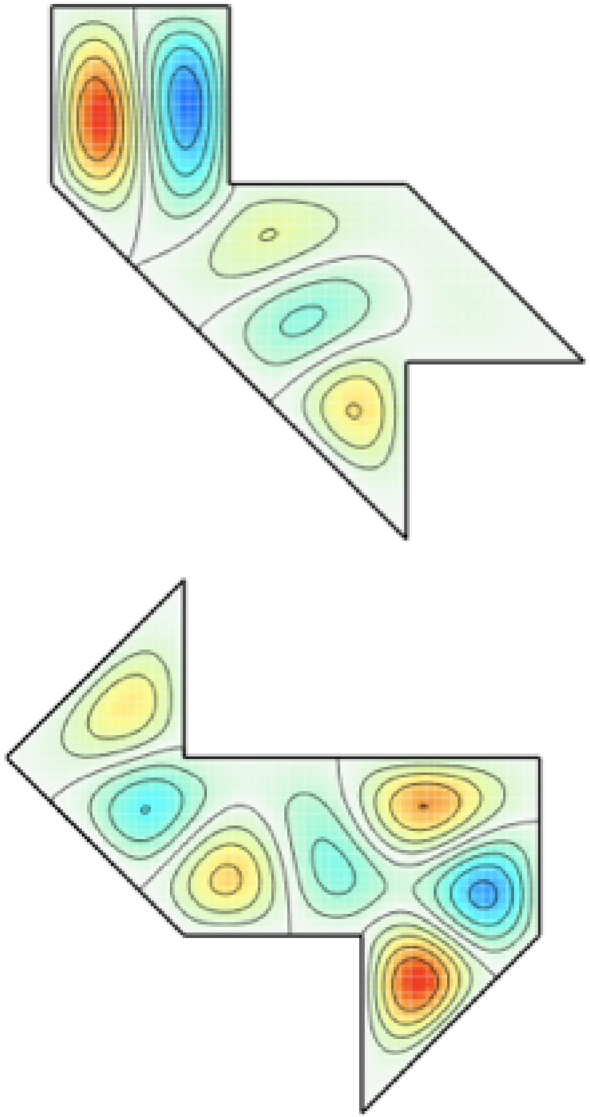
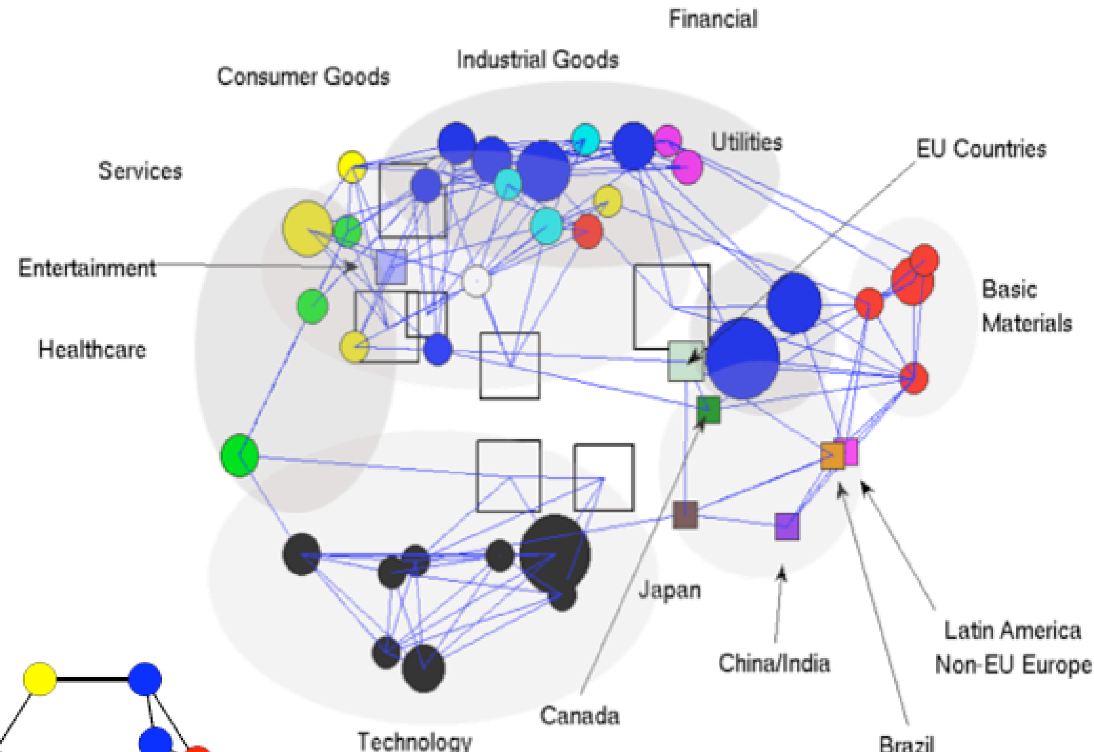
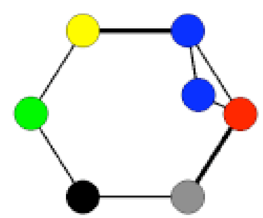
Can you hear the shape of the market?
Lecture 2, CSSS09





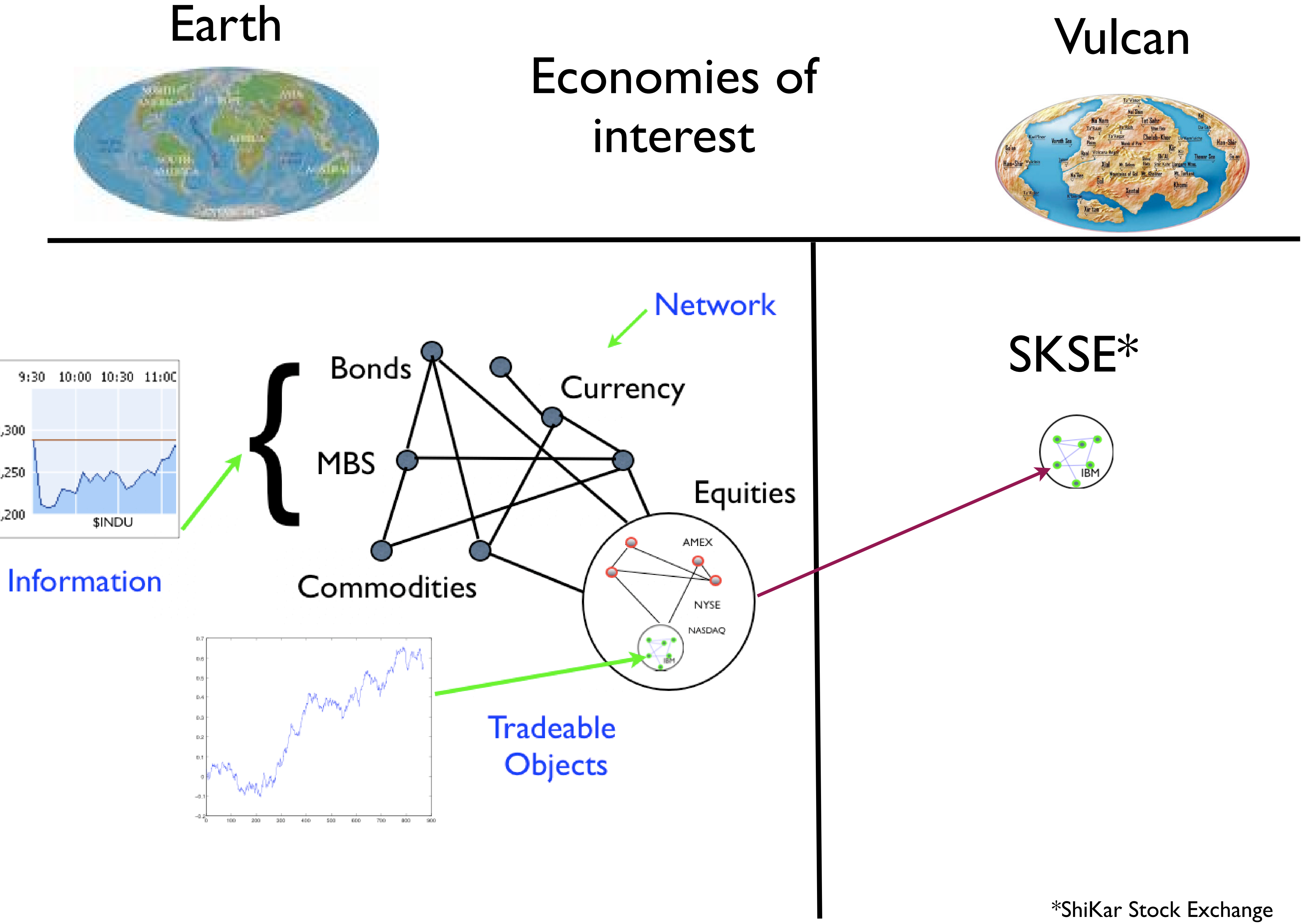


Greg Leibon
Memento, Inc
Dartmouth College

The Plan

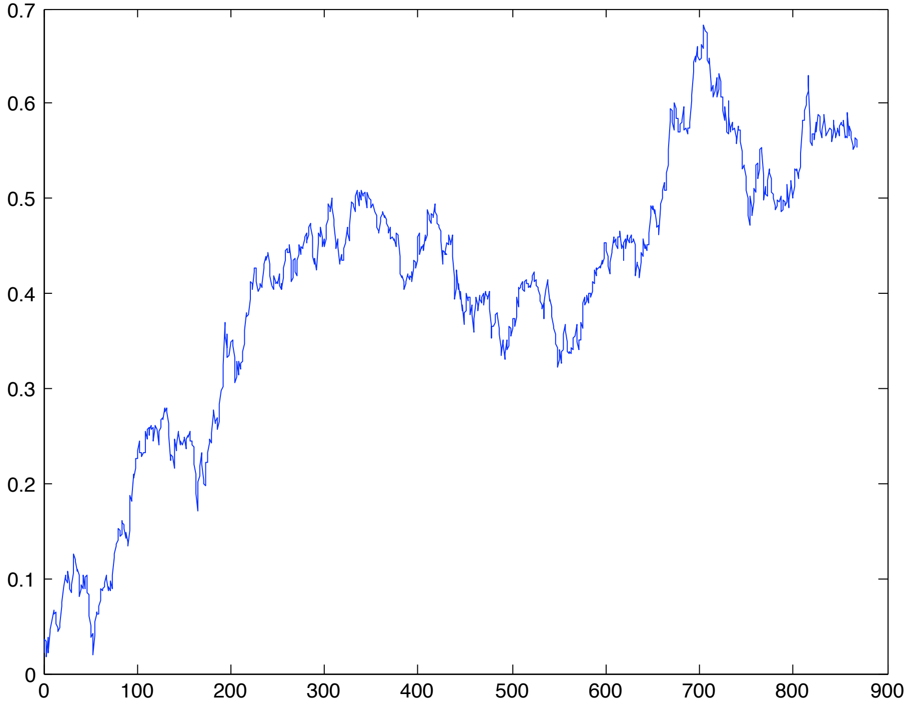
- Part 1: Listening to the Vulcan economy
- Part 2: Listening to the Earth economy

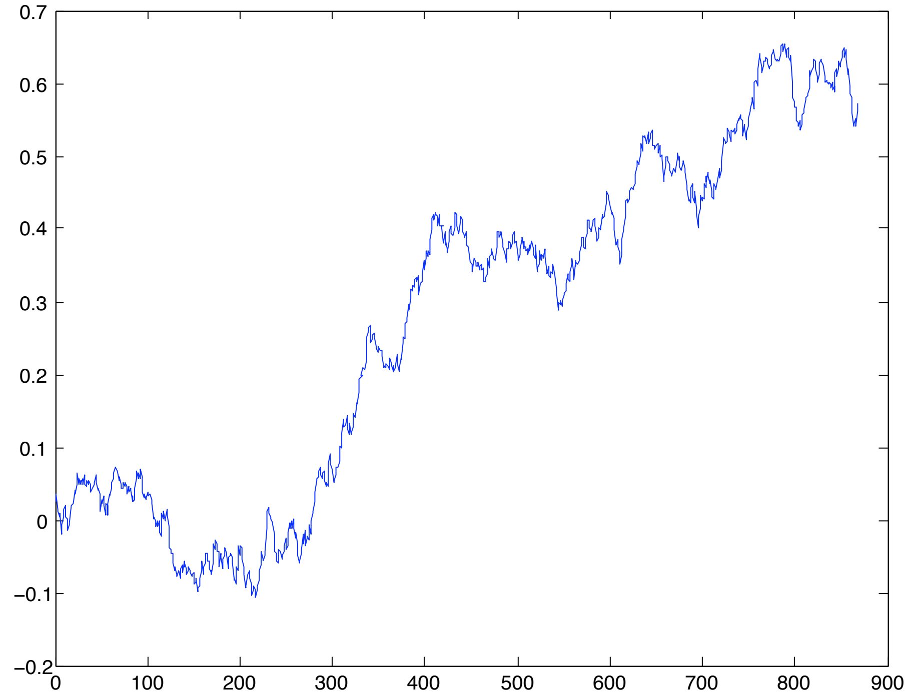
Part 1: The Vulcan Economy


Earth
Economies of interest
Vulcan
Network
Bonds
Currency
MBS
Equities
Commodities
AMEX
NYSE
NASDAQ
IBM
9:30 10:00 10:30 11:0C
,300
,250
,200
$INDU
Information
Tradeable Objects
SKSE*
IBM
*ShiKar Stock Exchange

Equities

$$X_t = \frac{S_t - S_{t-1}}{S_{t-1}} \approx d(\ln(S_t))$$





$$d(\ln(S_t)) = \sigma(t)dB_t + c(t)dt$$

Which is Vulcan and which is Earth?

Geometry of the Market

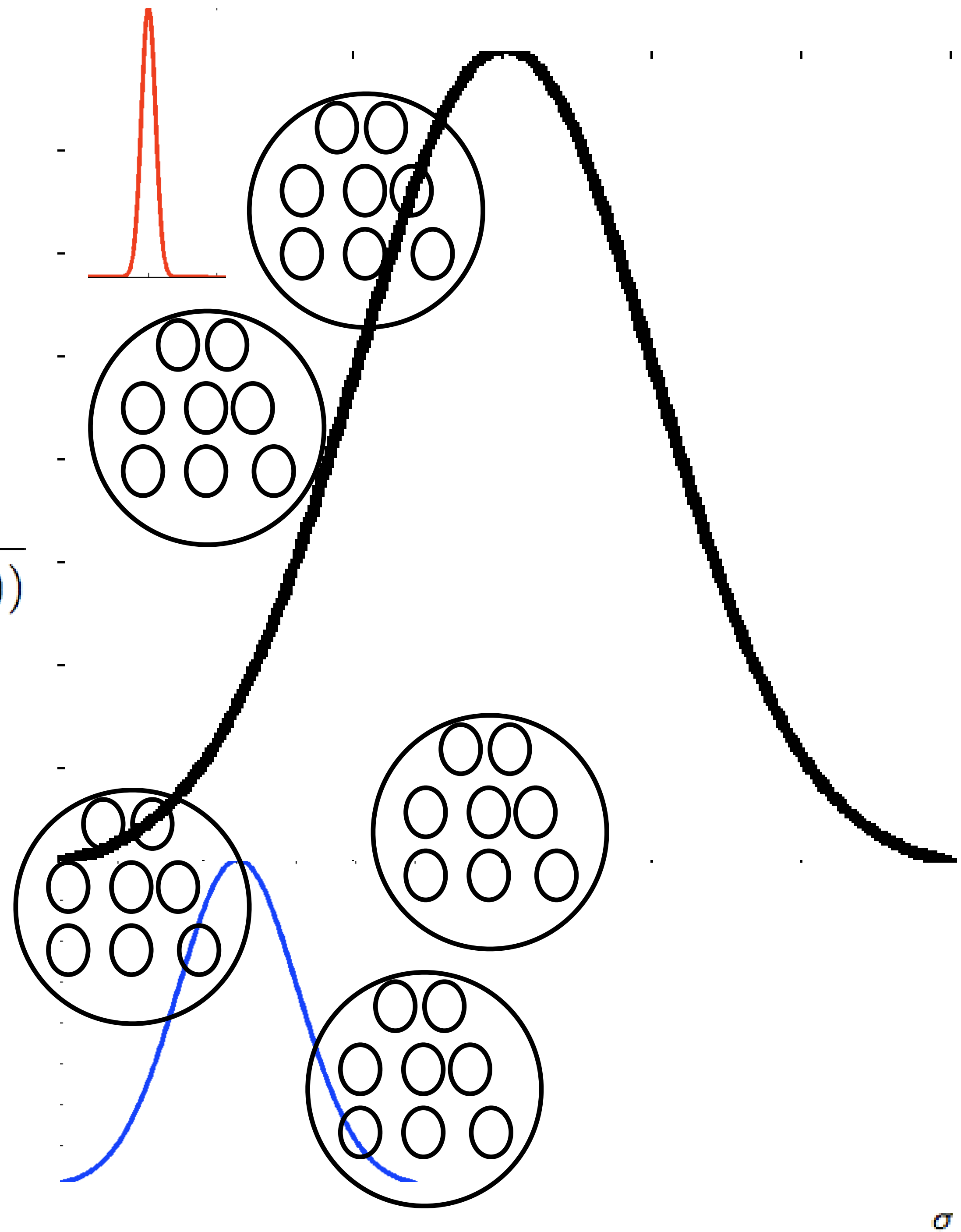
$$\hat{X} = \frac{X - \langle X \rangle}{\sqrt{\left\langle (X - \langle X \rangle)^2 \right\rangle}}$$

$$\rho(X,Y) = \hat{X} \cdot \hat{Y}$$

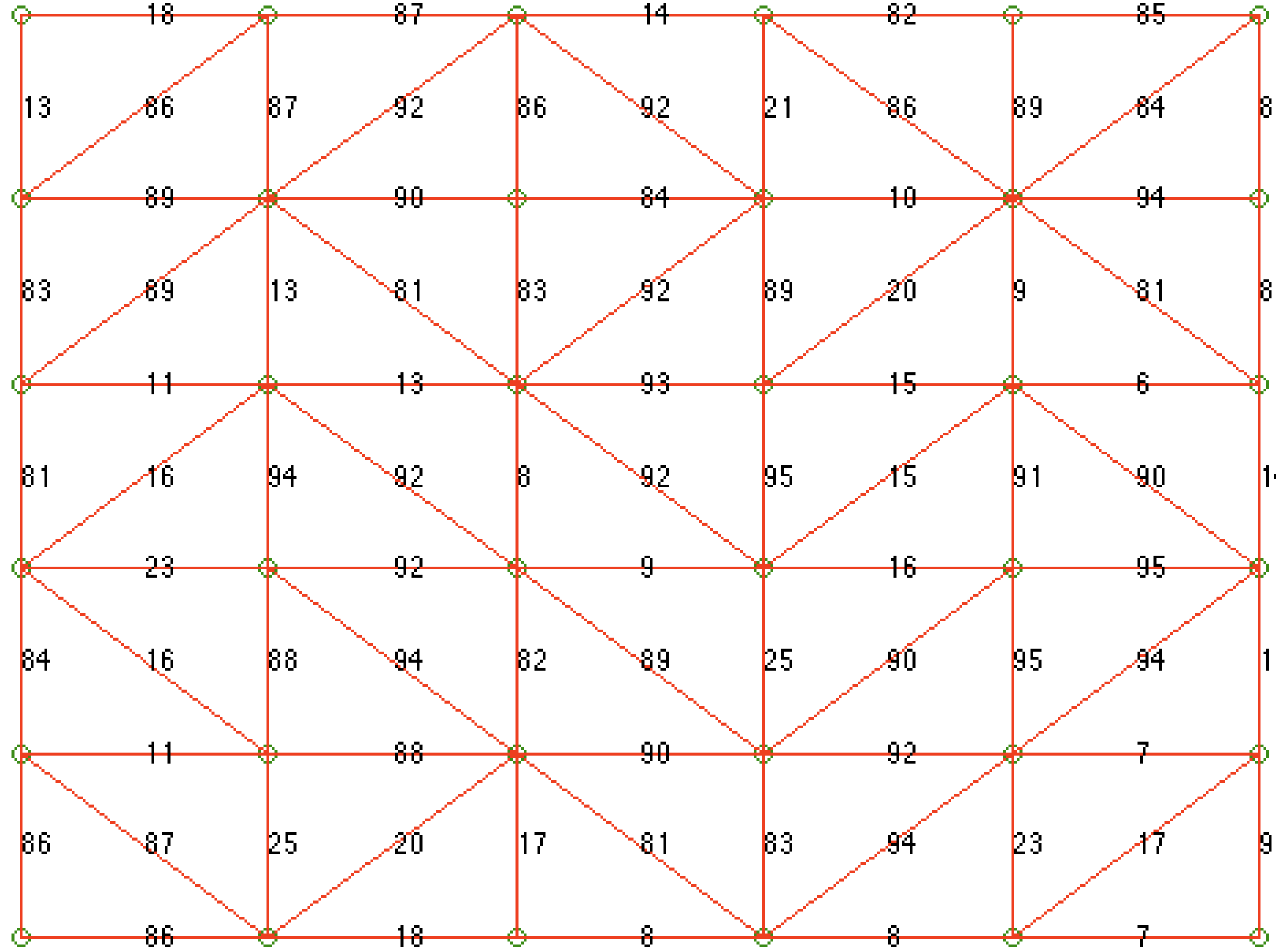
$$d(X,Y) = 2\sin(\theta/2) = \sqrt{2\,(1 - \rho(X,Y))}$$

$$Conductance_s = e^{\frac{-d(X,Y)^2}{\sigma^2}}$$

Scale is key...



Conductance (or similarity) Network.



$$C_i^j = \frac{W_i^j}{\sum_{i,j} W_i^j}$$

```
C=(1/sum(sum(W)))*W;
```

W=

0	86	0	0	0	0	90	0	0	0	0	0	0	0	0	0
86	0	10	0	0	0	96	6	13	0	0	0	0	0	0	0
0	10	0	18	0	0	0	0	6	0	0	0	0	0	0	0
0	0	18	0	8	0	0	0	97	92	93	0	0	0	0	0
0	0	0	8	0	16	0	0	0	0	15	6	0	0	0	0
0	0	0	0	16	0	0	0	0	0	0	17	0	0	0	0
90	96	0	0	0	0	0	17	0	0	0	0	96	0	0	0
0	6	0	0	0	0	17	0	94	0	0	0	13	86	0	0
0	13	6	97	0	0	0	94	0	91	0	0	0	100	96	0
0	0	0	92	0	0	0	0	91	0	95	0	0	0	99	19
0	0	0	93	15	0	0	0	0	95	0	12	0	0	0	0
0	0	0	0	6	17	0	0	0	0	12	0	0	0	0	0
0	0	0	0	0	0	96	13	0	0	0	0	0	8	0	0
0	0	0	0	0	0	0	86	100	0	0	0	8	0	98	0
0	0	0	0	0	0	0	0	96	99	0	0	0	98	0	11
0	0	0	0	0	0	0	0	0	19	0	0	0	0	11	0
0	0	0	0	0	0	0	0	0	90	93	0	0	0	0	11

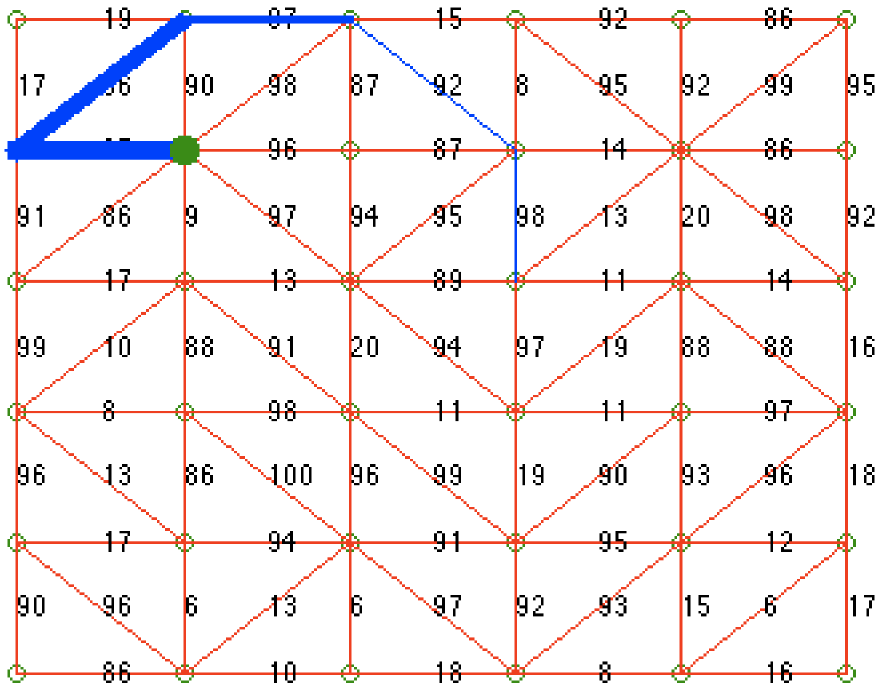
probability vector $$w^j = \sum_i C_i^j$$ w=sum(C,1);

transition matrix $$P_i^j = \frac{C_i^j}{w^j}$$ P=diag(1./w)*C;

Notice

$$diag(w)P = C$$

$$wP = w$$



`states=Snake(500,5,'No');`

Interpret as Random

$$P_i^j = P(X_{t+1} = j \mid X_t = i)$$

States

S(i)=[mod(i,6),floor(i/6)]

....26 32 26 19 25 32 26 21 28 21 28 22 28 33
26 21 15 16 21 16 21 26 27 33 26 25 26 33 32
33 27 26 32 33 26 20 14 20 15 9 4 9 8 9
15 20 14 9 10 15 9 14 8 14 15 9 4 3 2 7
13 7 2 9 8 9 8 9 15 9 4 11 18 17 23 16
22 21 16.....

