

STRUCTURE.PASSION CSSS-08

Tanja Gesell^{1*}, Alexander Moffet², YunQi Xue³, Walter Zesk⁴, Adam Campbell⁵, Meritxell Vinyals⁶, Laura Feeney⁷, David Papo⁸, Craig Hayenga⁹, Christopher Vitale¹⁰

¹Center for Integrative Bioinformatics Vienna, Max F. Perutz Laboratories, University of Vienna, Austria ²University of Texas at Austin, USA ³ Institute for Logic, Language and Computation (ILLC), Universiteit van Amsterdam, Netherlands ⁴Rhode Island School of Design, USA ⁵ University of Central Florida, USA ⁶ Artificial Intelligence Research Institute (IIIA-CSIC) Barcelona, Spain ⁷ Swedish Institute of Computer Science and Uppsala University, Sweden ⁸ Functional Brain Imaging Unit, Wohl Institute for Advanced Imaging, Tel Aviv, Israel ⁹ Lockheed Martin, USA, ¹⁰ Pratt Institute, New York, USA

Email:

*Corresponding author

Abstract

At first sight, it seems obvious what a structure is. Recently, there has been a lot of literature – e.g. in poststructuralist theory. However, it seems that there is no possibility in form or there is a lack of a clear, general definition. Indeed, the lack of consensus within this literature raises a number of questions regarding the possibility of providing a single general definition of structure. In our project, we have considered different definitions of structure as they are found in molecular biology, philosophy, logic, architecture, computer science, software engineering, information theory, physics and psychology. We started with a comparative analysis of this concept to elaborate the shared features of these definitions, likewise we have been studying the history of this concept as it has been evolving within these disciplines.

We have been discussing general concepts of structure, e.g. Koppel [1] and Wiener [2] have suggested that the structures of a given sequence might be associated with those Turing machines that are capable of producing the sequence as output. This computational approach offers a useful foundation upon which to base a general understanding of structure. In addition, we have discussed Category Theory as an alternative option to speak about an universal description of structure.

The global aim of this four-week summer school project was the try to clarify the meaning of structure in general and between scientists of different disciplines. Here we are giving a report about these four weeks. However, this is a long-term project. Following our detailed discussion and interpretation, we can conclude that on the one hand it is indeed possible to find definitions of structure in individual scientific disciplines, which have been evolving

over time: On the other hand, finding a satisfactory definition of the term "structure" that can be agreed upon on a cross-disciplinary basis requires further research. In the end, we presented a room installation and a video animation as a poster presentation, helping to stimulating questions and to take structure on higher human level into. Furthermore, we will continue our work with more people and will present an official WIKI page to collect and discuss structure definitions in the near future.

1 Introduction

At first sight, it seems obvious what a structure is. Recently, there has been a lot of literature – e.g. in poststructuralist theory [3]. However, it seems that there is no possibility in forming or there is a lack of a clear, general definition. Indeed, the lack of consensus within this literature raises a number of questions regarding the possibility of providing a single general definition of structure. Is there a single concept of structure or just different concepts of structure found in different disciplines? Are structures discovered or are they invented? Are they simply patterns, or do they offer a more profound understanding of the properties of different entities? Is there a single, correct structure with which each entity is associated, or is there a range of different potential structures, with pragmatic considerations determining the assignment of particular structures to particular entities? Is there a general challenge in thinking about structure? How do scientist apply structure today? What is a structure?

In our project, we have considered different definitions of structure as they are found in molecular biology, philosophy, logic, architecture, computer science, software engineering, information theory, physics and psychology. We started with a comparative analysis of this concept to elaborate the shared features of these definitions, likewise we have been studying the history of this concept as it has been evolving within these disciplines. In addition to the variance in concepts of structure from different disciplines, we are also interested in the similarity of structure, especially how scientists apply structures to conceptual modeling, on a higher level. We found Category Theory as an alternative concept. In section 2.2 we try to elucidate a general concept of structure. This might be done through reference to the capacity of a universal Turing machine operating upon sequences of characters. Koppel [1] and Wiener [2] have suggested that the structures of a given sequence might be associated with those Turing machines that are capable of producing the sequence as output. This computational approach offers a useful foundation upon which to base a general

understanding of structure. We have taken this further into account as a basis for some first observations about ourselves in subsection 3.6.

The global aim of this four-week summer school project was the try to clarify the meaning of structure in general and between scientists of different disciplines. Here we are giving a report about these four weeks. However, this is a long-term project. A discussion and interpretation about our first results is given in the end. Furthermore, we will continue our work with more people and will present an official WIKI page to collect and discuss structure definitions in the near future.

2 Methods

2.1 Comperative Analysis: Considering different disciplines

One way in which to improve our understanding of the concept of structure is to consider its use in and between a wide range of disciplines. In this short summer-school project, we have considered: molecular biology, philosophy, logic, architecture, computer science, software engineering, information theory, physics and psychology. We started to elaborate the shared features, attributes and themes of these definitions. Furthermore, we have been likewise studying the history of this concept as it has evolved within these different disciplines. Furthermore, we consider each disciplines under the following questions: Is there a single, correct structure with which each entity is associated, or is there a range of different potential structures, with pragmatic considerations determining the assignment of particular structures to particular entities? Is there a general challenge in thinking about structure? How do scientists in different disciplines apply structure today? In this paper, we are give a short summary of each discipline in the Appendix. However, we collected more information in the four weeks, which we will present on a official WIKI page in the near future. Finally, we used some unusual scientific methods to collect and work with our definitions. This is described in subsection 3.4.

2.2 Universal Description of Structure in the Literature

We hope to elucidate a general concept of structure. This might be done through reference: First, we looked simply at what dictionaries may have to say about structure. Although there has been recently a lot of literature in post-structuralist theory, we want to concentrate on a general clear definition of structure. However, a part of our group participated in the Theory Group CSSS-08 - this group mainly discussed this literature in another context. In addition, see Post-Structuralis Philosophy in the Appendix. We found Category Theory might be interesting. Furthermore, in combination with the SFI lectures we collected

material about Complexity theory, e.g. [4], [5], [6] and [7], which we have to consider in more detail in future research. In discussions about discipline hierarchy, we considered logic as a possible method to speak about a general concept of structure. Concepts which suggest that the structures of a given sequence might be associated with those Turing machines that are capable of producing the sequence as output seem to offer one of the most appropriate concept of structure at the moment. This computational approach offers a useful foundation upon which to base a general understanding of structure. Thus, we discussed a paper by Koppel in detail. This was extended into discussions about the concepts of Oswald Wiener.

2.2.1 Dictionary

This is the definition given by the online dictionary *De Mauro*¹. It might be interesting.

Structura from *struo*: to accumulate by stratification, to put one (object) close to another, then to connect an join; as a result: to arrange, to set up; to erect, to build.

1. a) set of constitutive elements of a given set in relation to their organization and distribution.
 b) mode, schemata in which the elements of a given discourse, text, are distributed and organized
2. ensemble of constant and systematic relationships between members of a group; ordering,
3. ensemble of elements of a construct, installation, with sustenance or connection/liaison functions and way in which these elements are ordered and linked to each other
4. complex or construct made of a set of elements

2.2.2 Category Theory

Category theory is a branch of mathematics that offers a set of general concepts with which to describe a wide range of mathematical operations [8].

The focus of study of category theory is the category. A *category* consists of a set O of objects and sets of morphisms between pairs of objects in O , where if $x, y \in O$ and f is a morphism between (x, y) , then $f \in \text{hom}(x, y)$ and $f : x \rightarrow y$. In addition, each morphism must satisfy the following conditions:

1. For all $x \in O$, there exists an identity morphism such that $f : x \rightarrow x$.
2. For any pair of morphisms f and g , if $f : x \rightarrow y$ and $g : y \rightarrow z$, then there exists a composite morphism $f \circ g$ such that $f \circ g : x \rightarrow z$.

¹<http://www.demauroparavia.it/>

3. For all morphisms f, g , and h , $f \circ (g \circ h) = (f \circ g) \circ h$.
4. If f is a morphism and g is an identity morphism, then $f \circ g = f = g \circ f$.

As the processes studied in many areas of mathematics can be described in terms of morphisms and objects, category theory offers a general conceptual framework with which to represent much of mathematics. In providing a general framework with which to represent mathematical systems, category theory thus allows for the analysis of the mathematical structure present both in these systems and in the objects that are present within them.

From a systems perspective, the concept of structure that emerges from within category theory is one defined by both the ontology of the system and by the functions that operate within it. That is, the structure of a system is given by the sets of objects and morphisms that are needed to describe it. One system can thus be said to have the same structure as another system if the two systems can be described by isomorphic categories.

However, when instead considering the object, the concept of structure that emerges from within category is instead that of invariance to transformation. The structure of an object is given by its functional role within the system, by the particular set of morphisms that act upon it. One object can thus be said to have the same structure as another object if each morphism, when input with these objects, yields the same output.

The use of mathematical modeling to identify the mathematical structure of a wide range of phenomena, coupled with the potential of category theory to elucidate mathematical structure, indicates the significant potential of category theory to contribute to the study of the concept of structure. Our preliminary application of category theory to the analysis of the concept of structure indicates that while it may be possible to use category theory to produce a formal definition of this concept, the account of structure that thus emerges may not be univocal but may depend upon the level of organization of the phenomena of study. While the conclusions to be drawn from the application of category theory to the study of the concept of structure remain provisional, it is clear that category theory has much to contribute to further research in this area.

