Optimal interdependence among power grids

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Vulnerability of this power grid?
Vulnerability of this power grid?

- Study robustness to cascading line outages hidden failures, etc.
  \[ P(\text{blackout of size } x) \]
- But what about those transmission lines to other grids?
How interdependent should grids be?

If regional grids were isolated: largest blackout $\leq \max\{\text{grid 1}, \text{grid 2}, \text{grid 3}, \text{grid 4}, \text{grid 5}\}$
Connections among grids enable blackouts

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We’re building lines among grids
We’re building lines among grids

Tradeoffs of interdependence:
- long distance trade, shares risk
- large cascading failures

What interdependence balances these tradeoffs?
How interdependent should X be?

Financial networks

Economies

Infrastructures

Figure 11. Results of the modified crisis spreading model with CAB. Displayed are (A) the cumulative counts of avalanche sizes, (B) the avalanche profile of countries with the ten largest avalanche sizes, and (C) the avalanche network of the modified model. One may note that similarity of the overall results to those of the original model (Figs. 2, 6, and 8B, respectively), despite some quantitative changes in the numeric values. In the modified model, we recover the power-law-like distribution at \( f/t < 7 \); the indirect avalanches constitute the dominant part of the avalanche profiles; and the avalanche network maintains the continental clustering pattern.
Nodes: “areas” of the southeastern USA grid
(PJM Regional Transmission Operator
⊂ Eastern Interconnection)
Two interdependent “power grids”
Our approach

Study $\geq 2$ “power grids” subject to certain dynamics.

What amount of interdependence most reduces risk (e.g., to a large blackout)?
Our approach

Goal 1: reduce risk of the whole system.  
Goal 2: each “grid” reduces its own risk.
Dynamics #1: sandpile model

systems possibly driven to “critical points”?

“All models are wrong; some models are useful.” – George E. P. Box

hypotheses

power grid outage models:
- sandpile
- topological networks
- flight simulators

infrastructure cascades:
- topological networks
- flight simulators

...
Dynamics #1: sandpile model

“All models are wrong; some models are useful.” – George E. P. Box

- Stylized model of overwhelmed nodes moving load to neighbors
- Result: an intermediate equilibrium in interdependence.

Sandpile model on networks

• Start with a network

• Drop units of load randomly on nodes

• Each node has a threshold. Here: degree.

• Load on a node ≥ threshold ⇒ node topples, moves sand to neighbors

• Neighbors may topple. Etc. Cascade (or avalanche) of topplings.

• Delete sand with probability $\varepsilon \ll 1$. 
Interdependent power grids

Power grids
(2 areas in SE USA)

Random 3-regular graphs
Equilibrium in interdependence

Suppose we own the power grid network $a$. Want to mitigate large cascades in our grid. Topple $\geq$ half network $a$.
Equilibrium in interdependence

Risk to the blue network

\[ \mathbb{P} \text{(cascade topples} \geq \frac{1}{2} \text{of blue network)} \]

Interdependence

\[ \mathbb{P} \text{(cascade topples} \geq \frac{1}{2} \text{of blue network)} \]

(simulations on random 3-regular graphs with $2 \times 10^3$ nodes each)
Equilibrium in interdependence

To mitigate large cascades, build some interconnections, but not too many.

and ≥ 10 followup papers.
Mitigate small cascades

To mitigate small cascades, seek isolation.

But isolation increases risk of large cascades.

Similar to forest fires, blackouts.
Risk of the whole system

More interconnections
→ More capacity
→ More load on average
→ Largest cascades in whole system become larger
Networks with the same total capacity

Networks with different total capacity

prefer the same interdependence $p^*$

prefer different interdependence $p^*$

Stable equilibrium

Frustrated equilibria
Sandpile model: complete picture

If all we want to do is mitigate risk of large cascades...

What grids want

What society wants
Dynamics #2: DC power flow

Interesting feature of sandpiles: equilibrium in interdependence.

Also in power grids?

If so, can we calculate it?

Collaboration with Anna Scaglione, Zhifang Wang (UC Davis)
DC power flow model
Linearization of nonlinear AC equations

- Flow problem
- Impedances on lines $x_{uv}$
- Power injection $\mathbf{P} \in \mathbb{R}^n$
  $p_u > 0$ generator, $< 0$ load
- Solve $\mathbf{P} = L \mathbf{\Theta}$ for phase angles $\mathbf{\Theta}$, where $L$ = weighted Laplacian.
- Power flows $\mathbf{f} = \mathbf{C A} \mathbf{\Theta}$.

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Cascading line outages on random 3-regular graphs

- Fraction $p$ of nodes have neighbor in other network.
- One or both networks are close to tripping: rescale $P$ s.t. $\|f\|_\infty = 0.99 \times$ trip point.
- Trip a randomly chosen line.

Repeatedly recompute power flows $f$ and trip lines above their trip point.

Cascading line outages on random 3-regular graphs

No intermediate equilibrium in interdependence

1. When both grids are close to tripping...
   “less interdependent!”

2. When just one grid is close to tripping...
   “more interdependent!”
   “less interdependent!”
Next steps

<table>
<thead>
<tr>
<th>Sandpile</th>
<th>local</th>
<th>nodes fail temporarily</th>
<th>identical nodes</th>
<th>nonlinear</th>
<th>tradeoffs</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC power flow</td>
<td>nonlocal</td>
<td>edges fail permanently</td>
<td>sources, sinks</td>
<td>linear</td>
<td>monotonic</td>
</tr>
</tbody>
</table>

More realistic dynamic for power generation?

<table>
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<tr>
<th>Linear algebra (Pepyne 2007)</th>
<th>Optimization (Carreras et al. 2002)</th>
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<td>Solve ( P = L \Theta ) for phase angles ( \Theta ).</td>
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Optimize cost here... and here?

Or does the ISO optimize for all grids in the region?
Modeling questions

- Need engineering practices in our model
- Who optimizes what?
  - ISOs mitigate their region’s risk?
  - Countries mitigate their own risk?
- The scale of each “network”?
  utilities ⊂ regions ⊂ ISOs ⊂ countries
- Which dynamical models?
  DC, AC, OPA, hidden failures, ...
- Timescales:
  A bad day for the power grid?
  Its evolution over decades?
Scale of the interdependent grids
Scale of the interdependent grids
determines the dynamics, timescales, who optimizes what

areas, utilities, zones

regions, ISOs
Scale of the interdependent grids
determines the dynamics, timescales, who optimizes what

areas, utilities, zones

regions, ISOs

who optimizes what?
what’s different about lines between grids?

who optimizes what?
sync at AC/DC links?
Questions
– Scales?
– Who optimizes what?
– What’s different about connections between grids?

Thanks!

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Sandpile paper: