




2500 large cascades

800 large cascades

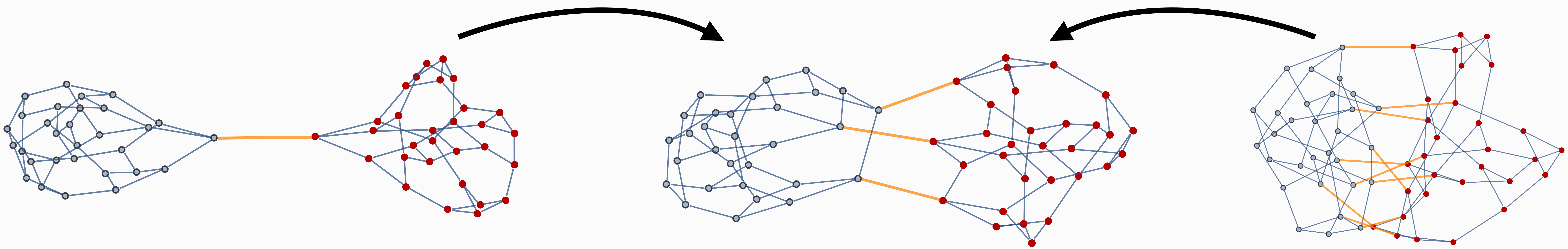
Interdependence
=
fraction p of nodes with an interconnection



(simulations on random 3-regular graphs with 2×10^3 nodes each)

Equilibrium in interdependence

To mitigate large cascades, build some interconnections, but not too many.



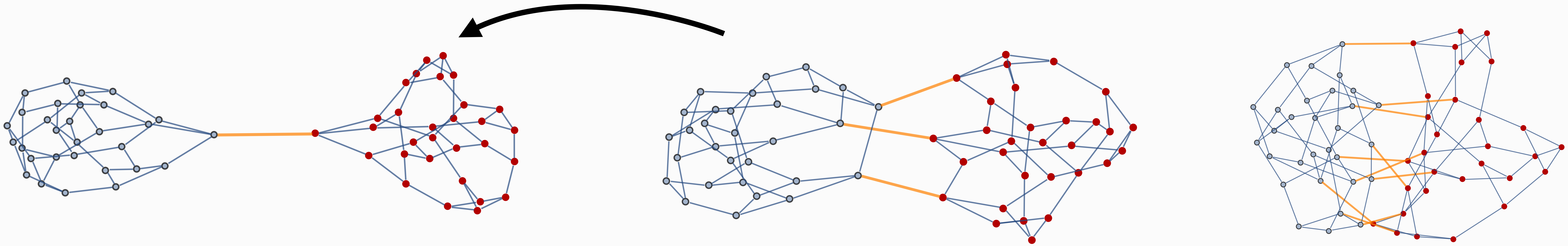
Contrasts with

Buldyrev et al. “Catastrophic cascade of failures in interdependent networks” *Nature* 2010.

and ≥ 10 followup papers.

Mitigate small cascades

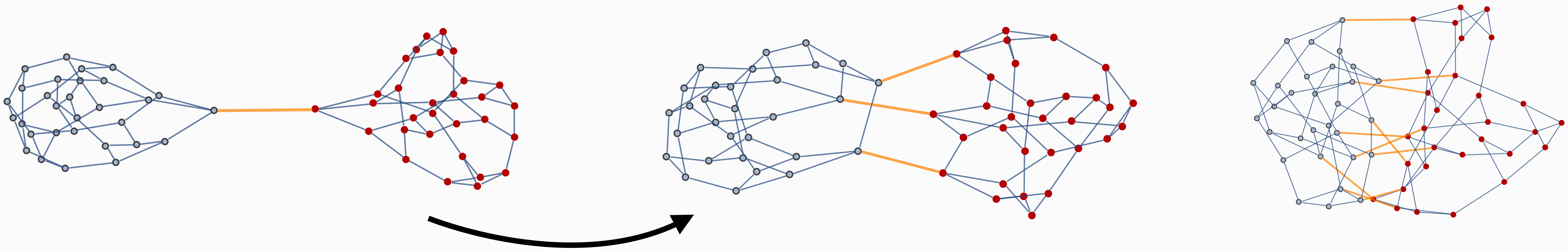
To mitigate small cascades, seek isolation.



But isolation increases risk of
large cascades.

Similar to forest fires, blackouts.