2.2.3 *Meta-Logic*

Furthermore, in discussions about discipline hierarchy, we considered logic as a possible method to speak about a general concept of structure, especially how scientists apply structures to conceptual modeling, on

a higher level [9]s. Logic, the formal reasoning language, is considered as a fundamental rule in building structure in different disciplines. For example, logical structure was used intensively by mathematicians to prove theorems, and later in all kinds of mathematical modeling. Do other disciplines also choose their modeling language, define the syntax and semantics of the language, and derive axioms in order to have a structural approach in finding new results? If they do, is it precisely like how a logician would do it or do we need to 'translate' them? If not, how far is this from the logic approach of structure?

2.2.4 Structure concepts using Turing machines (TM)

Moshe Koppel's structure concept uses the universal Turing machine (UTM) as a basis for a formal definition of structure. According to the Church-Turing thesis, the universal Turing machine, and any computationally equivalent formal mathematical model, can compute any effectively computable function, and so it is believed that UTMs can serve as a method of formally defining terms such as complexity and structure.

Give a universal Turing machine U , the minimal description length of a given string S is the length of the shortest input to U that produces S as its output [1]. Thus, a random string S has maximal complexity because, by definition, there is no shorter description of S other than the string itself. This definition of complexity is not accepted by everyone, as it seems counter-intuitive for a random string to have high complexity [10].

Koppel defines the sophistication of a string as "... the minimal amount of planning which must have gone into the generation of the string" [1]. Thus, random strings have high complexity, but low sophistication. Regular strings, such as 00000..., would also have low sophistication. To formally describe both sophistication and structure, Koppel reminds us that one of the major discoveries in computing is that both **the program**, P , and **the data**, D , can serve as input to a Turing machine. Given the minimal description (both P and D) of a string S , Koppel defines the structure of S as the **function** computed by P , and the sophistication of S is defined to be the length of P .

Oswald Wiener's structure concept has defined structure according to the Church-Turing thesis independent of Koppel: "A structure of a string is a TM that, with or without input, is capable of printing the string (a string, the, viewed as a trivial TM, is a structure of itself); alternatively: a (part of a) TM that "accepts" the string (that is, the function it computes is defined on the string); alternatively: a TM computing a Boolean function and affirming the string." [2]. To summarize: a structure is a

sequence of a TM, which generates or accepts this sequence. At a first glance, it seems that Koppel and Wiener have the same concept. However, Wiener goes further and combines this approach with self-introspection. Moshe Koppel is a mathematician and just mentions all philosophical and epistemological formulations in an informal introduction. There are also small formal differences: Wiener’s description seems to have more refinements, i.e. when Koppel is speaking of **the** structure, Wiener speaks of **a** structure, which is associated with an object that can have more than one structure (see subsection 3.1). Although, Oswald Wiener studied mathematics too, he is coming to the field as a poet, working with terms like “content”, “aspect”, “concepts”, “understandings”, etc. . Wiener is using Turing Machines like Koppel for his structure concept, but in “**Thinking Turing Machines**”. We do not repeat here the use of UTM as a basis for a formal definition of structure (see above), instead we give some examples of his further concepts. Following Wiener, structure (forms) exist in their colloquial sense, i.e. as physical machines which exist in peoples heads. For example, in comparison to the “two pattern types” of R. J. Nelson [11], Wiener follows that a sequence for a perceptual organism can have two different structures if two different TMs exist, which accept these sequences, in the repertoire of the organisms. Wiener’s structure concept gives answers to questions which stay open in Koppel: which types of entities could have structure(s) and where are they? In Wiener’s view structures are ”attributed” and the object with the “attributed” structures only exists in organisms capable of thought. We have taken these further into account as a basis for subsection 3.6.

3 Results

3.1 Collections of Attributes

In this short summer school project we have considered: molecular biology, philosophy, logic, architecture, computer science, software engineering, information theory, physics and psychology. In a first step, we started to elaborate the shared features, attributes and themes of these definitions, see (Table 1). However, our project is a cross-disciplinary issue, where we have to take the correlation between them into account. Structure can be related to the following concepts: Structure can be taken to reflect the possibility of formulating some compact **description** of observed facts. In this context, a structure is loosely speaking a scaled version of observed facts, with the scaling factor reflecting the properties of that structure.

The idea of compression is directly related to that of **randomness**: in this vein, a structure would be something that one would not expect to observe at random. Alternatively, a structure is what remains when randomness has been discounted. In this context, a structure reflects the usable **information** that

can be extracted from a given context/ set etc. In turn, non-randomness can be related to the concepts of **organisation/pattern, order**.

From a different point of view, a structure can also be seen as an **object** lato sensu (for instance some **time-varying 'reality'** – time can always be seen as an additional dimension of an object - or some relation between different "realities" can also be interpreted as an object). Following this line of reasoning, a structure is something that can be singled-out at some level, via some kind of segmentation.

An object generally has some properties (e.g. continuity) which define it as unitary (**atomicity** and **minimality**), at some level of description. At the same time, the properties defined on this object are descriptions of some form of **regularity** (**modularity** can be an example of that) which can be thought of as sub-objects – this leads us back to the concepts or description, non-randomness, order, etc. above. In this vein, a structure can be seen as a set (again, lato sensu) endowed with some kind of **relation among its elements**(where the relation itself is an element of the set). In a somehow complementary way, a structure can be defined on the basis of a **sets of constraints or boundary conditions**.

To be truly useful/informative the relation has to show some degree of **generality** or, equivalently, some **stability/robustness**. One way to conceive of stability emerges rather naturally when considering systems' structure-dynamics-function relationships, time becomes essential to the definition of structure. Structure is identified with **minimal meaningful static building blocks**. In other words, a structure is the limit of time-varying activity where dynamics is slower than either the precision of the measurement instrument or than the smallest time-scale captured by the analysis. Along these lines, structure is the space onto which dynamics can take place.

3.2 Towards General Challenges in Definitions

We have been likewise been studying the history of this concept as it evolved within these different disciplines. One result of this comparative analysis is that the definition of STRUCTURE is continually evolving in each discipline, indicating that our project is in essence a never-ending one. One general trend that can be observed across disciplines (e.g. molecular biology, psychology, logic and physics) is the incorporation of **time** as an essential dimension in the definition of structure. In some contexts, particularly when considering systems' **structure-dynamics-function relationships**, time becomes essential to the definition of structure. Structure is identified with minimal meaningful static building blocks. In other words, structure is the limit of time-varying activity where dynamics is slower than either the precision of the measurement instrument or the smallest time-scale captured by the analysis. Along these lines, structure is the space onto

<i>Attributes/Themes/Concepts</i>	Molecular Biology	Philosophy	Logic	Architecture	Computer Science	Software Engineering	Information Theory	Physics	Psychology	Philosophy <small>Post-Structuralis</small>
Atomicity										
Compression										
Constraints										
Decomposition										
Description <small>of observed facts</small>										
Dynamics										
Enforcement										
Functionality										
Generality										
Levels of abstraction+										
Minimality										
Modularity										
Organisation										
Pattern										
Perception										
Randomness										
Relation <small>among its sites</small>										
Regularity										
Related concepts										
Reusability										
Re-Use (Recursion)										
Robustness										
Sequence & Machine										
Stability										
Time varying reality										
Variability										

Table 1: Examples of some attributes, concepts and themes from our discussion, which we want to extend and overworked to related concepts. (*Post-Structuralis Philosophie, + object lato sensu)

which dynamics can take place. Structure can also be taken to reflect the possibility of formulating some compact description of observed facts. In this context, a structure is loosely speaking a scaled version of observed facts, with the scaling factor reflecting the properties of that structure.

In future research, we hope to produce an underlying geometry or ontology of structure definitions over time through our new wiki page.

3.3 Universal Definition of Structure

However, it is clear, that the question "What is a STRUCTURE?" cannot be an democratic issue. We have started to look for a hierarchy of the disciplines. Doing this we have been considering Category Theory and Meta-Logic. We have discussed in detail two important paper [1,2] about a universal description of structure. They have suggested that the structures of a given sequence might be associated with those Turing machines that are capable of producing the sequence as output. On the first glance, it seems that Koppel and Wiener have the same concepts. However, Wiener goes further and combines this approach with self-introspection. However, so far none of them provided a universally acceptable definition of the term for all members of the project. It seems, that we have concluded in this paper that structure is a multi-furcated concept. However, in our discussions we developed a closer understanding of a general definition of structure, which we have not been able to define in general yet.

3.4 Multi-scale representation

Based on the short amount of time and the amount of information from different people, we have chosen to display the key word and conceptions in each discipline and from our discussion and individual work on yellow post-its on the wall, see Figure 1. In addition, we did a survey among summer-school participants about the question of defining structure (What is a structure?) and we presented their definitions and key words (concepts). For this we used the survey of the social network group. Via animations we show simultaneously our and the survey answers in different random modes. This shows all the structure definitions simultaneously and could give of each recipient the chance to think about structure on an higher level, helping and stimulating questions. We think, that this is indeed an interesting result and could be the basis for further deeper individual and general research about structure both in each discipline and in general.

