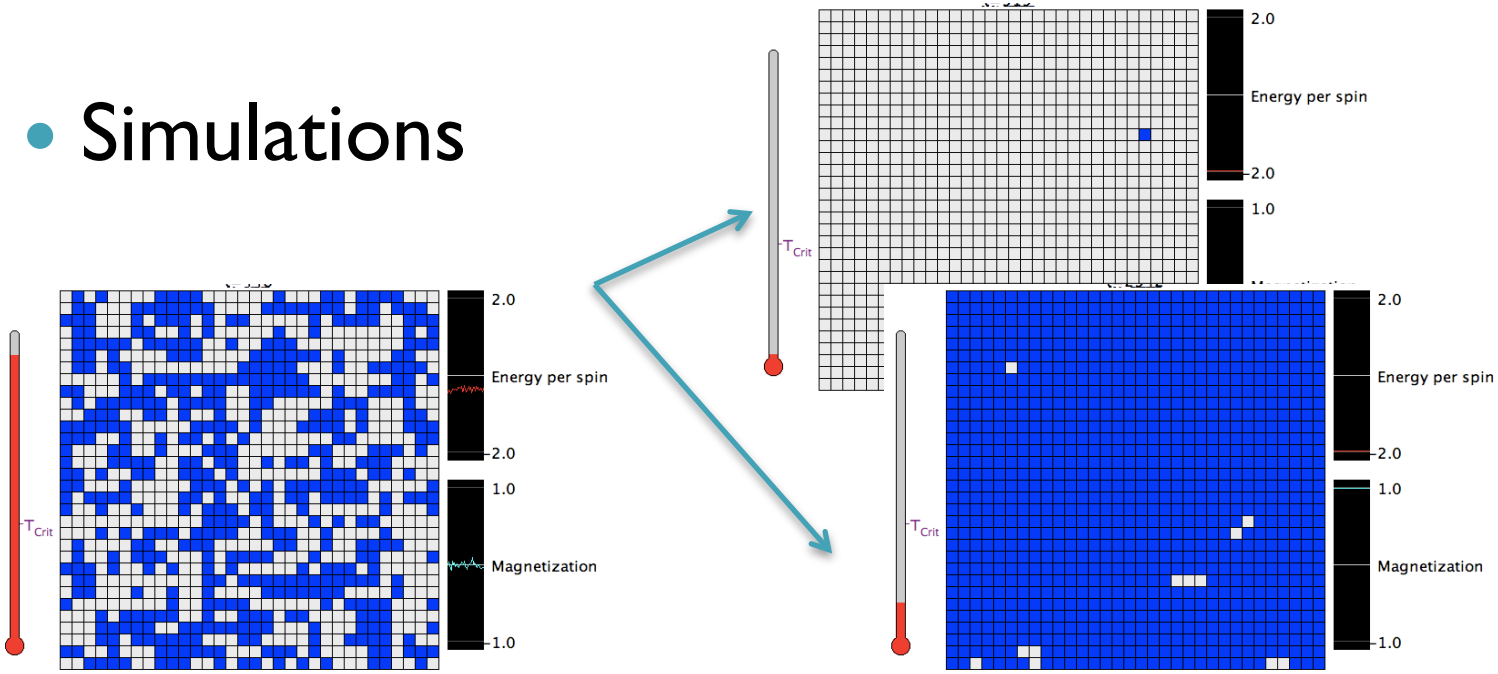
Emergence

Part IV: Coarse graining, Renormalization and all that real world stuff

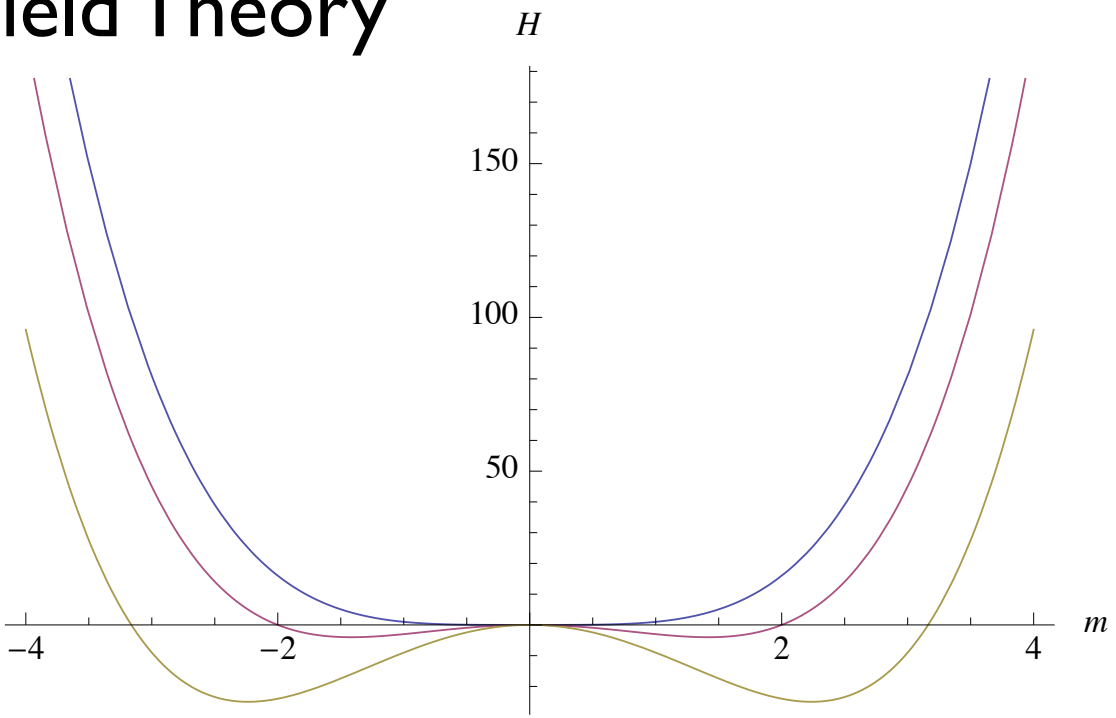
Summary of Yesterday

- (1) Phase Transitions
- Simulations



Summary of Yesterday

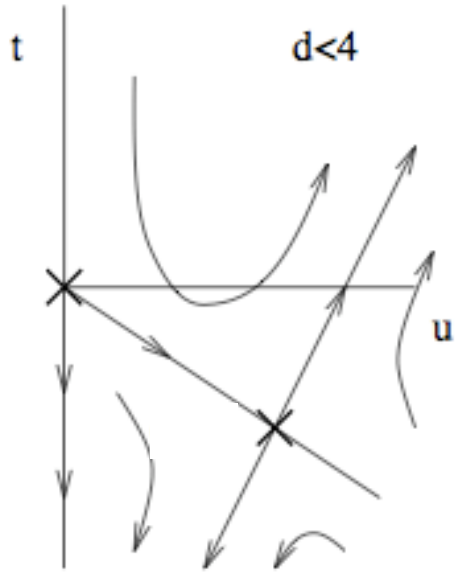
- (1) Phase Transitions
- Mean Field Theory



Summary of Yesterday

- (2) Universality

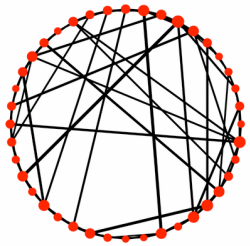
- Renormalization Group Flow



What about the real world?