Risk of the whole system



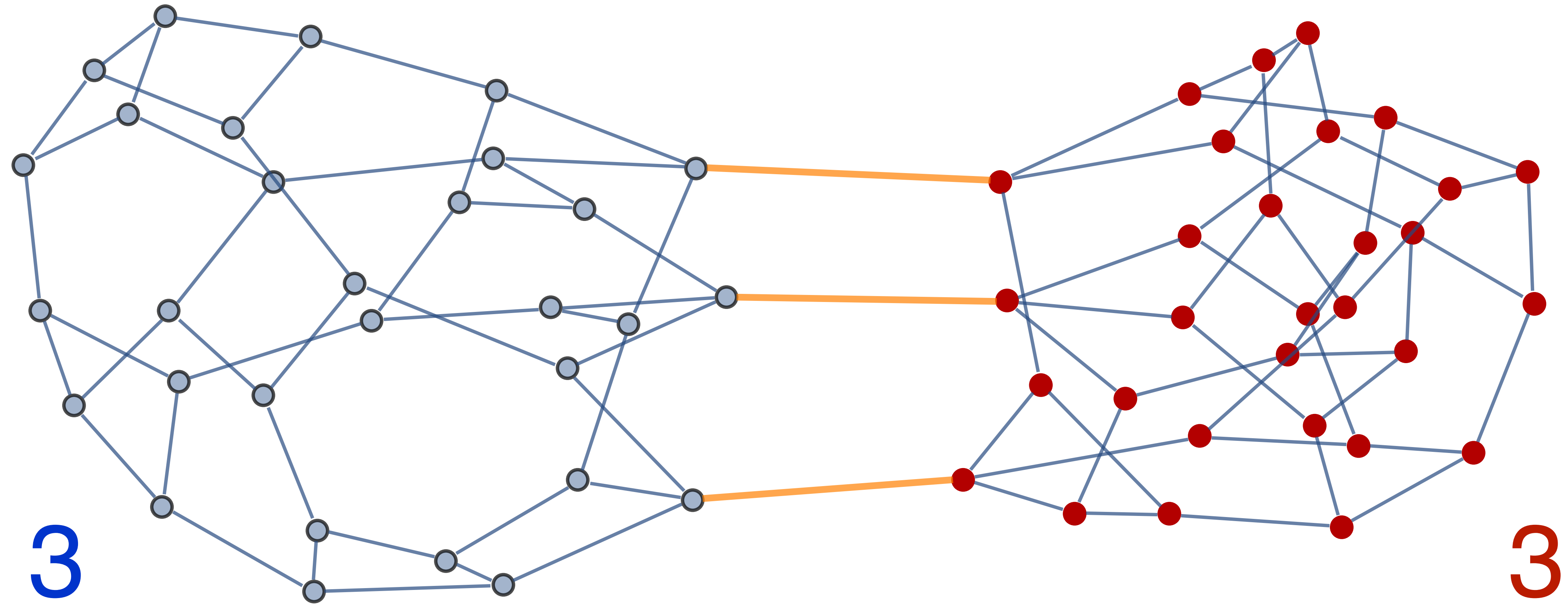
More interconnections

→ More capacity

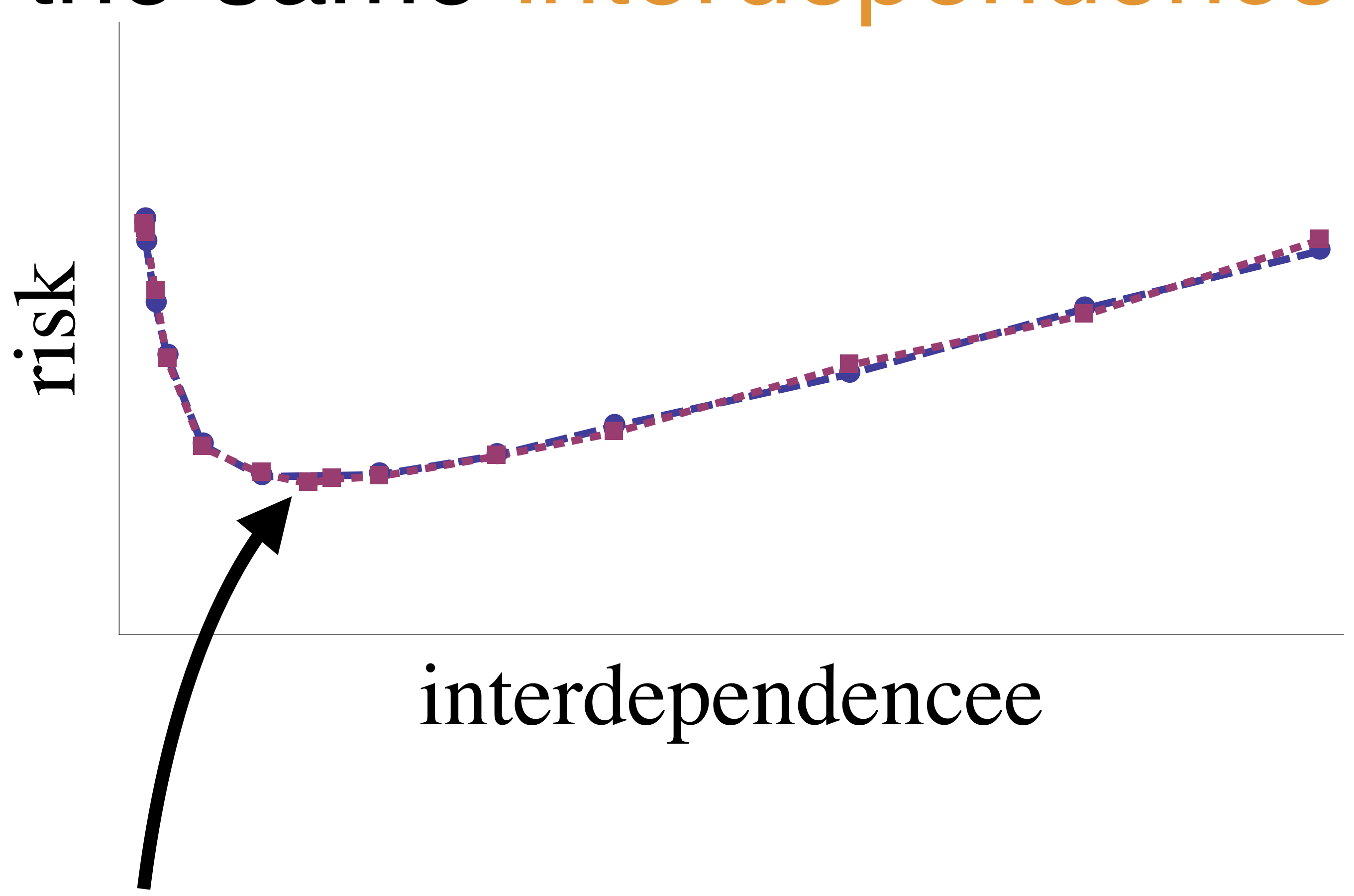
→ More load on average

→ Largest cascades in whole system become larger

Networks with the same total capacity

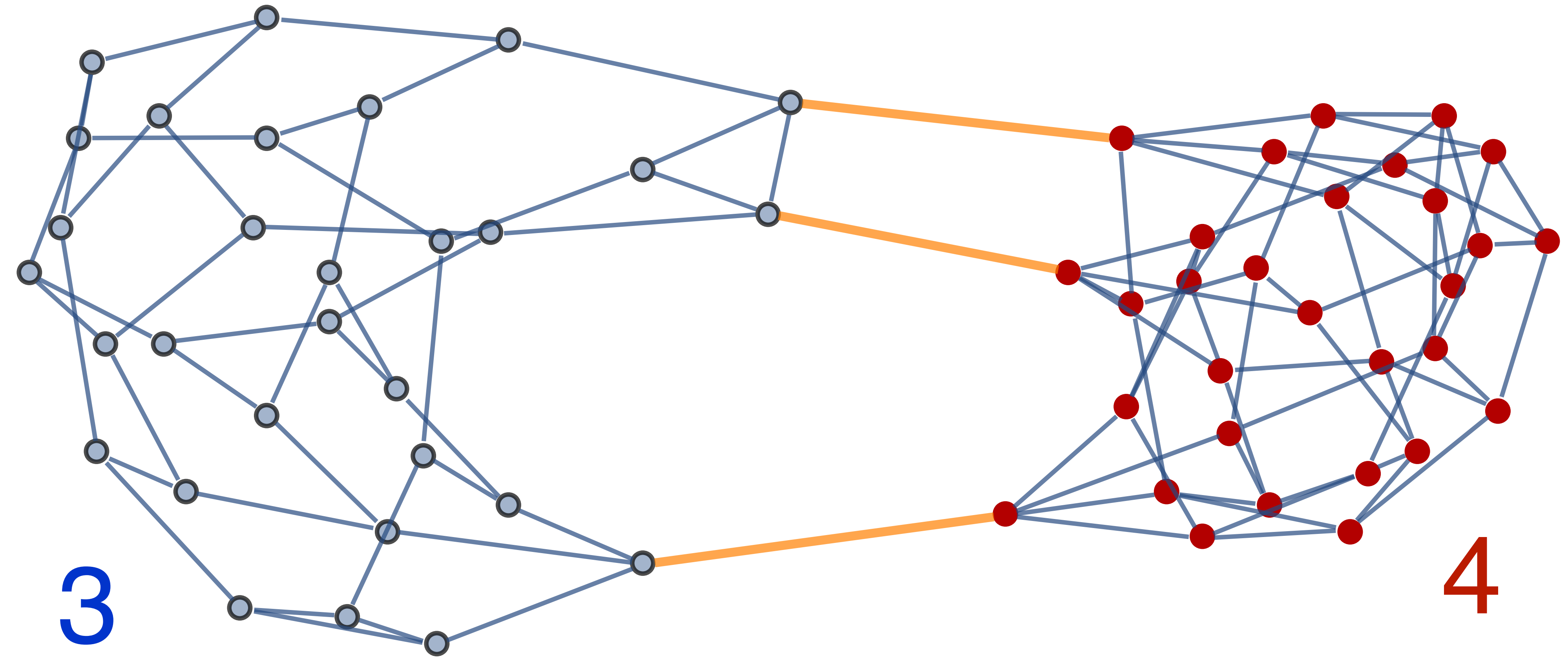


prefer the same interdependence *p**