3.5 Questions

Here is a selection of true questions generated during our discussions and displayed on our wall:

What is a structure? Types of structure? What makes a structure a structure? How do you get structure? How do we detect pattern? Structure reality? What is an underlying structure? Structure as Babushka Dolls? What is a sequence by nature? Are we doing the structure of theory or structures that are from the application of disciplines? How do we find the ones that generate structures rather than assume a pre-core? How do we remove boundary conditions such that structures are generated by a theory? What is the dynamic structure of lightning and what theories create its structure? Do constructed artefacts have a different structure than natural ones? Scale matters? To describe a system we need to define the structure over multiple scales? R or N ? TM operates on integers $(0,1)$. If phenomena in the physical world are continuous (i.e. real nature), can a TM describe the physical world? 1) Is the physical world real? 2) alternative TM? Structure of natural and engineered entities are these different? Do complex engineered entities have structure that is separate from the structure(s) that its builders used to create it? Is structure present or isn't? Or should we ask: What is the degree of structure? Is structure inter-subjective or participatory? Is there a contrast to an empirical or scientific argument for structure that can be objectively identified and expected to persist through time?

The questions are a value of themselves. However, all these questions suggest we may need to develop new terminology in this general field. As Heisenberg pointed out in his lecture that the development of new terms is one of the most difficult, but one of the most important aspects of research.

3.6 Self-Introspection: Why are we discussing structure (an answer)?

Finally we end up in personal question and self-introspection: "I like things other people have made. I like them more than things that have never been manipulated and, remarkably, I can usually tell the difference between the two." It is equally pleasurable to make things in the hopes that other people will see them and get the same enjoyment. There is a connection formed through our synthetic cultural objects. Most people enjoy this connection that arises from the things that we make and, more importantly, this enjoyment is based on our need to find social reinforcement of our individual perception of the world. In a sense, our culture of making, including popular media, helps coordinate and structure our disparate, individual perspectives into an intersubjective, communal worldview. This reinforcement therefore is no trivial pleasure. If we were unable to coordinate our perception of pattern as a social group it would be far more difficult to communicate or organize effective social structure. This could be a primary reason we make art. "Art objects" – paintings, sculpture, music, literature, architecture – along with mass media help

coordinate individuals into a functional social group. This argument is based on a belief about art-making: artists use the tools of metaphor and symbol to reflect their environments and completely novel or random creations have little worth (if it is even possible for them to exist). Implicit is the idea that synthetic things are solely a product of the artist's environment processed by his/her machinery of perception. As machines of perception, artists are able to organize, translate and compress their environment; recognizing pattern inherent in the environment or the machine. The output, the art-making, is a process of continued translation and compression as the patterns are reconstructed for social digestion. The artist seems to be a little Turing machine that reads the ones and zeros written in the sky and then registers them with the material at hand. So what does this contribute to a definition? The process of making reminds of the physicist's black box analogy. The box is some aspect of the physical world. It can't be seen (it is in a black box), but every once in a while a little bit pops out of the box, potentially offering information about what is going on inside. Artists peer intently at their black box gathering the scant output. The artist registers what is emitted and decides to relate his observation with the rest of us. Lets say the bits are all ones – thousands of them. Any well intentioned artist would not waste time saying "one, one, one ..." thousands of times. He would simply say "ones come out of that box". He recognized a pattern and structured the information to minimize the description for the rest of us. If the pattern is more complex, our artist will have to use more and more careful observation and insight to discern patterns and structure/compress his description for the rest of us. This discussion becomes more significant when we examine what we mean by synthetic structure. It is easy to discuss the possibility of persistent structure outside of our subjective experience, but the prior example should hint at the difficulty therein. Any descriptive statement made by one individual to another falls squarely in the synthetic category. Since the synthetic structure is dependent on the structure of the viewer's perceptive machinery as we as any objective structure in the environment any claim made for objective structure independent of human observation is called into question. Even if we all perceive the same patterns and structures around us they could be artifacts of our neural machinery. The Structure-Passion CSSS-08 group is composed primarily of scientists. We discussed, that structure in science is something to be discovered or divined through careful observation and analysis and much of the discussion focused on discerning "ideal structure – objective, persistent structure independent of human experience" (some doubts that these exist follow below). There are other members of the group, however whose work emphasizes making as well as observing. Programmers, artists and architects are all constructors in addition to observers and structure is synthesized in their products. Scientist are trained to give precise definitions of their results; artists are encouraged to represent their work subjectively. However, also scientists have to

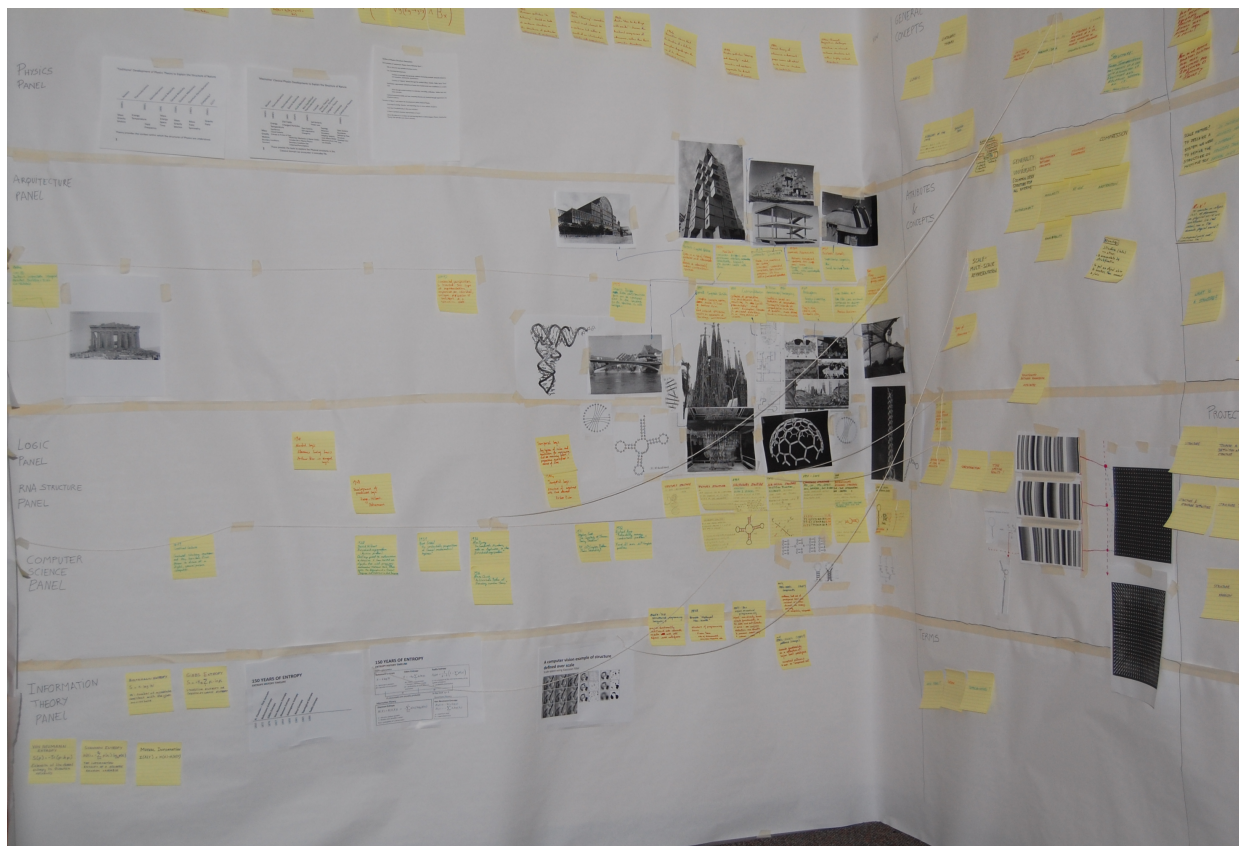


Figure 1: Location St. John's College: suite 3, number 39, upper floor. Our display wall with yellow Post-Its, waste paper, print outs and string. The installation consists of information from people of different disciplines about structure and structure definitions.

choose the parameters, making them “collectively subjective”. We considered to say we are not machines; we are not perfect; in contrast to algorithms we have mistakes in our recursions, as well as the biomolecular machinery. Adding Wiener to this, can we do research, e.g. about biomolecular structure, “in Thinking Turing Machines”? This emphasises that a structure also as a scientific result exist in our heads.

4 Discussion & Future Work

This project is only a starting point to collect definitions of structure in different fields and to go in a deeper discussion to define structure in a general viewpoint. In this short summer school project we have considered: molecular biology, philosophy, logic, architecture, computer science, software engineering, information theory, physics and psychology. We started to elaborate the shared features, attributes and themes of these definitions. Furthermore, we have been likewise studying the history of this concept as it has evolved within these different disciplines. One result of this comparative analysis is that the definition of structure

is continually evolving also in each discipline, meaning the project is a never-ending story. At the moment, we could infer a general challenge between some disciplines that times becomes essential to the definition of structure. Anyway, it is clear, that the question "What is a structure?" cannot be an democratic issue. We have started to look for a hierarchy of the disciplines. Doing this we have been considering Category Theory and Meta-Logic. We have discussed in detail two important paper [1,2] about a universal description of structure. They have suggested that the structures of a given sequence might be associated with those Turing machines that are capable of producing the sequence as output. On the first glance, it seems that Koppel and Wiener have the same concepts, that structures of a given sequence might be associated with those Turing machines that are capable of producing the sequence as output. However, Wiener goes further and combines this approach with self-introspection.