- Asymmetry?

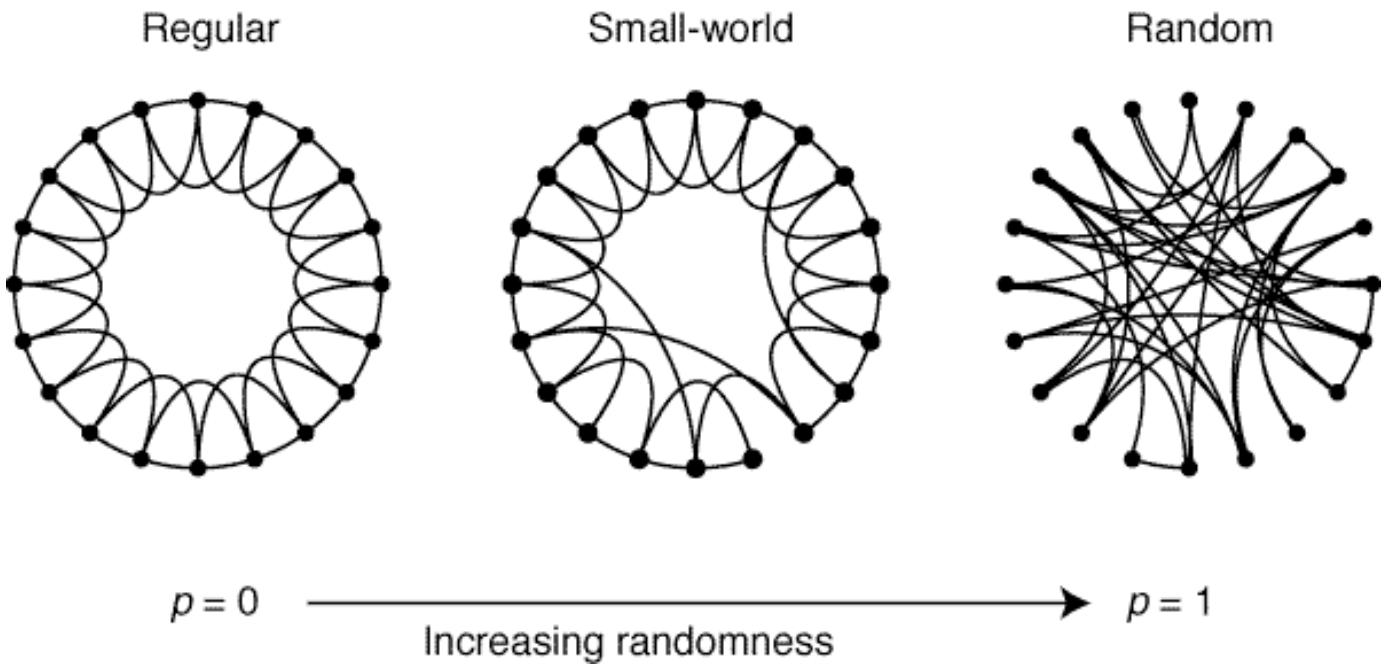
- Complex interactions



- Non-equilibrium systems

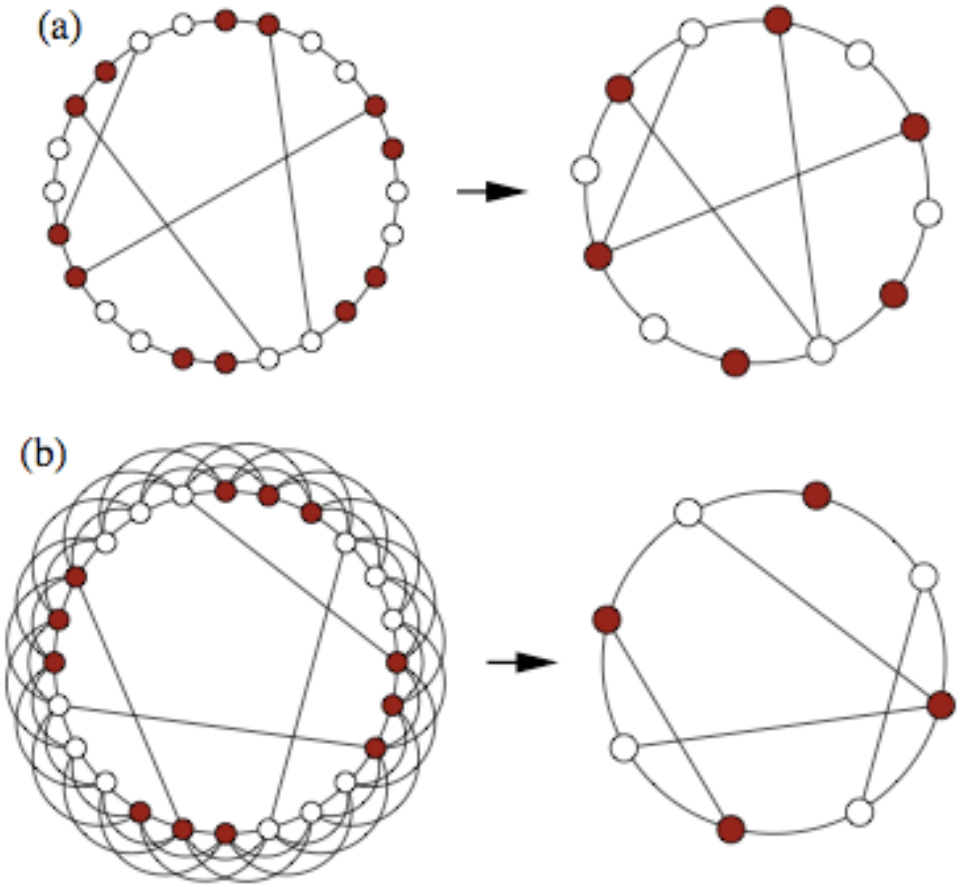
  - Steady-state vs thermal equilibrium

Small World Networks



Watts & Strogatz, 1998

Small World Networks



Newman & Watts, 1999

Small World Networks

- 'Continuous' phase transition as $p \to 0$
- Much like the one-dimensional Ising model, but control parameter is p
- What are analogues of
  - Higher dimensional lattice models?
  - Landau-Ginzburg theory?