MatLab:

```
E=repmat(1/36,36,36);
[seq,states] = hmmgenerate(N,P,E);
```

Called a Markov Chain

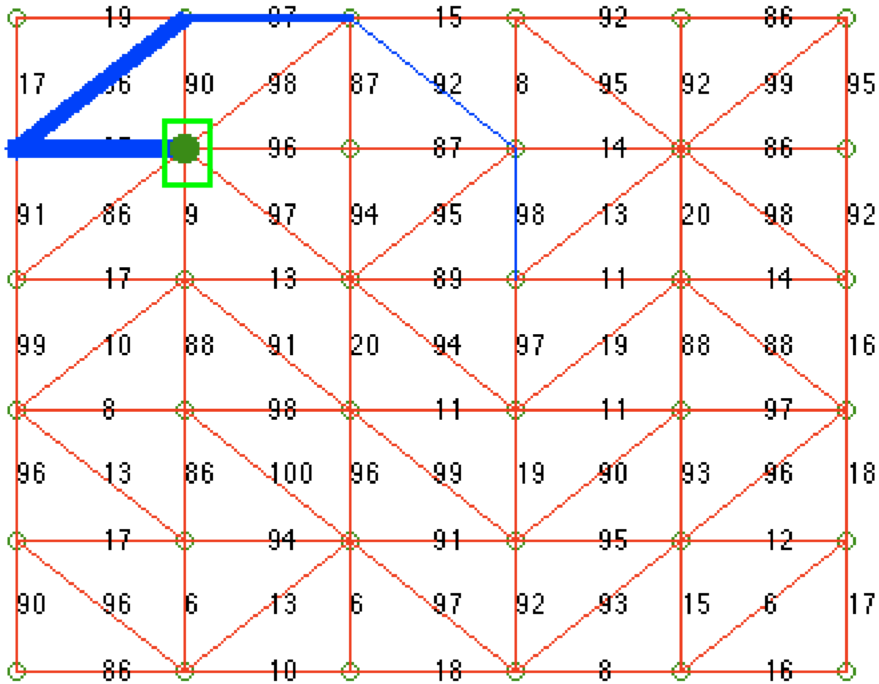
Question: Given I'm in state i what is the probability that I'm in state j after 2 steps?

$$P(X_{t+2} = j \mid X_t = i) = \sum_{k=1}^{|S|} P(X_{t+2} = j \mid X_{t+1} = k) P(X_{t+1} = k \mid X_t = i)$$

$$= \sum_k P_j{}^k P_k{}^i$$

Hey! that's matrix multiplication! In general....

$$P(X_{t+n} = j \mid X_t = i) = (P^n)_i{}^j$$



Equilibrium measure is the average

Text

Law of Large Numbers

$$\lim_{T\to\infty} \frac{\sum_{k=1}^{T} 1_k(i)}{T} = w^j$$

$$1_k(i) = \begin{cases} 1 & X_k = i \\ 0 & X_k \neq i \end{cases}$$

....26 32 26 19 25 32 26 21 28 21 28 22 28 33 26 21 15 16 21
16 21 26 27 33 26 25 26 33 32 33 27 26 32 33 26 20 14 20 15
9 4 9 8 9 15 20 14 9 10 15 9 14 8 14 15 9 4 3 2 7
13 7 2 9 8 9 8 9 15 9 4 11 18 17 23 16 22 21 16.....

....provide the chain is *ergodic*, *meaning* it is possible to get from every state to every other state (eventually)

MECHANIQUE CELESTE

OF

P. S. LAPLACE,

Member of the Institute and of the Bureau of Longitude of France, &c. &c.

When the motions are very small; we may neglect the squares and the products of u, v, and v; the equation (H) then becomes

$$\delta V - \frac{\delta p}{\rho} = \left(\frac{du}{dt}\right).\delta x + \left(\frac{dv}{dt}\right).\delta y + \left(\frac{d\text{v}}{dt}\right).\delta z;$$

therefore in this case $u.\delta x + v.\delta y + \text{v}.\delta z$ is an exact variation, if, as we have supposed, p be a function of ρ; by naming this differential $\delta\varphi$, we shall have

$$V - \int \frac{\delta p}{\rho} = \left(\frac{d\varphi}{dt}\right)^{*};$$

and if the fluid be homogeneous, the equation of continuity will become

$$0 = \left(\frac{d^2\varphi}{dx^2}\right) + \left(\frac{d^2\varphi}{dy^2}\right) + \left(\frac{d^2\varphi}{dz^2}\right).$$

These two equations contain the whole of the theory of very small undulations of homogeneous fluids.

Circe 1799

The Laplacian

$$\Delta f = f(x) - av_{S(x)}(f)$$

$$\Delta f = \lim_{\varepsilon \to 0} \frac{1}{\varepsilon^2}\left(f(x) - \frac{1}{4\pi\varepsilon^2} \int_{S^2(x,\varepsilon)} f(y)\,dA \right)$$

$$= -\left(\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} \right)$$

$$\Delta f = (I - P)f$$

Key property 1: letting $\langle f, g\rangle = \sum_i f_i w^i g_i = f^{tr} diag(w) g$

$$< f, \Delta g > = < \Delta f, g >$$

proof:

$$\langle f, \Delta g\rangle = f^{tr} diag(w)(I - P)g$$

KEY: weights are symmetric

$$= f^{tr}(diag(w) - C)g$$

$$= ((diag(w) - C)f)^{tr} g$$

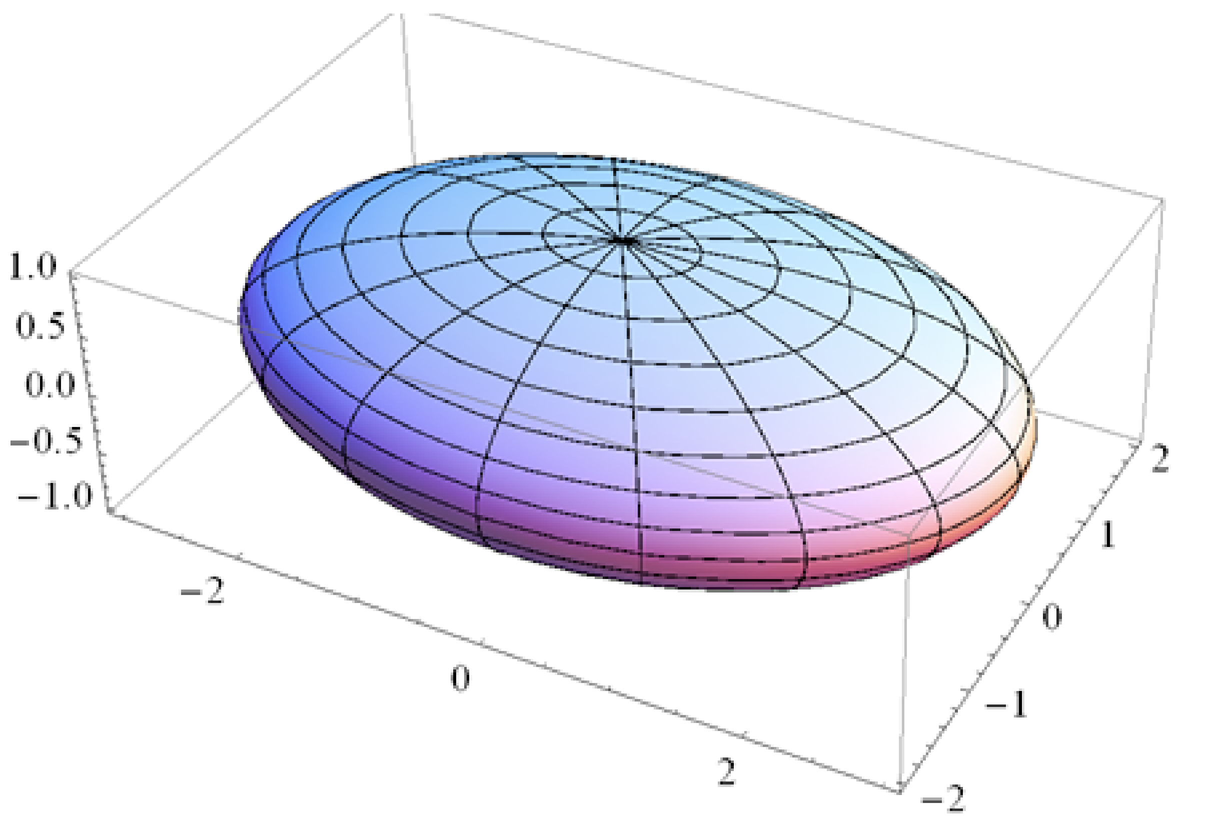
$$= (diag(w)(I - C)f)^{tr} g$$

$$= ((I - C)f)^{tr} diag(w) g = \langle \Delta f, g\rangle$$

Q.E.D

$$< f,\Delta g >=< \Delta f, g >$$

Spectral Theorem (Real): If <,> is an inner product and <Av,w>=<v,Aw>, then there is an orthonormal basis of A eigenvectors.



Find this basis with MatLab....

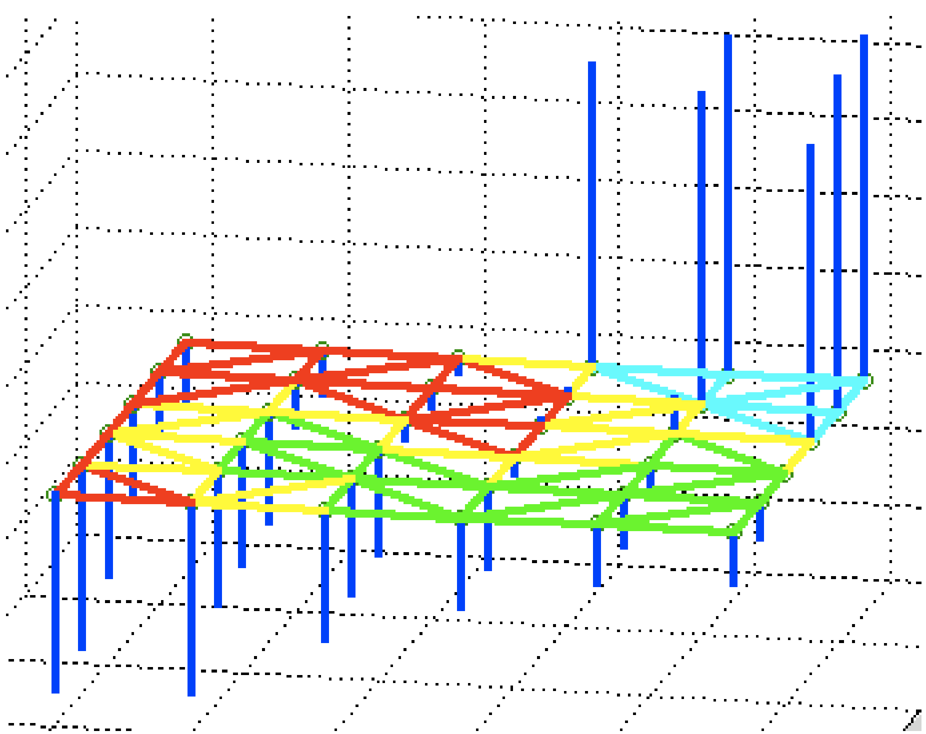
```
    %Create Laplacian and find eigenbasis
Lap=diag(ones(length(P),1)) - P;
[B Eig]=eig(Lap);
    % (Check: Lap=B*Eig*B^(-1))
    %Note: better algorithms for symmetric matrix
                              O=[];
for i=1:length(Lap)
    norm=sqrt((B(:,i))'.*mu*B(:,i));
    O=[O B(:,i)/norm ];
end
    % (Check:  O'*diag(mu)*O)
```

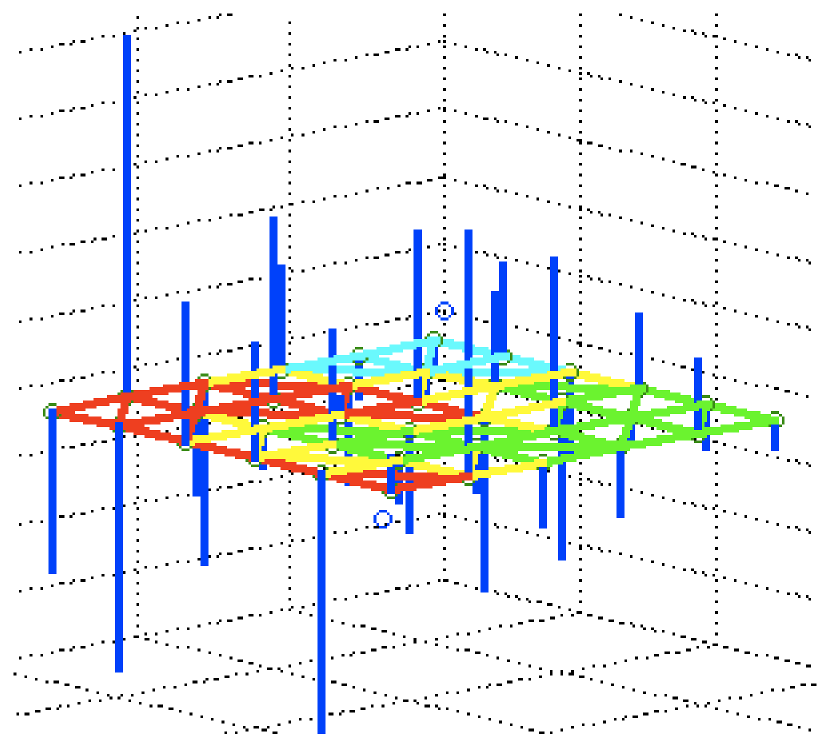
The Green-Kelvin Identity

$$\langle f, \Delta f \rangle = \int f \Delta f d\vec{x} = \int |\nabla f|^2 d\vec{x} \qquad \langle f, g \rangle = \int_M f(x) g(x) dVol(x)$$

$$\langle f, g \rangle = \sum_i f_i w^i g_i$$

$$\langle f, \Delta f \rangle = \sum_{i,j} f_j w^i (I_i^{\ j} - P_i^{\ j}) f_j = \frac{1}{2} \sum_{i,j} w^i P_i^{\ j} \left(f_i - f_j \right)^2$$





Key Property 2: Proof

$$\langle f, \Delta f \rangle = \frac{1}{2} \sum_{i,j} w^i P_i^j (f_i - f_j)^2 >= 0$$

Proof:

$$\langle f, \Delta f \rangle = \sum_{i,j} f_i w^i (I_{ij} - P_i^j) f_j$$

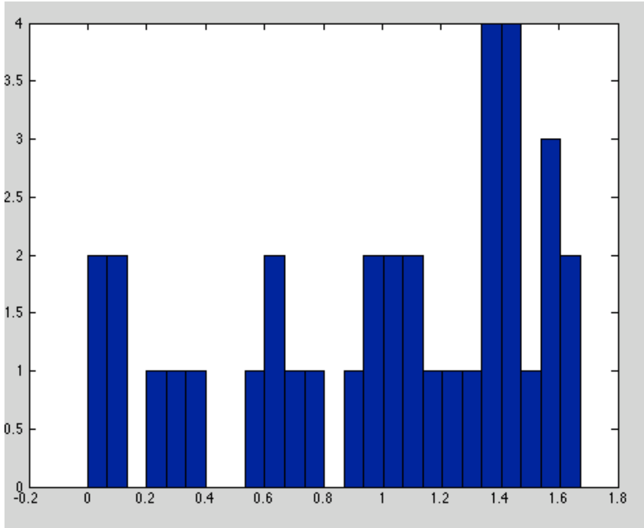
$$= \frac{1}{2} \sum_i w^i f_i^2 - \sum_{i,j} f_i w^i P_i^j f_j + \frac{1}{2} \sum_j w^i f_j^2$$

$$= \frac{1}{2} \left(\sum_{ij} w^i P_i^j f_i^2 - 2 \sum_{i,j} w^i P_i^j f_i f_j + \sum_{i,j} w^j P_i^j f_j^2 \right)$$

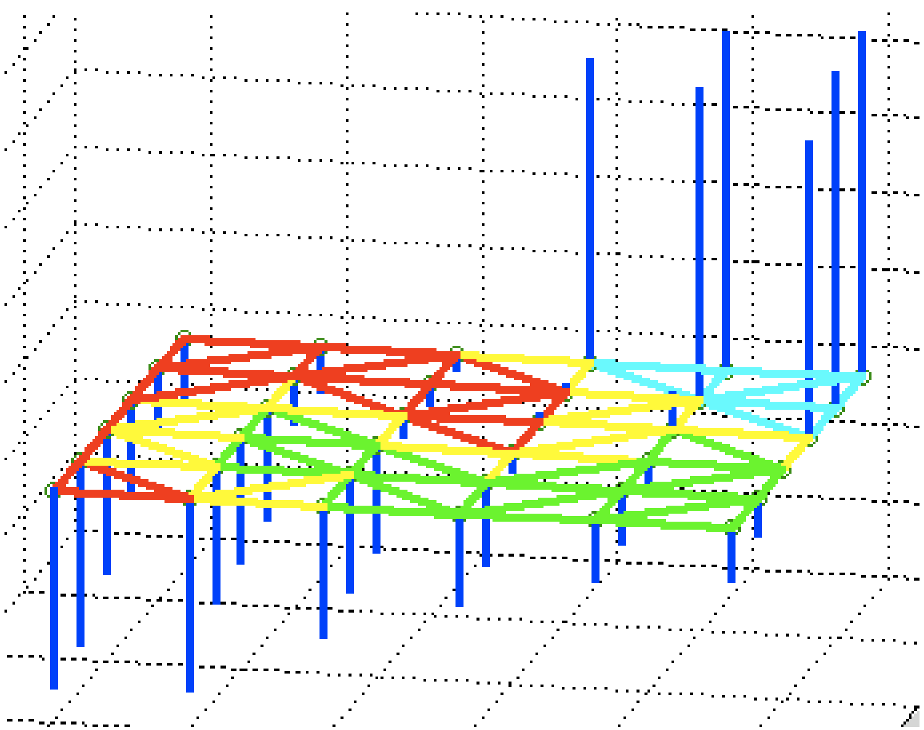
$$= \frac{1}{2} \sum_{ij} w^i P_i^j (f_i^2 - 2 f_i f_j + f_j^2)$$

row sum is one

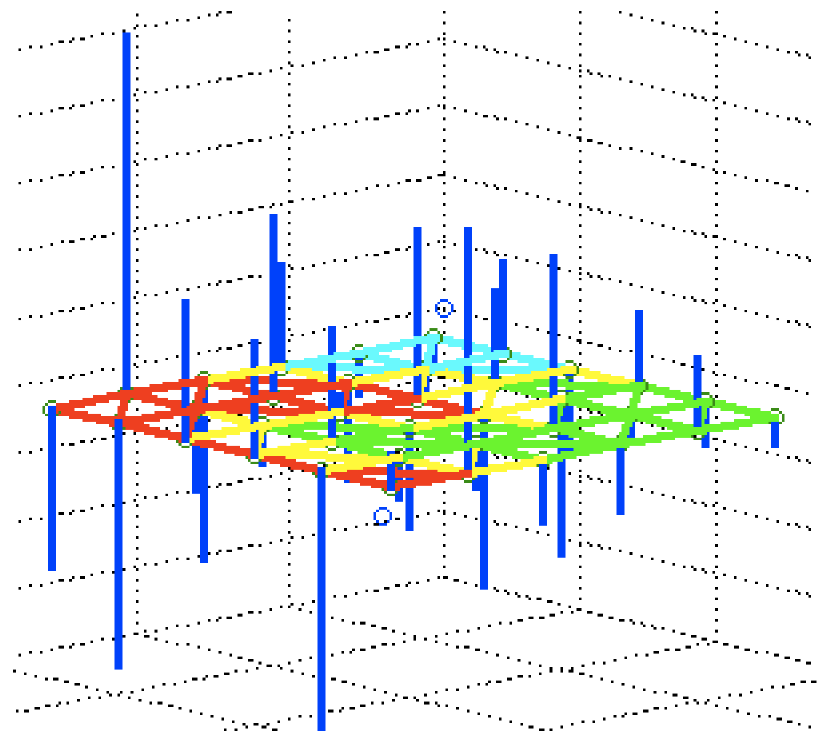
is equilibrium measure



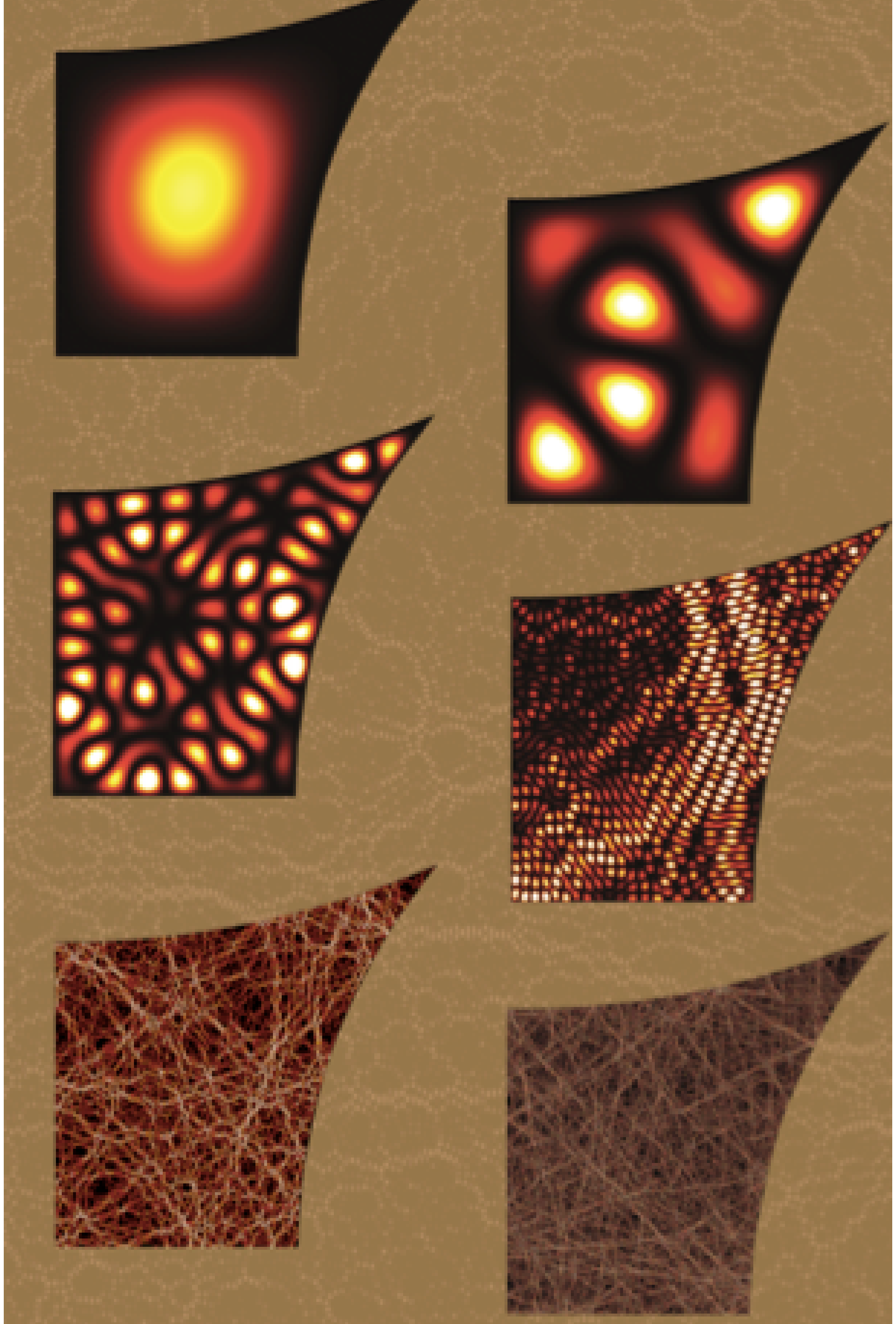
Eigenvalues are the frequency of oscillation



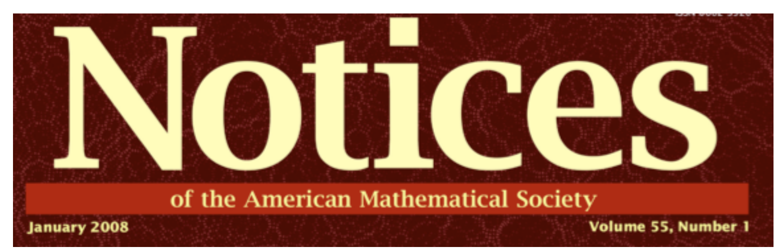
λ_2



λ_{30}



Eigenvalues are the
"frequency of oscillation"
of the eigenfunctions.
...look at nodal lines show



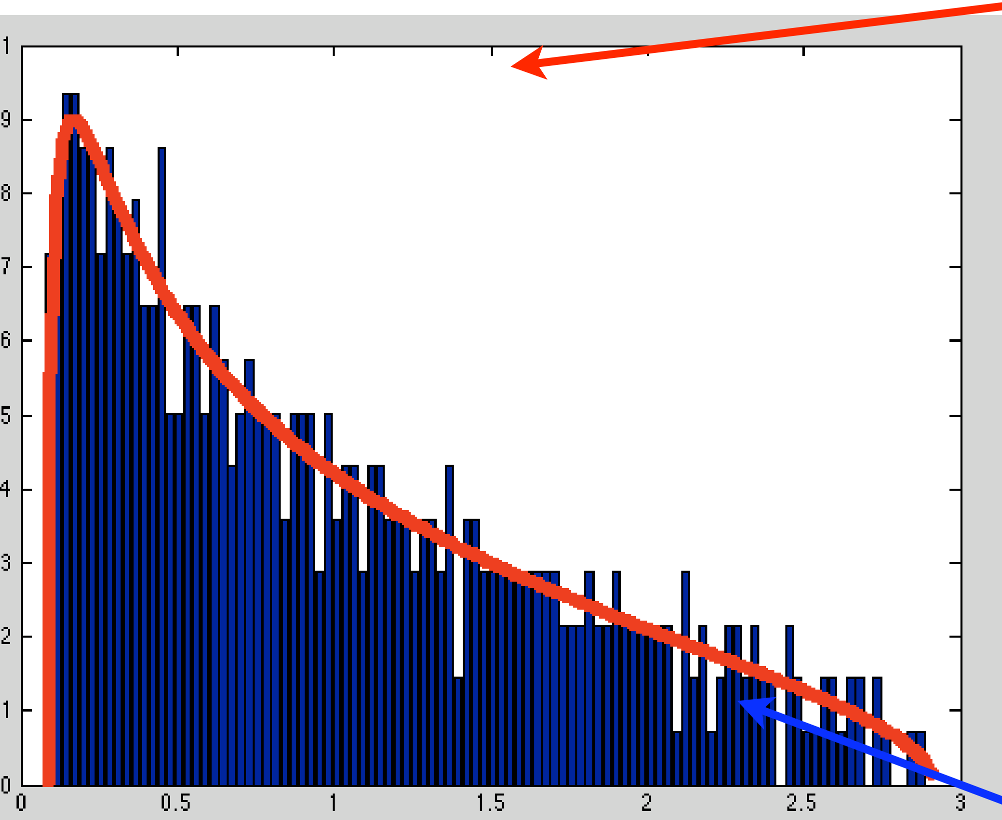
Alex Barnett

Back to the Vulcan market

No Structure (null model)

Universal and Nonuniversal Properties of Cross Correlations in Financial Time Series

Vasiliki Plerou,[1,2] Parameswaran Gopikrishnan,[1] Bernd Rosenow,[3] Luís A. Nunes Amaral,[1] and H. Eugene Stanley[1]