So far none of them provided a universally acceptable definition of the term for all members of the project. It seems, that we have concluded in this paper that structure is a multi-furcated concept. However, in our discussions we developed a closer understanding of a general definition of structure, which we have not been able to define in general yet. In a first step, we tried to point this out in an unusual scientific way: with an installation in our room and video animation for the poster presentation. This shows all the structure definitions simultaneously and could give of each recipient the chance to think about structure on an higher level, helping and stimulating questions. We think, that this is indeed an interesting result and could be the basis for further deeper individual and general research about structure in each discipline and in general.

A fundamental part of life science is the search for structure definitions. However, in the humanities, interest in the definitions of structure has traditionally lagged due to life science, but has expanded exponentially in recent years, e.g. poststructuralist theory (see Appendix). Having said this, there is still a lack of a clear, general definition. Although computer scientist have provided some interesting methods, e.g. TM, for developing general structure definitions. This contrast lack shows that our structure project is a cross-disciplinary issue between life science and humanities. We hope we can include more social scientist in our network. But, we also need new methods to express our definition of structure as from the arts. In the last centuries there has been increasing seperation between art and science, where history shows us art works have inspired scientists and vice versa. Could personal introspection be a life scientific method today like in the past? The reason why we have chosen to call our paper STRUCTURE.PASSION CSSS-08 is to examine structure in all possible ways, reflecting the limitiations of modern scientific methods.

Following our detailed discussion and interpretation, we can conclude that on the one hand, it is indeed possible to find definitions of structure in individual scientific disciplines, which evolved over time: However,

finding a satisfactory definition on the term "structure" that can be agreed upon on a cross-disciplinary basis requires further research, including helpful unusual scientific methods, e.g. from the field of art. We will continue with our discussions internally on our GOOGLE site. Furthermore, some of us will continue our work in future projects. Some artists, galleries and scientist have shown interest in helping with the project in the future. We will continue our internal discussion on a google page and we will set up an official WIKI page in the near future.

Acknowledgement

We thank the Santa Fe Institute for the financial support for attending the Complex Systems Summer School 2008 as well as the participants of CSSS8 for inspiring discussions. Especially the Social Network Group for taking our question in their email survey and all the others for the answering, including Tom Carter for his interest in our project. TG wants specially thanks to Oswald Wiener for stimulating discussions.

References

1. Koppel M: **Structure**. In *The Universal Turing Machine, A Half-Century Survey*. Edited by Herken R, New York: Springer-Verlag 1994:403–419.
2. Wiener O: **Form and Content in Thinking Turing Machines**. In *The Universal Turing Machine, A Half-Century Survey*. Edited by Herken R, New York: Springer-Verlag 1994:583–607.
3. Deleuze G: *Woran erkennt man den Strukturalismus?* Berlin: Merve Verlag 1973.
4. Parisi G: **Complexity in Biology: The Point of View of a Physicist**. *Talk given at Thinkin Science for Teaching: the Case of Physics* 1994, 1:1–5.
5. Wallace D: **Everett and Structure**. *Studies in the History and Philosophy of Modern Physics* 2001, 34:87–105.
6. Bejan A: **Constructal theory of pattern formation**. *Hydrol. Earth Sci.*, 11, 753-768 2007, 11:753–768.
7. Atay F, Jalan S, Jost J: **Randomness, chaos, and structure**. *Chaotic Dynamics* 2008, :xx.
8. Mac Lane S: *Categories for the Working Mathematician*. Berlin: Springer-Verlag 1971.
9. Dalen D: *Logic and Structure (Universitext)*. xxx: Kindle Edition 2004.

10. Gell-Mann M: **What is Complexity?** *Complexity* 1995, **1**.
11. Nelson RJ: **On Mechanical Recognition.** *Philosophy of Science* 1976, **43**:24–52.
12. Fontana W: **Modelling 'evo-devo' with RNA.** *Bioessays* 2002, **24**:1164–1177.
13. Pipas JM, Mahon JEM: **Method for predicting RNA secondary structure.** *Proc Natl Acad Sci USA* 1975, **72**:2017–21.
14. Nussinov R, Pieczenik, G J Griggs, Kleitman D: **Algorithm for loop matchings.** *SIAM J. Appl. Math.* 1978, **35**:68–82.
15. Zuker M, Stiegler P: **Optimal computer folding of large RNA sequences using thermodynamics and auxiliary information.** *Nucleic Acids Res.* 1981, **9**:133–148.
16. McCaskill J: **The equilibrium partition function and base pair binding probabilities for RNA secondary structure.** *Biopolymers* 1990, **29**:1105–1119.
17. Mückstein U, Tafer H, Hackermüller J, Bernhart S, Stadler P, Hofacker I: **Thermodynamics of RNA-RNA binding.** *Bioinformatics* 2006, **22**:1177–1182.
18. Bernhart S, Tafer H, Mückstein U, Flamm C, Stadler P, Hofacker I: **Partition function and base pairing probabilities of RNA heterodimers.** *Algorithms Mol Biol* 2006, **1**:3.
19. Gesell T: **A Phylogenetic Definition of Structure.** *PhD thesis*, University of Vienna, in prep.
20. Gesell T, von Haeseler A: **In silico sequence evolution with site-specific interactions along phylogenetic trees.** *Bioinformatics* 2006, **22**:716–722.
21. Miescher JF: **Über die chemische Zusammensetzung der Eiterzelle.** *Medezinisch-Chemische Untersuchungen* 1969, **4**:441–460.
22. Choudhur S: **The path from nuclein to human genome: A brief history of DNA with a note on human gen sequencing and its impact on future research in biology.** *Bulletin of Science Technology Society* 2003, **23**:360–367.
23. Caspersson T: **Studien bei dem Eiweissumsatz der Zelle.** *Naturwissenschaften* 1941, **29**:xxx.
24. Brachet J: **La Localisation des acides pentosenucliques dans les tissus animaux es les oeligufs d'amphibiens en voie de l'evolppment.** *Arch. Biol.* 1941, **53**:1941.

25. Watson JD, Crick F: **The structure of DNA.** *Cold Spring Harb Symp Quantl Biol* 1953, **18**:123–31.
26. Rich A, Watson JD: **Some relations between DNA and RNA.** *Proc Natl Accad Sci* 1954, **40**:759–64.
27. Hoagland M, Stephenson ML, Scott JF, Hecht LI, Zamecnik P: **A soluble ribonucleic acid intermediate in protein synthesis.** *J Biol Chem* 1958, **231**:241–57.
28. Hoagland M, Zamecnik P, Stephenson ML: **Intermediate reactions in protein biosynthesis.** *Biochim Biophys Acta* 1957, **24**:215–6.
29. Nirenberg M, Leder P, Bernfield M, Brimacombe R, Trupin J, Rottman F, O’Neal C: **RNA codewords and protein synthesis, vii. on the general nature of the RNA code.** *Proc Natl Acad Sci* 1965, **53**:1161–8.
30. Crick FH: **Central dogma of molecular biology.** *Nature* 1970, **227**:561–563.
31. Crick FH: **On protein synthesis.** *Symp. Soc. Exp. Biol.* 1958, :139–163.
32. Nussinov R, Jacobson A: **Fast algorithm for predicting the secondary structure of single-stranded RNA.** *Proc. Natl. Acad. Sci. U.S.A.* 1980, **77**:6309–6313.
33. Guerrier-Takada C, Gardiner K, Marsh T, Pace N, Altman S: **The RNA moiety of ribonuclease P is the catalytic subunit of the enzyme.** *Cell* 1983, **35**:849–857.
34. Kruger K, Grabowski P, Zaug A, Sands J, Gottschling D, Cech T: **Self-splicing RNA: autoexcision and autocyclization of the ribosomal RNA intervening sequence of Tetrahymena.** *Cell* 1982, **31**:147–157.
35. Russell B: **On Denoting.** *Mind* 1905, **14**:479–493.
36. Frege G: *Begriffsschrift, eine der arithmetischen nachgebildete Formelsprache des reinen Denkens.* Halle: Verlag von Louis Nebert 1879.
37. Frege G: **Über Sinn und Bedeutung.** *Zeitschrift für Philosophie und philosophische Kritik* 1892, **100**:25–50.
38. Strawson PF: **On Referring.** *Mind* 1950, **59**:320–334.
39. Grice P: **Meaning.** *The Philosophical Review* 1957, **66**:377–388.

40. Donnellan KS: **Reference and Definite Descriptions**. *Philosophical Review* 1966, **77**:281–304.
41. Kripke S: *Naming and Necessity*. Cambridge: Harvard University Press 1980.
42. Soames S: *Beyond Rigidity: The Unfinished Semantic Agenda of Naming and Necessity*. Oxford: Oxford University Press 2002.
43. Szabó ZG (Ed): *Semantics Versus Pragmatics*. Oxford: Oxford University Press 2005.
44. Gideon S: *Space, Time and Architecture*. Cambridge, Mass: Harvard University Press 1967.
45. Church A: **An unsolvable problem of elementary number theory**. *American Journal of Mathematics* 1936, **58**:345–363.
46. Turing A: **On computable numbers, with an application to the Entscheidungsproblem**. *Proceedings of the London Mathematical Society* 1937, **42**:230–265.
47. Cook S: **The complexity of theorem-proving procedures**. *Proceedings of the third annual ACM symposium on Theory of computing* 1971, :151 – 158.
48. Karp RM: **Reducibility Among Combinatorial Problems**. *Complexity of Computer Computations* 1971, :85–103.
49. SSambasivam, VDBodas: **Entropy: Form follows function**. *Issues in informing science and information technology* 2006, **3**.