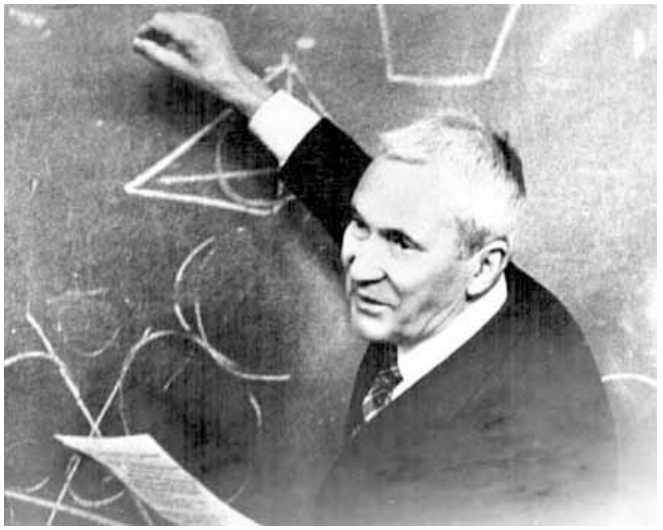
Non-equilibrium Systems

- Central object is called a Master equation

- Tells us how probabilities change in time

Non-equilibrium Systems

- "Every mathematician believes he is ahead over all others. The reason why they don't say this in public, is because they are intelligent people"

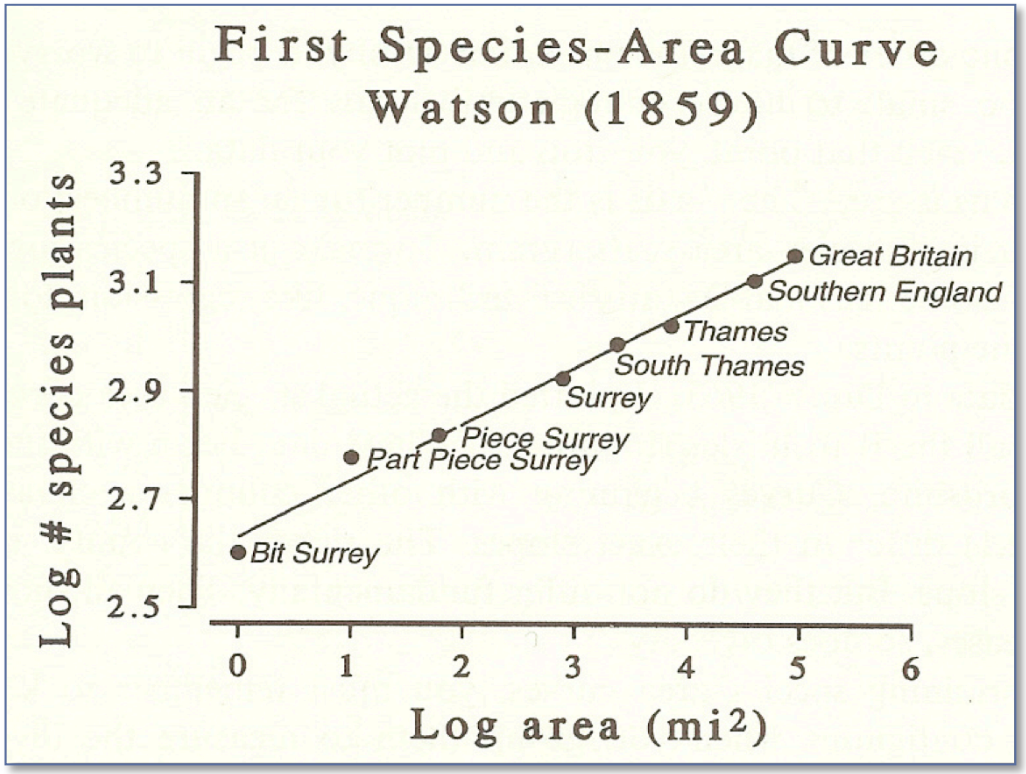


Kolmogorov

Non-equilibrium Systems

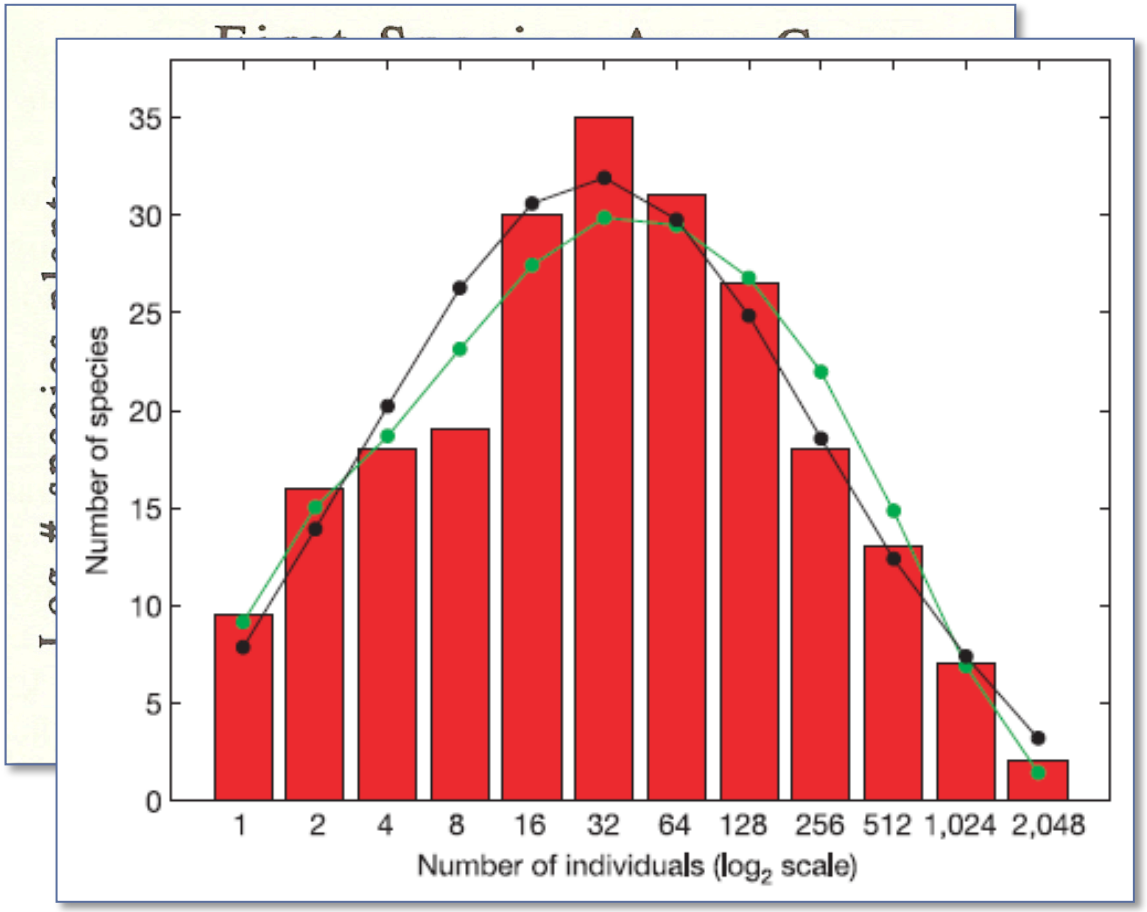
- What are universal, emergent phenomena in ecology?
- What's the right Master equation?