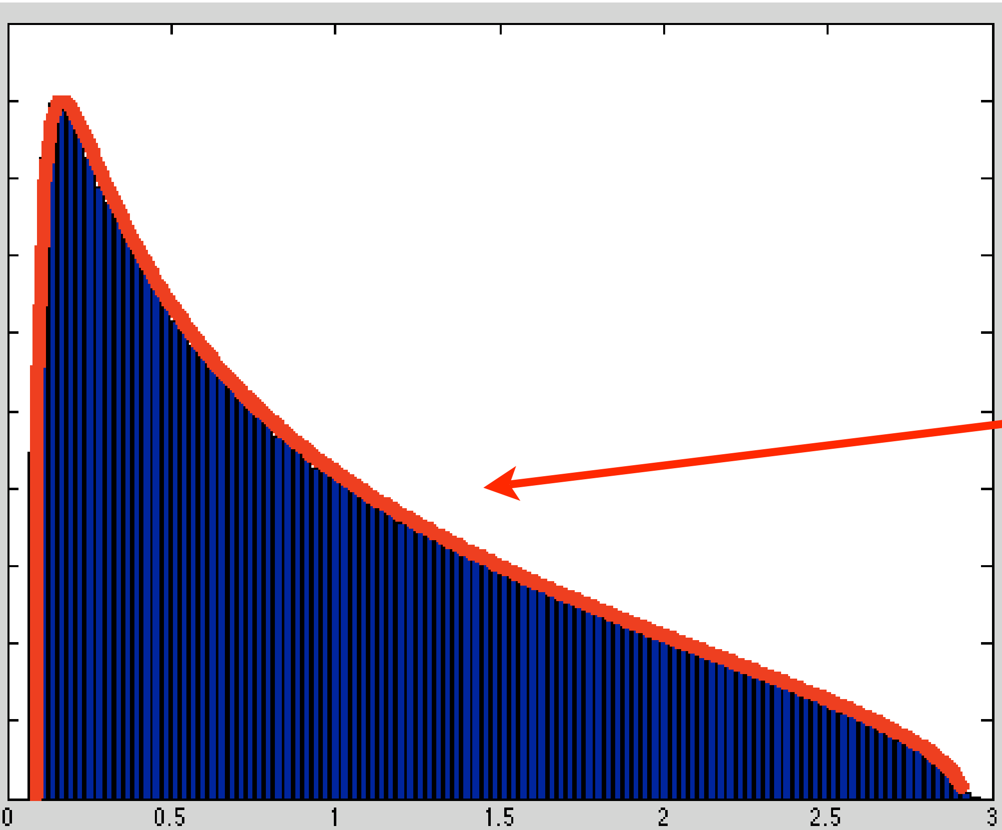
Statistical properties of random matrices such as R are known [26,27]. Particularly, in the limit $N \to \infty$, $L \to \infty$, such that $Q \equiv L/N$ (>1) is fixed, it was shown analytically [27] that the probability density function $P_{rm}(\lambda)$ of eigenvalues λ of the random correlation matrix R is given by

$$P_{rm}(\lambda) = \frac{Q}{2\pi} \frac{\sqrt{(\lambda_+ - \lambda)(\lambda - \lambda_-)}}{\lambda}, \quad (6)$$

for λ within the bounds $\lambda_- \leqslant \lambda_i \leqslant \lambda_+$, where λ_- and λ_+ are the minimum and maximum eigenvalues of R, respectively, given by

$$\lambda_\pm = 1 + \frac{1}{Q} \pm 2\sqrt{\frac{1}{Q}}. \quad (7)$$

Histogram,
1 runs,
N=500,
L=1000



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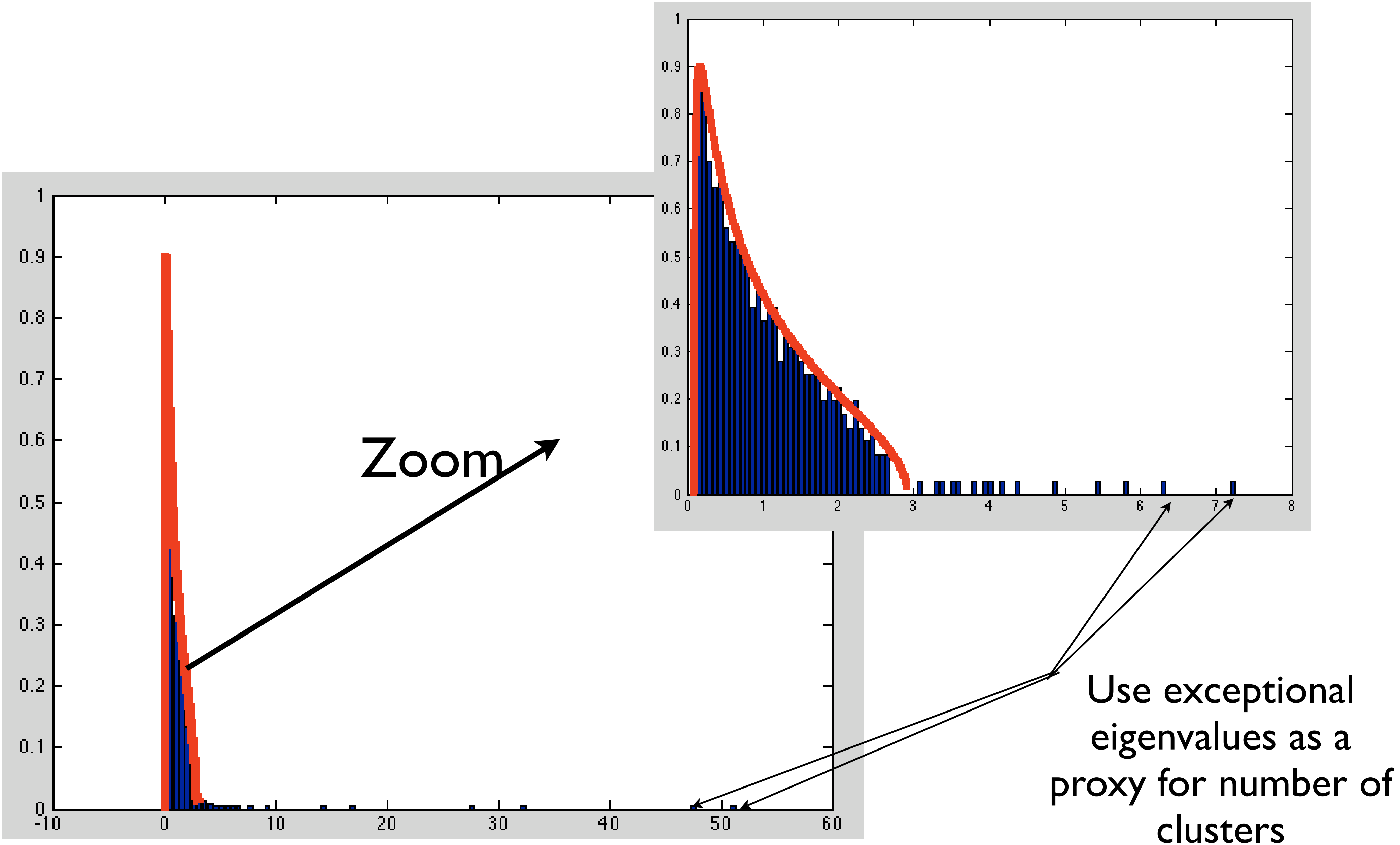
$$\lambda_\pm = 1 + \frac{1}{Q} \pm 2\sqrt{\frac{1}{Q}}. \tag{7}$$

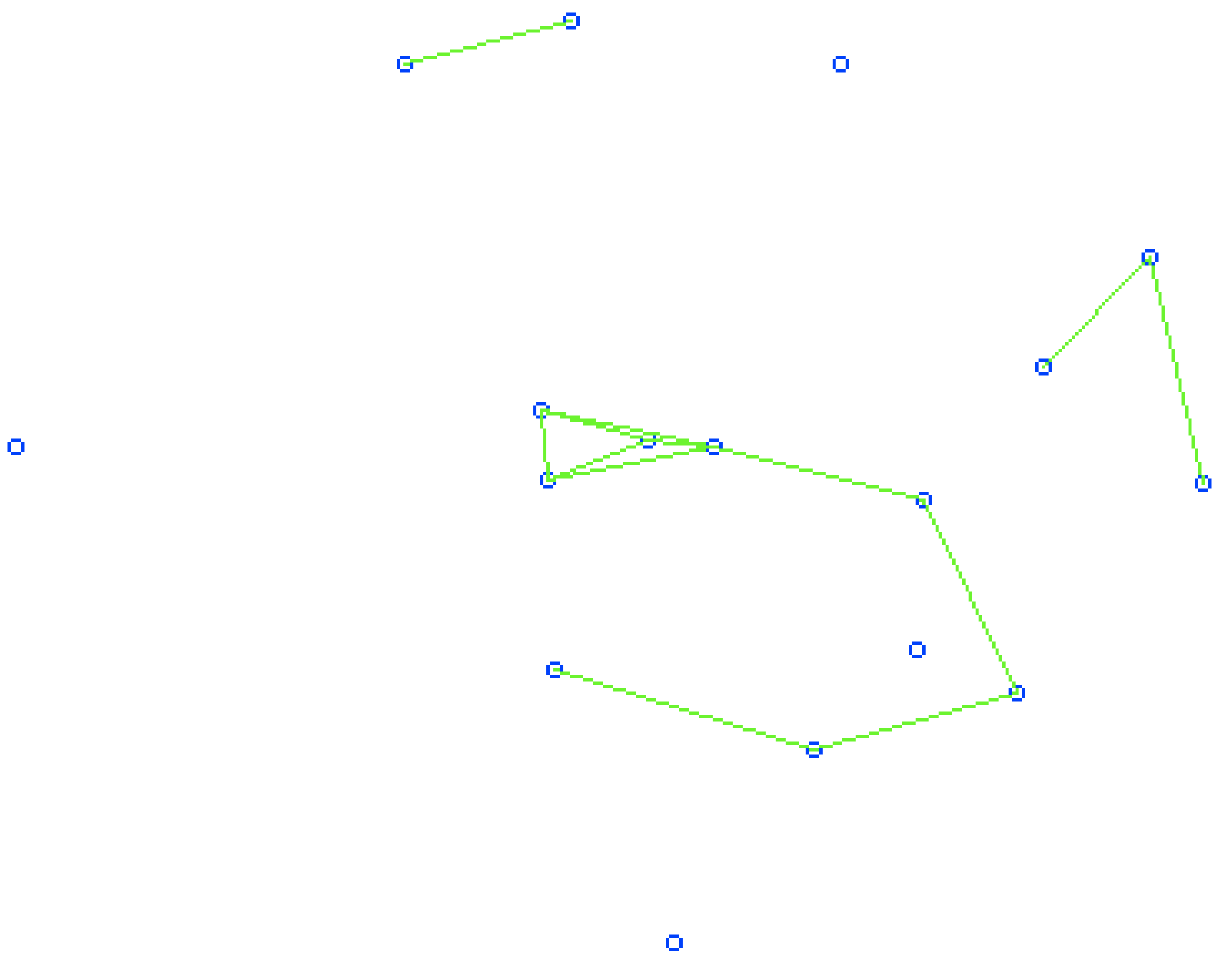
Histogram,
200 runs,
LN=500,
L=1000

The Vulcan Market

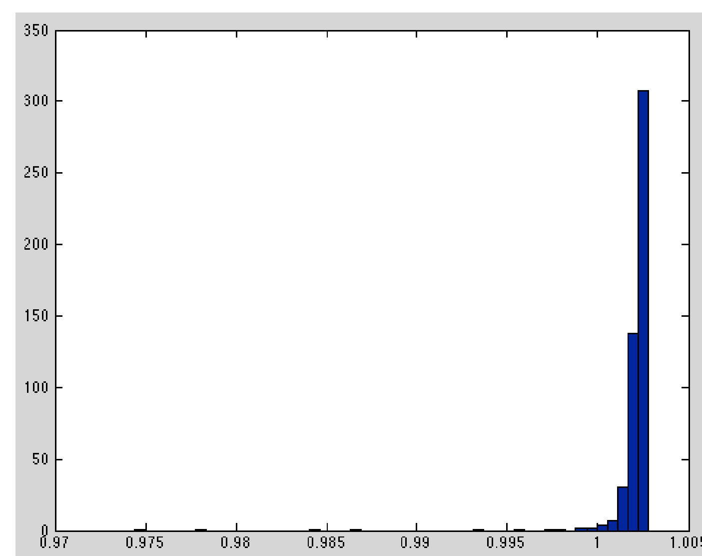


Find the find geometry.....

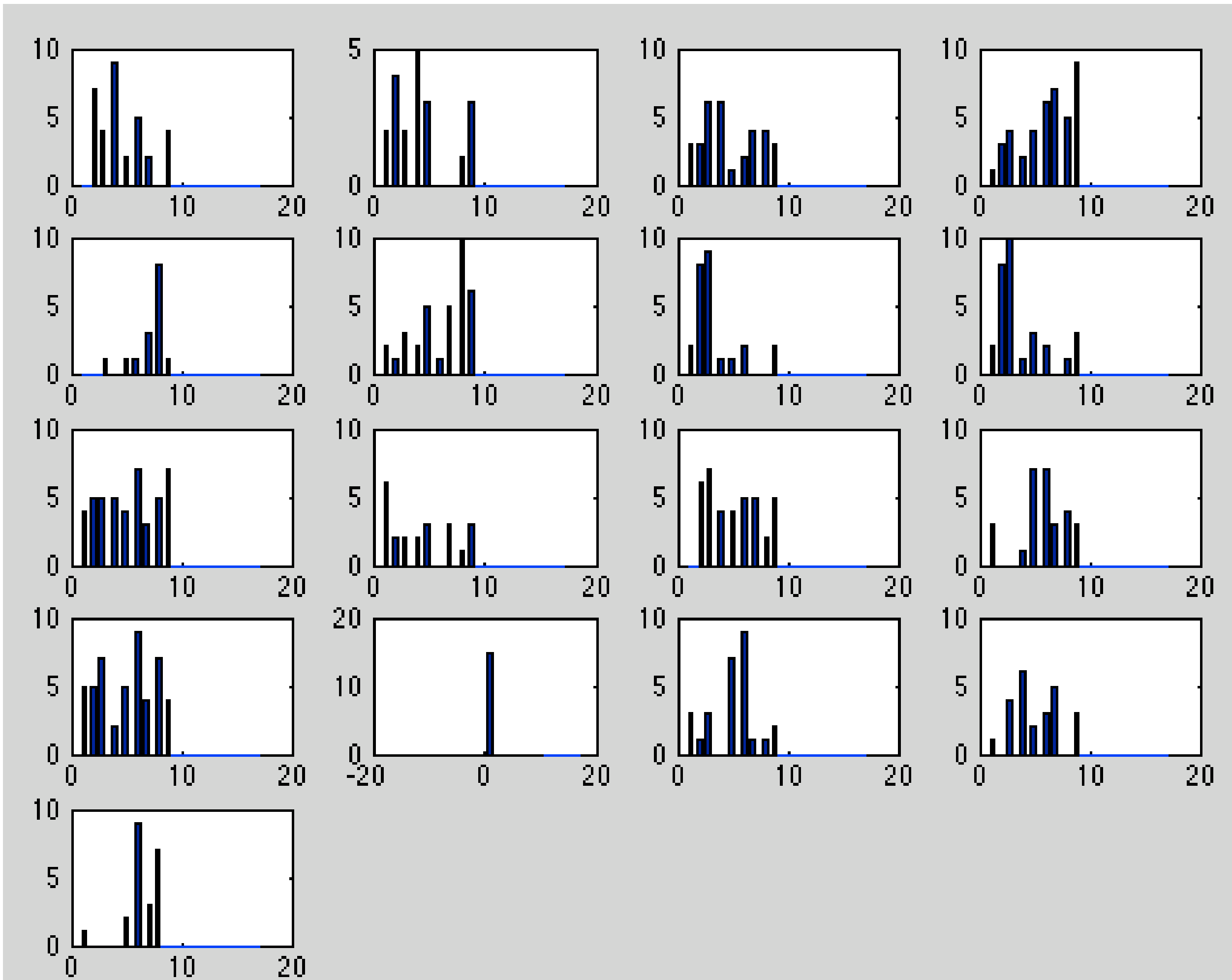
17 Clusters



Original Distribution

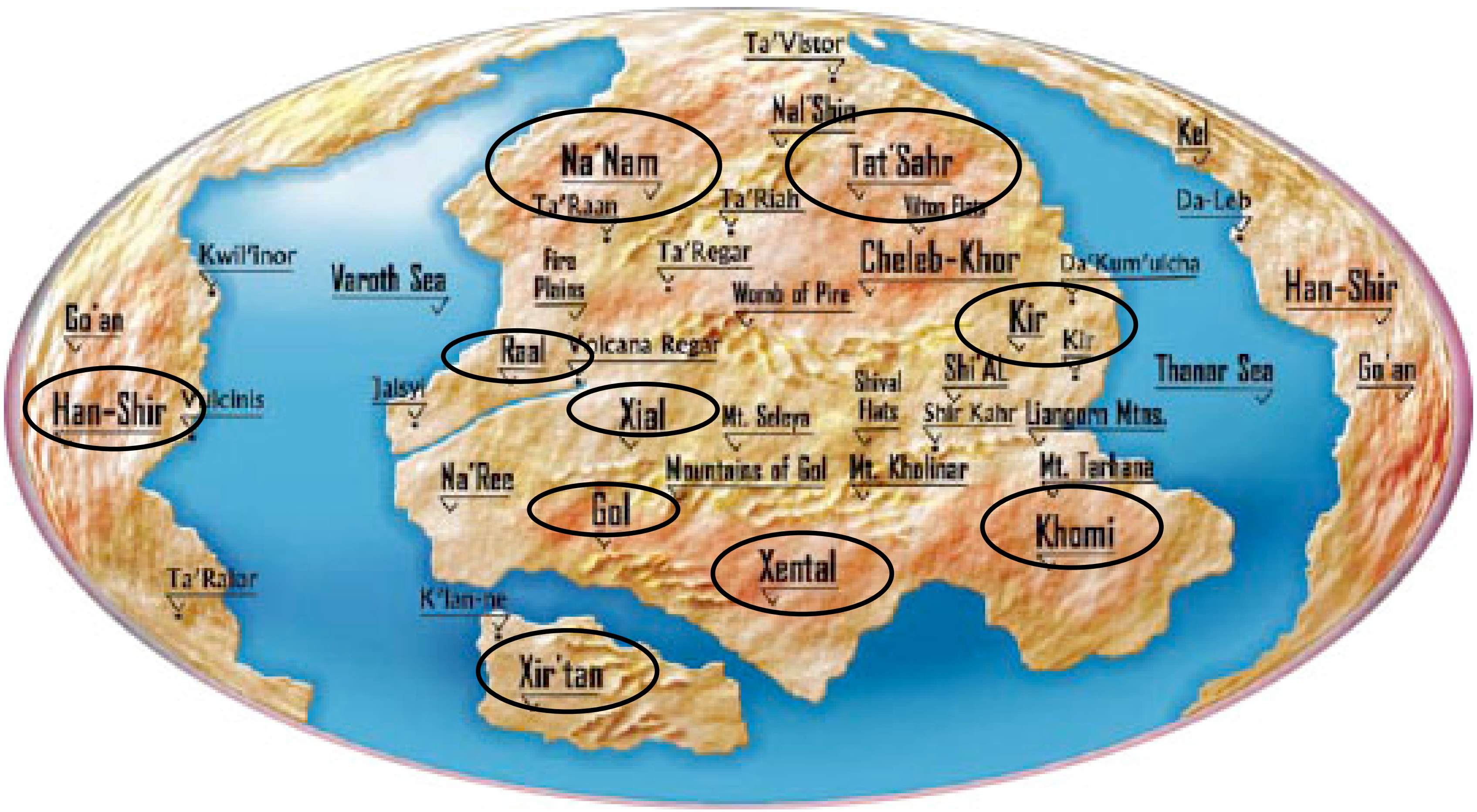


Not Sectors....



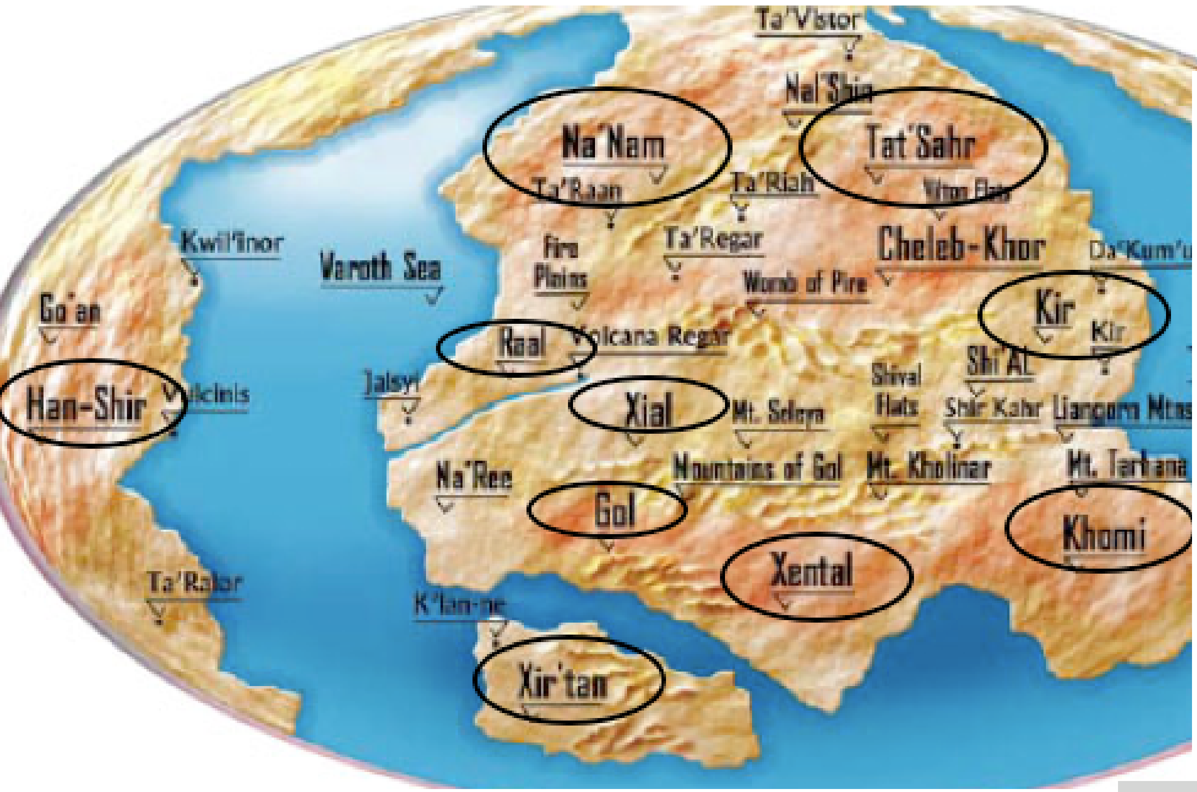
1 Basic Materials
2 Conglomerates
3 Consumer Goods
4 Financial
5 Healthcare
6 Industrial Goods
7 Services
8 Technology
9 Utilities

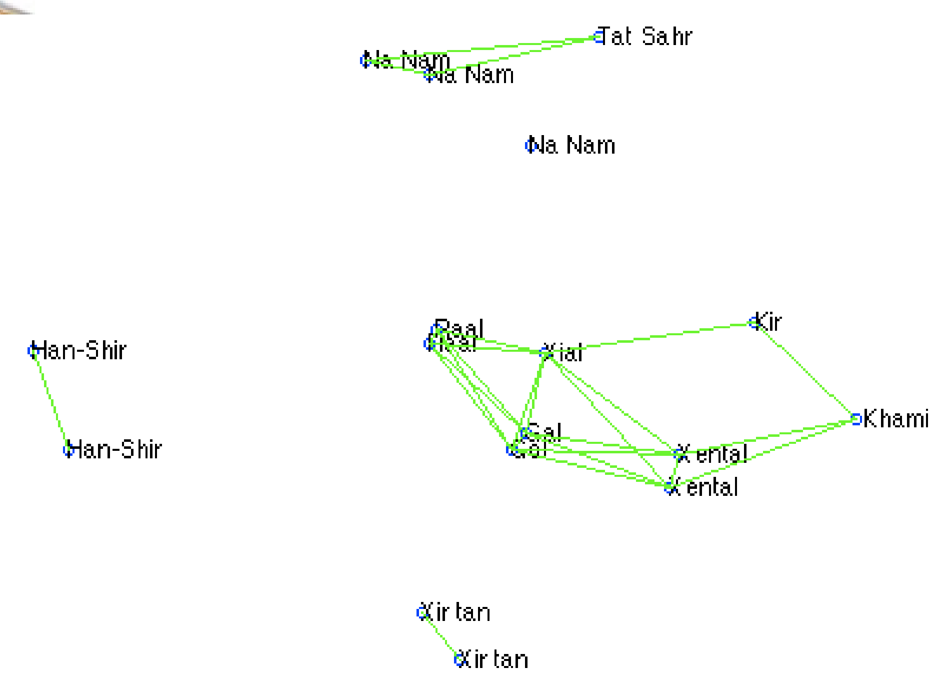
Geographic Isolation

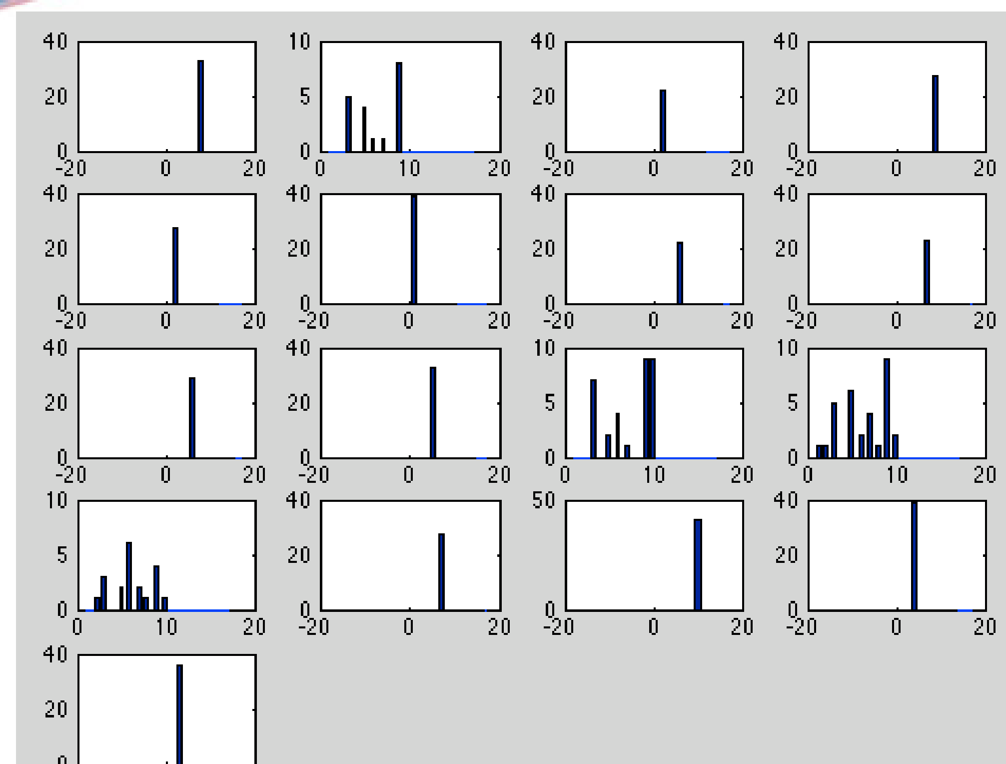


Vulcan Economic Hubs

"Seems logical to me captain"