5 Appendix

In this short summer school project we have considered: molecular biology, philosophy, logic, architecture, computer science, software engineering, information theory, physics and psychology. Here are some examples in progress. The order is simple how the topics came into the project.

5.1 Molecular Biology: RNA as an Example

Traditionally, **structural biology** has a clear focus on protein structures. This does not come as surprise, given the high complexity of protein structure and their diverse functional spectrum, e.g. catalytic activities. However, today RNA is in the focus of attention of many scientists and **structures of RNA molecules**, are also of functional interest. Different methods have been developed to detect the structure, e.g. crystallography or NMR. But recently the field of Bioinformatics has developed **algorithm**, which give precise

definitions of RNA structure, which can be defined at several levels of resolution. The level known as secondary structure is presently the best **compromise between theoretical tractability and empirical accessibility** on a large scale [12].

Primary structure is defined as the succession of the four different bases: adenine (A), cytosine (C), guanine (G), and uracil (U). RNA is a polymer made of **nucleotide units** consisting of a ribose group, a phosphate on of the four different bases above.

Secondary structure is conventional² a listing of base pairs that can be visualized by a planar graph.

With an adjacency matrix $A = (a_{ij})$ a conventional secondary structure fulfil the following conditions:

1. $a_{i,i+1} = 1$ for $1 \leq i \leq n - 1$
2. For each i , $1 \leq i \leq N$ there is at most one $a_{ij} = 1$ where $j \neq i + 1$ and $j \neq i - 1$.
3. If $a_{ij} = a_{kl} = 1$ and $i < k < j$ then $i < l < j$

The first part defines the continuous “backbone” of the primary structure of the molecule. The second part defines the secondary structure interactions with at most one other nucleotide not immediately adjacent in the backbone. The third part of the definition, excludes interactions that are (some what arbitrarily) classified as tertiary structure interactions.

Tertiary Structure is the three dimensional structure, which is formed through arrangements of the secondary structure elements in space.

Minimum free energy structure (MFE) Based on the definition of secondary structure that can be calculated since 1981 [15], using a dynamic programming algorithm. Zuker’s **k-loop decomposition** identify and classify basic building blocks of a structure, e.g. hairpin and interior loops. The MFE is the structure with the most favourable folding energies. Usually the free energy of folding ΔG relative to the unfolded sequence is considered.

Suboptimal structures accompany the mfe structure and contribute to the molecular properties in the sense of a Boltzmann ensemble (see subsection 5.7). The partition function by Mc Caskill [16] is the proper description of the RNA molecule at thermodynamic equilibrium or in the limit of infinite time.

²“Conventional” means here that the structure is free of pseudoknots. Some other definitions include certain or all classes of pseudoknots.

Consensus Structure infer a common structure for two or more different RNA sequences. Sometimes it is known “a priori” that the aligned sequences should fold into a common secondary structure. This is the case e.g. for rRNAs, tRNAs, and many other small non-coding RNA molecules. In this case it makes sense to ask, what is the most stable structure that can be formed simultaneously by all input sequences. A common method is to use the mutual Information content (Chiu 1991, Gutell 1992), however, recent work has shown that this measure does not detect RNA structure covariation very well (Lindgreen 2006). Some alternative approaches using a sum of energetic and covariance terms (Hofacker 2002) or evolutionary SCFGs (Knudsen 2003) have been implemented.

RNA-RNA Structure Base pairs may be formed within or between molecules. The function of an RNA molecule often depends on its interaction with other RNAs. Straightforward, recently extensions of standard thermodynamic and kinetic folding algorithms was developed to predict structures formed by two RNA molecules upon hybridization [17, 18].

A phylogenetic definition of structure consists of three aspects: The substitution matrix, a neighbourhood system and the phylogenetic tree. The substitution matrix specifies the evolutionary dynamics of nucleotide evolution. However, the matrix is influenced by the neighbourhood system, that defines the interactions among sites in a DNA sequence. The phylogenetic tree introduces an additional dependency pattern in the observed sequences [19]. Based on a concept of a well defined neighbourhood system, SISSI (**SI**multing **SI**te-**SI**pecific-**SI**nteractions) [20] was developed. SISSI simulates the evolution of a nucleotide sequence along a phylogenetic tree taking into account user defined site-specific interactions. Thus, it is possible to mimic **sequence evolution under structural constraints**. Based on these simulations, it is possible to discuss the different definitions of structure, for example single minimum free energy and consensus structure, as well as the definition of structure from a phylogenetic point of view and towards defining **lineage specific neighbourhood systems**.

Timeline

1869 Nucleic acids were first described by Miescher [21].

1909 Some nucleic acids contain ribose founded by Phoebus Levene [22].

1941 Cellular sites of protein synthesis are rich in RNA [23, 24]

1953 Description of the **DNA double helix** [25].

- 1954** RNA was suspected to play a role in information transfer from DNA to proteins [26].
- 1957** Discovery of tRNA [27,28].
- 1964** Deciphering of the genetic code [29].
- 1970** The Central dogma of molecular biology formulated by Francis Crick [30,31]. The central Dogma is a set of rules about the direction that information can flow in a cell. It basically states that there can not be an information transfer originating from protein. The central dogma is not concerned with the role of RNA (DNA, Proteins) in the cell. However, interpretations of the scientific community at large changed the meaning of the central dogma, and RNA was reduced to a carrier of information.
- 1975** First algorithms was developed to predict RNA secondary structure [13].
- 1978** Fast algorithm to predict secondary structure [14,32]
- 1981** Based on the definition of secondary structure the mfe structure can be calculated [15] .
- 1982** Discovery of catalytic activity of RNA in modern cells. For that, Sidney Altman and Thomas Cech won a Nobel prize [33,34], but was not entirely sufficient to remove the carrier of information label from RNA.
- 1990** Discovery of RNA inference as well as the miRNA pathways in animals and plants. This led to a paradigm shift of our understanding of gene regulation and the scientist's perception of RNA, 20 years after the central Dogma of molecular biology.
- 1990** The partition function by Mc Caskill [16] is the proper description of the RNA molecule at thermodynamic equilibrium or in the limit of infinite time.
- 2002** Ribosome is essential a Ribozyme. At least, since this discovery RNA is highly attended by molecular biologists.

5.2 The Philosophy of Language

Advancements in mathematical logic in late nineteenth century proved to have an important impact on the concept of structure as it was applied within philosophy. First order logic offered philosophers with a rich formal language in which to represent the logical forms of various entities, ranging from sentences to scientific theories, thus allowing reasoning involving these entities to be represented within a logic.

In the philosophy of language, the importance of the concept of structure can be seen in the importance attributed to the representation of sentences by their logical form. Such representation was suggested by Bertrand Russell in his classic paper "On Denoting" [35], which proved to have an enormous impact on the subsequent practice of philosophy.

Russell's paper was motivated by concerns regarding the truth values of sentences that contain empty definite descriptions. As an example, consider the following sentence:

1. The present king of France is bald.

The truth or falsity of this sentence would appear to depend upon the properties of the referent of the definite description 'the present king of France.' The logical structure of the sentence appears to be that of

2. Bk

where 'B' is a once place predicate signifying baldness and 'k' is a constant referring to the present King of France. But of course France does not have a King, and so the referent of 'k' is null. If (2) is the logical form of (1) it thus follows that the truth value of (1) is undecided. This means that if we propose to determine the meaning of a sentence in terms of its truth conditions, the sentence is meaningless. And this then results in a number of problems, discussed at length by Russell in his paper.

In "On Denoting," Russell offers a way to avoid these problems through the assignment of meanings to sentences such as (1) via the the assignment of logical forms to these sentences that differ from that of (2). That is, Russell proposes that the logical form of (1) is not indicated by its surface grammar. Instead, Russell proposes that the logical form of (1), and of other sentences that include definite descriptions, is instead

3. $\forall x(Kx \wedge \forall y(Ky \rightarrow y = x) \wedge Bx)$

where 'K' and 'B' are one place predicates stating that an entity is the King of France and is bald, respectively, and ' $\forall y(Ky \rightarrow y = x)$ ' states that any individual that satisfies the 'K' predicate is unique (as required of a definite description). According to Russell, the logical form of (1) thus differs dramatically from the surface grammar of the sentence. Rather than simply assume the existence of an individual 'k', the sentence instead takes the form of an existence claim. And as there is no individual that satisfies this claim, the sentence is thus false.

Within philosophy, the logical structure of a sentence can thus be thought of as its logical form. This form offers a distillation of the fundamental semantic properties of the sentence, determining both its meaning and the role that it plays in logical reasoning.

This concept of logical form found within philosophy thus appears to resemble quite closely the concept of structure proposed by Koppel, who holds that the structure of a sentence, S , is a function, F , such that, when F is input particular data D ,

4. $F(D) = S$.

Considering (4) in the context of the above discussion, we can see that (3) plays the role of 'F', the interpretations of 'K' and 'B', and the specification of the domain of quantification in (3) play the role of 'D', and 'S' plays the role of (1).