‘Universality’ in Ecology



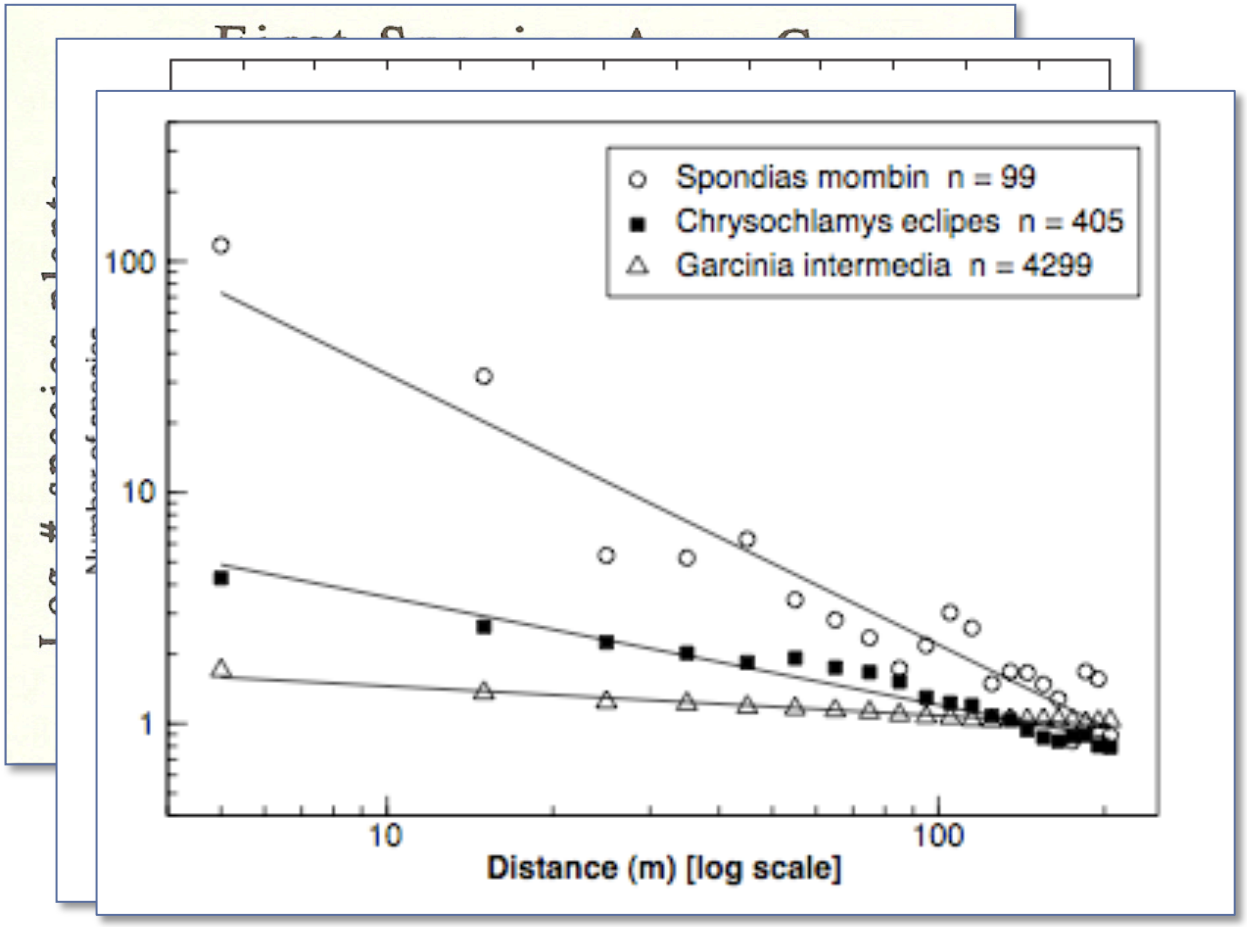
Species-Area Relationships

‘Universality’ in Ecology



Species-Abundance Distributions

‘Universality’ in Ecology

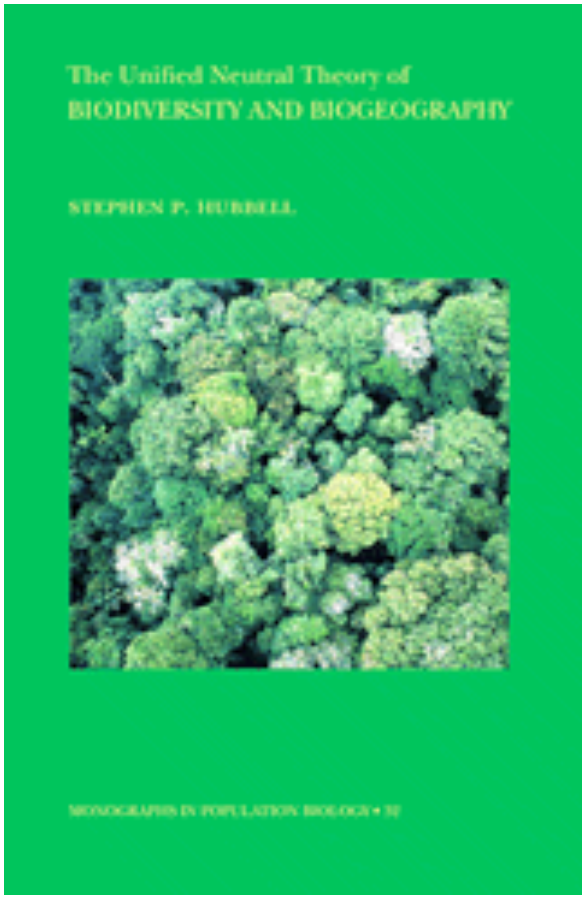


Spatial Correlations

Neutral Ecology

- Is there a *structural* or an *evolutionary* reason for basic features of these patterns? Or...?
- Neutral ecology assumes that coexisting species are close to equally fit
- All that remains is 'drift'