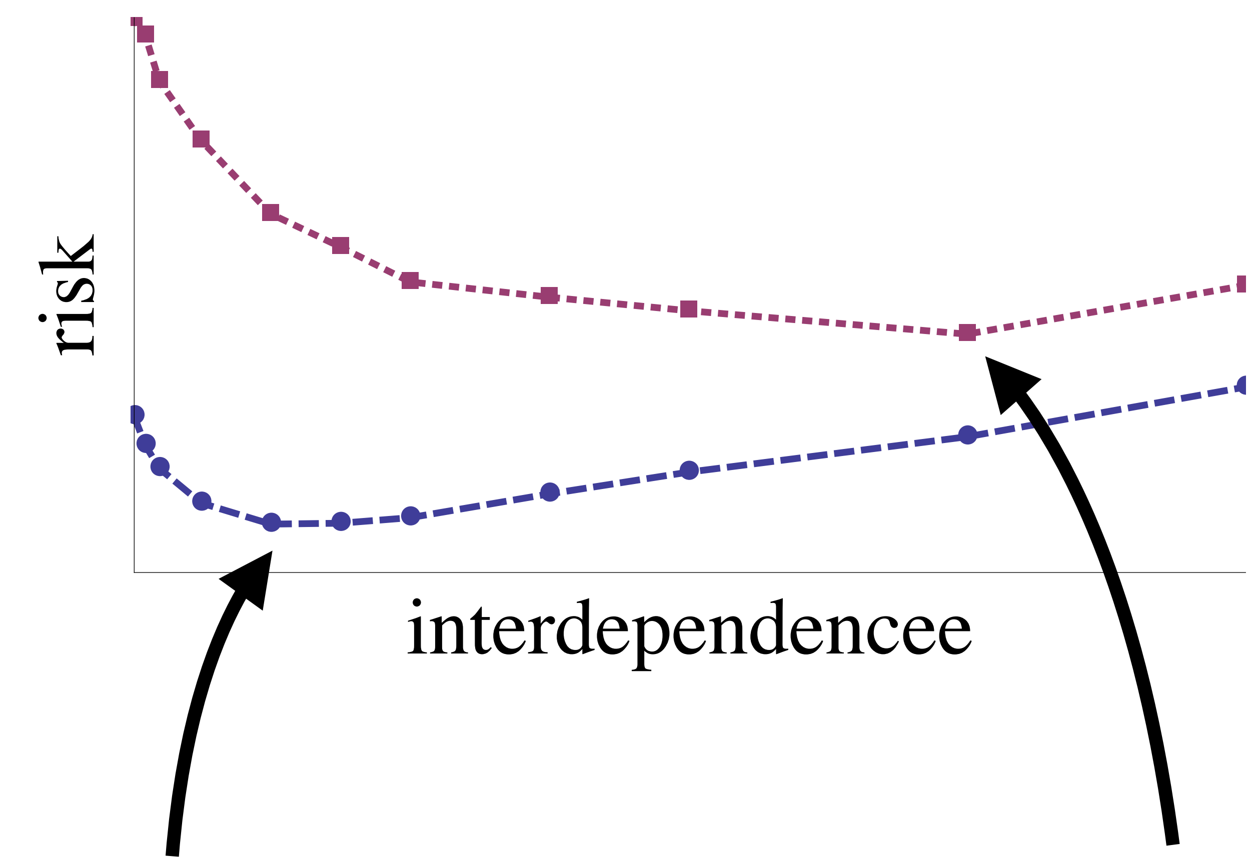


Stable equilibrium

Networks with **different** total capacity



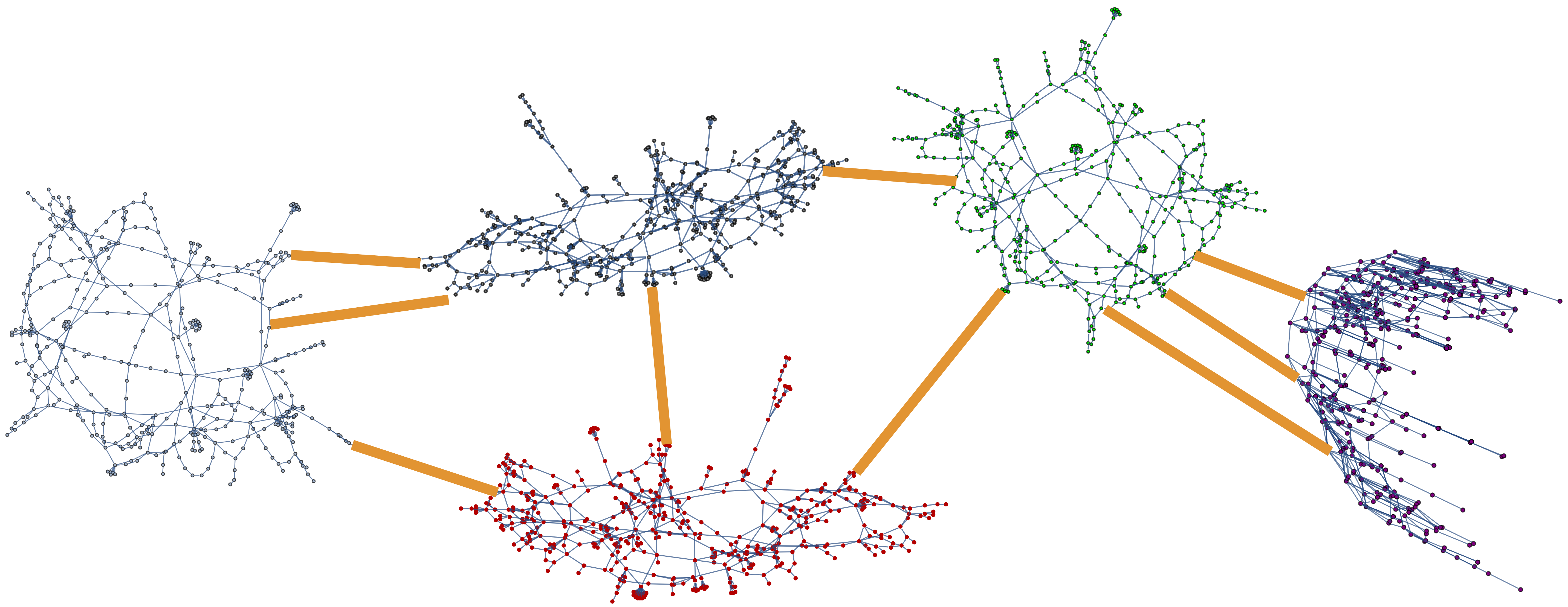
prefer different interdependence *p**



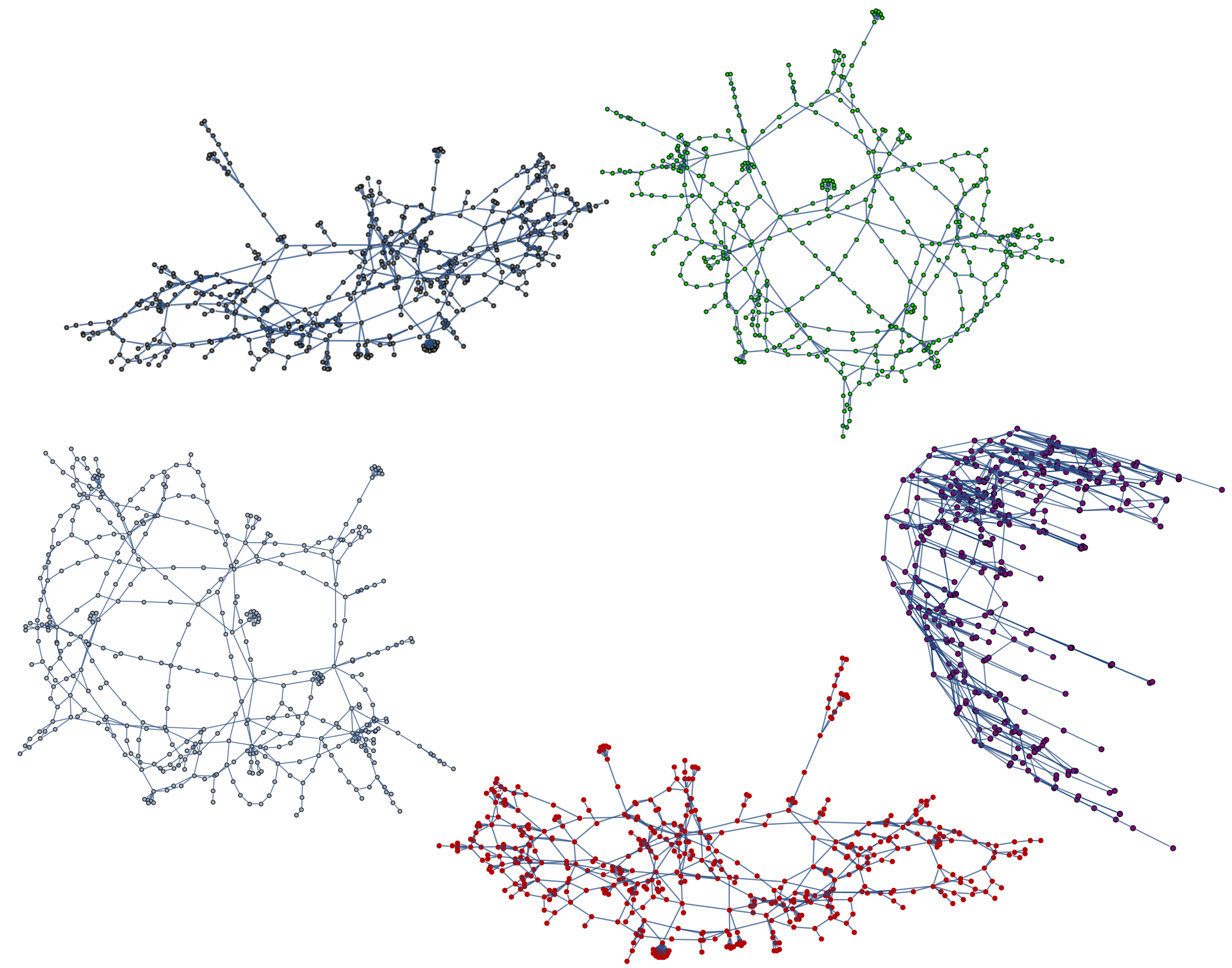
Frustrated equilibria

Sandpile model: complete picture

If all we want to do is mitigate risk of large cascades...