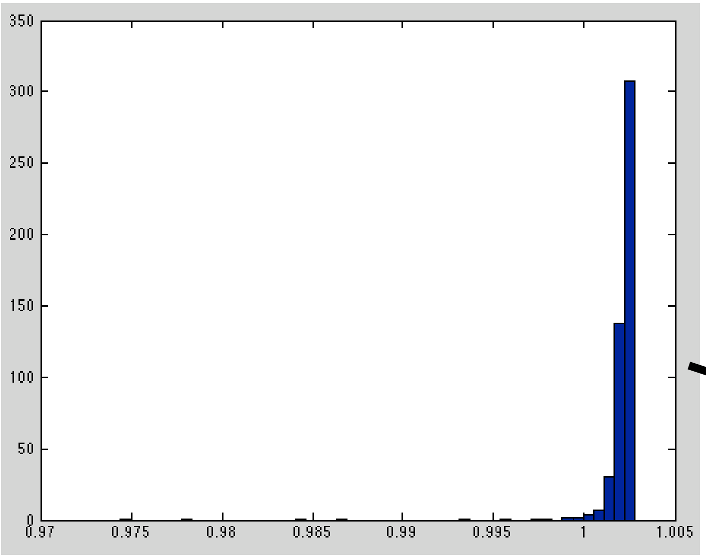


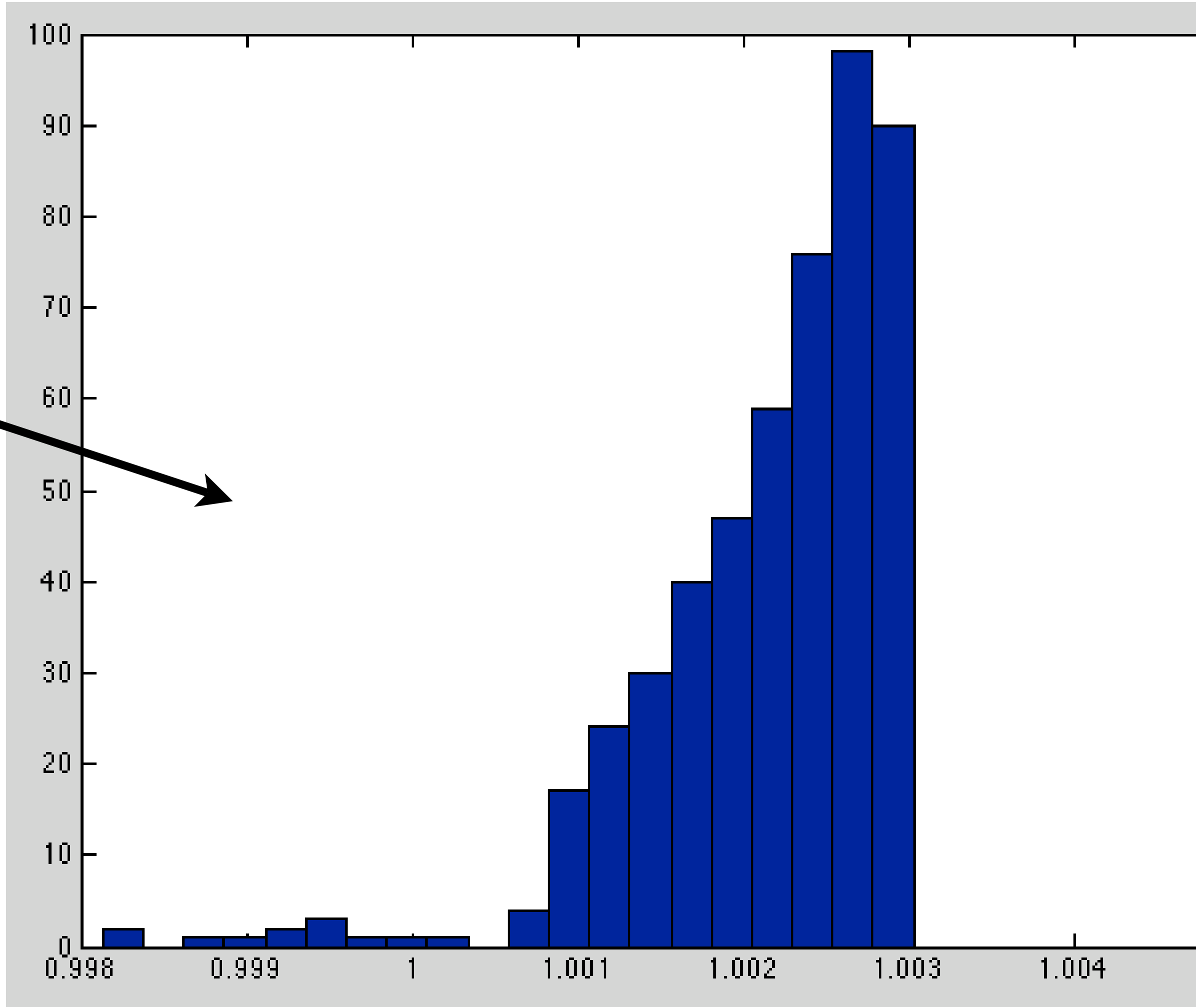




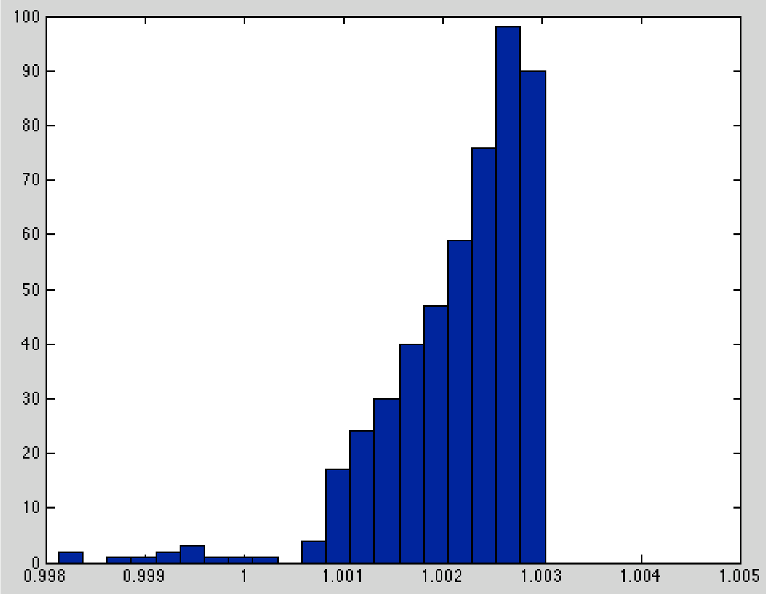
Cluster Break Down
By Region

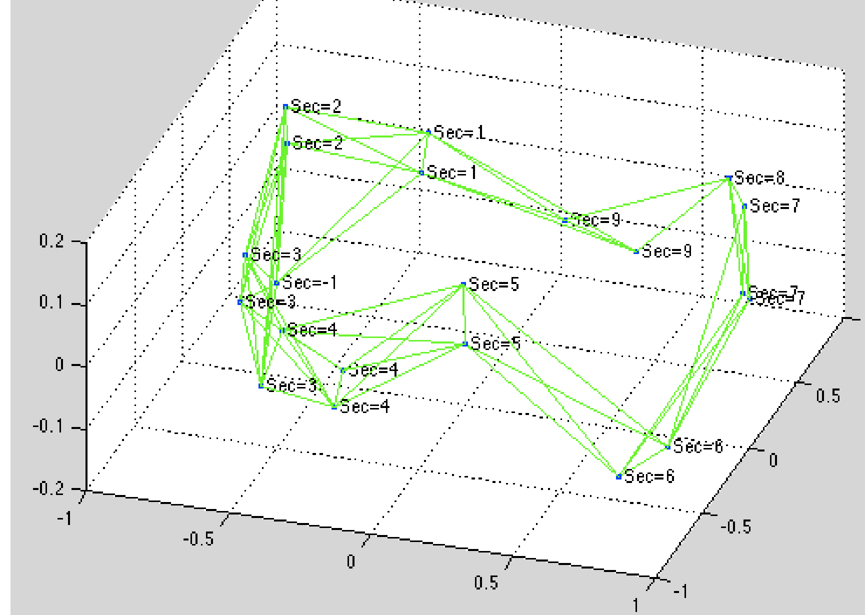
How much information do these eigenvalues contain? Well we "scrub them out" and see!

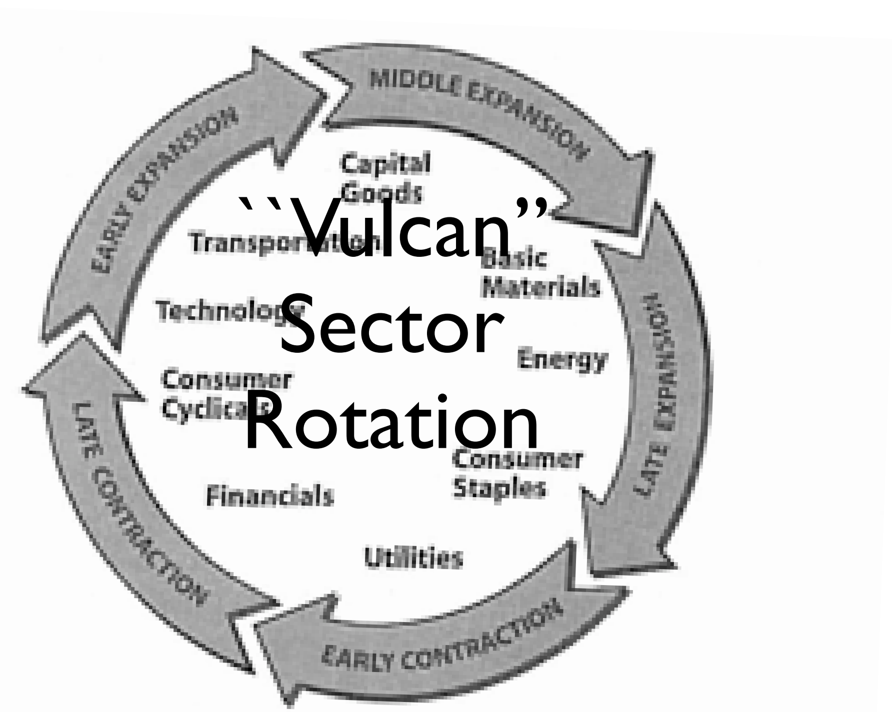




What is left captain?





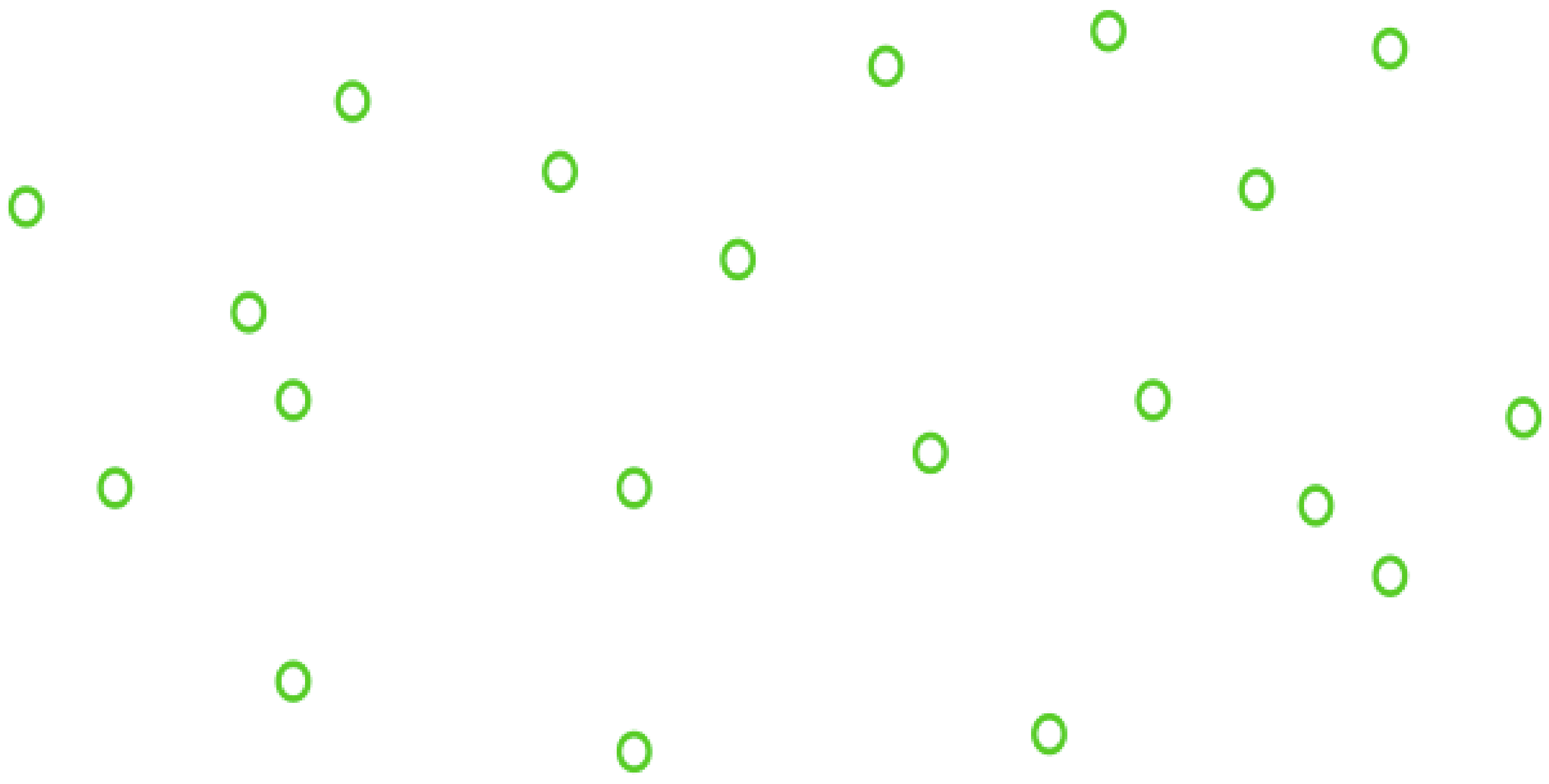


GURE 13.1 Technology and transportation leadership during 2003 fits Early Expansion phase.

Is the topology real?

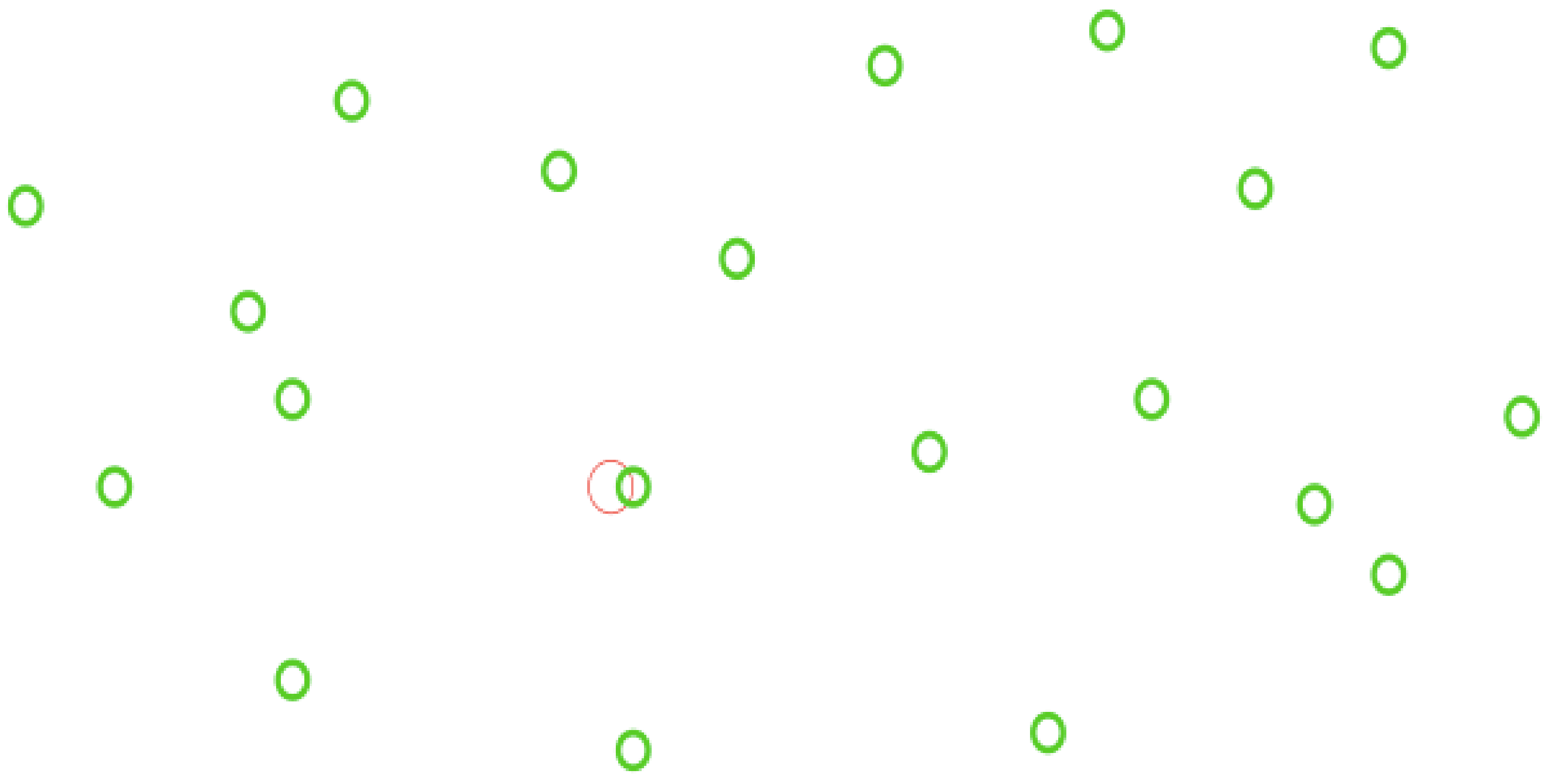
Empty Sphere Method

Take a bunch of point...



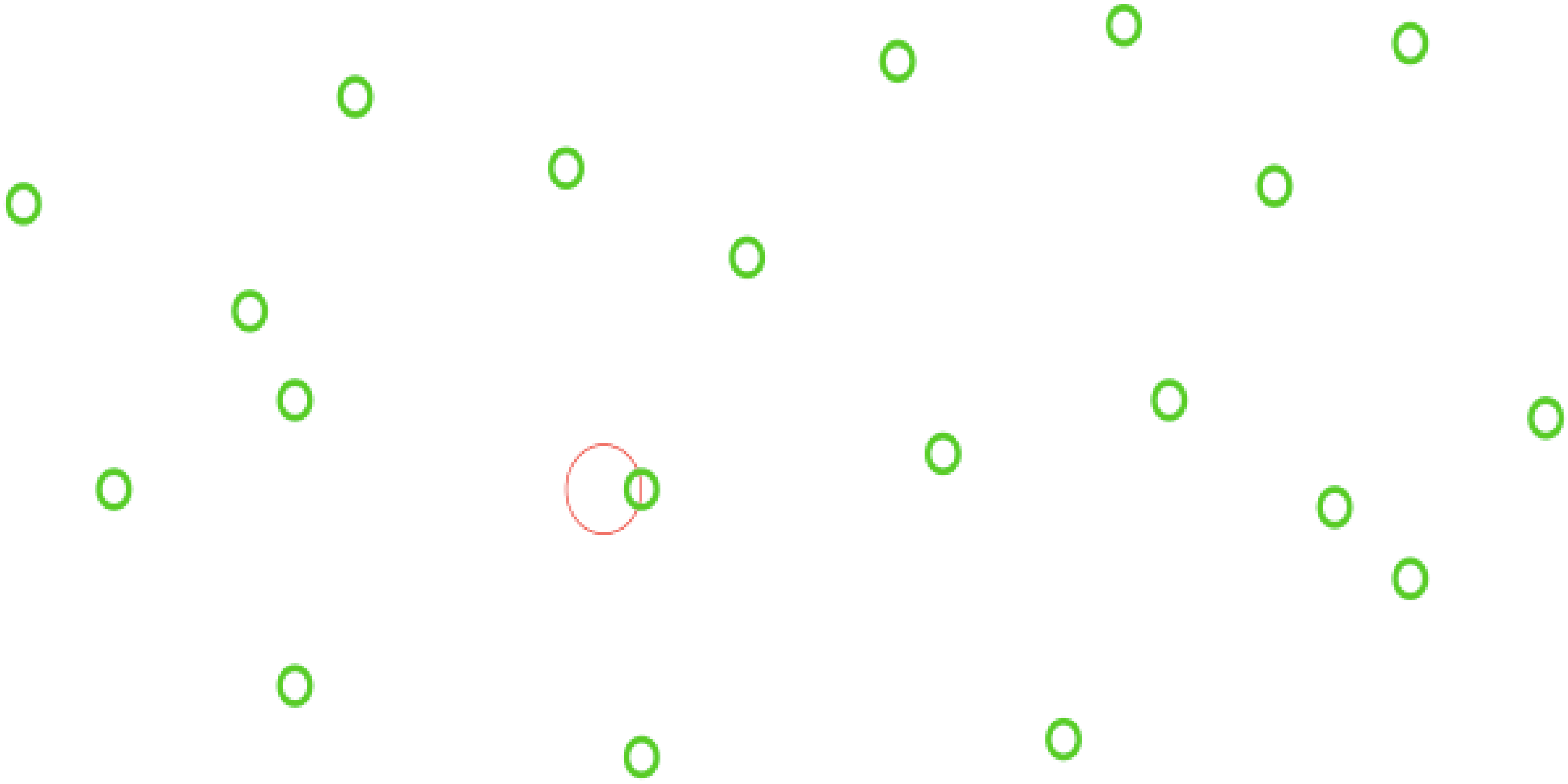
Empty Sphere Method

Search for empty spheres...



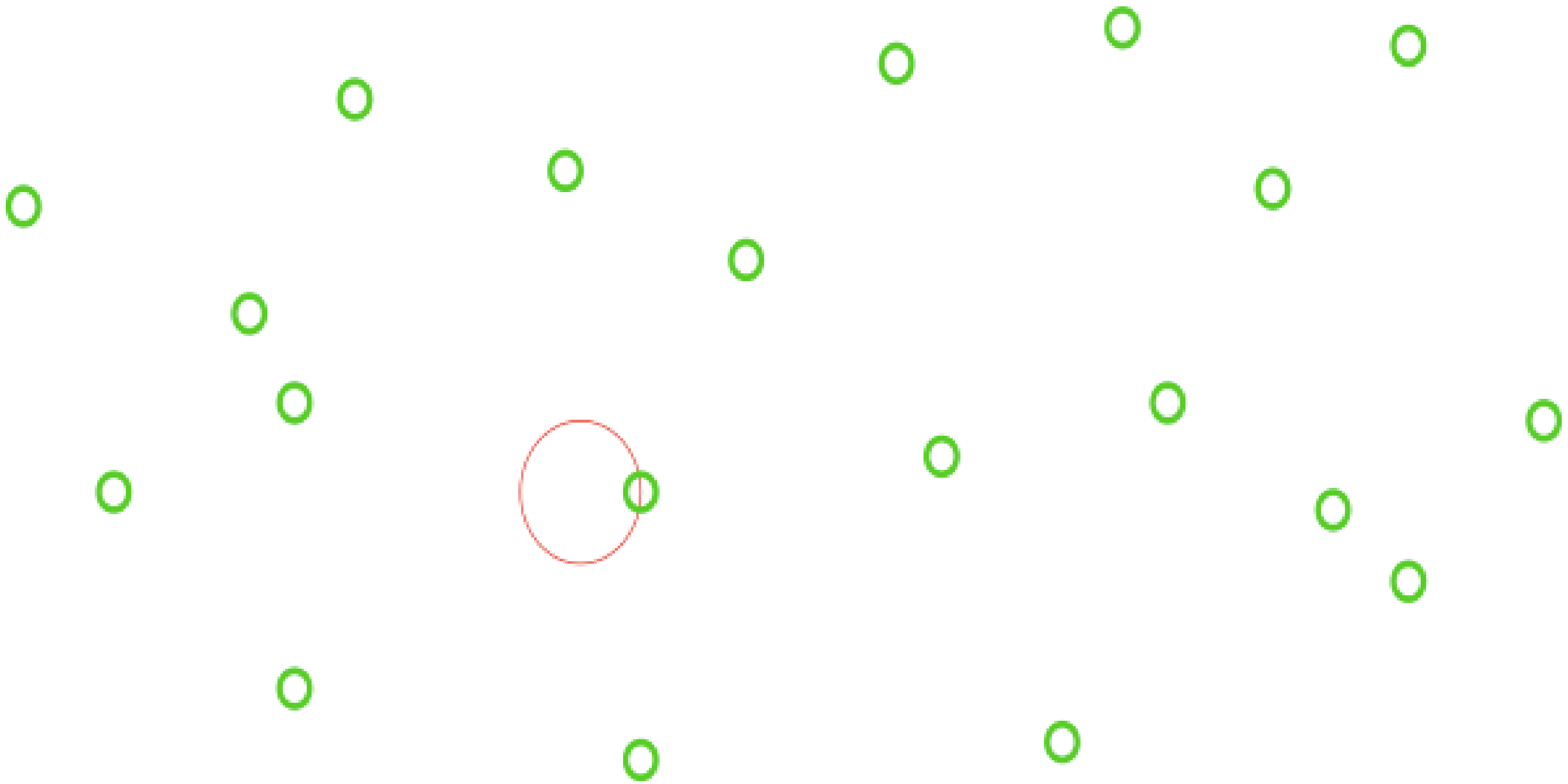
Empty Sphere Method

Search for empty spheres...



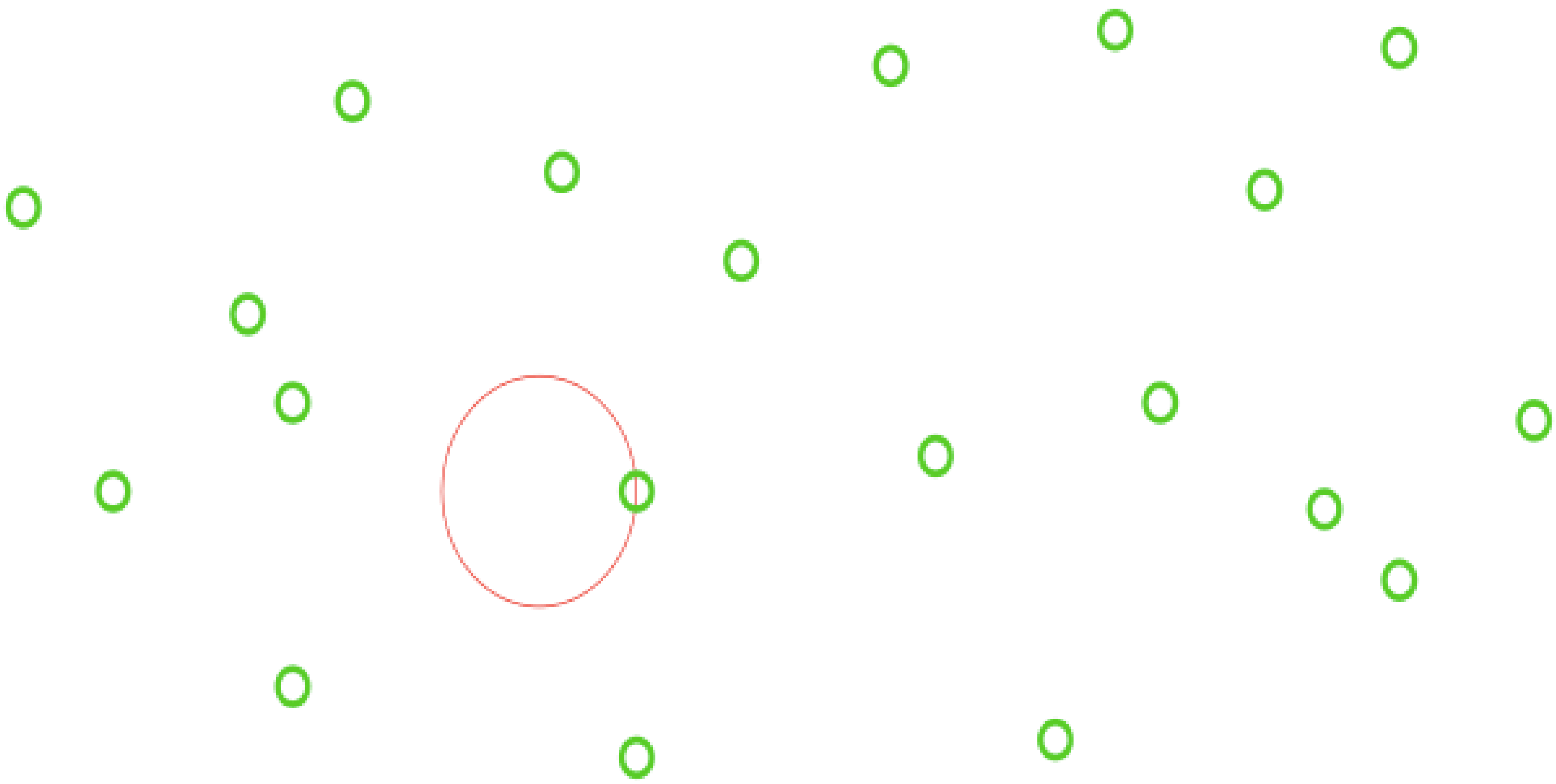
Empty Sphere Method

Search for empty spheres...