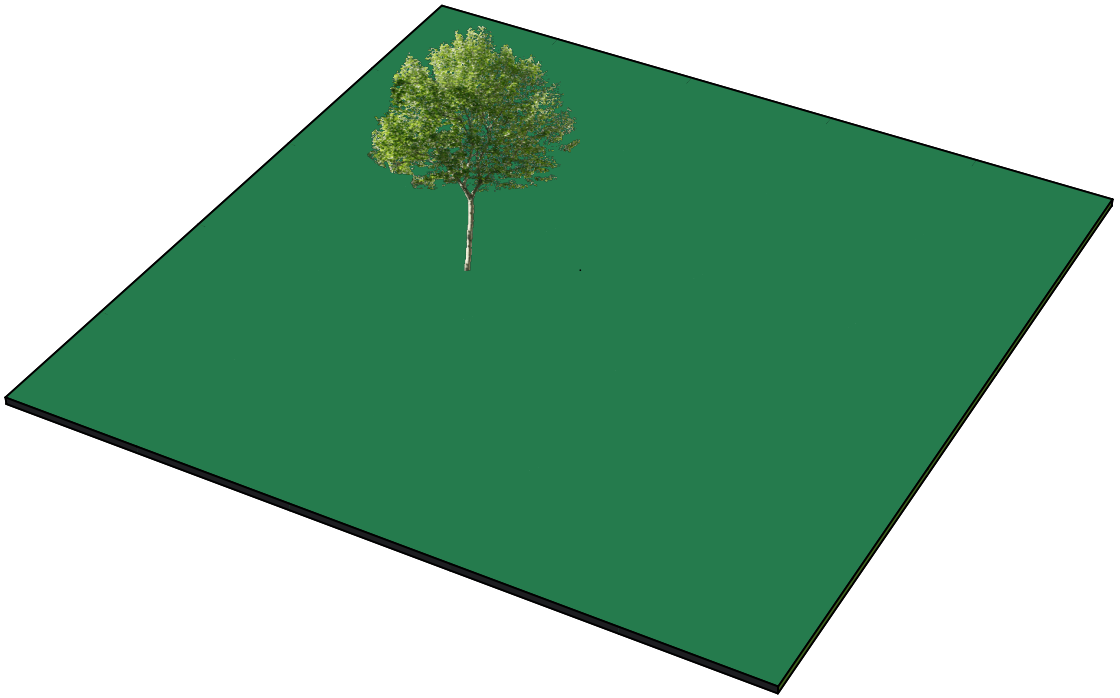
Neutral Ecology



Steve Hubbell

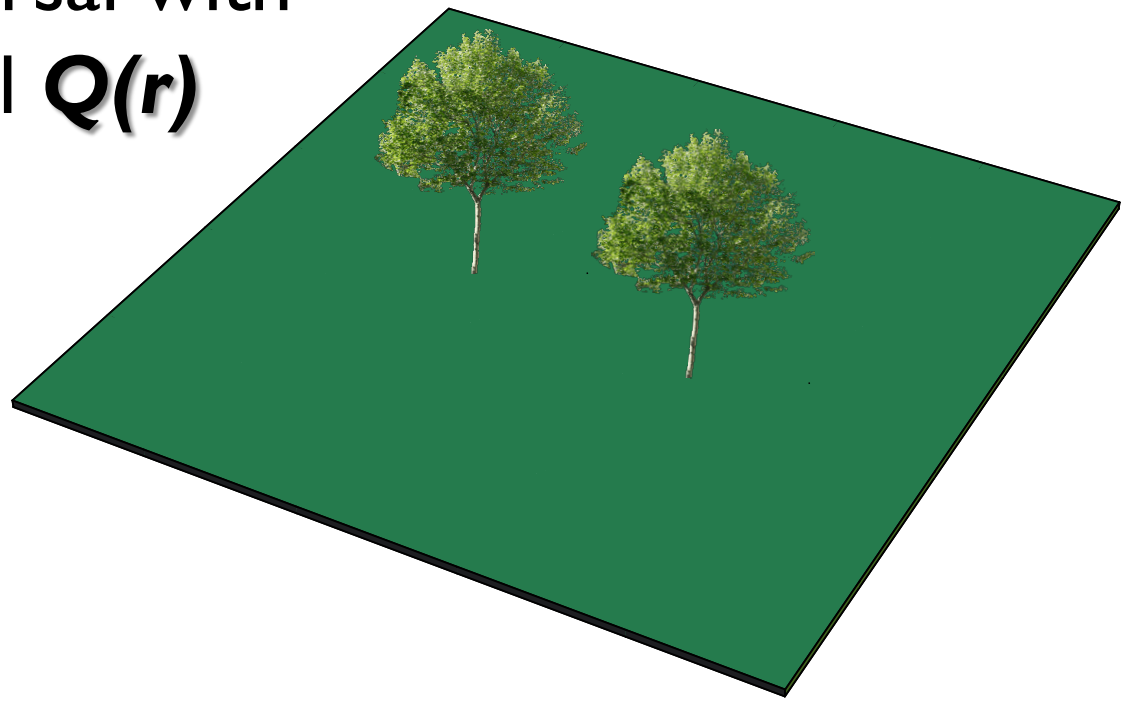
Spatial Neutral Ecology

- Birth at rate ***b***



Spatial Neutral Ecology

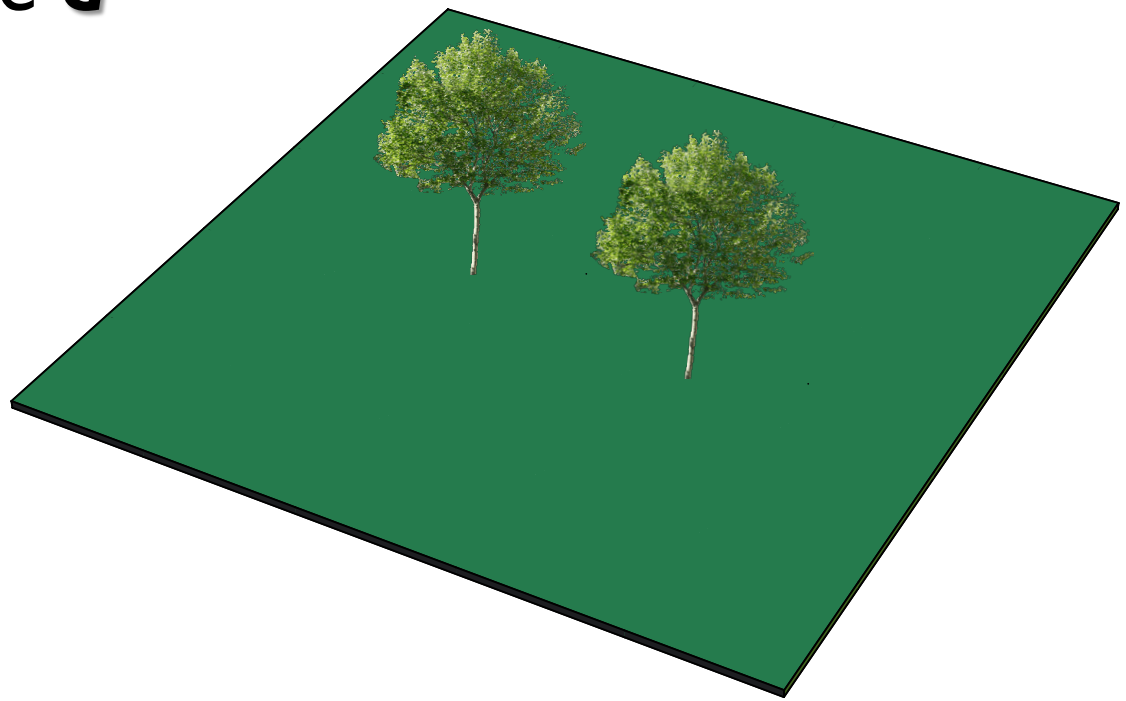
- Birth at rate ***b***
- Dispersal with kernel ***Q(r)***