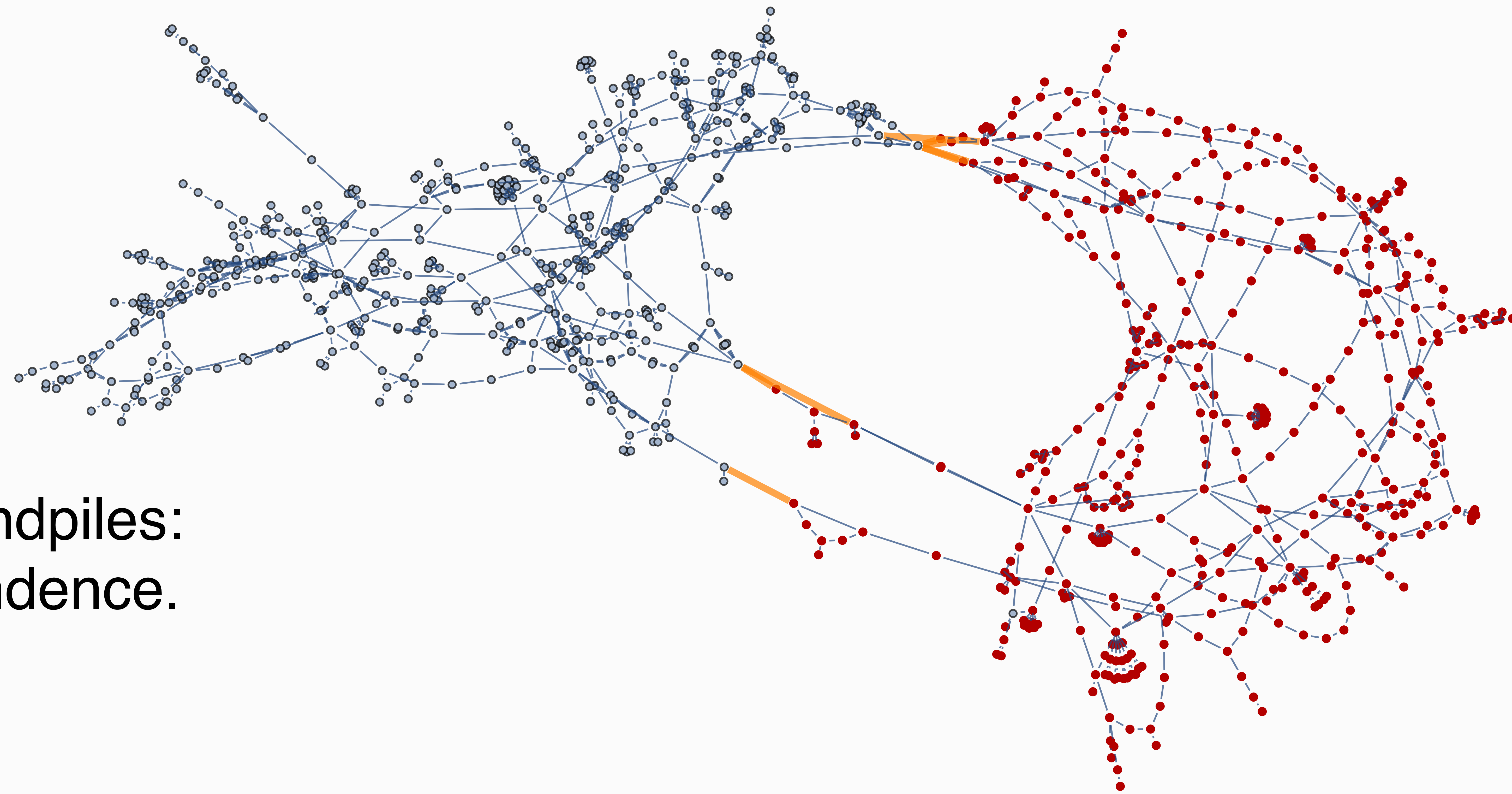


What grids want



What society wants

Dynamics #2: DC power flow



Interesting feature of sandpiles:
equilibrium in interdependence.

Also in power grids?

If so, can we calculate it?

Collaboration with
Anna Scaglione,
Zhifang Wang
(UC Davis)

DC power flow model

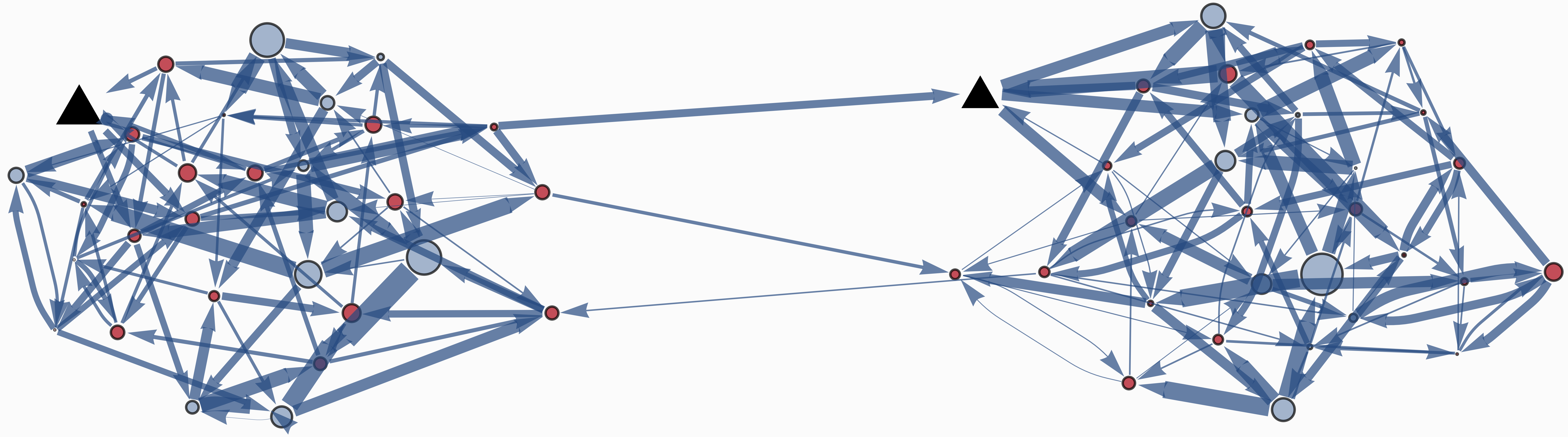
Linearization of nonlinear AC equations

- Flow problem
- Impedances on lines x_{uv}
- Power injection $\mathbf{P} \in \mathbb{R}^n$
  $p_u > 0$ generator, < 0 load
- Solve $\mathbf{P} = L\,\boldsymbol{\Theta}$ for phase angles $\boldsymbol{\Theta}$,
  where L = weighted Laplacian.
- Power flows $\boldsymbol{f} = C\,A\,\boldsymbol{\Theta}$.

D. L. Pepyne, *Topology and cascading line outages in power grids*, Journal of Systems Science and Systems Engineering, **16** (2007), pp. 202–221.

M. Li, Q. Zhao, and P. Luh, *DC power flow in systems with dynamic topology*, Power and Energy Society General Meeting-Conversion and Delivery of Electrical Energy in the 21st Century, IEEE, (2008), pp. 1–8.

Cascading line outages on random 3-regular graphs



- Fraction p of nodes have neighbor in other network.
- One or both networks are close to tripping: rescale **P** s.t. $\|\boldsymbol{f}\|_\infty = .99 \times$ trip point.
- Trip a randomly chosen line.

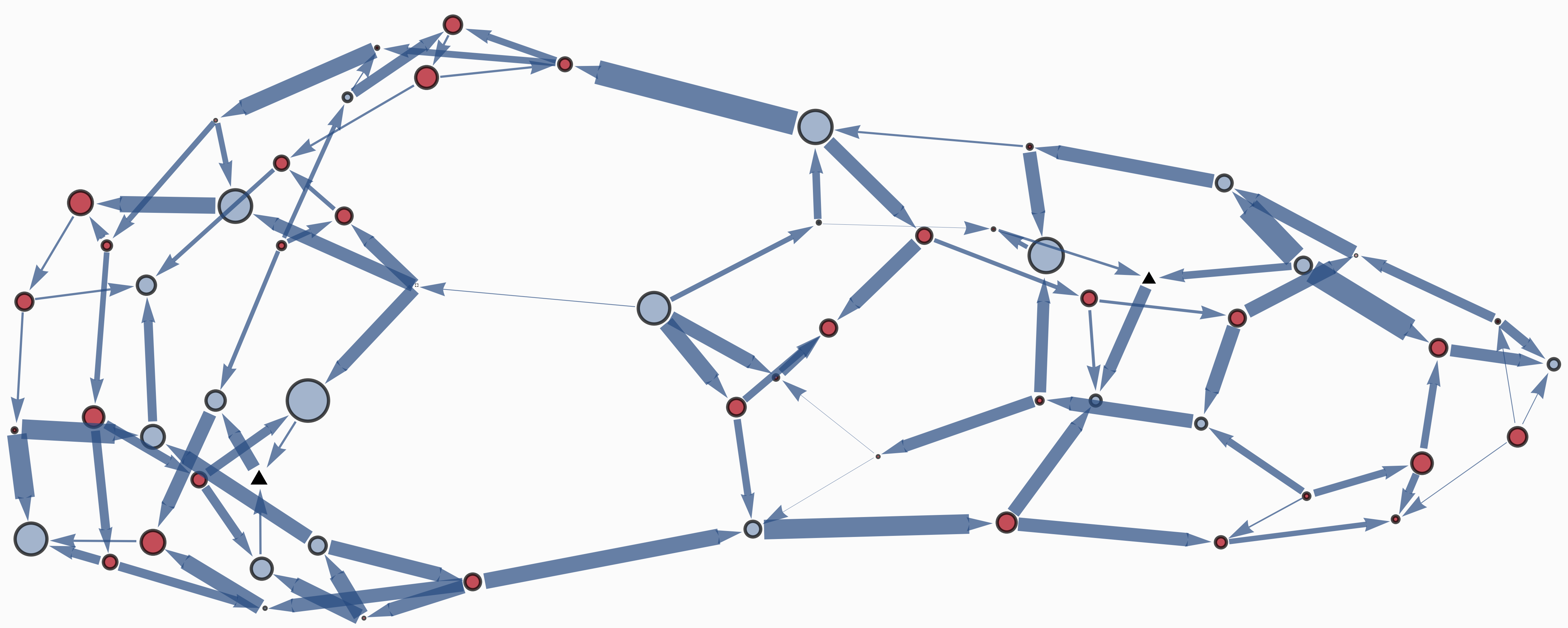
  Repeatedly recompute power flows $\boldsymbol{f}$ and trip lines above their trip point.