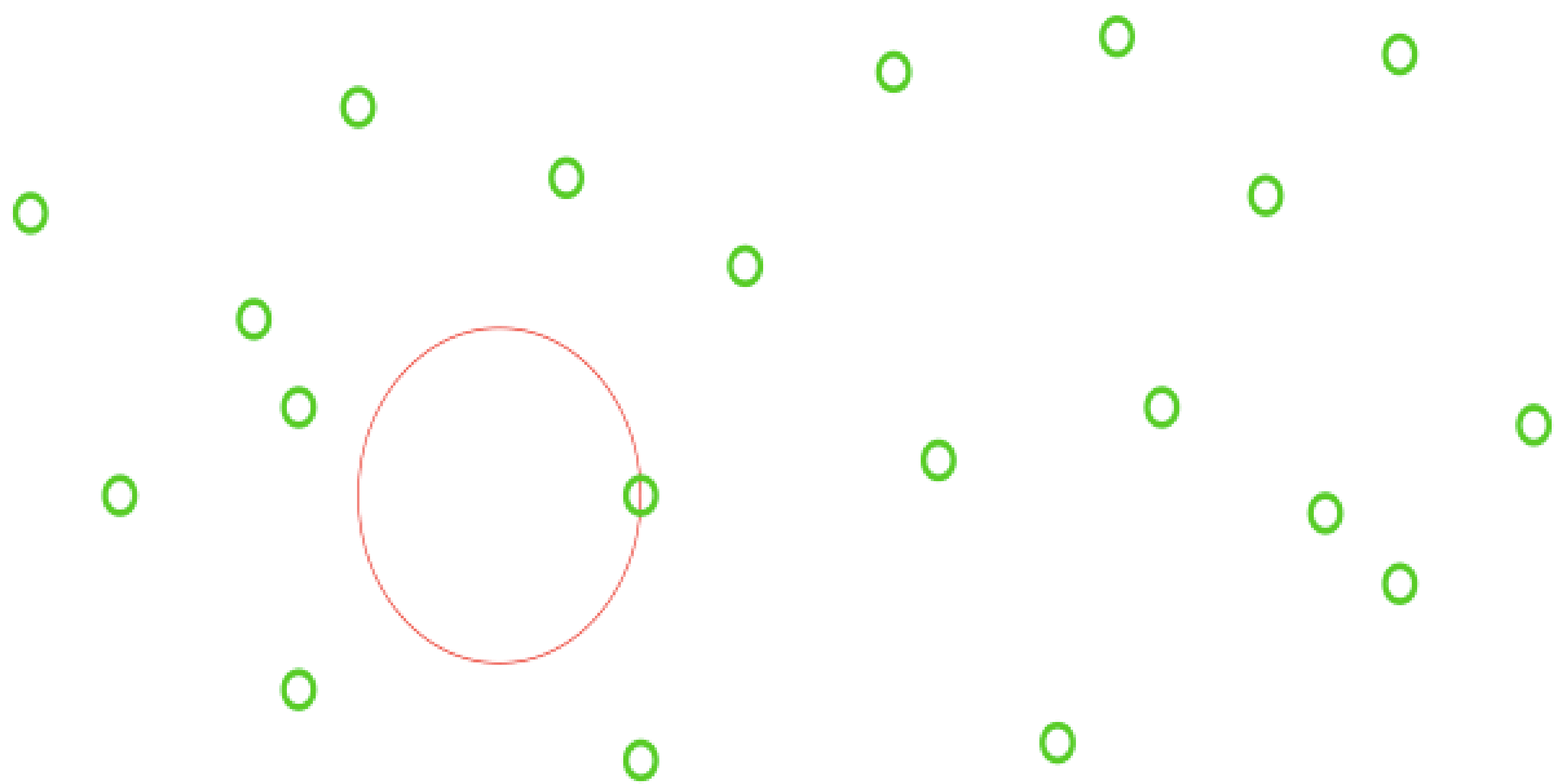
Empty Sphere Method

Search for empty spheres...



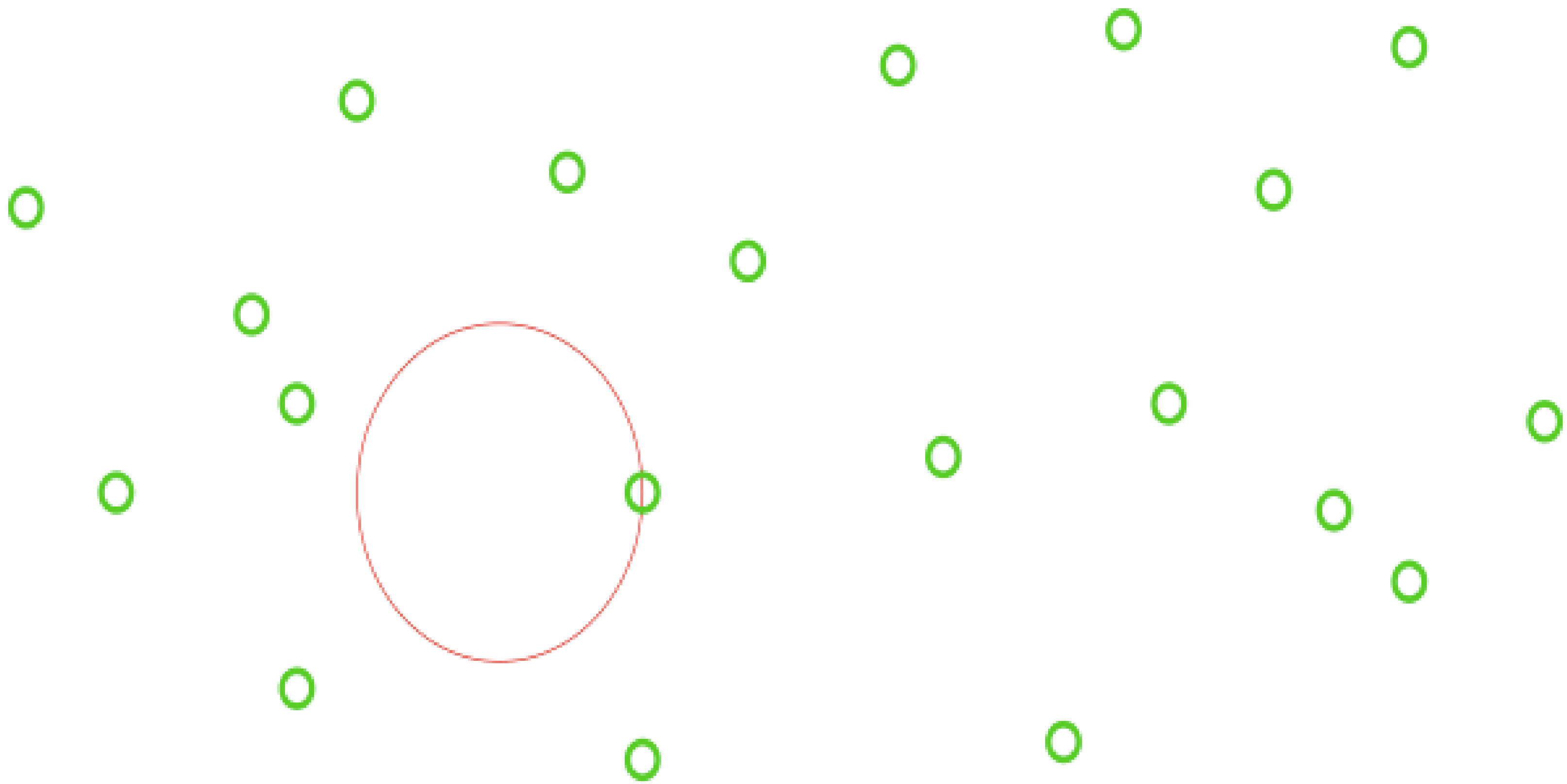
Empty Sphere Method

Search for empty spheres...



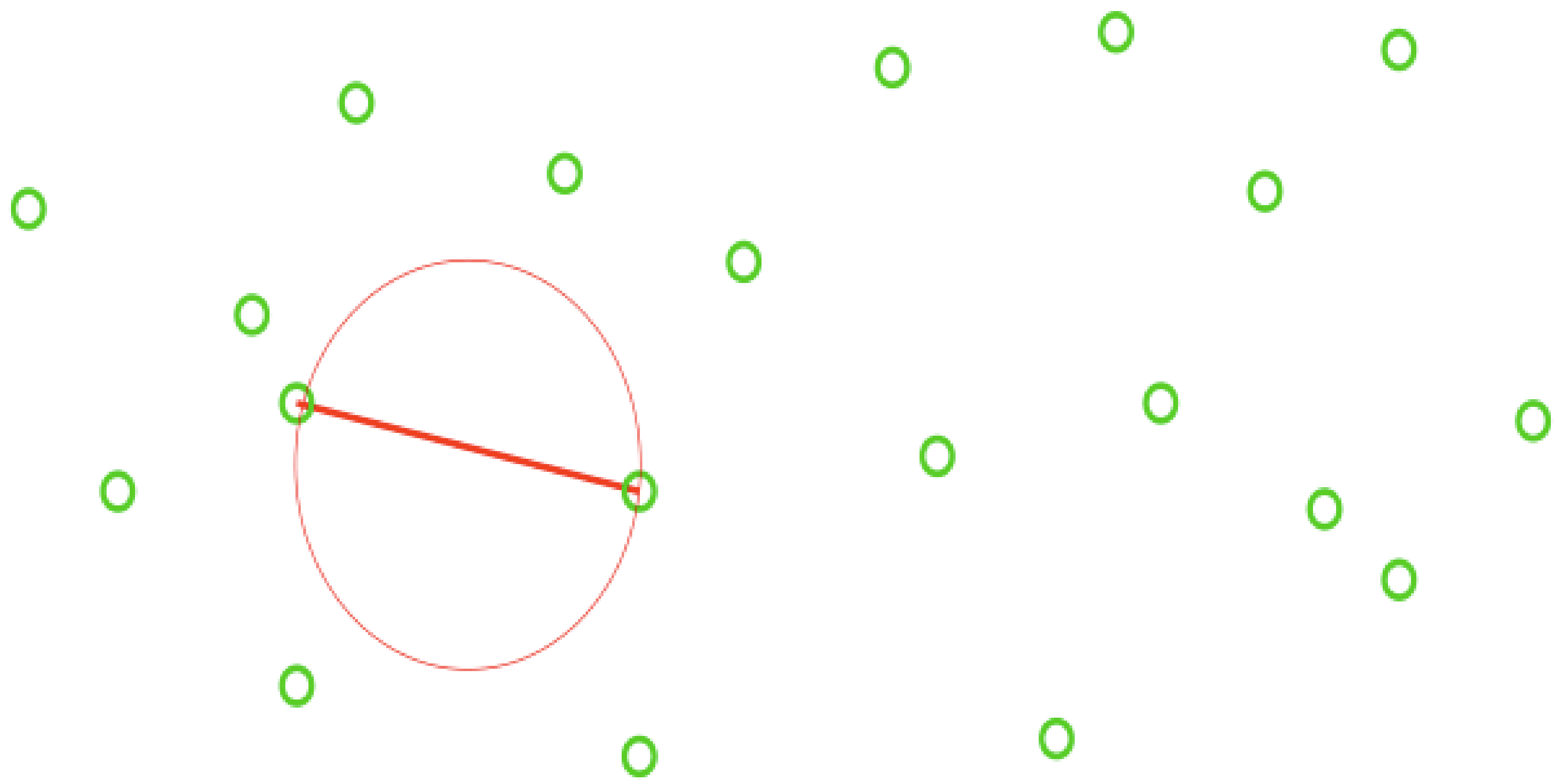
Empty Sphere Method

Search for empty spheres...



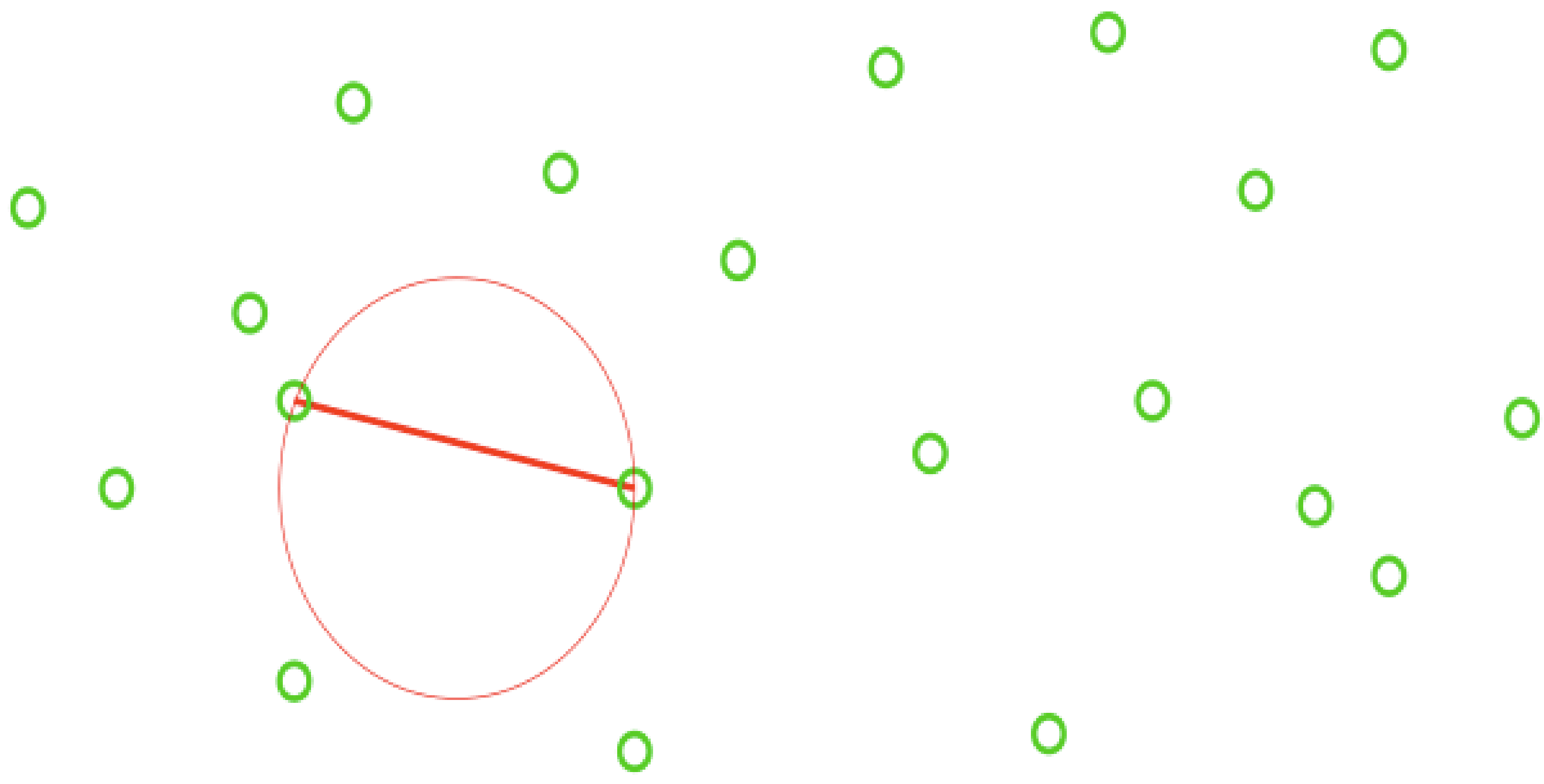
Empty Sphere Method

Search for empty spheres...



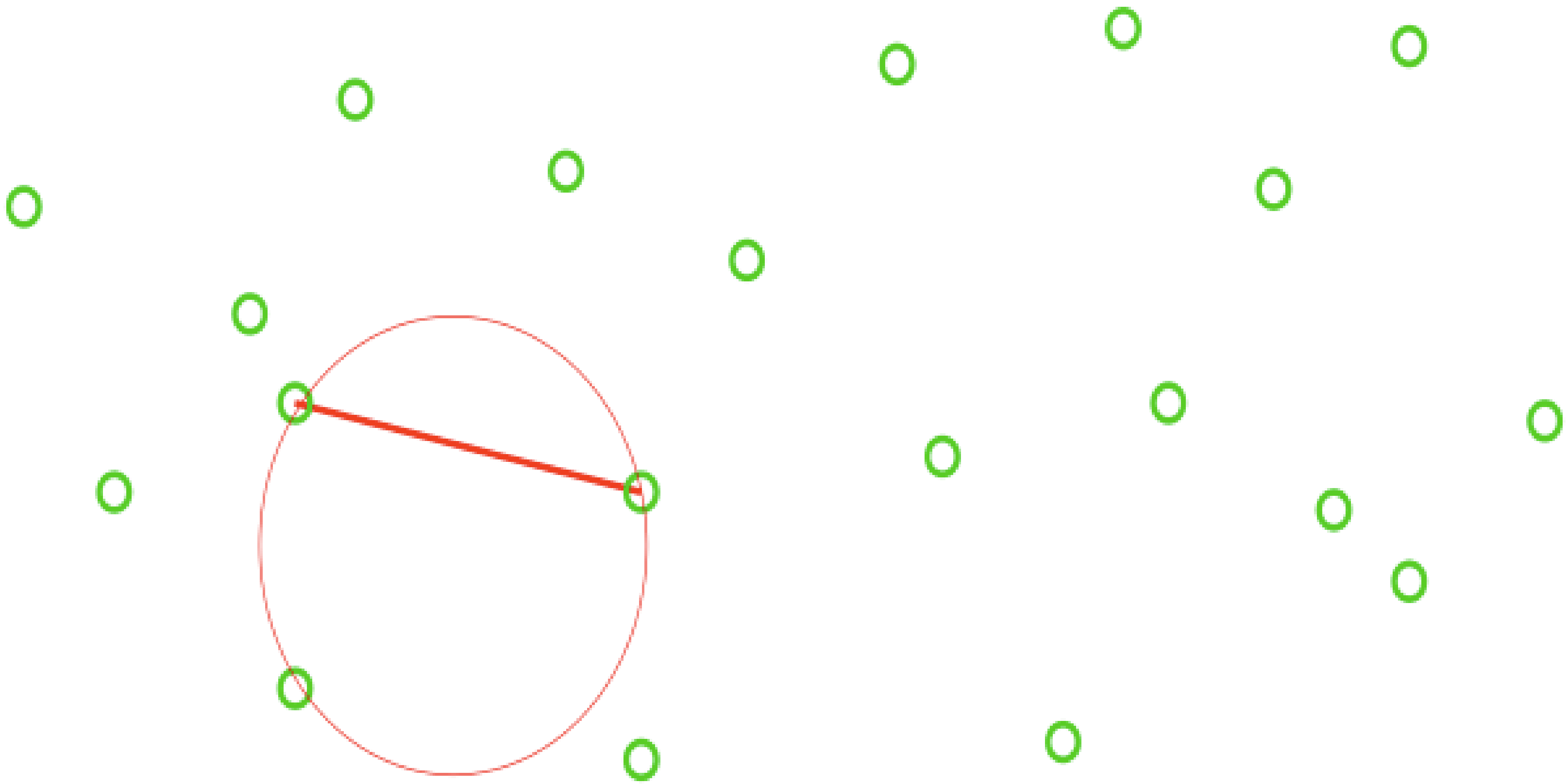
Empty Sphere Method

Search for empty spheres...



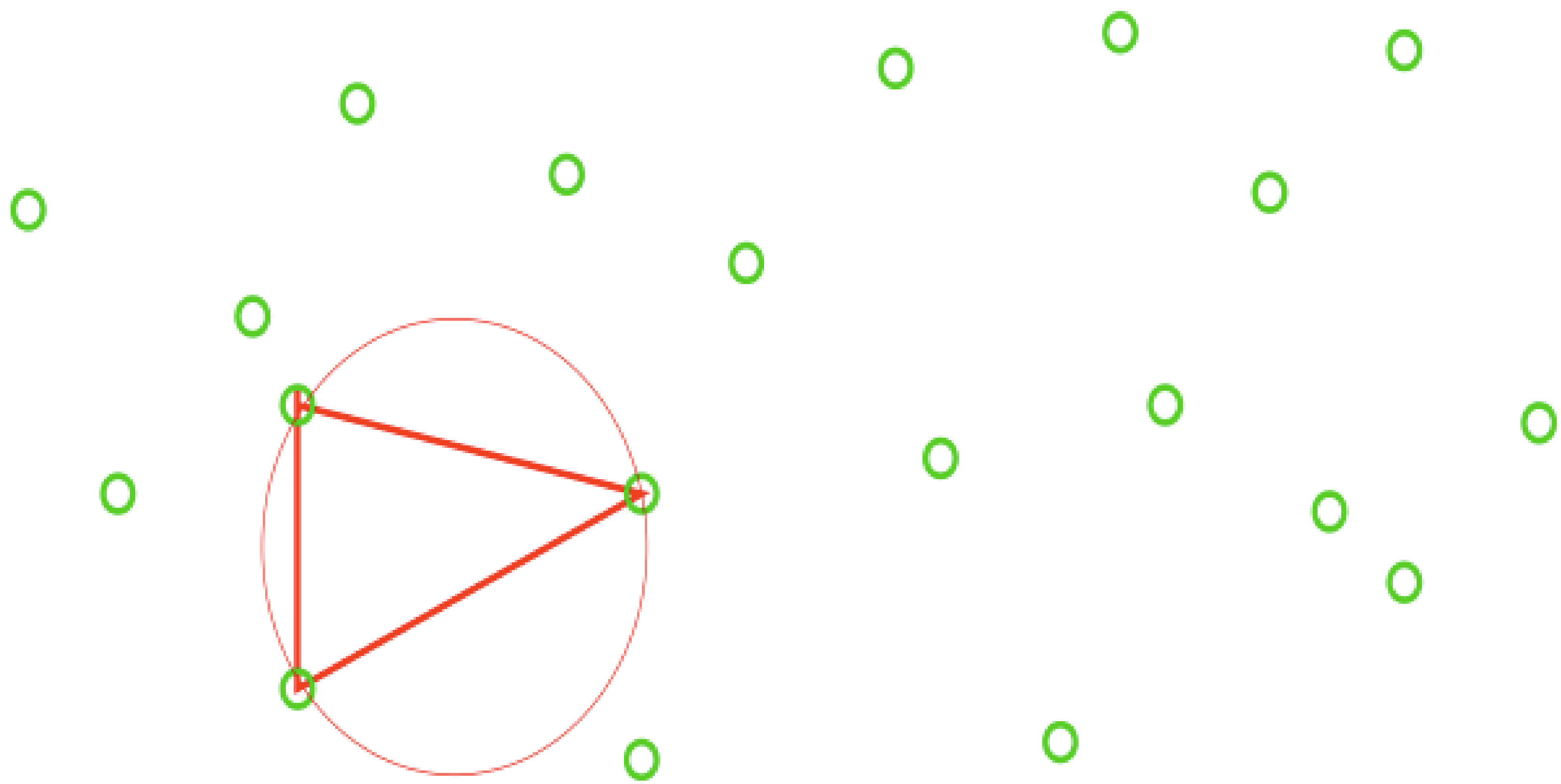
Empty Sphere Method

Search for empty spheres...



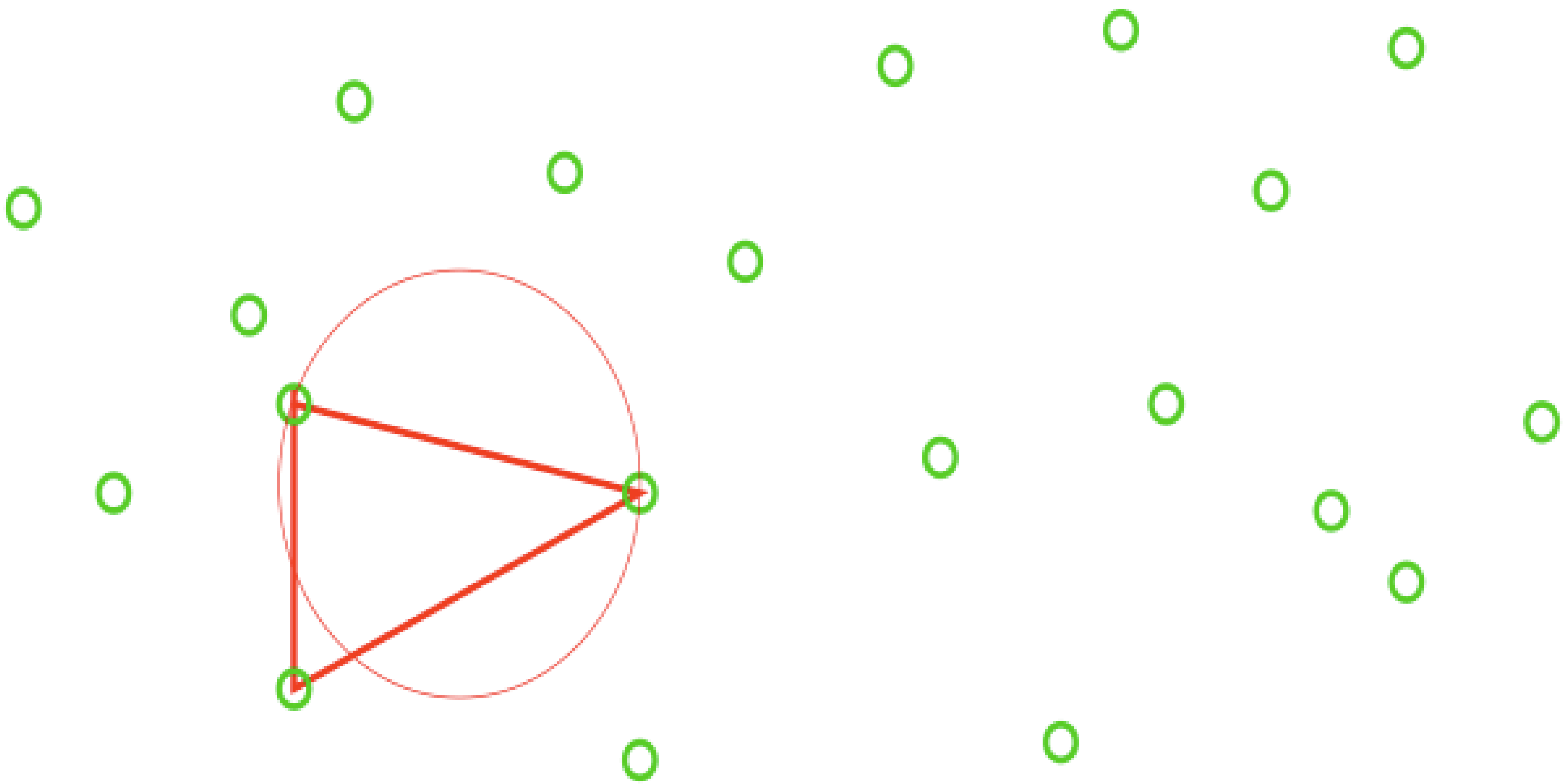
Empty Sphere Method

Search for empty spheres...