Spatial Neutral Ecology

- Mortality

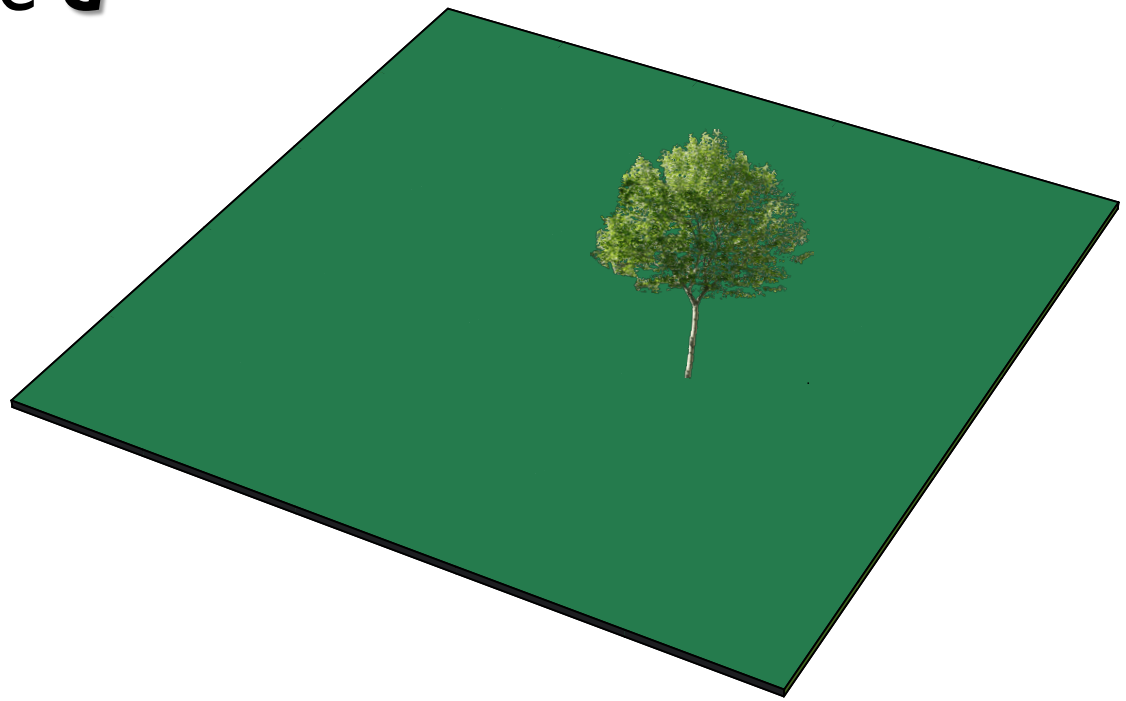
  at rate ***d***



Spatial Neutral Ecology

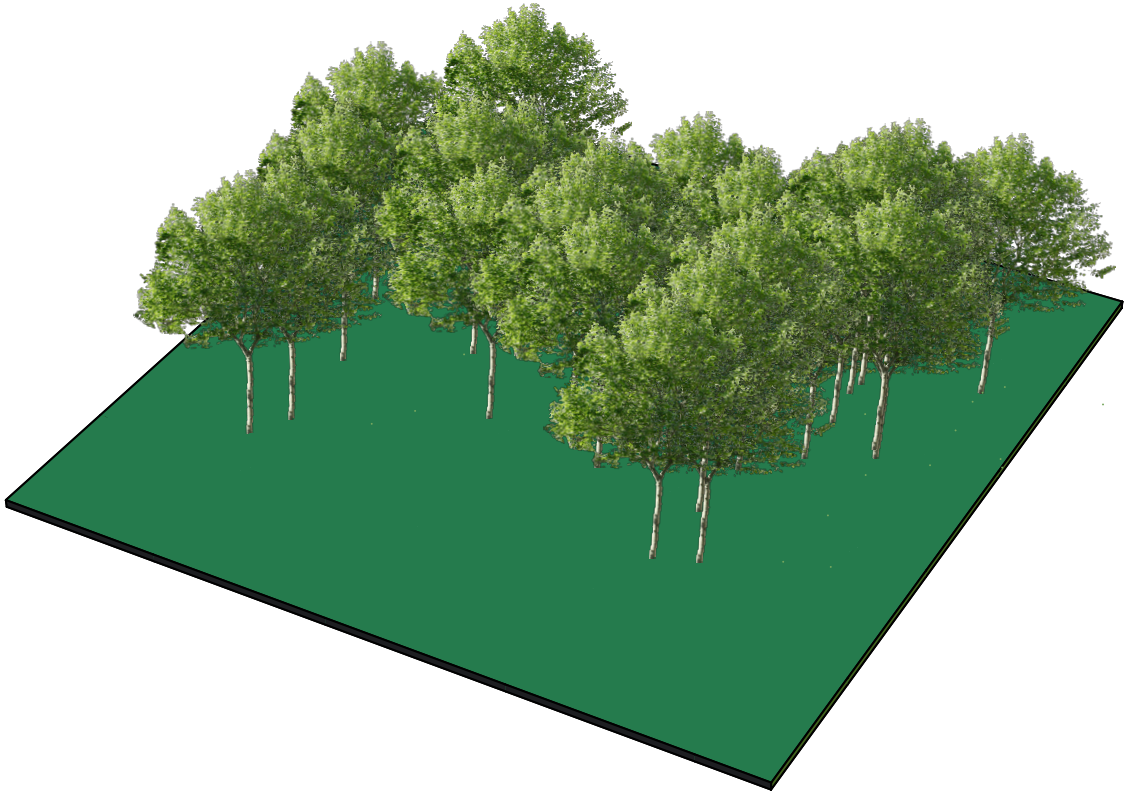
- Mortality

  at rate ***d***



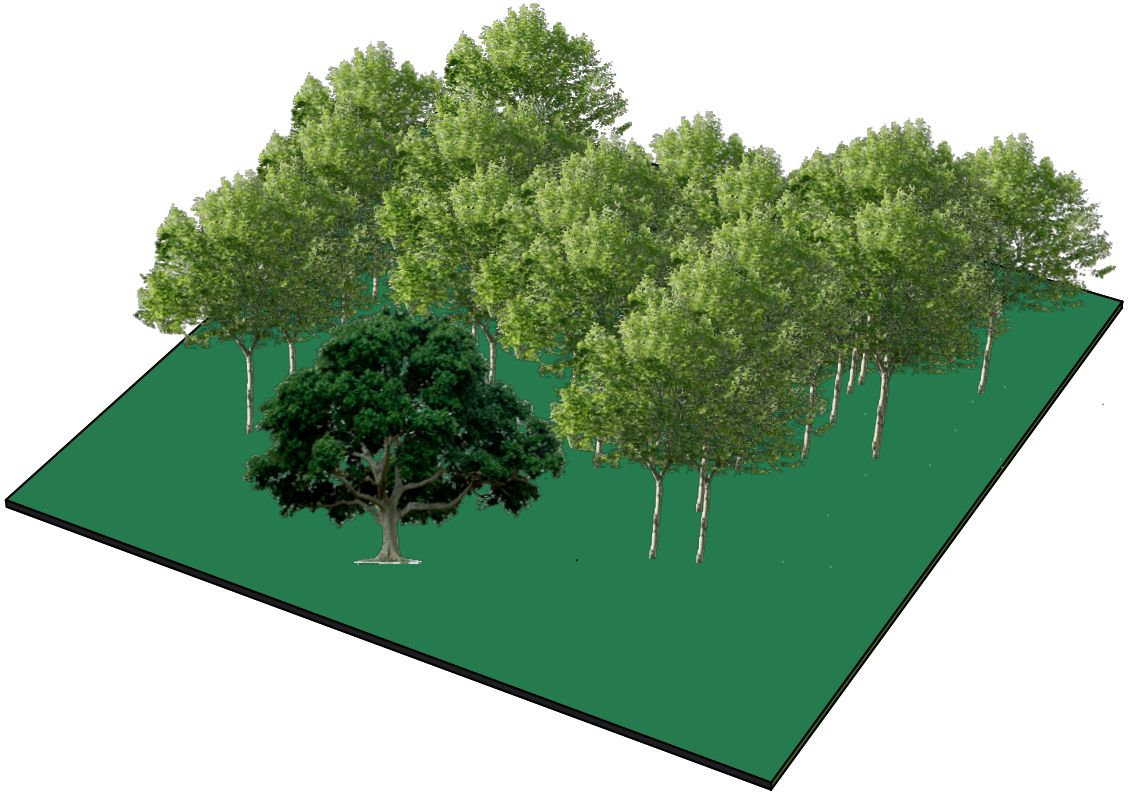
Spatial Neutral Ecology

- Speciation at rate α



Spatial Neutral Ecology

- Speciation at rate α



Mathematical Formulation

- Generalize the Species Abundance Distribution

$$P(n,t) \rightarrow P(\ldots, n_{i-1}, n_i, n_{i+1} \ldots, t)$$

- Probability that species has a given distribution of individuals on a discrete lattice