D. L. Pepyne, *Topology and cascading line outages in power grids*, Journal of Systems Science and Systems Engineering, **16** (2007), pp. 202–221.

Cascading line outages on random 3-regular graphs

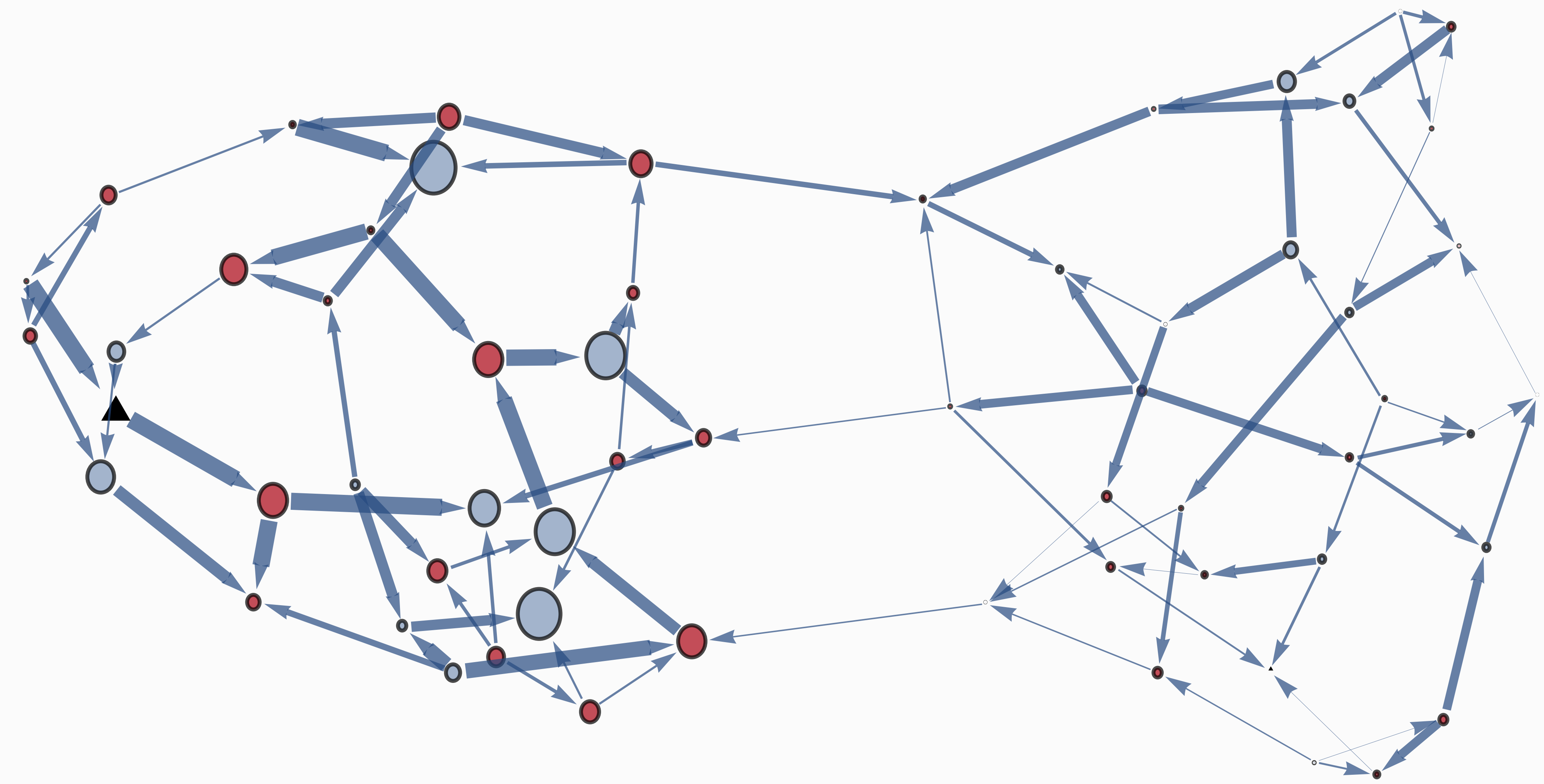
No intermediate equilibrium in interdependence

1. When both grids are close to tripping...



"less interdependent!" "less interdependent!"

2. When just one grid is close to tripping...



"more interdependent!" "less interdependent!"

Next steps

Sandpile	local	nodes fail temporarily	identical nodes	nonlinear	*tradeoffs*
DC power flow	nonlocal	edges fail permanently	sources, sinks	linear	*monotonic*

More realistic dynamic for power generation?

Linear algebra (Pepyne 2007)	**Optimization** (Carreras et al. 2002)
Solve $\mathbf{P} = L\,\boldsymbol{\Theta}$ for phase angles $\boldsymbol{\Theta}$.	Minimize *Cost* = (power generated) – constant × (power shed) subject to constraints.

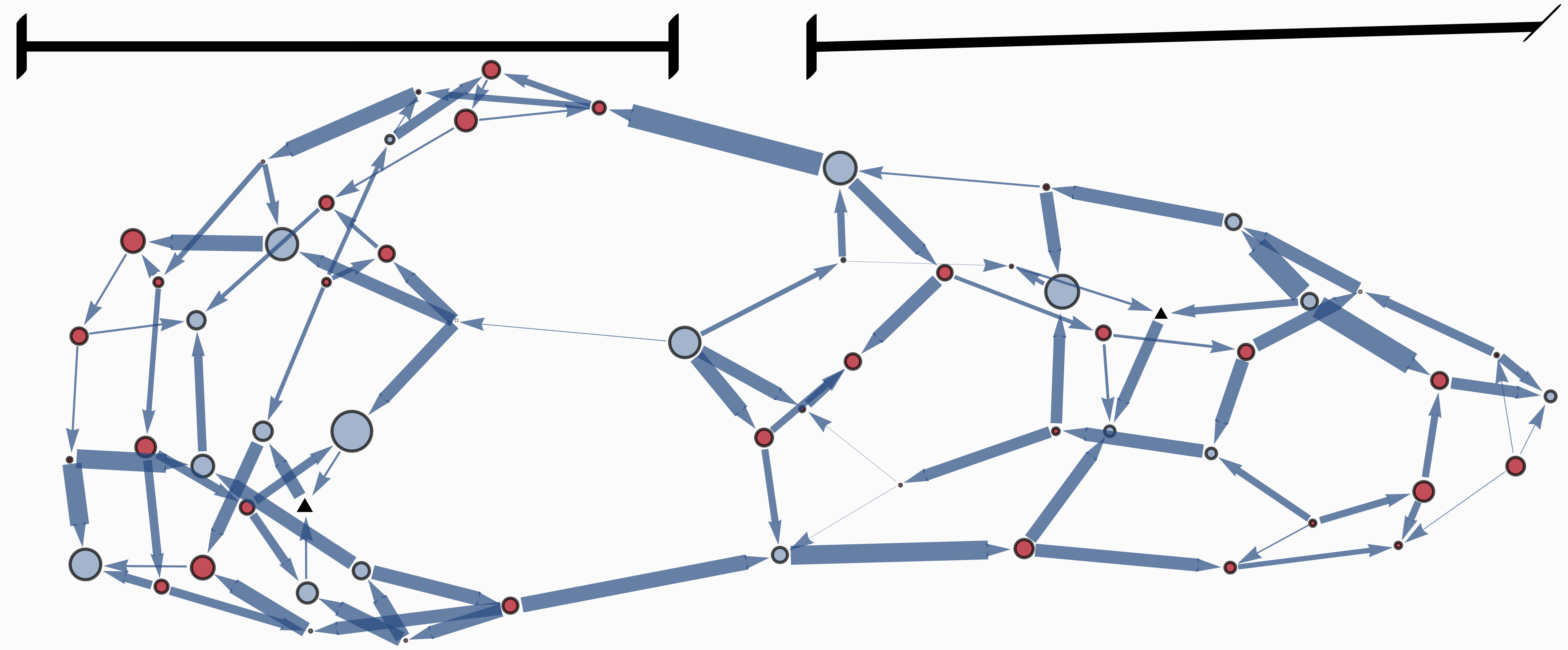
D. L. Pepyne, *Topology and cascading line outages in power grids*, Journal of Systems Science and Systems Engineering, **16** (2007), pp. 202–221.