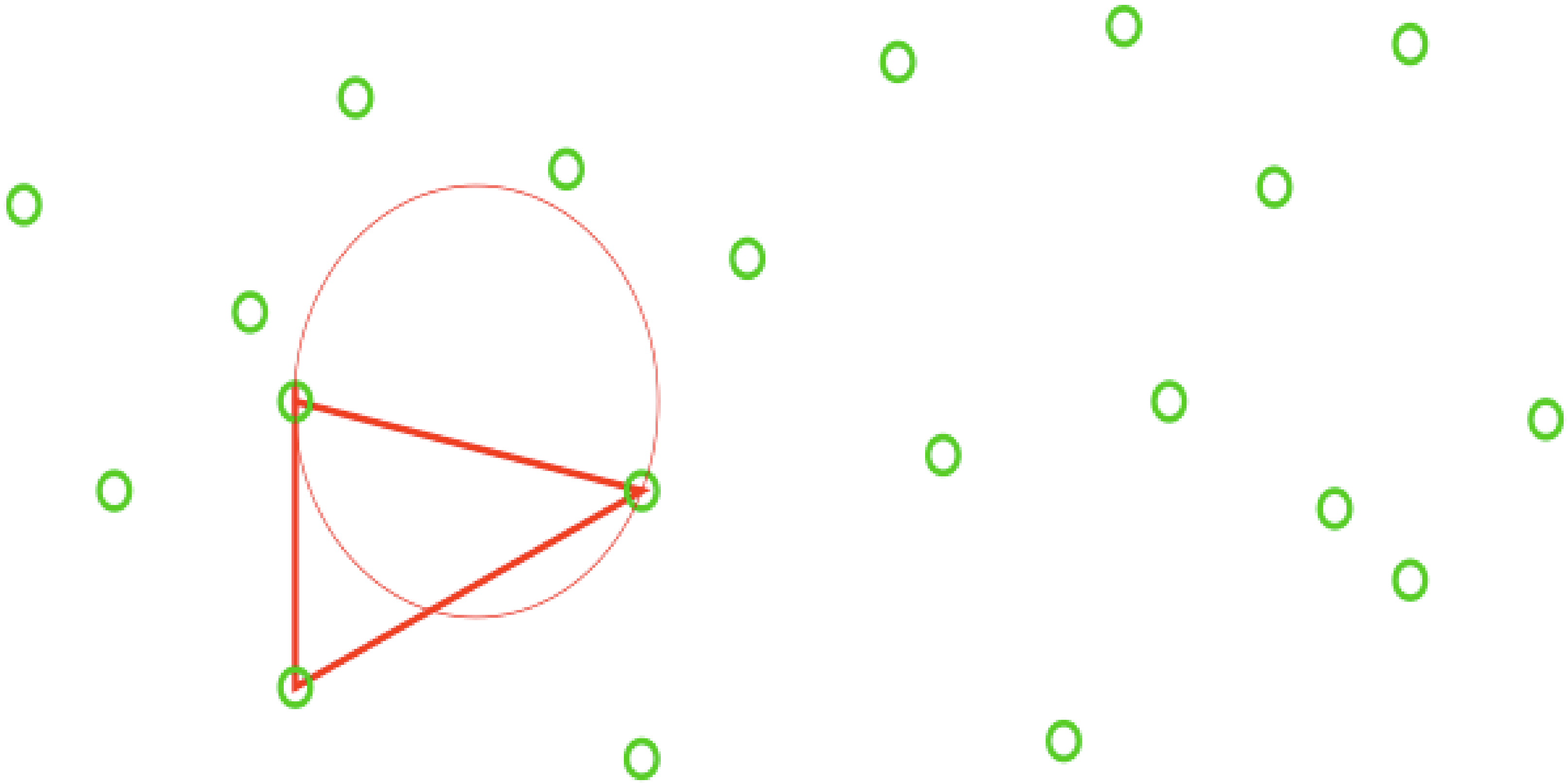
Empty Sphere Method

Search for empty spheres...



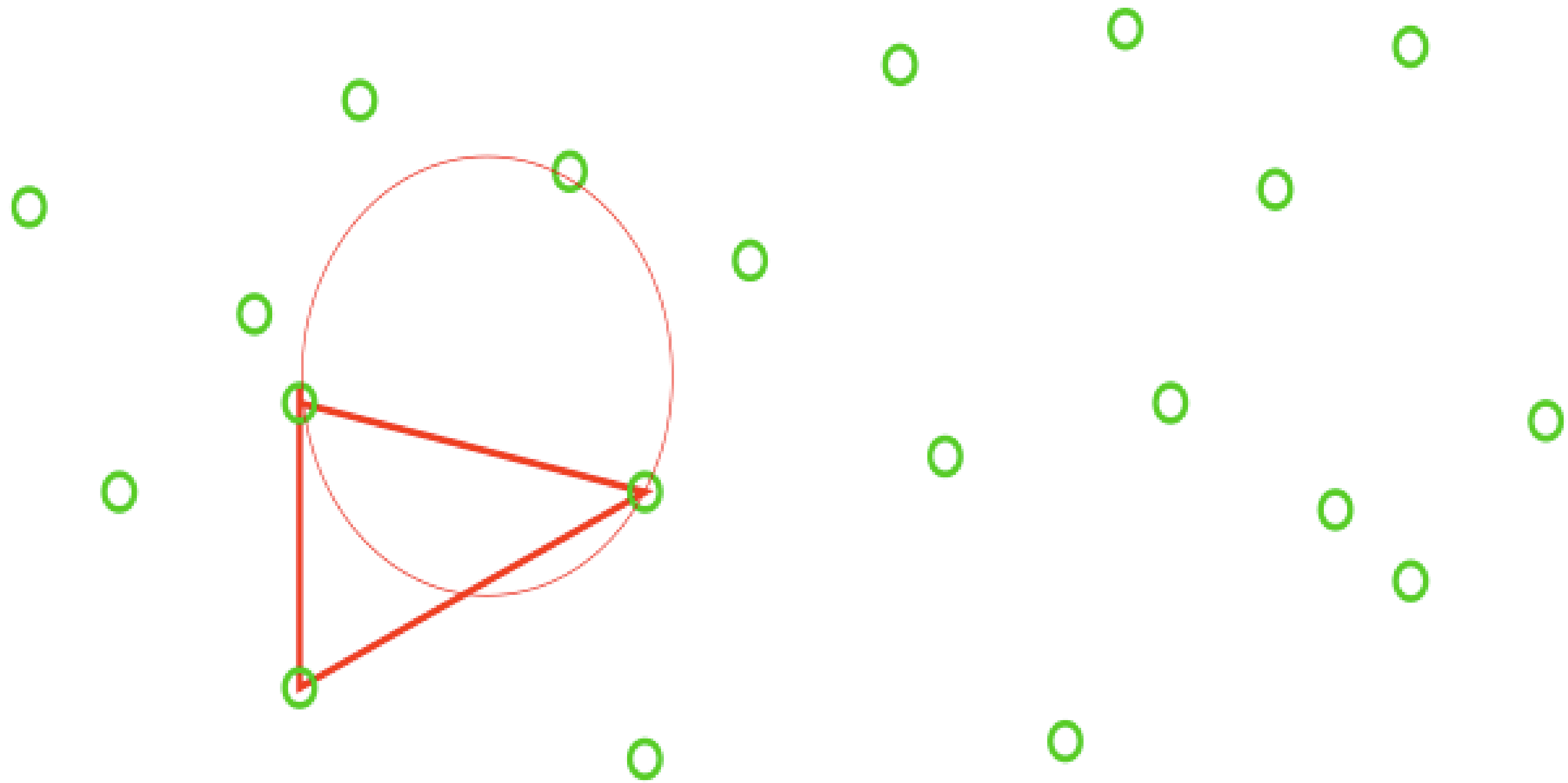
Empty Sphere Method

Search for empty spheres...



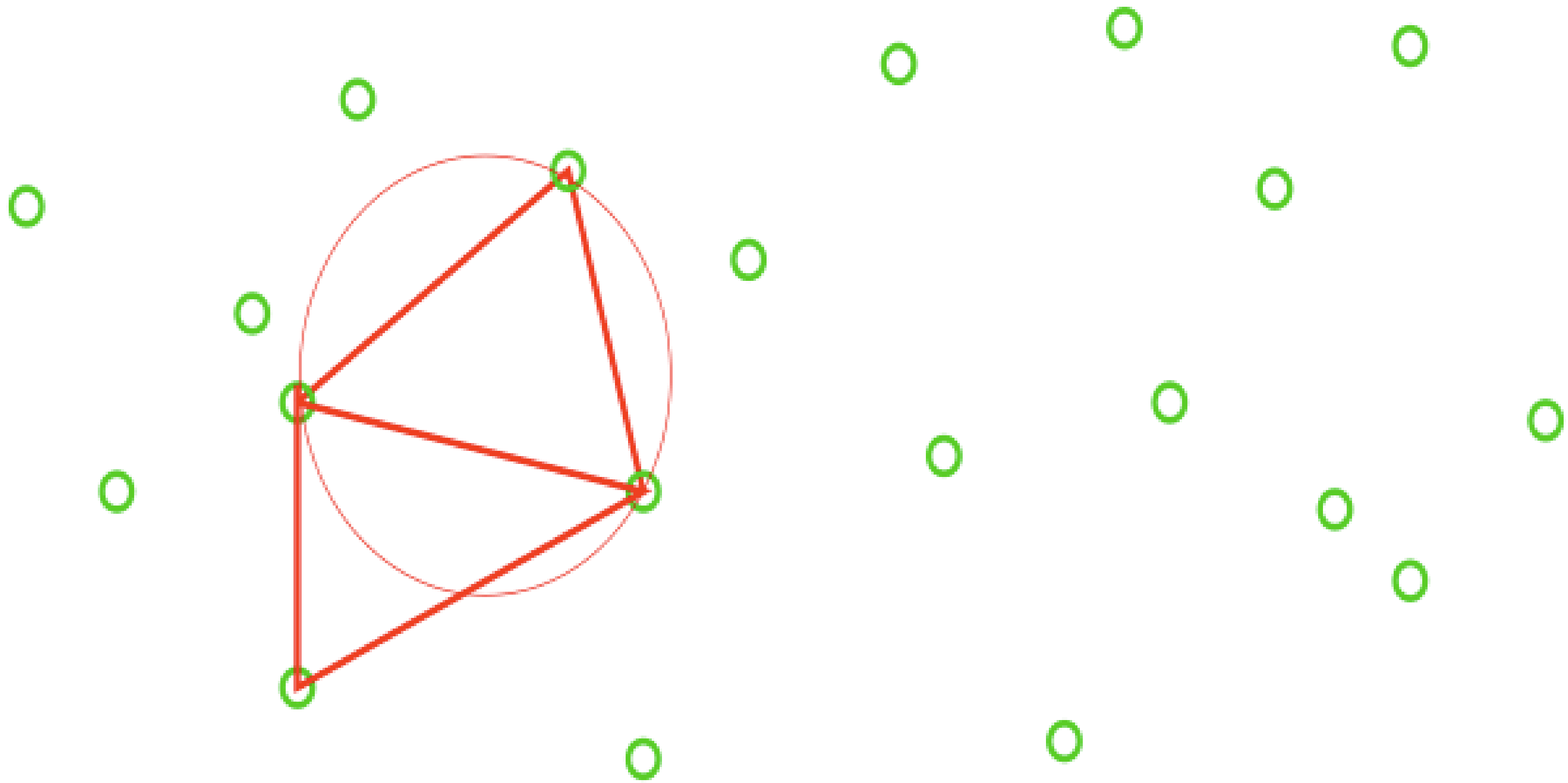
Empty Sphere Method

Search for empty spheres...



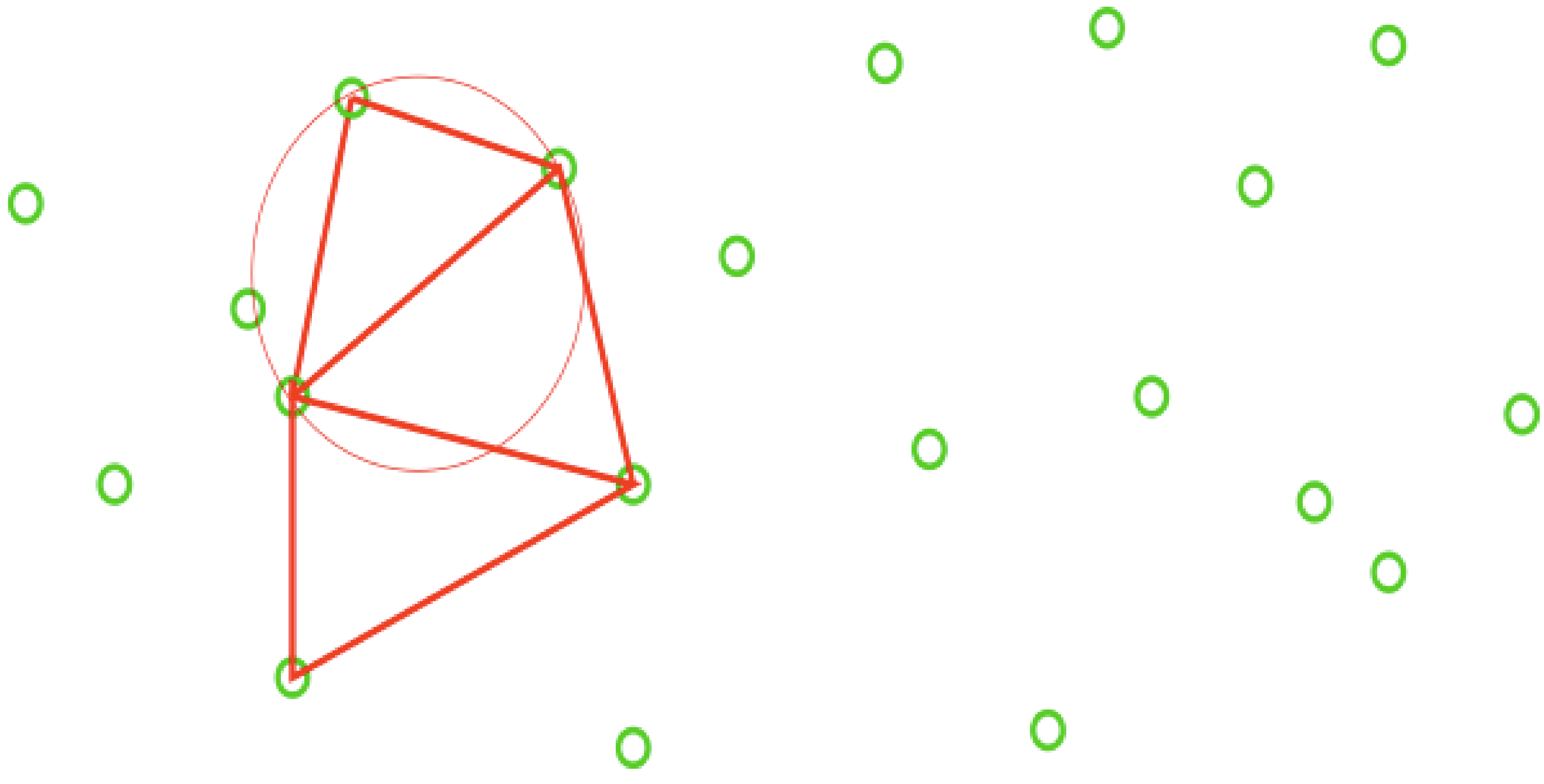
Empty Sphere Method

Search for empty spheres...



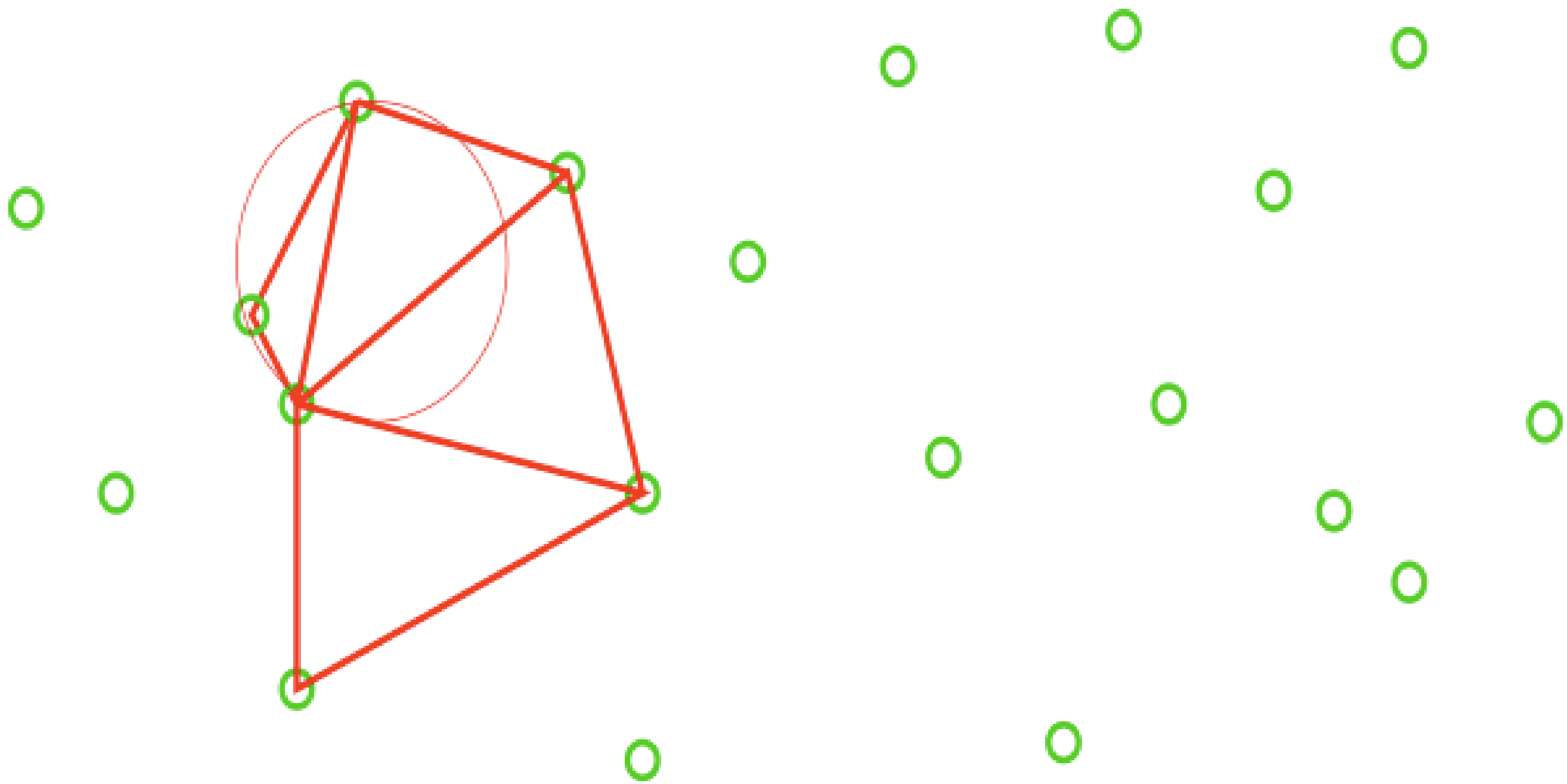
Empty Sphere Method

Search for empty spheres...



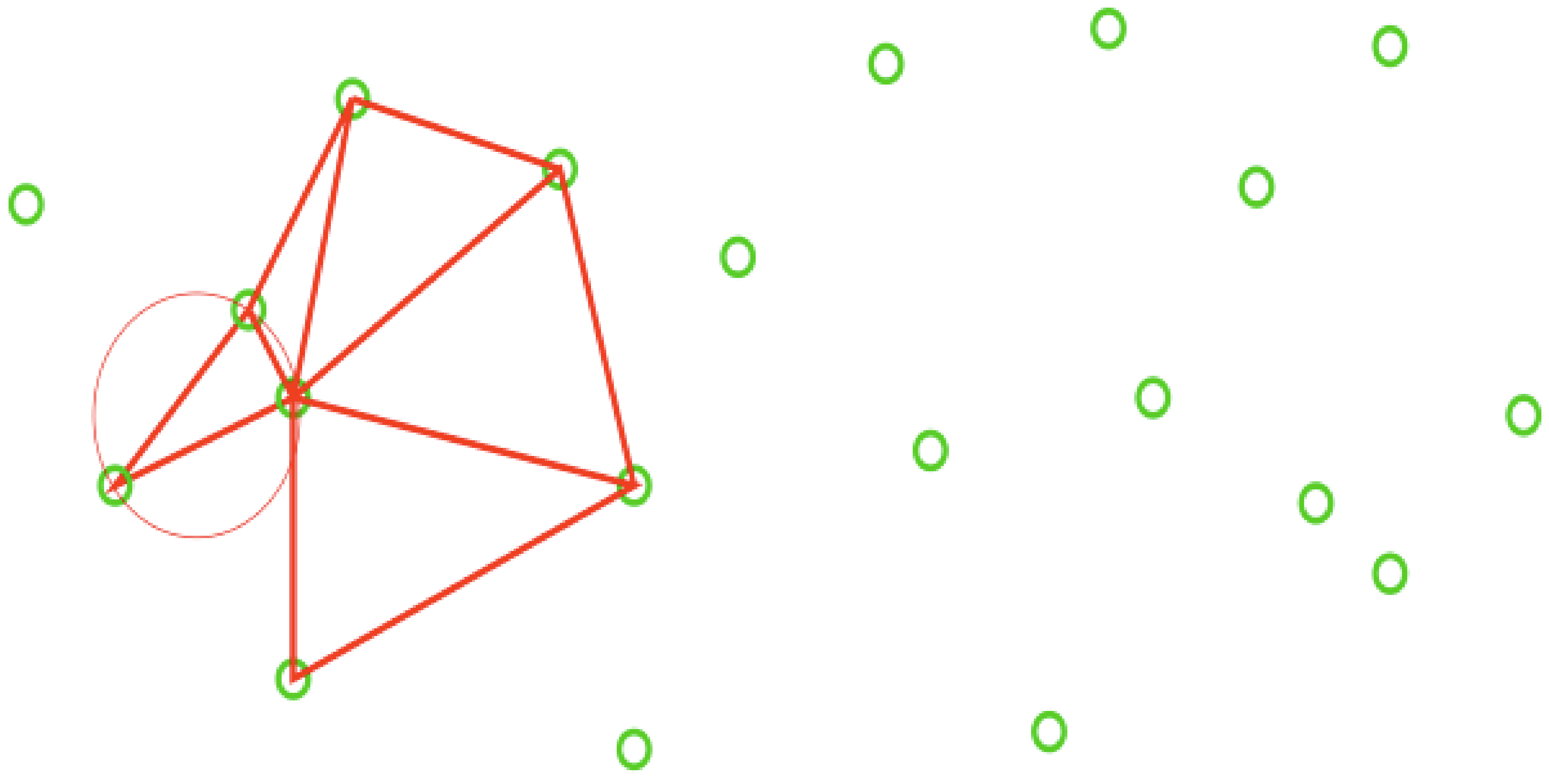
Empty Sphere Method

Search for empty spheres...



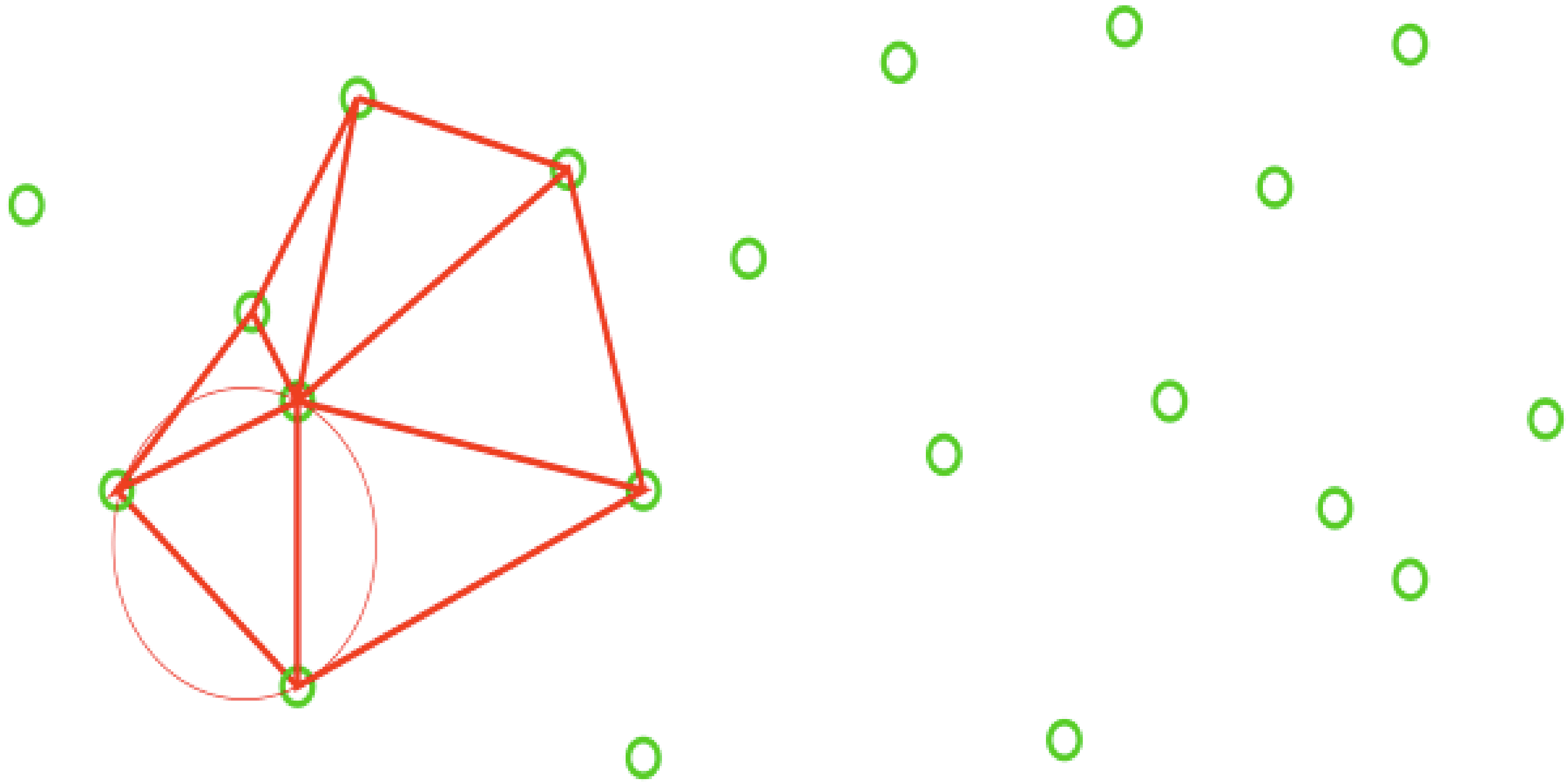
Empty Sphere Method

Search for empty spheres...