Historical Outline

1880's : Frege [36] lays the foundations of first order logic in his *Begriffsschrift*. This amounts to a substantial improvement over past logical systems in that it allows for the representation of subsentential semantic structure.

1880's : Frege [37] lays the foundation for the linguistic turn in philosophy by suggesting that proper names and definite descriptions refer to entities by way of a semantic intermediary, which he terms the sense of the referring expression. The supposition of such senses allows Frege to explain why the semantic content of "The Morning Star is the Morning Star" differs from that of "The Morning Star is the Evening Star." It likewise allows him to explain the role of empty names such as "The present king of France." Frege suggests that the unit of semantic analysis is the sentence and that the referent of a sentence is its truth value.

1905 : Russell [35] publishes his essay "On Denoting" in which he combines the logic and semantic advances of Frege into a single theory of reference. Like Frege, Russell posits the role of a kind of intermediary in determining the referent of a referring expression. However, rather than suppose this intermediary has the structure of some kind of abstract Platonic form, Russell suggests that, when considering the role played by sentences, the intermediary takes the form of a rephrasing of the referent in first order logic.

1950's : Strawson [38] and others [39] challenge Russell's theory for its failure to consider the context in which a sentence is asserted.

1970's : Donnellan [40] and Kripke [41] challenge whether there exists an intermediary that determines the referent of sentence, or if proper names and other referring expression instead refer directly to their referents. This alternate approach to semantics comes to be known as the causal theory of reference and poses a direct challenge to the semantic tradition represented by Russell and Frege.

2000's : Philosophers begin to worry whether meaning should be approached through a formal semantic theory, or instead by way of pragmatic analysis. Soames [42] and others [43] suggest that many problems in the theory of reference can be solved in this way.

5.3 Architecture

Following [44]:

VITRUVIUS THE FUNDAMENTAL PRINCIPLES OF ARCHITECTURE

1. Architecture depends on Order (in Greek [Greek: taxis]), Arrangement (in Greek [Greek: diathesis]), Eurythmy, Symmetry, Propriety, and Economy (in Greek [Greek: oikonomia]).
2. Order gives due measure to the members of a work considered separately, and symmetrical agreement to the proportions of the whole. It is an adjustment according to quantity (in Greek [Greek: posotês]). By this I mean the selection of modules from the members of the work itself and, starting from these individual parts of members, constructing the whole work to correspond. Arrangement includes the putting of things in their proper places and the elegance of effect which is due to adjustments appropriate to the character of the work. Its forms of expression (Greek [Greek: ideai]) are these: groundplan, elevation, and perspective. A groundplan is made by the proper successive use of compasses and rule, through which we get outlines for the plane surfaces of buildings. An elevation is a picture of the front of a building, set upright and properly drawn in the proportions of the contemplated work. Perspective is the method of sketching a front with the sides withdrawing into the background, the lines all meeting in the centre of a circle. All three come of reflexion and invention. Reflexion is careful and laborious thought, and watchful attention directed to the agreeable effect of one's plan. Invention, on the other hand, is the solving of intricate problems and the discovery of new principles by means of brilliancy and versatility. These are the departments belonging under Arrangement.
3. Eurythmy is beauty and fitness in the adjustments of the members. This is found when the members of a work are of a height suited to their breadth, of a breadth suited to their length, and, in a word, when they all correspond symmetrically.

Timeline

StoneHenge 3000 B.C.

Ancient evidence of man's perception of structure in the environment (be it metaphysical or astronomical) and an impulse to reconstruct it. Excellent example of trabeated structure (still most common architectural structure) which is hierarchical and highly redundant.

Romans 75 B.C.

VITRUVIUS

THE FUNDAMENTAL PRINCIPLES OF ARCHITECTURE

1. Architecture depends on Order (in Greek [Greek: taxis]), Arrangement (in Greek [Greek: diathesis]), Eurythmy, Symmetry, Propriety, and Economy (in Greek [Greek: oikonomia]).

The Romans undertook large scale restructuring of their environment (Roads, aqueduct ect, Their ideas about order and arrangement reflects a perceived structure in their world view reflective of Western philosophy (Plato, Aristotle ect.) of the time

Gothic Architecture 1000 AD or thereabouts

Builders attempt to manifest liturgical structure. Civic architecture responds primarily to pragmatic concerns but also incorporate the local, concentric social hierarchy of the period.

1400 Renaissance

Perspective representation is developed and leads to modern, individualist understanding of how people relate to buildings.

1775 Severne Bridge

Iron Bridge modular segments brought to site. Industries ability to "stamp" out identical elements contributes to a conception of structure as modular and abstract (in that it is repeatable and independent from a particular instance).

1851 Crystal Palace, Joseph Paxton truly modern building glass and iron, modular extendable ect.

1890 Gaudi's Sagrada Familia

With hanging models, Gaudi engineers perfectly efficient catenary structure. The complex interactions of the weights in the model let the form and structure emerge from programmatic requirements interacting with gravity.

1910 Cubism (collapse of perspective) Corbusier's promenade, Loos' raumplan. Painting and architecture of motion is a response to Einstein's linking of space and time. Designer's conceive a building as a sequence of experiences. The structure is perceived in little pieces over time, distorted by perspective and never to be revealed as a static whole

1905-1940 Structural Form - concrete bridges of Maillart. The bridges express their loads as continuous catenary structure. The form/structure is a direct response to asymmetry in the environment (gravity) – similar to Gaudi

1915 Dom-ino universal structure that allows free programmatic and plastic expression. "Pure" structure separated from function – a lattice of habitation

1940 Buckminster Fuller's geodesic dome and Tensegrity. Fuller was obsessed with the tetrahedron as a perfect structure and the idea of tensegrity as ultimate structural efficiency. In some respects this harkens back to the ideas of Plato and perfect forms but, unlike most of the platonic forms, Fuller's tensegrity structures continue to be discovered in molecule bonds and atomic structure.

1950 Japanese metabolists. Response to the structure perceived in living things and an understanding of evolution and adaptation as progressive (leading to something better). Capsule living (Kenzo Tange). Housing with exchangeable elements to adapts as people move in and out.

1964 Archigram Fabric and floating architecture Plug-in City, Walking City, Instant City. Architecture changes rapidly through time – light and ephemeral. Like the metabolists, this is an architecture of rapid change and motion based on developments in biology, computer science and the emergence of mass media.

5.4 Computer Science

These are my own definitions and intuitions about structures of human-written programs and are not rigorously defined. Typically, a program's structure is defined by its modularity, consistency, and level of abstraction.

Modularity describes how redundancy is eliminated from the program. For example, if I have to sort an

array of numbers 10 times, then I shouldn't write the sorting code 10 different times. I should write it one time in a function and call that function from the 10 locations. This way, if there is something wrong with my sorting routine, I only have to modify it once. Also, I can reuse the code in any other place in my program. This example is obvious, but sometimes it's not so clear when a new function should be defined, or if code should just be "copied and pasted".

Consistent programming practices, such as indentation guidelines, variable naming, etc., can make programs more readable by others or even the author of the program. Maintainability becomes an issue when naming conventions are not followed.

The right level of abstraction within a program can make maintaining the code much easier. If the program is designed and structured well, then adding new features can be easy. However, if the programmer did not plan for future modifications to the code, then additions to the program may be very difficult.

Data structures are important in the description of structure in software engineering, and they're so important I figured I'd mention it here too. Using the correct data structures can greatly increase the efficiency of a program. For example, searching to find an item in an unsorted list of N elements could take up to N steps. But with hash tables, finding an item can be done in almost constant time. If N is large, or if you have to search for items many times, the benefits of using hash tables will be great. Data structures are crucial for the implementation of algorithms also. Depth first search uses stacks to keep track of backtracking procedures, and queues are used in breadth first searches to determine where to go next. There are times when it seems like a problem cannot be solved efficiently, but with the correct use of data structures, a seemingly intractable problem becomes tractable.

Timeline

300 B.C. Euclid's Greatest Common Divisor algorithm becomes first known algorithm

1679 Gottfried Leibniz may have been first to dream up the concept of a digital computer

1928 David Hilbert proposed the *Entscheidungsproblem* (decision problem)

1936 Alonzo Church's λ -calculus [45]

1937 Alan Turing's Turing Machine [46]

1971 Stephen Cook shows boolean satisfiability problem to be first NP-Complete problem [47]

1972 Richard Karp gives 21 more NP-Complete problems [48]

5.5 Software Engineering

STRUCTURE, ABSTRACTION AND MODULARITY

Structure is used in computer systems to provide abstractions that allow humans to develop and work with large computer programs. Because computer hardware and software are human constructs, the structures associated with computer systems are intrinsic: they are present in the system because we have built them into the system. This discussion focuses on computer software, but many principles apply to computer hardware and computer networks as well.

Very early software was co-designed for specific hardware at the machine instruction (i.e. bit) level. As computers became more powerful and more complex, this became impractical. Programming languages were developed in order to formally describe the sequence of hardware level instructions performed by a computer in a way that allows humans to manage this complexity.

The study of programming languages and the underlying grammatical structures that allow a programming language to be unambiguously translated into a description of the sequence of instructions executed by a program is an important branch of computer science. The practical goal is to develop programming languages that allow programmers to express their program's functionality with minimum risk of mis-expression (aka a bug!), while still ensuring that the language is unambiguous and can be translated into an efficient sequence of bit-level computer instructions.