Carreras, B. A., Lynch, V. E., Dobson, I. & Newman, D. E. Critical points and transitions in an electric power transmission model for cascading failure blackouts. *Chaos: An Interdisciplinary Journal of Nonlinear Science* **12**, 985 (2002).

Linear algebra (Pepyne 2007)	**Optimization** (Carreras et al. 2002)
Solve **P** = L **Θ** for phase angles **Θ**.	Minimize *Cost* = (power generated) – constant × (power shed) subject to constraints.

Optimize cost here...

and here?

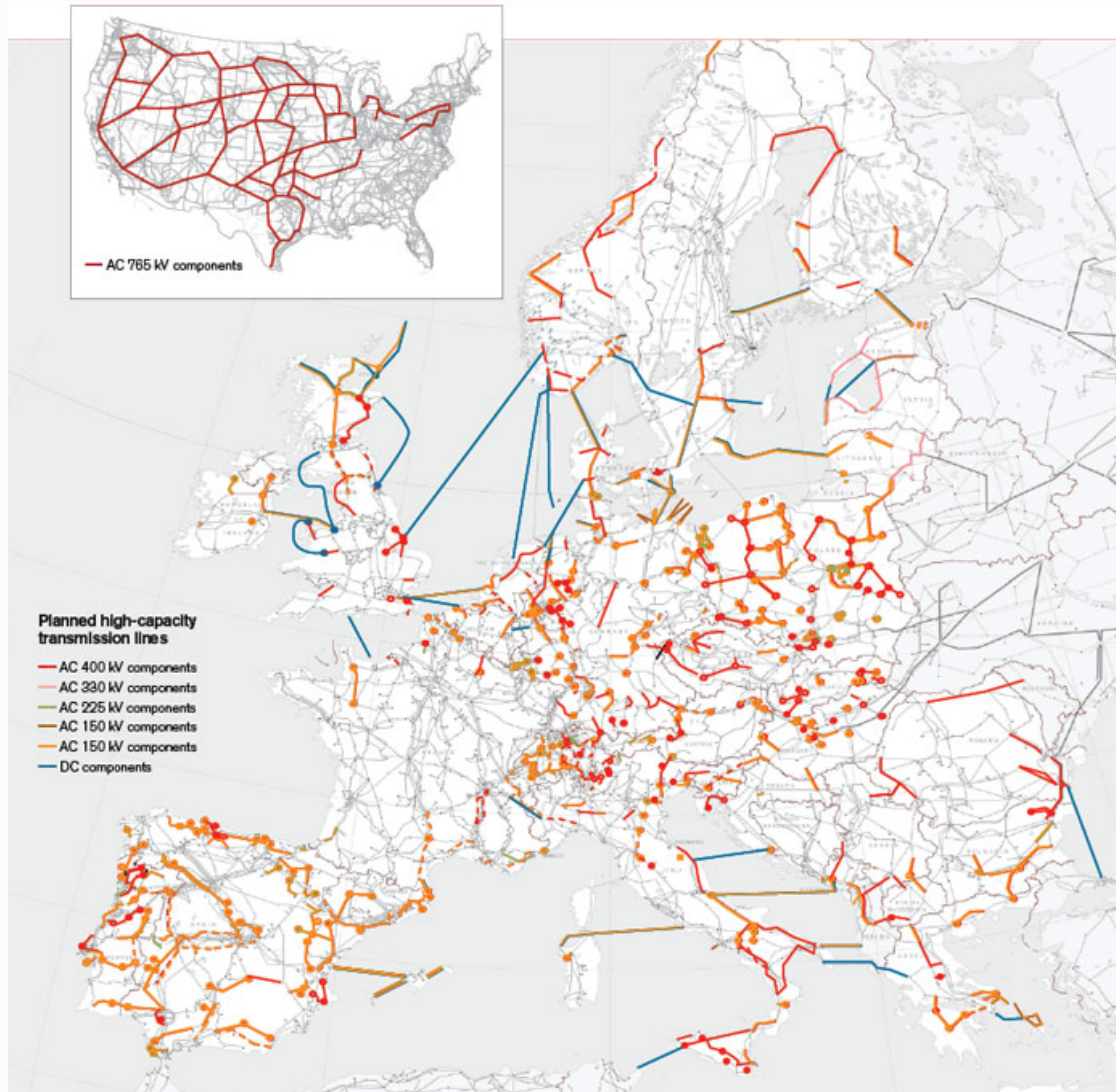


Or does the ISO optimize for all grids in the region?

Modeling questions

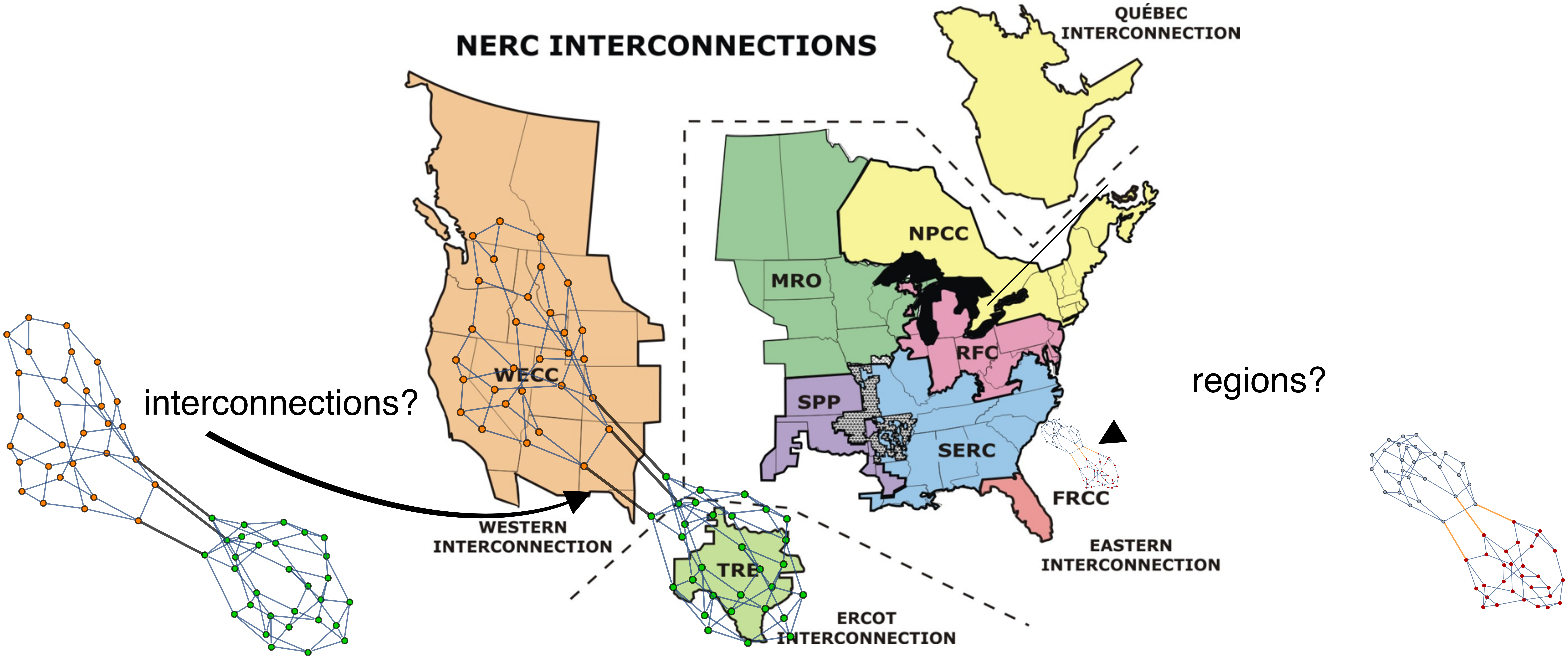
- Need <u>engineering practices</u> in our model
- Who optimizes what?

  – ISOs mitigate their region’s risk?
  – Countries mitigate their own risk?
- The scale of each “network”?
  utilities ⊂ regions ⊂ ISOs ⊂ countries
- Which dynamical models?
  DC, AC, OPA, hidden failures, ...
- Timescales:
  A bad day for the power grid?
  Its evolution over decades?



Technology Review, “Joining the Dots” Jan. 2011.