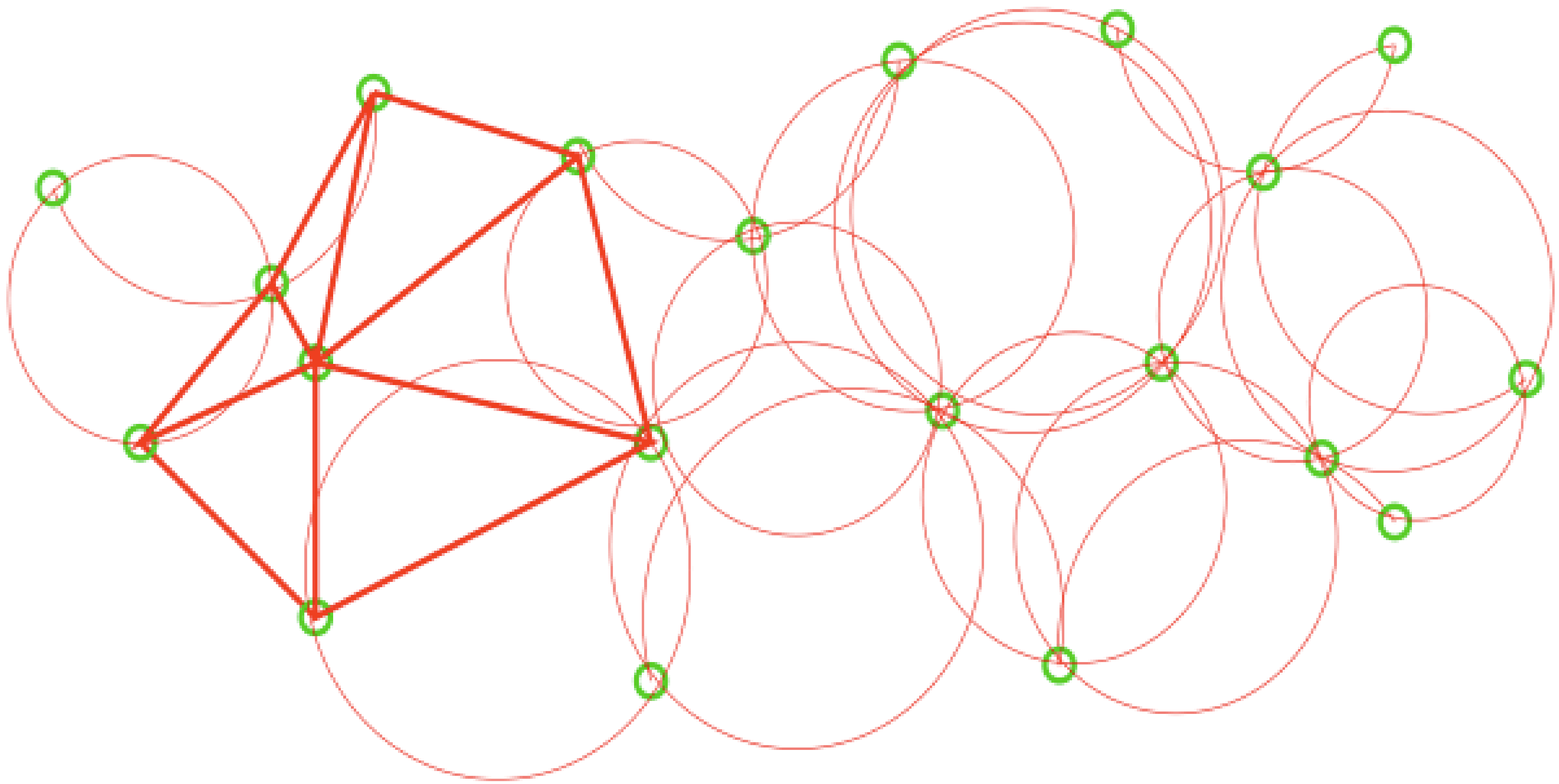
Empty Sphere Method

Search for empty spheres...



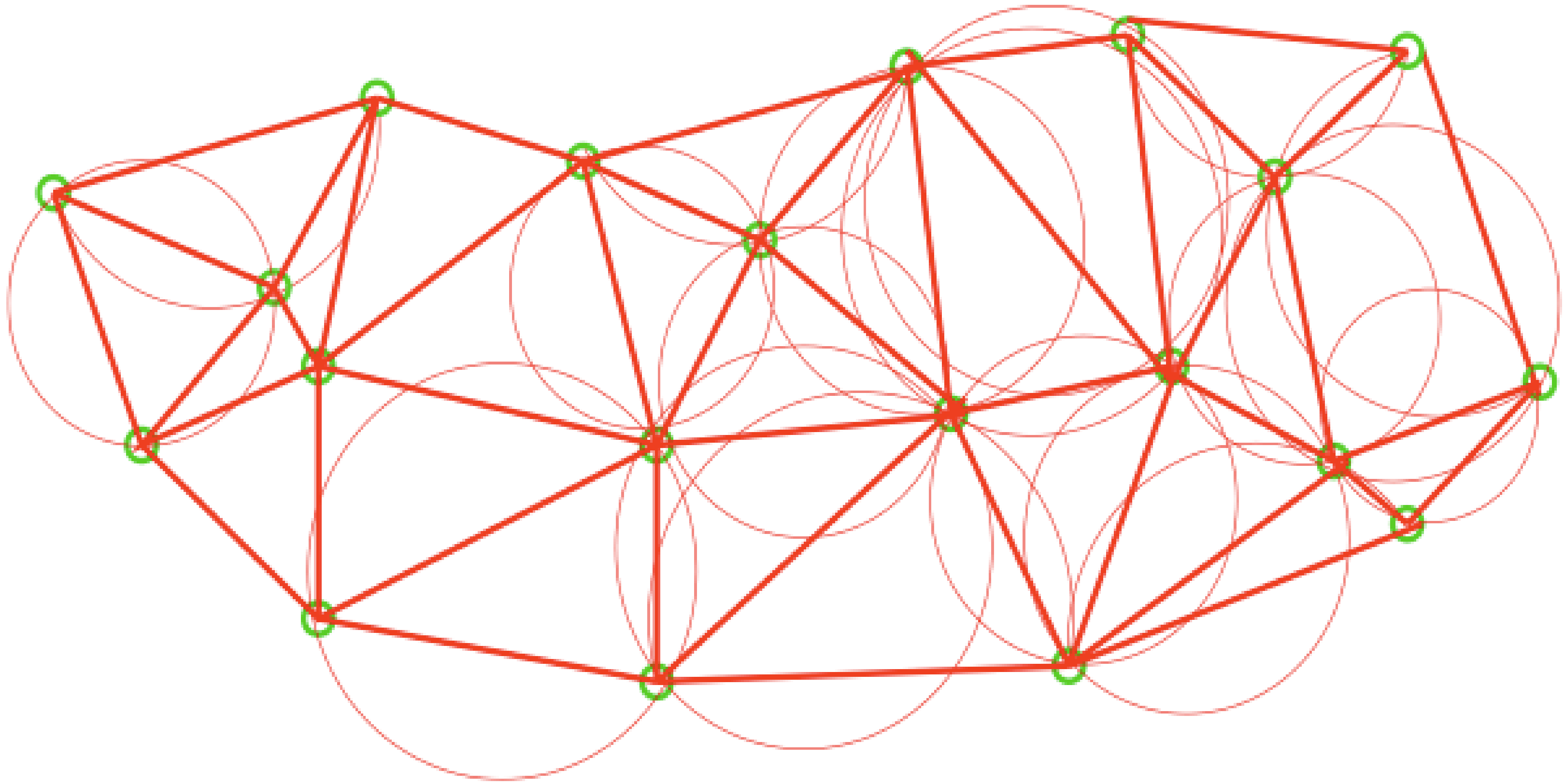
Empty Sphere Method

Search for empty spheres...

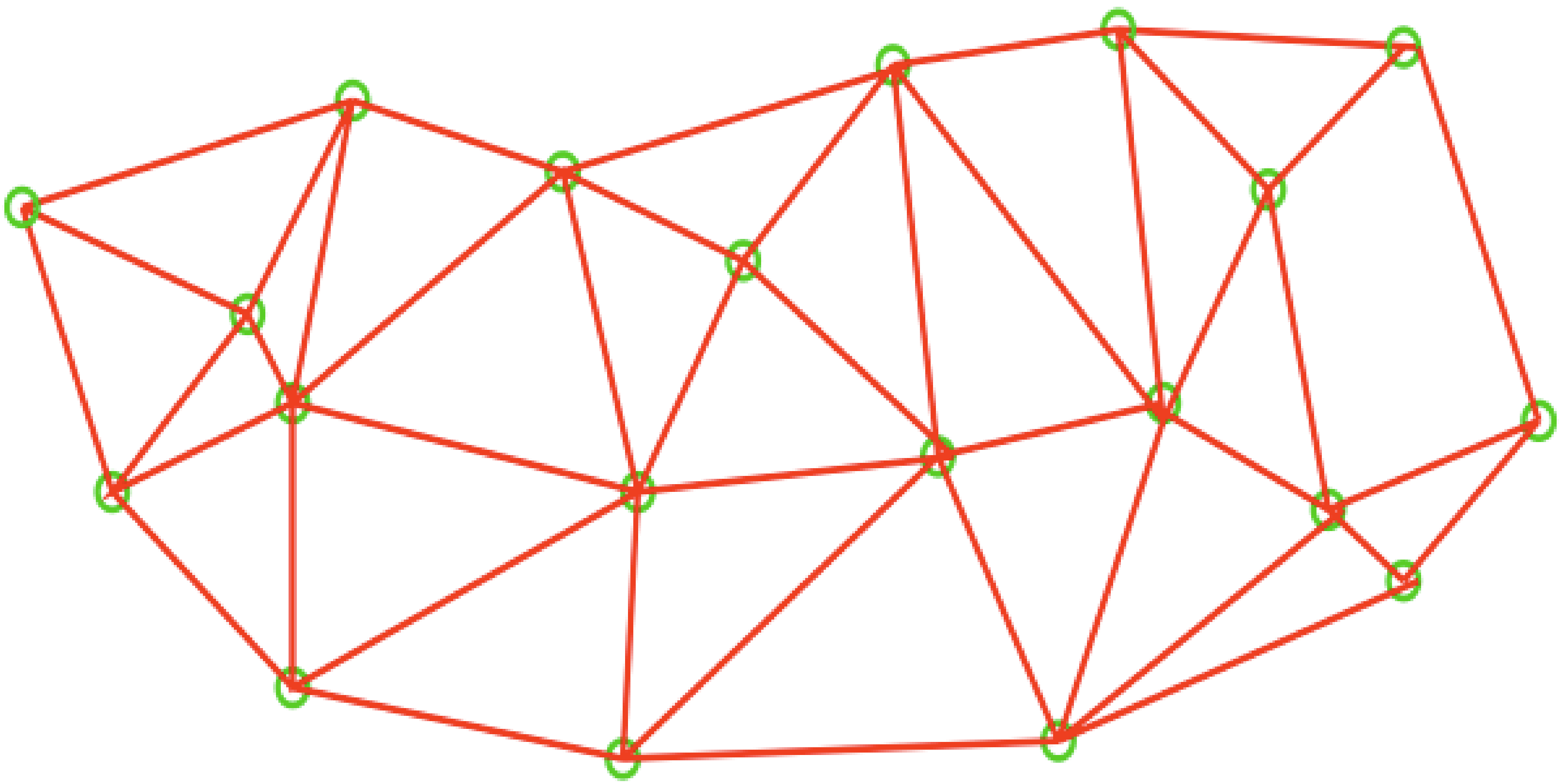


Empty Sphere Method

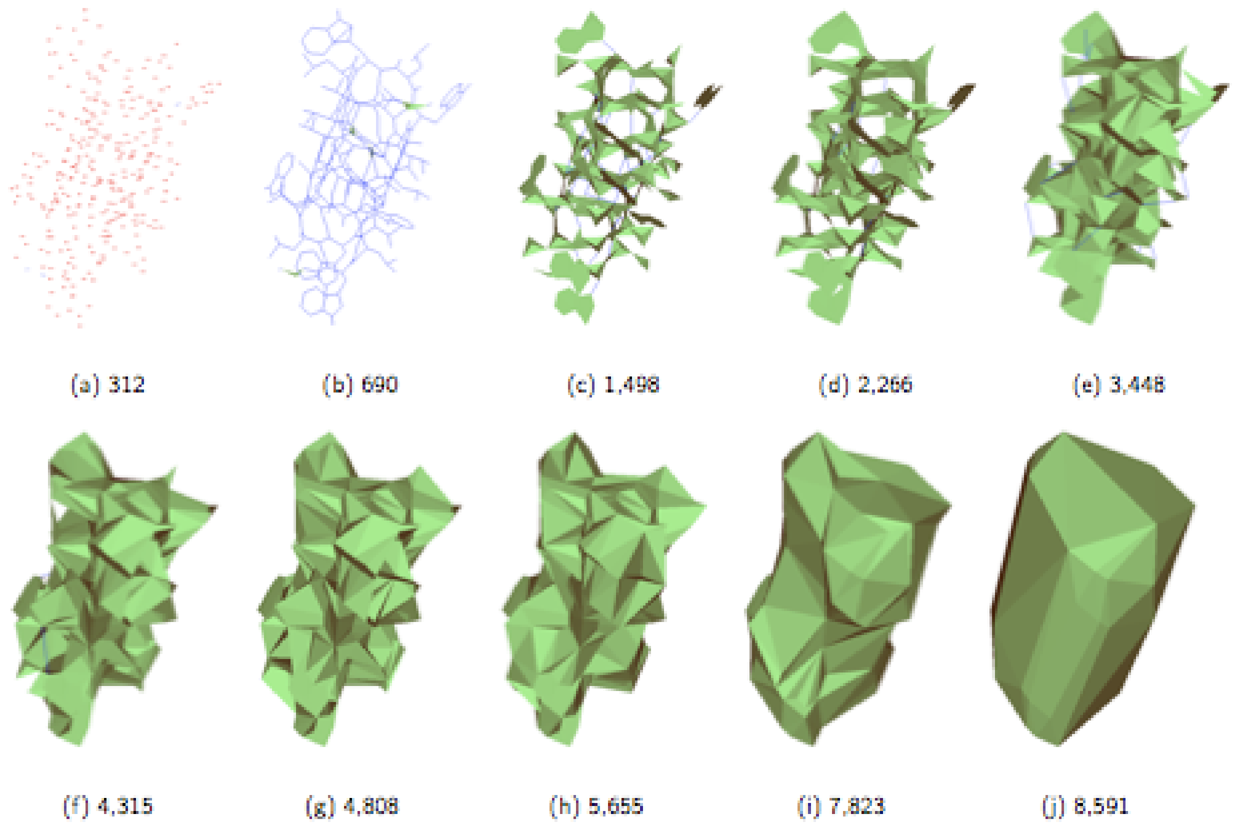
Search for empty spheres...



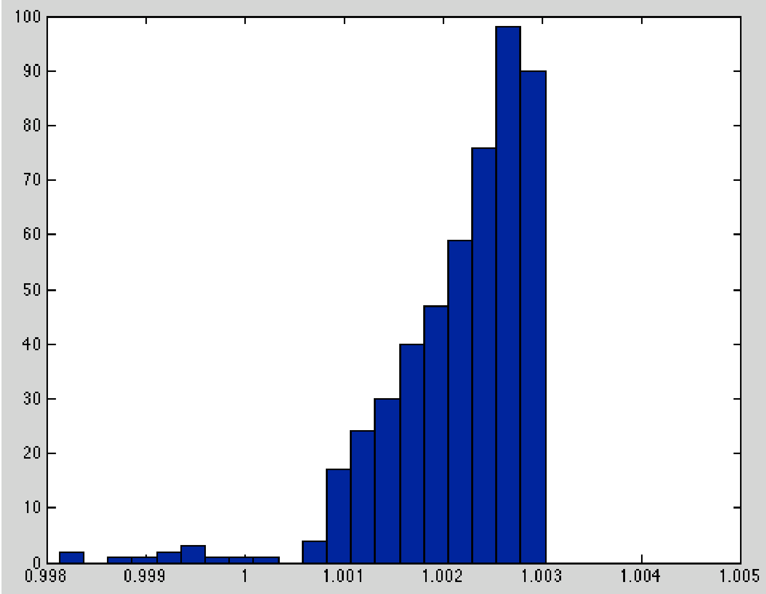
Viola, le *Delaunay Decomposition*.

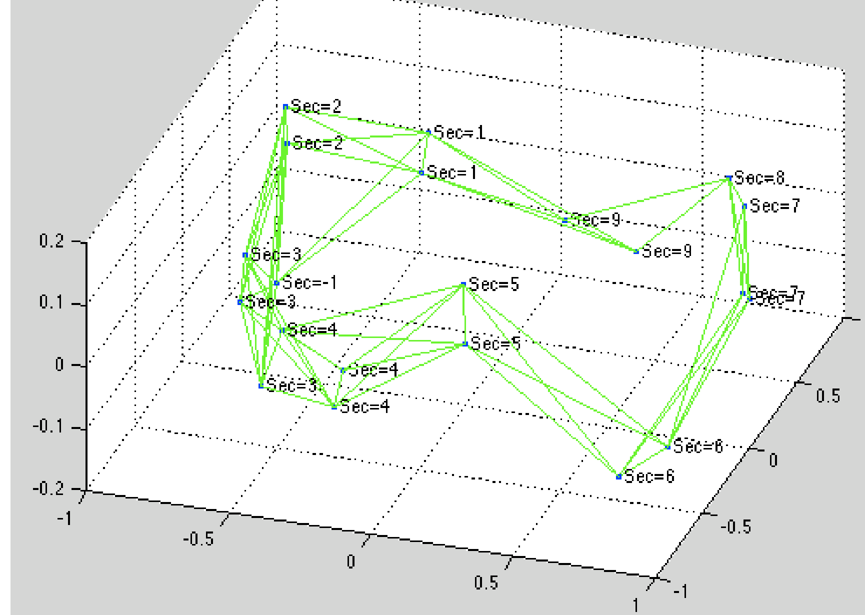


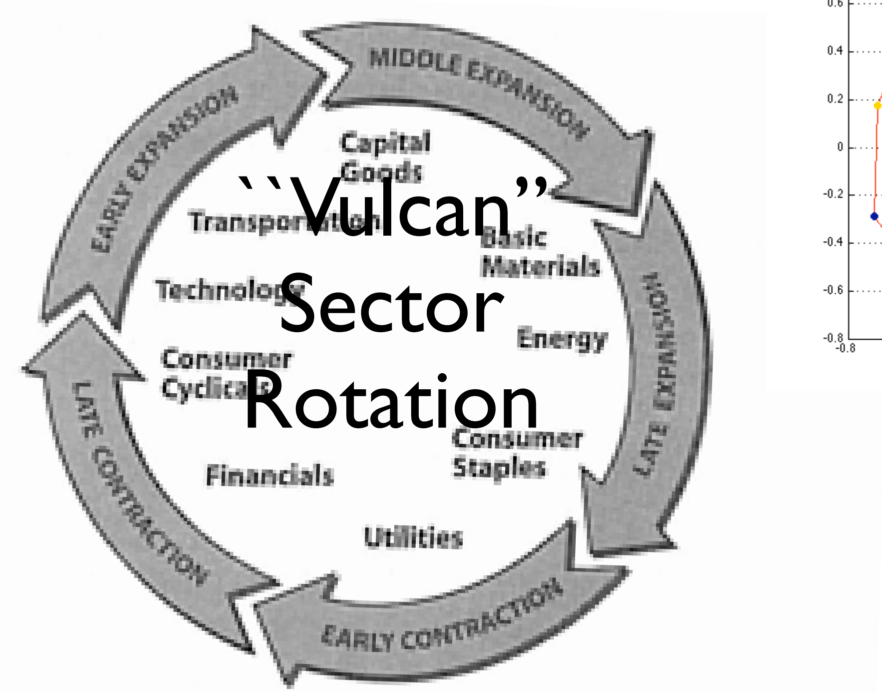
- Manifold Learning: Loads of current research!
- "Most Fun"=persistent homology. Detects the emergence of topology. (See: *Topology for Computing* by Zomorodian)



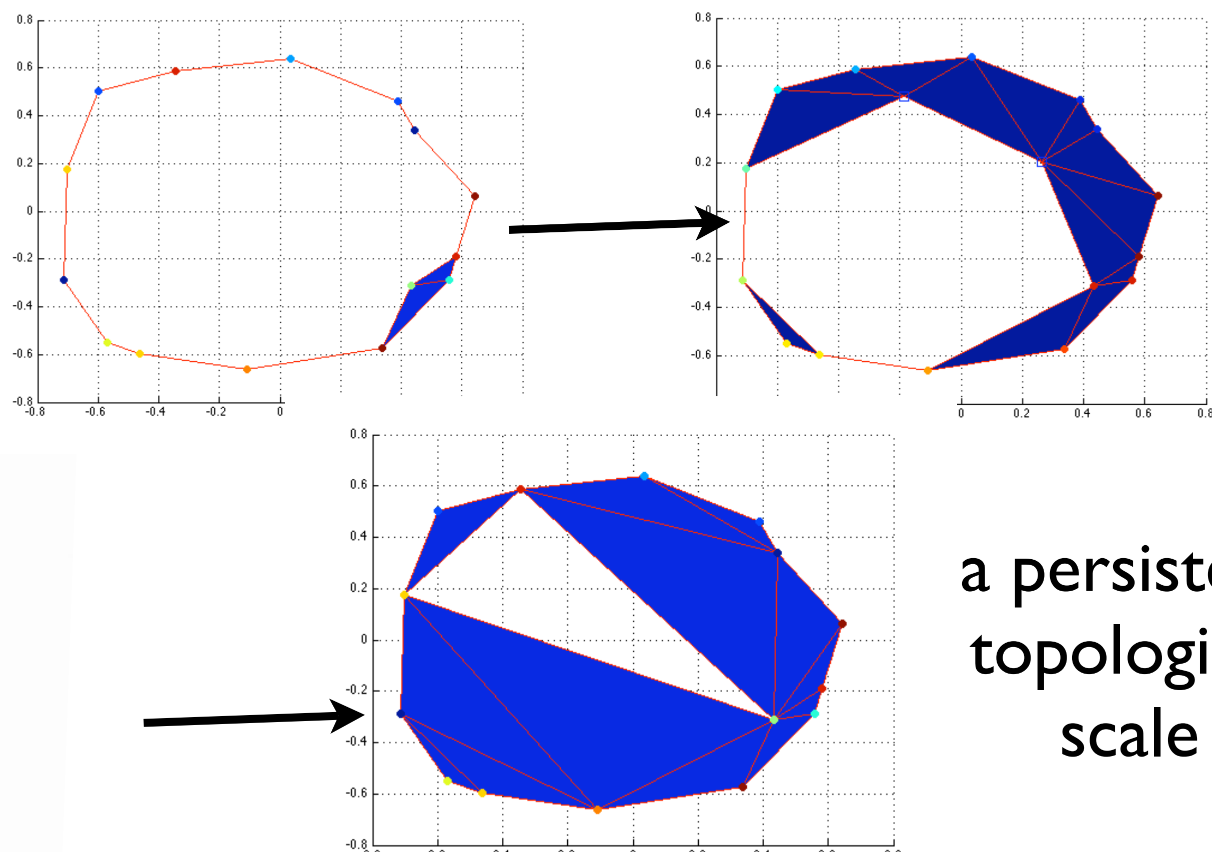
"Bottom up" method





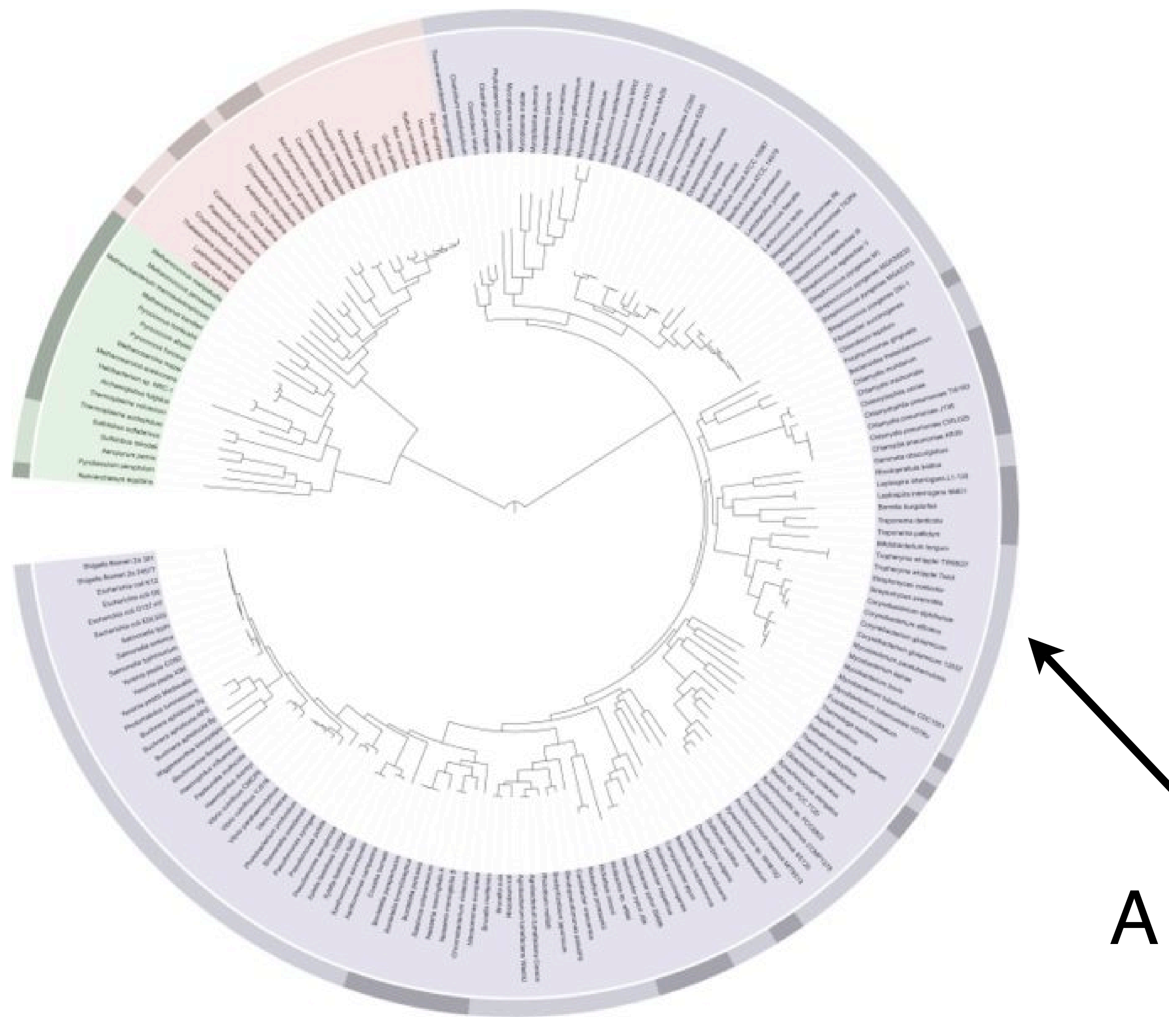


GURE 13.1 Technology and transportation leadership during 2003 fits Early Expansion phase.