Mathematical Formulation

- $P(\ldots, n_{i-1}, n_i, n_{i+1} \ldots, t)$ satisfies Master equation

- Re-express in terms of a generating function

$$P(\ldots, n_{i-1}, n_i, n_{i+1} \ldots) \rightarrow Z(\ldots, j_{i-1}, j_i, j_{i+1} \ldots)$$

Mathematical Formulation

$$\frac{\partial \mathcal{Z}[J,t]}{\partial t} = b \int dx\ J(x) \left(Q \star \left[(J(x)+1) \frac{\delta \mathcal{Z}}{\delta J(x)} \right] - \frac{d}{b} \frac{\delta \mathcal{Z}}{\delta J(x)} + \alpha \right)$$

Generating Function

Dispersal Kernel

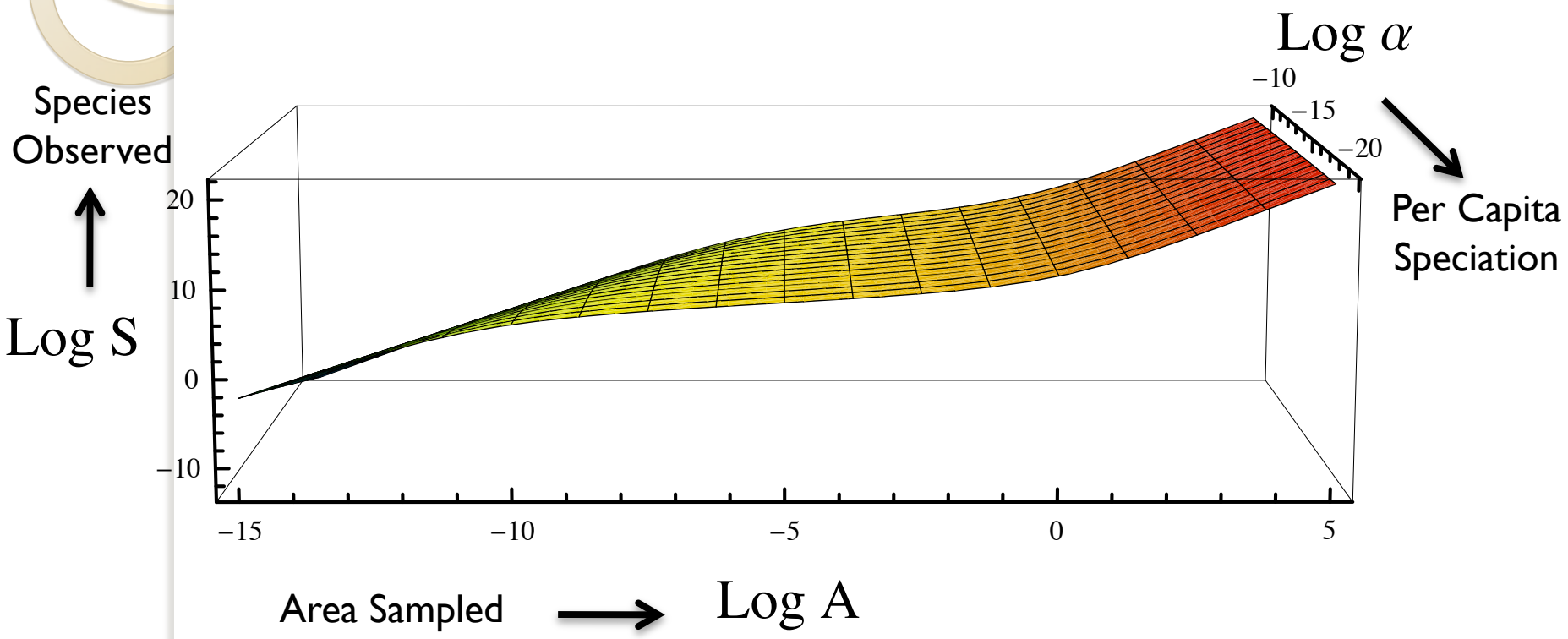
Birth and Mortality

Speciation

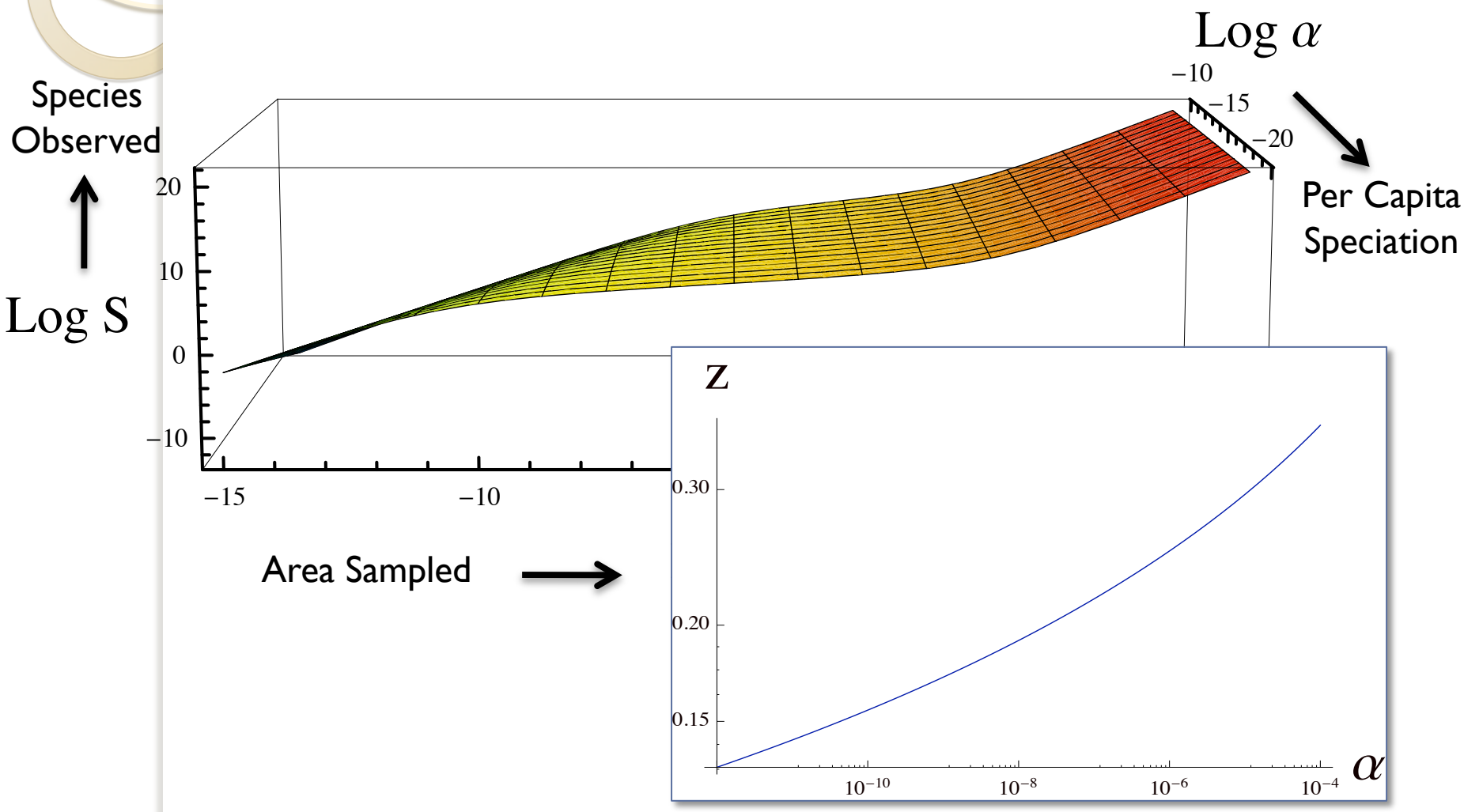
The Species-Area Relationship

- The SAR depends on:
  - Dispersal Length-scale σ
  - Per capita speciation rate α