Two important kinds of structures developed very early: One was the data structure which allowed related pieces of data to be treated as a single logical entity.. An example of a simple data structure is the array (e.g. `x[1],x[2],x[3]`) which allows a sequence of data to be treated as a logical entity. Another example is a data type, which describes (but may or may not enforce) the intended semantics of a piece of data, as in `struct date int year, month, day`. The other key structure was the function, which described a re-usable sequence of instructions with well defined input and output. In addition to allowing code to be re-used, this also allows programs to be composed of sub-structures that are combined in well defined ways.

As programs became larger and more complex, new structures were developed to help manage the complexity. In particular, objects or classes, which abstracts and encapsulates functionality along with the data on which it operates.

In parallel with technical structures of software engineering, there has in recent years been considerable study of the structure of programming teams. Though this topic has been studied even from the 1950's, the increasingly widespread use of computer systems in a range of areas has made human-oriented topics such as ease of program development and the organizational structures increasingly important.

Timeline

1950's imperative programming languages

1960's-70's structured programming (sub-procedures related in a limited set of ways)

1980's-90's object oriented programming (encapsulation of data and functionality)

2000's emphasis on usability and organizational structures (though studied even from 1950's)

5.6 Information Theory

Following [49] and some web-links:³

From an information theory point of view, structure is a property of a system S with an original description X that allows to encode it in a more simplified description, namely Y . Therefore, Y is a new description of X that exploits its structure. If given the description Y we have some function that maps to description X then there is no loss of information in this mapping and the information of X is totally encoded in Y . Then, we can also say that Y is a better description for the original system than X . In information theory, the measure that relates two descriptions of the same system is the mutual information:

$$I(X|Y) = H(X) - H(X|Y) \tag{1}$$

Where $H(X)$ is the information necessary to describe X and the conditional information $H(X|Y)$ is the information necessary to describe Y given X . If $H(X|Y)$ is equal to 0 it means that there is no loss of information in the description Y respect to description X . However this kind of reduction without information loss is not always possible and usually when we get the simplest version of X description exploiting its structure we lose information in such mapping. Hence, in general, the seek for a good description Y is a trade off between the length of Y ($H(Y)$) and the amount of information loss ($H(X|Y)$). This trade-off depends on the specific domain. However, we can affirm in general that given all the descriptions of the same length for the system X , the best description among this set is the one that minimizes $H(X|Y)$. However is not all about compression ... In the section about we describe structure as a property of a system that allows to encode it in a more simple, compressed description. If we focus on a single system it is true but in general, structures (although no compressing the description) are also useful to compare and generalize multiple systems.

³<http://www.scholarpedia.org/article/Entropy>, <http://en.wikipedia.org/wiki/Entropy> and http://en.wikipedia.org/wiki/Information_entropy.

To do that we define an structure as a pair that contains a function and an input for these function. This is a more generalized description of structure because the same function with different inputs can be used as a description of multiple systems.

$$d_s1 = F(I_s1)$$

Where F is a function that fixes an structure for a system and I is the input of this function for the specific system. In this case, make sense to use d_s1 although it is longer than its original description

$$(L(X) < L(d_s1))$$

for two main reasons. This description is more general and allows to compare multiple systems. Thus is:

$$\begin{aligned} d_s1 &= F(l_s1) \\ d_s2 &= F(l_s2) \\ d_s3 &= F(l_s3) \quad \dots \end{aligned}$$

The function F is well studied and we get a lot of knowledge from the system if we can express it in this form

1. Different definitions of entropy . Entropy timeline
2. In classical physics, the entropy of a physical system is proportional to the quantity of energy no longer available to do physical work. Entropy is central to the second law of thermodynamics, which states that in an isolated system any activity increases the entropy
3. In quantum mechanics, von Neumann entropy extends the notion of entropy to quantum systems by means of the density matrix
4. In probability theory, the entropy of a random variable measures the uncertainty about the value that might be assumed by the variable
5. In information theory, the compression entropy of a message (e.g. a computer file) quantifies the information content carried by the message in terms of the best lossless compression rate
6. In the theory of dynamical systems, entropy quantifies the exponential complexity of a dynamical system or the average flow of information per unit of time.
7. In the common sense, entropy means disorder or chaos

Entropy definitions/formulas (150 years of entropy)

1860 Rudolf Clausius Develops first concepts related to the entropy (interior work/external work)

1872 Boltzmann Entropy a type of Gibbs entropy, which neglects internal statistical correlations in the overall particle distribution

1876 Gibbs entropy the usual statistical mechanical entropy of a thermodynamic system

1927 Von Neumann entropy a quantum statistical mechanics definition of entropy, that extends the classical entropy concepts to the field of quantum mechanics by introducing the density matrix

1944 Shannon entropy an information theory definition of entropy that measures the uncertainty associated with a random variable. It quantifies the information contained in a message, usually in bits or bits/symbol. It is the minimum message length necessary to communicate information

1958 Kolmogorov Entropy, Kolmogorov-Sinai Entropy, or KS entropy a mathematical type of entropy in dynamical systems related to measures of partitions

1961 Rényi entropy a generalized entropy measure for fractal systems

1965 Topological entropy a way of defining entropy in an iterated function map in ergodic theory

1988 Tsallis entropy a generalization of the standard Boltzmann-Gibbs entropy

‘Traditional’ Development of Physics Theory to Explain the Structure of Nature

Newton	Thermodynamics	Electrodynamics	Special Relativity	General Relativity	Quantum Mechanics	Quantum Electrodynamics
1600's	1800's	1905	1920's	1960's	2000's	
Mass Gravity Motion	Energy Temperature Field Frequency	Mass Energy Space Time	Mass Gravity Motion	Mass Field Symmetry	Gravity Field	

Theory provides the context within which the structures of Physics are understood.

1

‘Alternative’ Classical Physics Developments to Explain the Structure of Nature

Newton	Thermodynamics	Electrodynamics	Special Relativity	Fluid Mechanics	Non-Equilibrium Thermodynamics	Fractals	Chaos Theory/ Nonlinear Dynamics	Category Theory	Complexity	Constructal Theory
1600's	1800's	1905	1920's	1960's						2000's
Mass Gravity Motion Boundary Conditions Assumed	Energy Temperature Equilibrium Closed Systems Entropy as Arrow of Time	EM Fields Charged Particles	Newtonian Mechanics in Navier-Stokes Includes EM in Plasma Physics Boundary Conditions Set Linearized/Perturbations	Open Systems Self-organization Emergence	Self-Similarity Power Laws		Topology Attractors Simulation Adaptive Systems Local affects Global Macrostructure may not Simplify			Open Systems Boundaries defined by Flow Structure Develops over Time

2

These provide the tools to explore the Physical structures in the Classical domain we encounter in everyday life.

5.7 Post-Structuralis Philosophy

Set/Category theory and observation of structure

PLATO, DESCARTES, KANT, HEGEL on STRUCTURE Philosophy has always searched for structures. Plato would argue that our discussions at today's meeting were great, leading to our admission that we don't know anything about what structure is! In the dialogues, Socrates would always lead his interlocutors to that stage, so that he could then lead them to a 'deeper' structure - transcendent Ideas. Rene Descartes used radical doubt to get to 'clear and distinct ideas' that could not be successfully doubted. Immanuel Kant argued that our very thoughts were conditioned by the built-in categories (ie: number, space) that determine how we experience things, and Hegel thought these structures didn't come built-in by our creator, but evolved through time and history, and much of contemporary philosophy's approach to ideas of structure are footnotes to that insight.

POST-STRUCTURALIST PHILOSOPHY The most important philosophical movement (at least in the continental tradition) of the last 50 years, however, is often known as POST-STRUCTURALISM. Can there be a post-structuralist account of structure, or is that a contradiction? While there are many post-structuralist approaches to the question of structure, the work of French philosopher and mathematician ALAIN BADIOU is likely to help us most in our endeavors. Badiou uses SET THEORY and CATEGORY THEORY to ground his philosophy.

ALAIN BADIOU AND SET THEORY For Badiou, the simplest structure in existence is a SET. A set is anything in which there is a 'count-for-one'. The set of negative numbers contains -5, -3, -5572, and since each of these is a negative number, EACH gets to 'count for one' in the set. What structures a set? Simply the list of its members. In other words, there isn't some eternal definition of negative numbers at work here, but rather, the very name of the set 'negative numbers' is given meaning only by what is included in the set it defines. Set names and their elements define each other. So, if you have the set of 'green objects' and you include one which is kinda off green, to Badiou/set theory this isn't so much an error, but a change of the use of the set called 'green objects.' Structuring sets this way allows for set theory to avoid placing the definition of a set outside of the set. It is both inside and outside the set, or, to use the philosophical term, the definition is 'extimate' to the set, both inside and outside, the very condition of the determination of a boundary between outside and in.

BADIOU AND CATEGORY THEORY For Badiou, SETS only determine that a group of things

exist, each of which gets to 'count for one,' each of which gets called by a name x . In order to think about CHANGE in these elements, Badiou moves to CATEGORY THEORY. This is simply a different type of structure, a new lens through which to view the same set elements. A category is defined, in category theory, as containing all the things within a set which are unchanged by a given morphism. For example, the category of 'mass' in physics allows us to include in that category the ASPECT of things which are unaffected by transformations of displacement in space. The CATEGORY of mass can then be applied, for example, to the SET of 'green things,' but also to many other things. Where the set concept allows us to think membership/being, categories help us to think transformation, as well as which things remain the same/different under transformation.