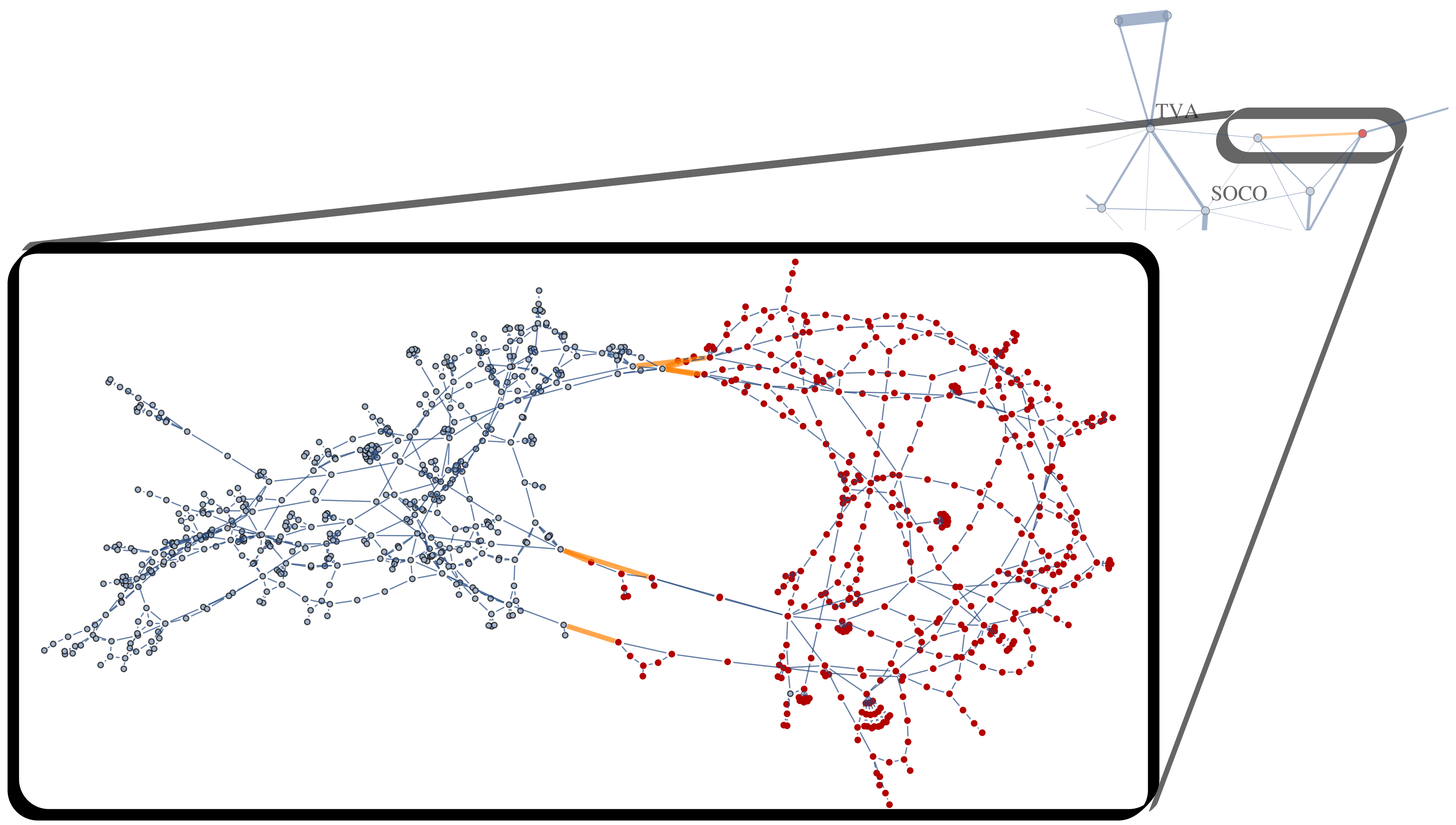
Scale of the interdependent grids

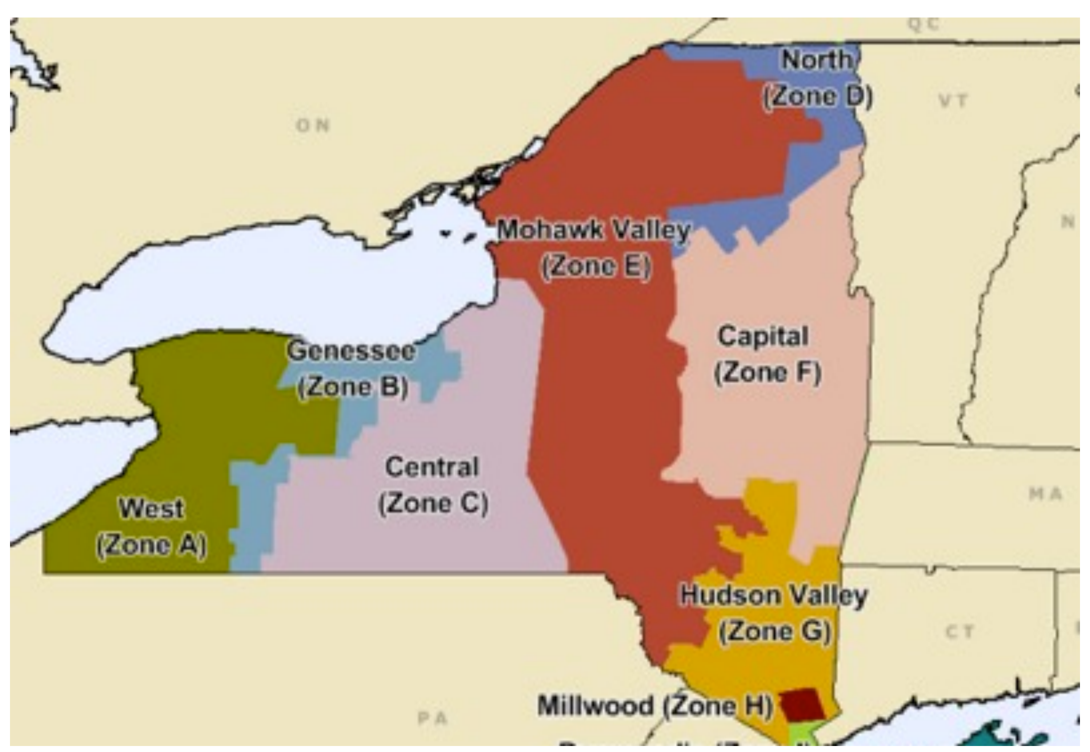


Scale of the interdependent grids

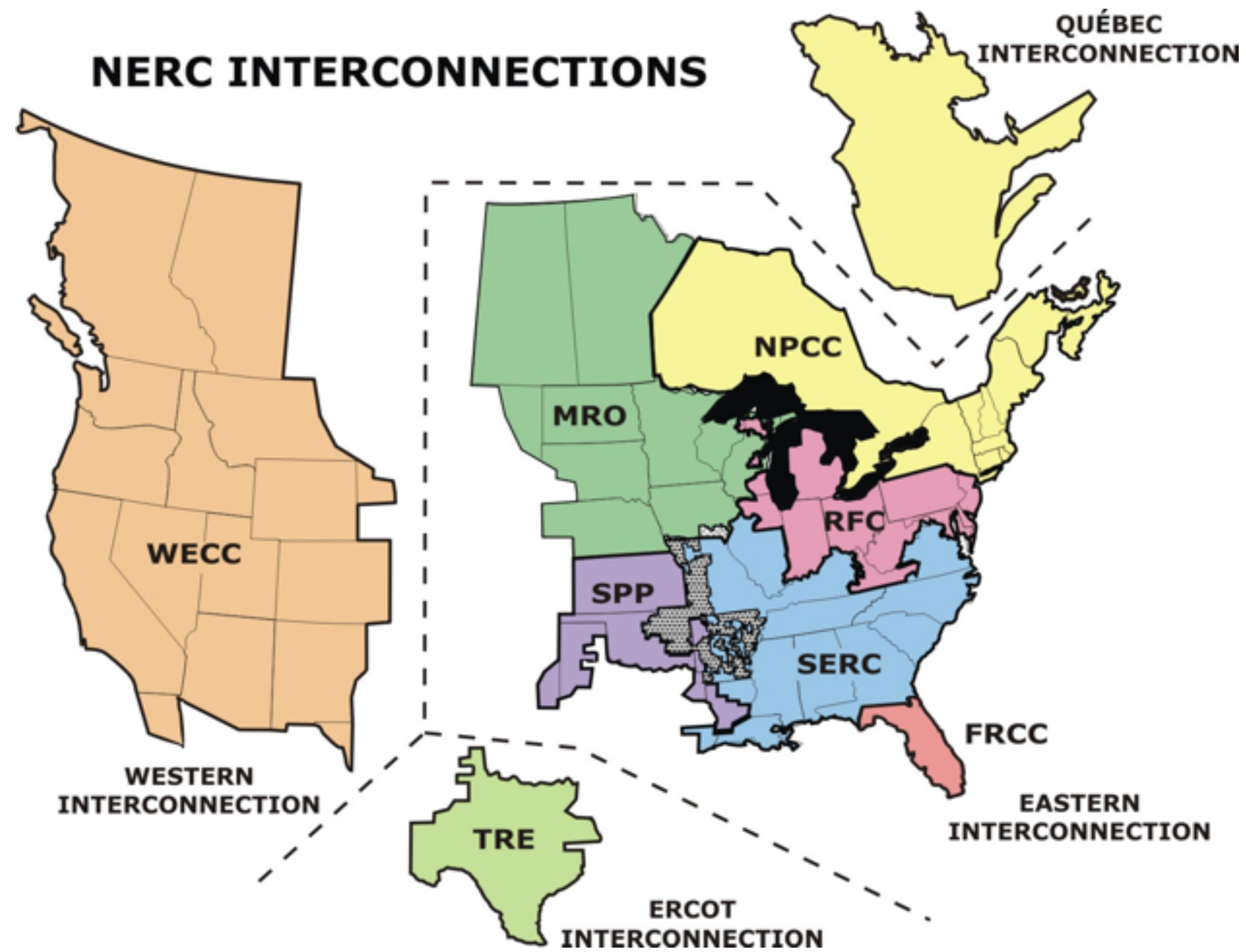
determines the dynamics, timescales, who optimizes what