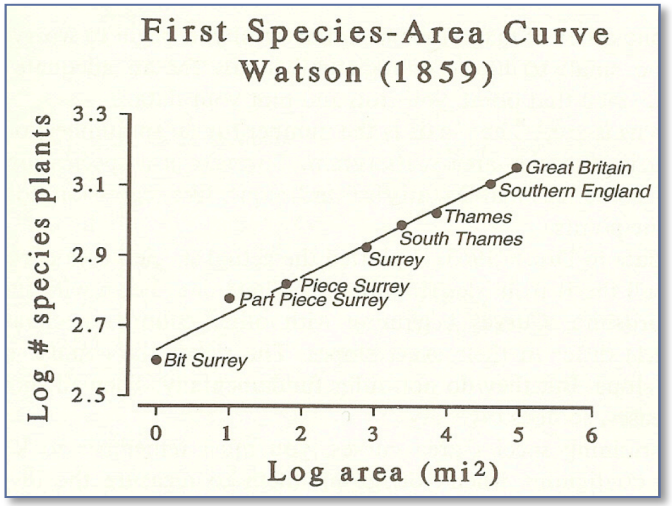
The Species-Area Relationship



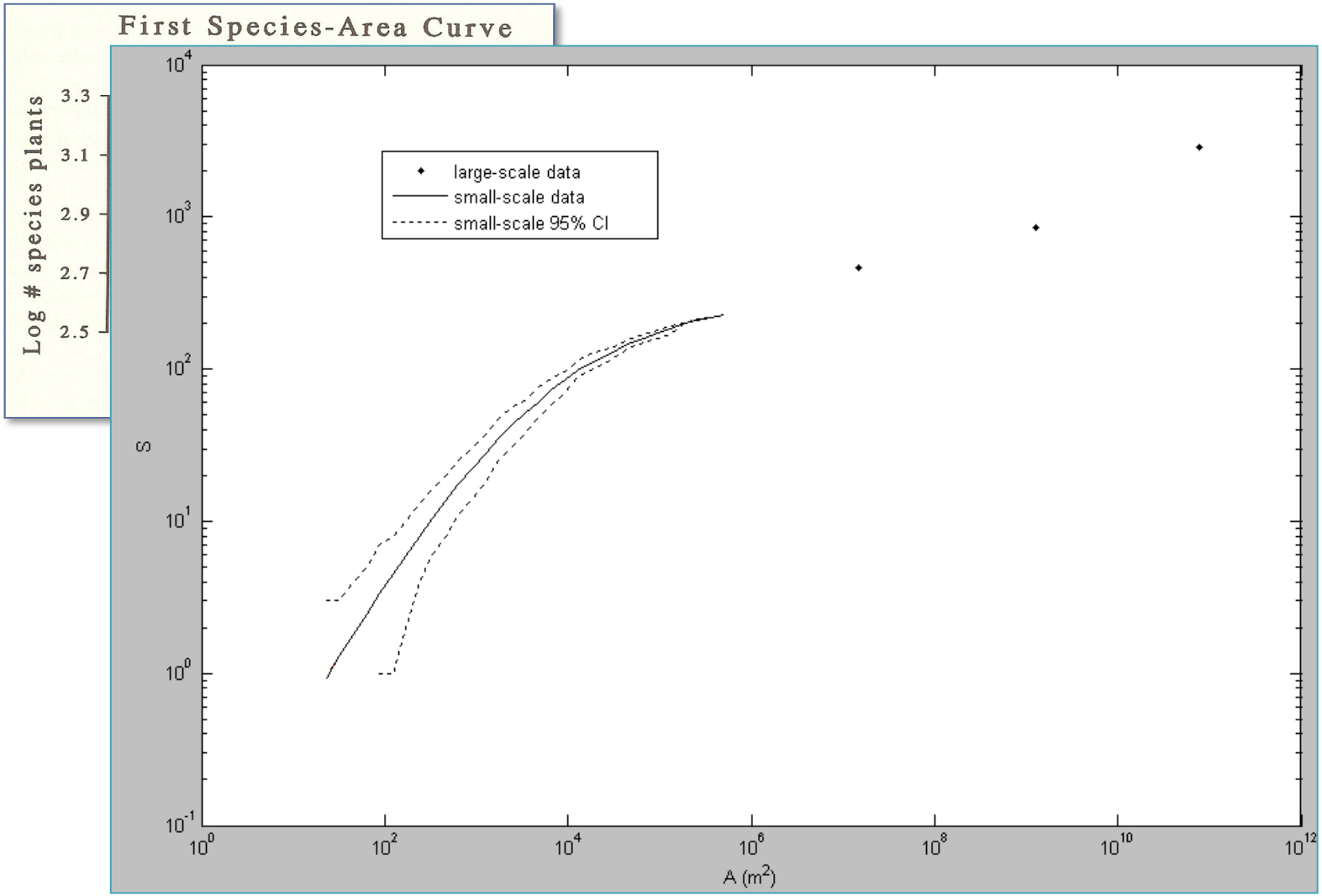
The Species-Area Relationship



The Species-Area Relationship



The Species-Area Relationship

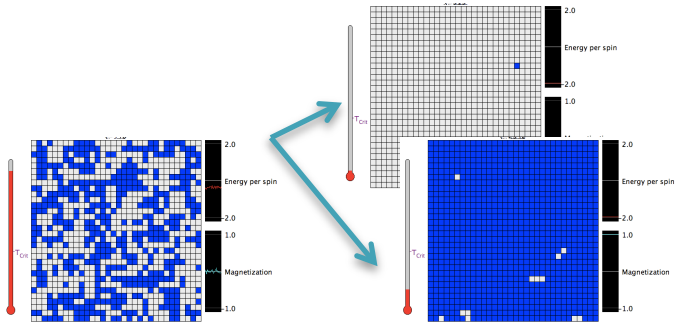


Where's the Emergence?

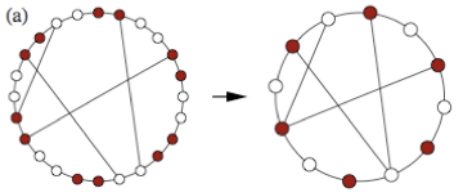
- We saw in physics-world that there was a reason that higher-order interactions went away
- Have neutral theorists been making a clever (but unjustified) step forward?
- We need a theory of universality for non-equilibrium theories

Summary

- Equilibrium, Lattice
  - ‘Simplicity from complexity’



- Real World Systems
  - Renormalization & Networks



  - Non-equilibrium systems