CATEGORY'S COROLLARIES Paraphrasing Badiou's take on category theory, each CATEGORY implies a GROUP, OBJECT, FIELD, CRITERIA, DECISIONS, and SUBJECT. In the case of mass, we'd say that 'mass' indicates the CATEGORY of objects which remain invariant under the morphism/function 'displacement in space.' In this case, the OBJECT of the category is 'mass', and the implied GROUP is the set of objects with mass. The FIELD is implied in all this - namely, 'space'. Without the implied standard against which invariance/variance under transformation can be judged, it would be impossible to observe change or lack thereof. The field can be uniform (ie: metric space) or locally constructed, potentially up to and including for each object. All of which implies an observing SUBJECT which decides whether or not an object is variant or invariant under a transformation, and these DECISIONS are made by means of CRITERIA. For Badiou, there can be no categories without a subject. In fact, object, subject, group, criteria, category, decisions, and field are MULTIPLE ASPECTS OF THE SAME PROCESS of reciprocal structuration.

OBSERVING STRUCTURE WITH GOEDEL By setting up his system this way, Badiou is able to reconcile his work with RUSSEL'S PARADOX, and its 'solution' in math, namely, GOEDEL'S INCOMPLETENESS THEOREM. Each set and its categories are determined by an observing subject. That subject makes a CUT in the world when it makes decisions. Here another philosopher can help us. Sociologist Niklas Luhman, inventor of 'systems sociology,' has argued that there are two types of ways to 'observe' the world. The first is to observe something out in the world, and in doing so, one distinguishes that thing from all the other 'stuff' one is not currently observing (foreground/background). So, when one is observing a cat, one is not observing 'not-cat'. Observing a cat is an example of what Luhman would call 'first order observation', which always involves producing a 'cut' in the world via

observation. However, if one wants to observe oneself observing, one takes oneself as the object of observation, thereby splitting the world into 'me (that is observing)' and 'not-me.' Luhman calls this 'second order observation.' Luhman argues that what second-order observation gains in complexity, it loses in clarity. In observing oneself observe, the original observation (ie: 'that's a cat') is shown to depend on the ability of a subject to judge that correctly. Its status becomes dependent upon a subject which is itself now an object of observation, leading to an infinite regress. That is, the status of the first observation is now, technically speaking, undecideable, based on an abyss. Going back to Badiou and the issue of the incompleteness theorem, we see how to reconcile Badiou's use of category theory with Goedel's theorem. For Goedel (built in his answer to Russell's 'liar's paradox'), it is impossible for any closed axiomatic system to 'prove' its own foundations. This is actually part of the definition of an axiom. An axiom is always asserted by a subject (ie: a mathematician), who in a sense 'observes' the axiom by delineating it from that to which the axiom does not apply. However, if the subject then wants to observe itself postulating an axiom, the axiom is now based upon an infinite regress (posulated by a subject which postulates itself postulating . . .). It becomes undecideable.

THE PARADOX OF OBSERVATION Based on these ideas, we can say that when one is in the mode of 'first order observation,' there is an IMPLIED STRUCTURE, which acts as the condition whereby one can observe at all, and an EXPLICIT STRUCTURE which is the content of the observation. That is, if you say 'that is a cat', the EXPLICIT structure would be all the things you see that allow you to determine that what you see is truly a cat. But there is another level of structure, namely, the IMPLIED structure, or the conditions which allowed you to make that judgement, say, your existence as a person on planet earth, with standard conditions of gravity, time, space, with decent mental state, etc. If you leave that structure IMPLIED, your observation is based on the circularity of the definition of cat-ness and the furry thing standing in front of you, so your picture of structure here is DECIDEABLE. But is, unfortunately, INCOMPLETE. In order for your picture to be complete, you'd need to do some second- order observation, and include the fact that you were the one to make this determination, and that in order to do this you needed to include all the structure of the universe that allowed this to happen. But as soon as you do this, you hit an infinite regress, because you can always say that even this observation of yourself needs to be observed, ad infinitum. Once you include this infinite regress, your initial observation of a cat is now COMPLETE, BUT UNDECIDEABLE or INCOHERENT as to its certainty and grounding. This is Goedel's insight - we get either incompleteness or incoherence,

the choice is ours.

(ANTI)STRUCTURED SYSTEMS All of which is to say that it is possible to develop a POST-STRUCTURALIST notion of STRUCTURE, so long as structure defined as something which is EX-TIMATE to that which it structures. That is, any system that is structured cannot fully take into account its own structure at the risk of incoherence, though it can PARTIALLY take into account that structure by means of oscillation between first and second order observation. Like quantum entanglement, as soon as you are required to be either complete or coherent, you will suffer from Goedel's curse. But if you can exist in a state of mental 'superposition', then it becomes possible to think of structure as (anti)structure. For Lacan, who was a major influence on Badiou, this sort of complex (anti)structure is best given a name, which he calls the 'object a', or simply [a]. For Lacan, all structure is accomplished via the agency of some structuring term (ie: cat) which occupies the position in a set/category indicated by the notion [a]. The object [a] is a position, not a thing, and many structures can act as an object [a] to a system in need of structure, so long as it acts as I have described, namely, as an (anti)structure. Lacan's naming of the object [a] is simply an act of nomination whereby uncertainty and incompleteness get a form within the domains of completeness and certainty. Badiou keeps this notion, but simply renames it that of an 'event.' An event is both inside and outside a set/category, providing its (anti)structure, giving shape to the system thereby determined. The event is behind every object observable within a given system. Thus, lurking behind the observation 'that is a cat' is the event of my own unique existence in the universe at this time and place. Thus, any object within a system (ie: cat) is always DOUBLE, both inside and outside the system, extimate, in that its own 'legibility' within the system cannot be thought without thinking its relation to the 'event//object [a]' which gives it meaning. Ultimately, the 'event/object [a] isn't quite an object at all, but more a process, but one which has been halted, substantivized as an object by the act of observation. Still, calling it an 'object' or 'event' allows us to think in hypostatized, spatial terms, which is often the form in which we encounter such structure. But it is important to remember that all structure is, in such a context, always in process, a STRUCTURING, related to the past and present of an observer. Taking the past and present out of this, moving us back from category theory to set theory, unfortunately puts us back at the level of first-order observation, with all its inherent paradoxes. Structure is, like an axiom, extimate to its own definition, both process and state, neither and both. Welcome to post-structuralist notions of structure.

5.8 Survey among summer-school participants

We included in the survey among summer-school participants the question:

WHAT IS A STRUCTURE?

A reoccurring pattern of relationships – the arrangement and relations between the parts or elements of something complex – trying to find that out... — an organizing concept – Er, too many definitions! – It's a class for variables in C... it's an organization of things... – it depends. in networks could be a collection of nodes and the connections between them. if in the context of a model is the relationship between variables . the idea is that a structure has a property of 'permanence', even if transitory... – It is a (any) departure from randomness. Or, for instance if we are talking about the correlation structure among a set of variables, or the network structure (among, say, people), then it's the general term describing what we're going to look for—regardless of what we find (e.g., which links are actually present). – something consistent that aims at gathering people/logistics in a definite goal – a framework which controls how a system works – A set of relations between some collection of objects that a) clearly delimits this collection from the collection of all objects and b) organizes the interactions and dynamics within the collection of objects and between that collection and the extra-structural universe. – A structure is any system which defines the way in which objects are grouped or related. It is a specific reasoning framework. – – Good question – Do not understand the question. – simulation – an object in some programming languages; an edifice; – – A description for regularities in the world – describes how parts of a system are related/(inter)dependent and maybe how they fit together to add up to something that is greater than just the sum of the parts – no idea – Something that allows you to describe a system in a more general or a more compact way – How am I supposed to answer this in one line ? – a coherent entity – The connections between individual units (that compose a larger whole) that determine the stability of the whole. – the result of the interlinking of nodes in a network, where dynamics can travel on. The parameters of these structures are from significant importance for the dynamics in such networks. – abstraction that allows us to model something in terms of pieces that are easier to understand. – A structure is a recognized pattern that serves some function. – Structure can also refer to the general properties that a set of items have in common. For example, the structure of a tree would be roots, trunk, branches, leaves, etc. – a entity containing information – A framework to allocate and manage workload within a system – something that defines the nature of interaction and behaviour of "elements" in a system. For example, social structure. – Any organizing macro-event. – that's hard! – A collection of rules or constraints; a object that demonstrates a given set of rules or satisfies a set of constraints. – complex systems with certain elements... – where the interactions between entities in the

system are different from some null hypothesis (exactly what depends on what questions you are asking - e.g. no interactions, random, every interaction exists) – Word with different meanings. – If you know the answer, tell me. – a form, law, institution or norm that bounds activity or behavior – i'd wish you tell me :) – This question is way too ambiguous – a menhir – a type of formal interpretation which consists of a set, functions, and relations defined on the set. – A mathematical object or a data storage object in programming. – heterogenous parts that form patterns of interaction in some stable environment? – nothing – An underlying form that allows for the understanding of an object without explicit reference to its superficial qualities.