areas, utilities, zones





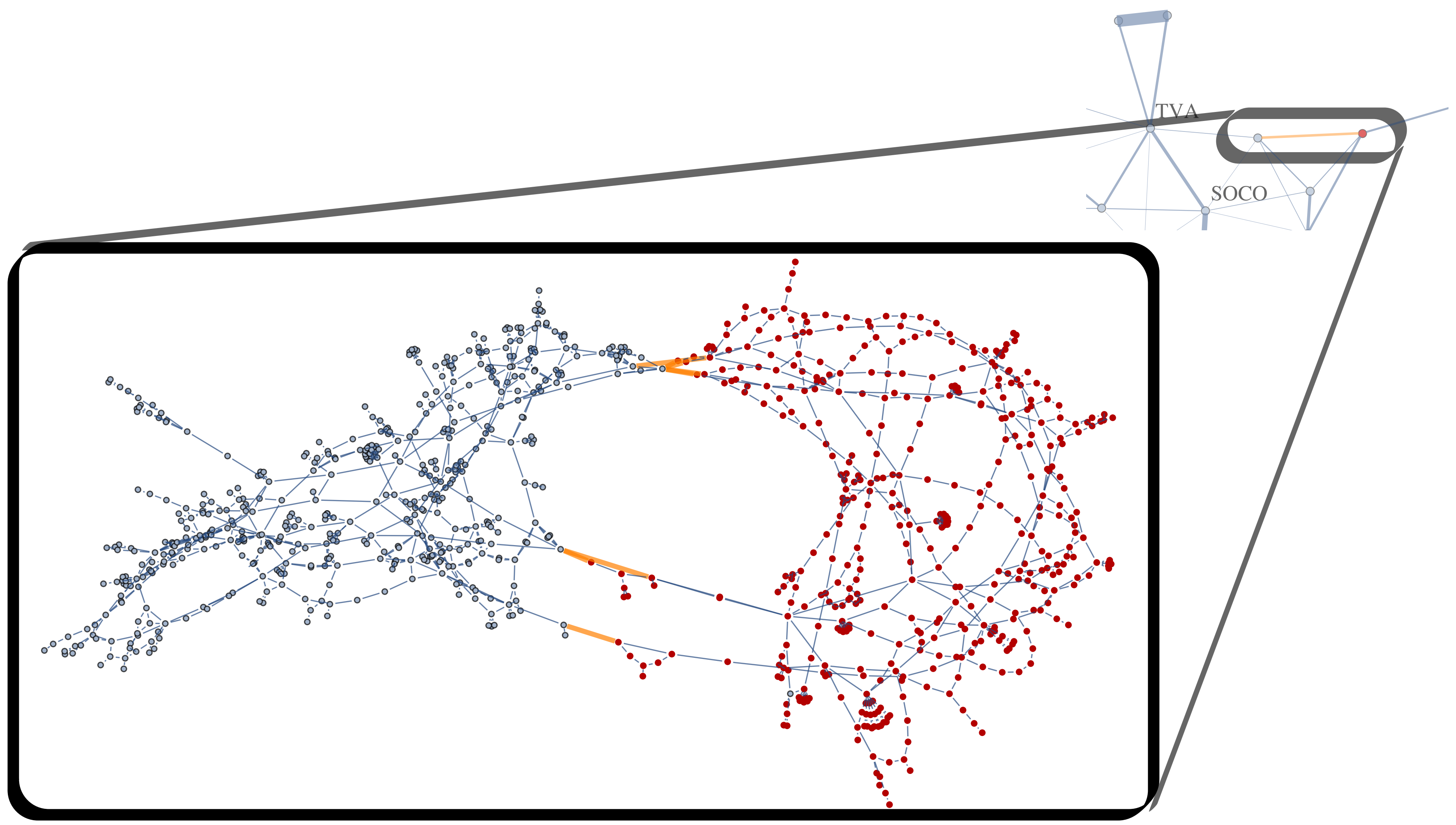
regions, ISOs



Scale of the interdependent grids

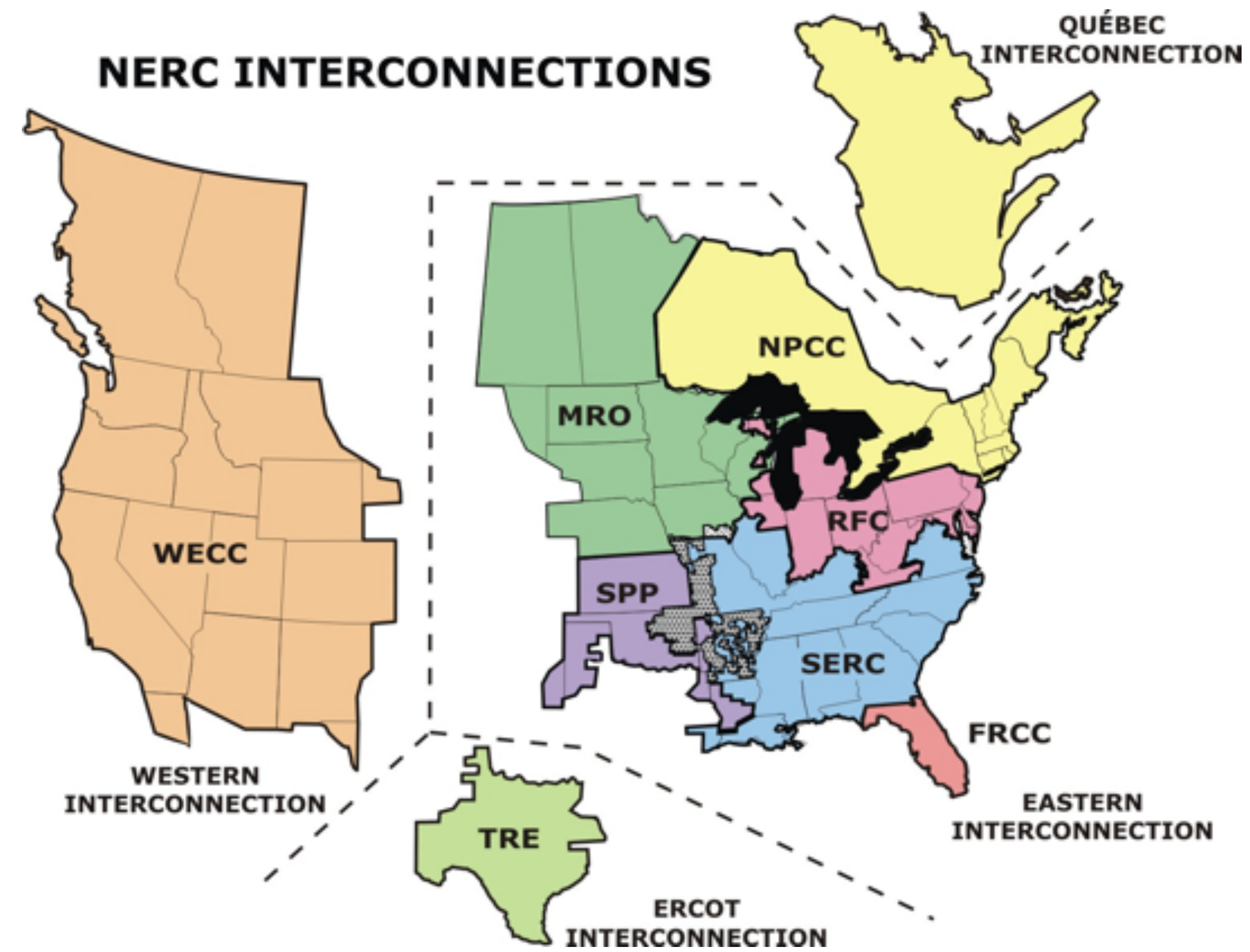
determines the dynamics, timescales, who optimizes what

areas, utilities, zones



who optimizes what?
what's different about lines between grids?

regions, ISOs



who optimizes what?
sync at AC/DC links?

Questions

– Scales?
– Who optimizes what?
– What's different about connections between grids?

Sandpile paper:
C. Brummitt, R. D'Souza, E. Leicht. Suppressing cascades of load in interdependent networks. *PNAS* **109** (12), E680–689 (2012).

Thanks!

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