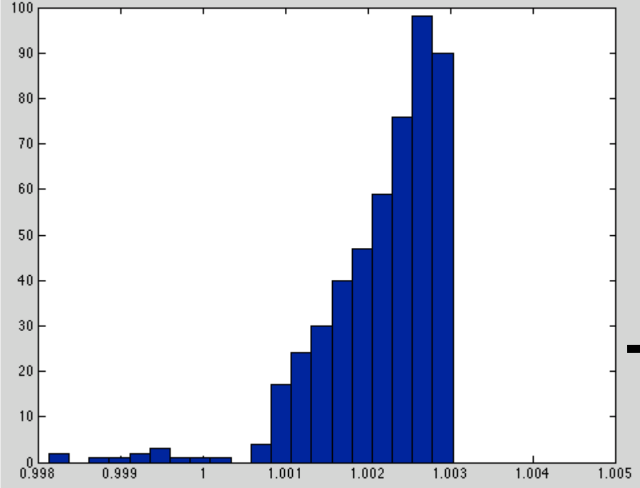


a persistent topological scale

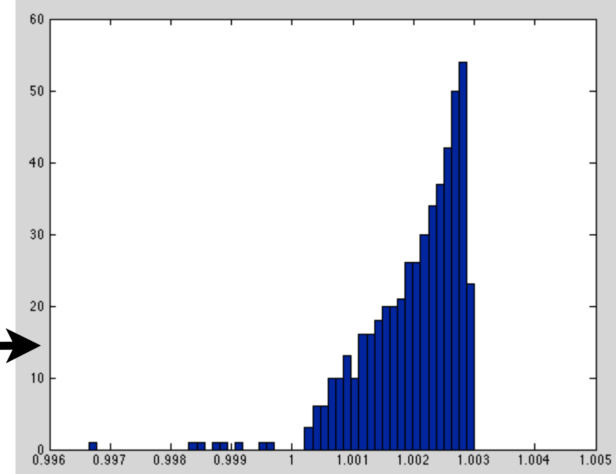
Tree of Life
"bottom up methods"



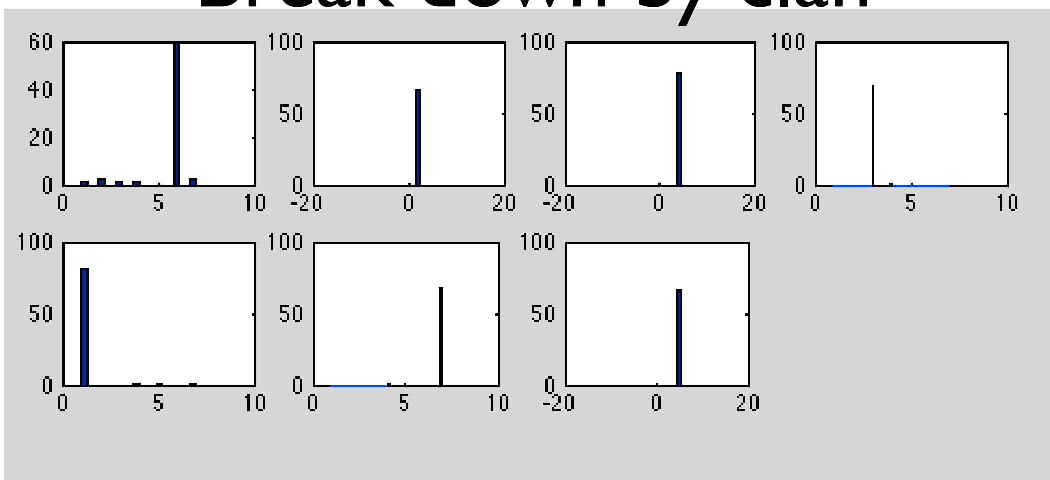
A dendogram



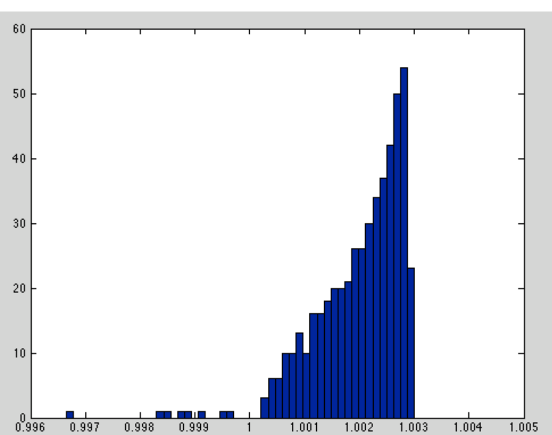
Scrub out the “sectors”

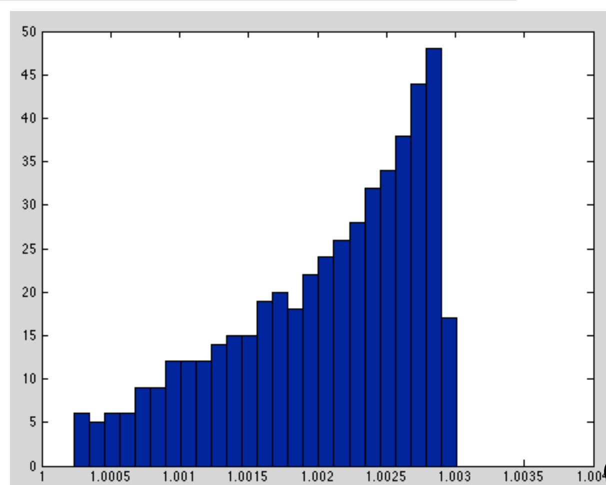


Break down by clan



One last scrub

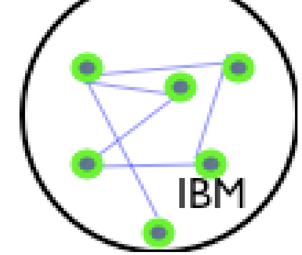


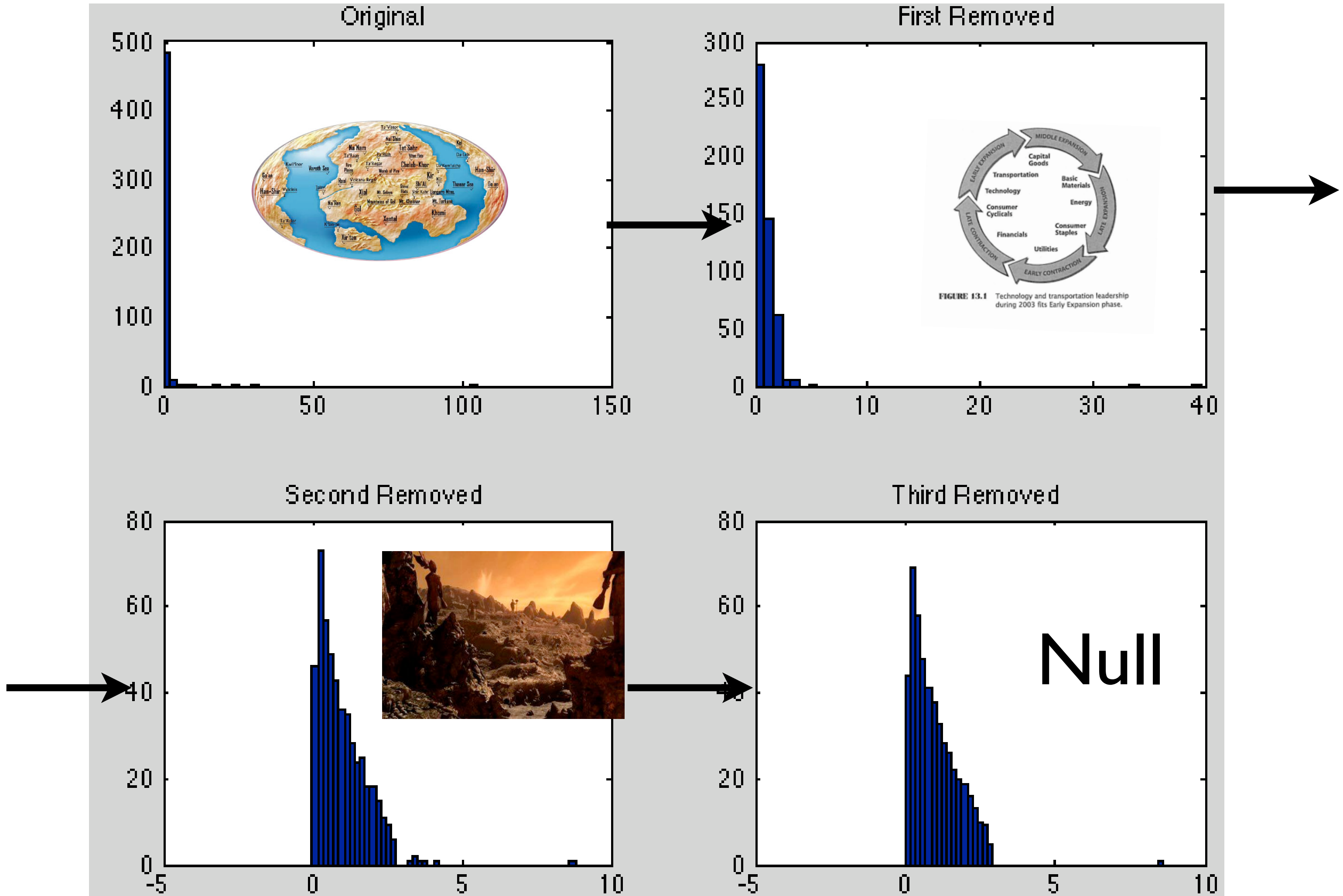


Have we found the null model yet?

So it would seem.

Partition Decoupling Method: Cluster, Scrub, Iterate.





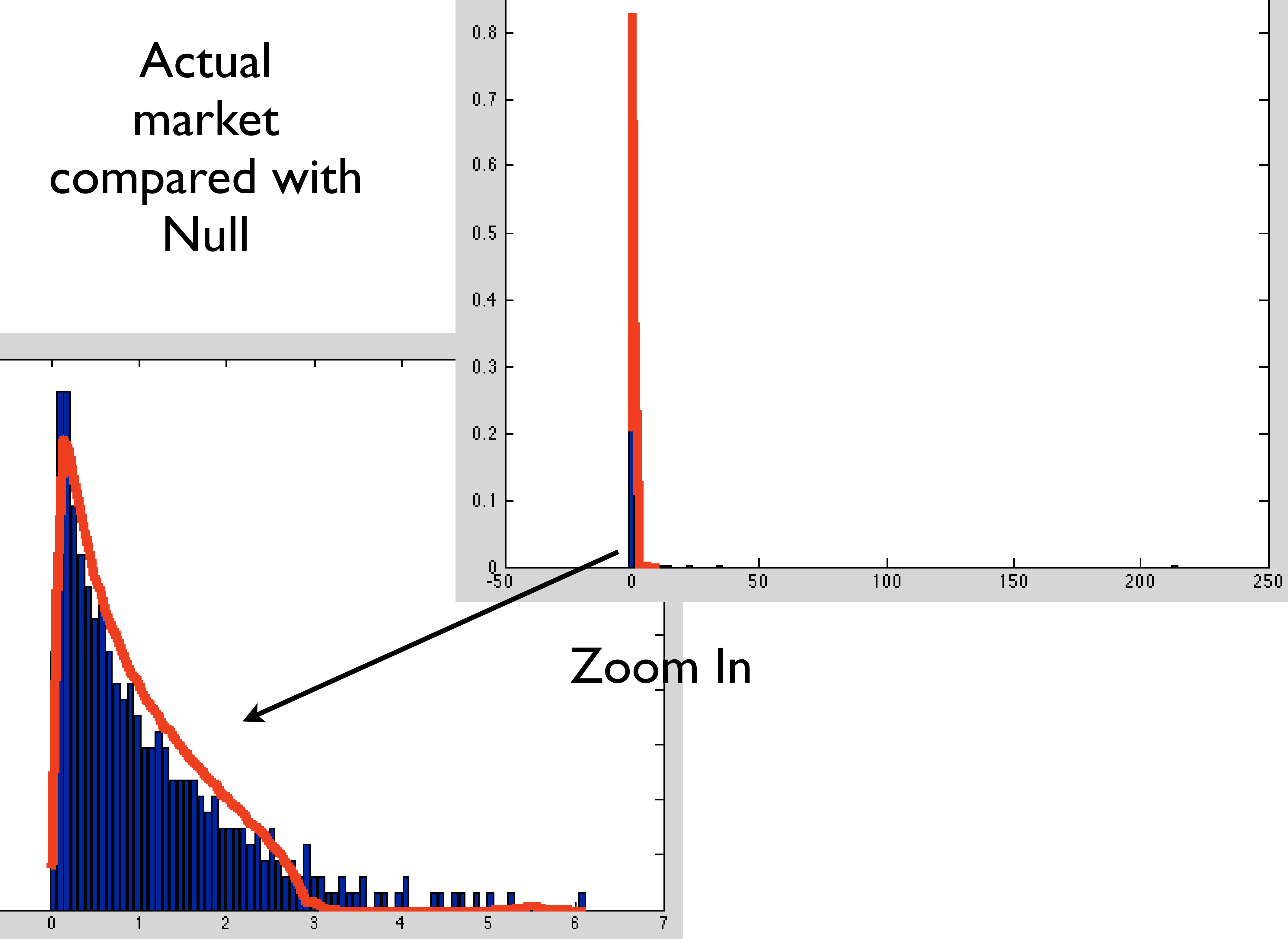
From the Vulcan market we can

1. explore the robustness of the tool
2. explore the effects of correlation scales
3. explore the effects of geometry.

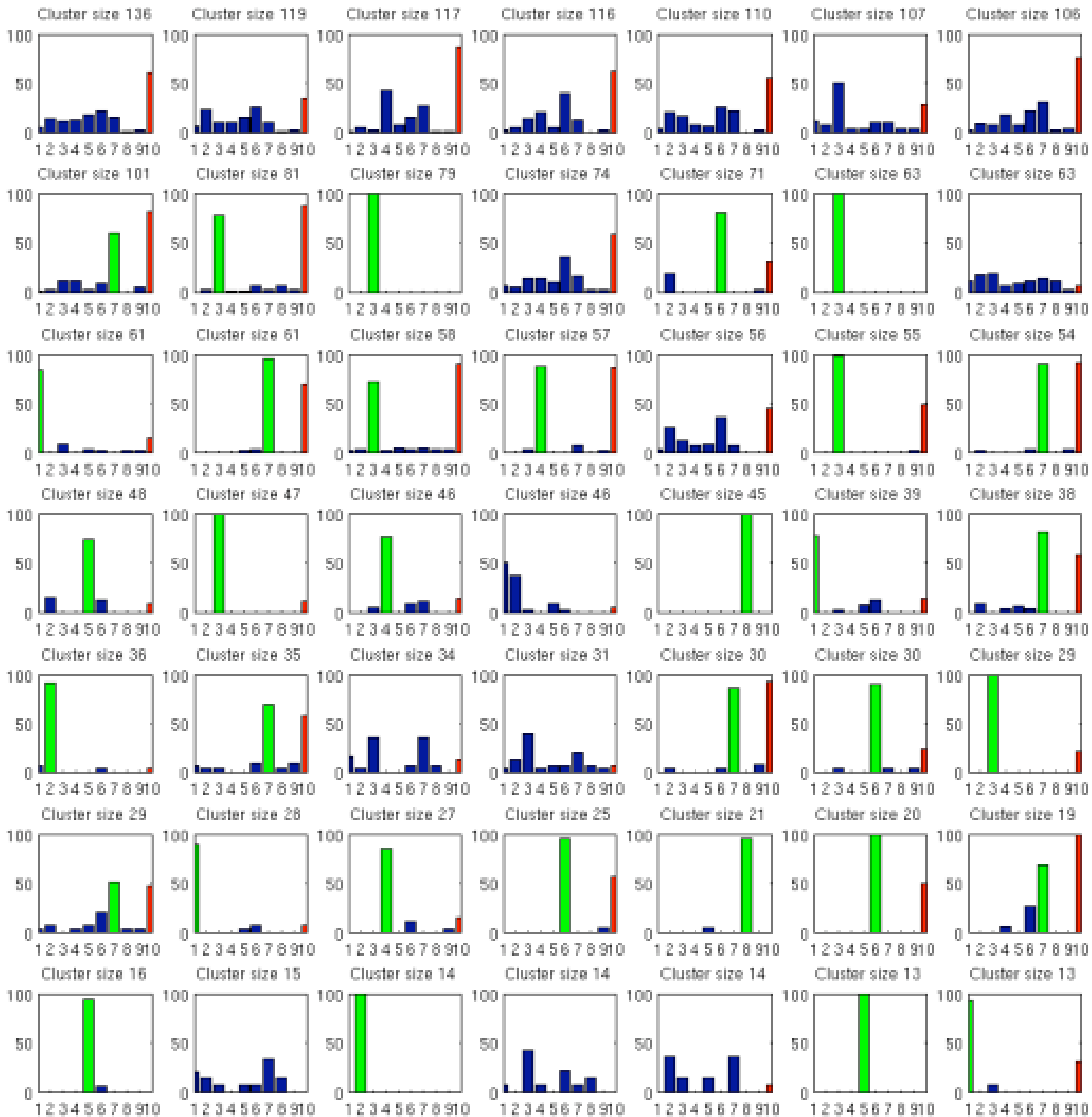
Part 2: The Earth Economy

G. Leibon, S. Pauls, D Rockmore, R Savell, Topological Structures in the equities market network, Proceedings of the National Academy of Sciences of the United States of America, Vol. 105, No. 52. (30 December 2008), pp. 20589-20594.

Actual market compared with Null

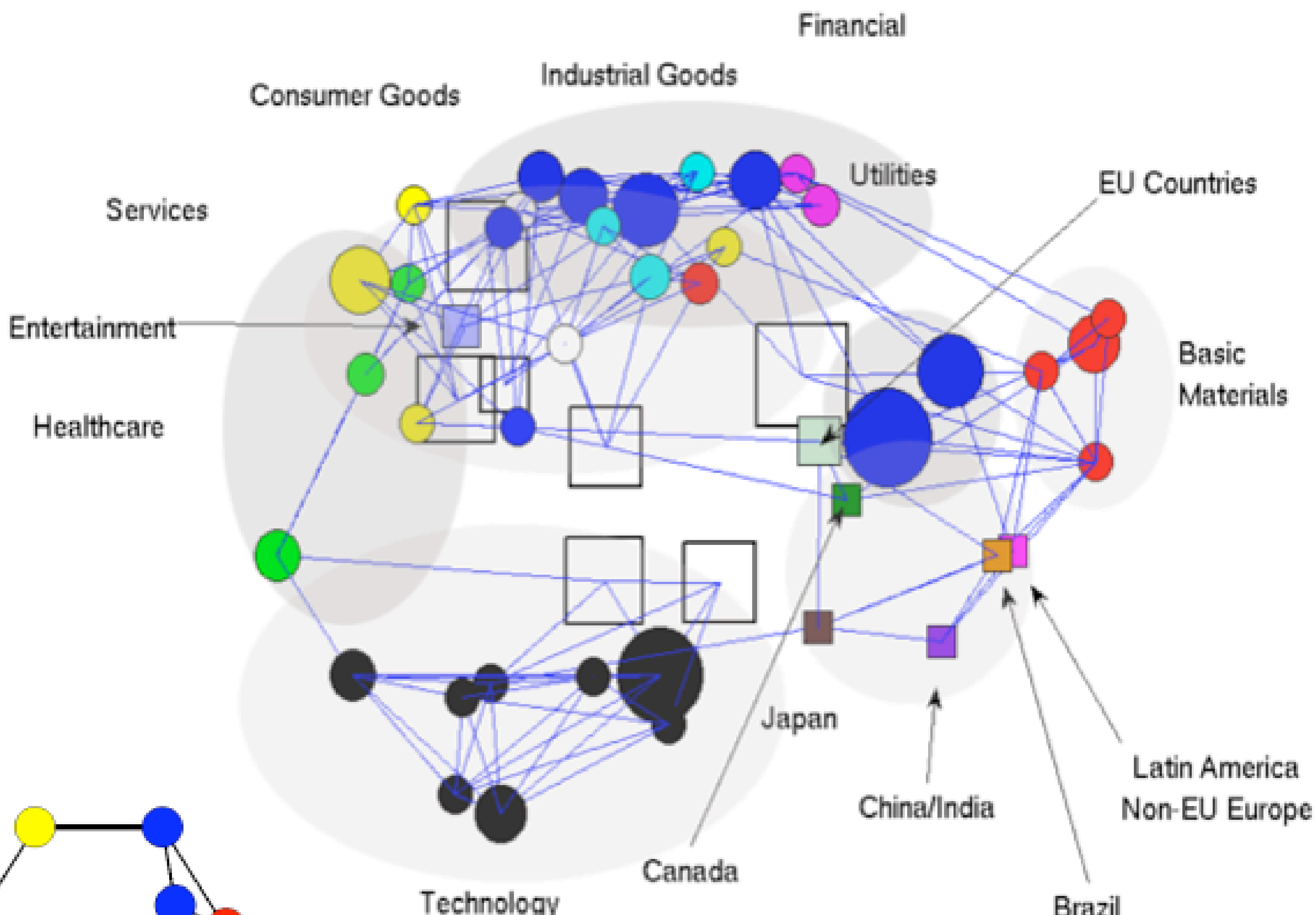


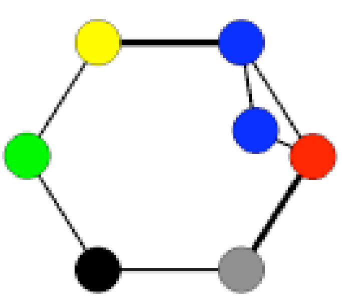
Zoom In

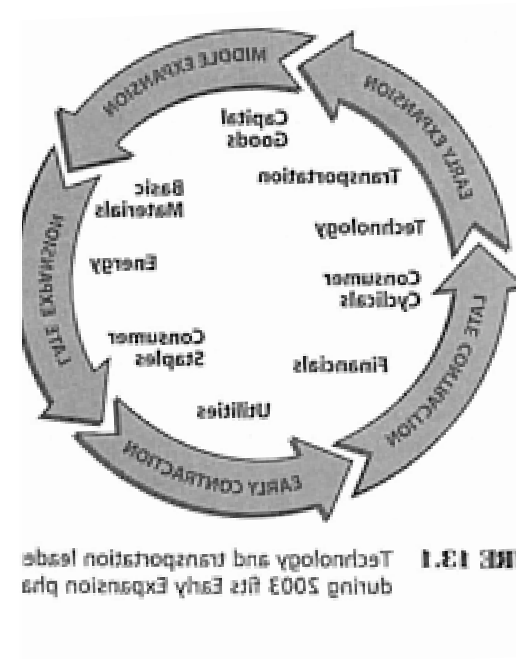


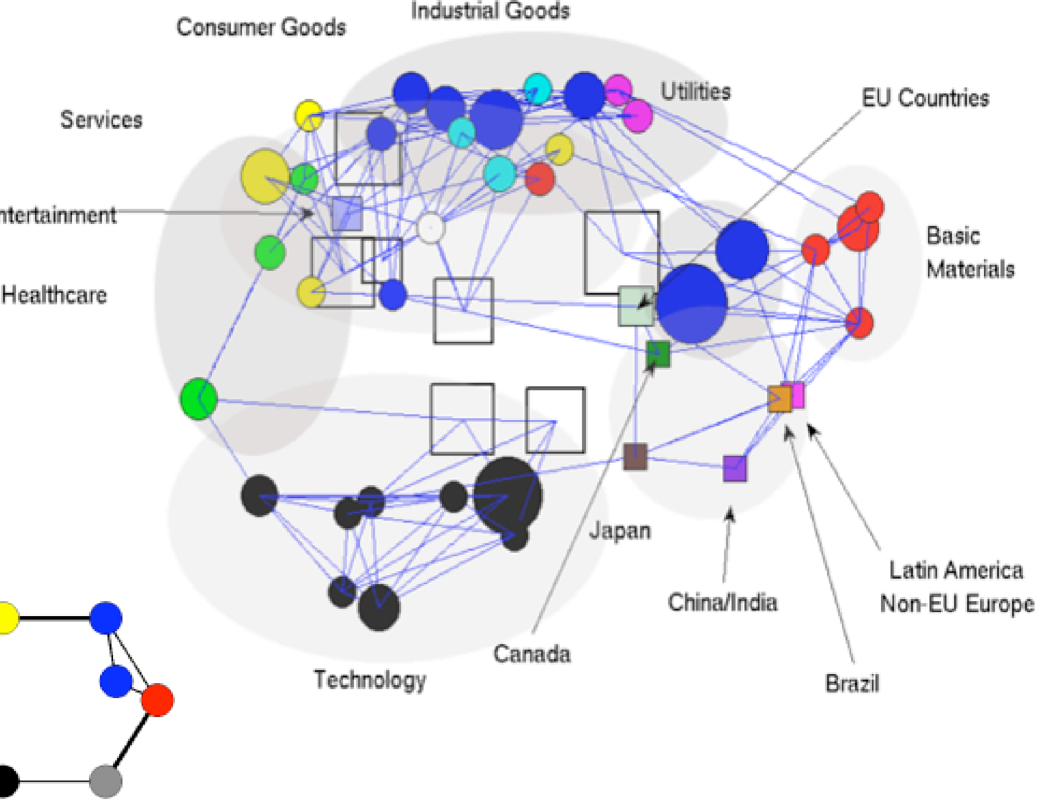
1 Basic Materials
2 Conglomerates
3 Consumer Goods
4 Financial
5 Healthcare
6 Industrial Goods
7 Services
8 Technology
9 Utilities

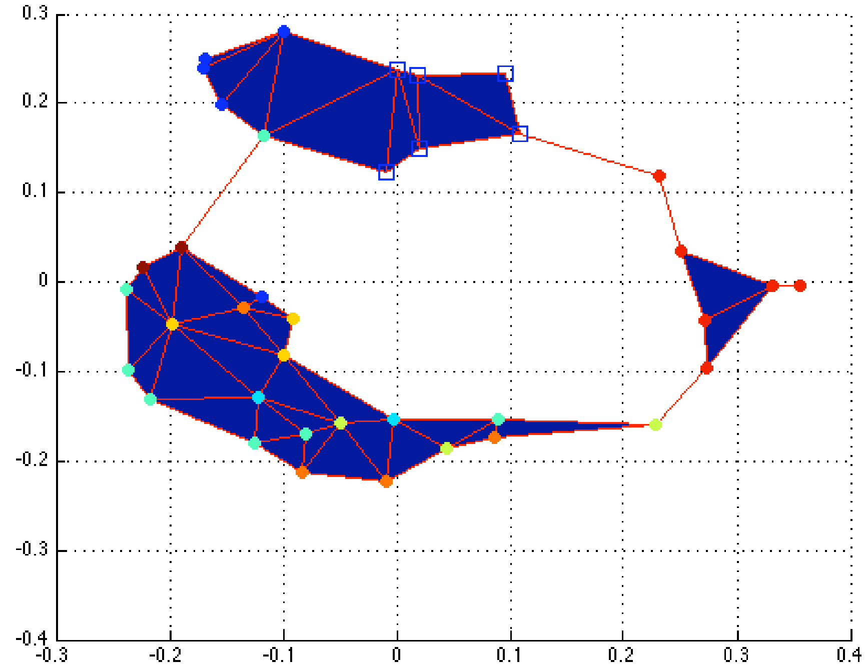
The Spectral Market








Consumer Goods
Industrial Goods
Utilities
EU Countries
Services
ntertainment
Basic
Materials
Healthcare
Japan
China/India
Latin America
Non-EU Europe
Canada
Technology
Brazil


0.3
0.2
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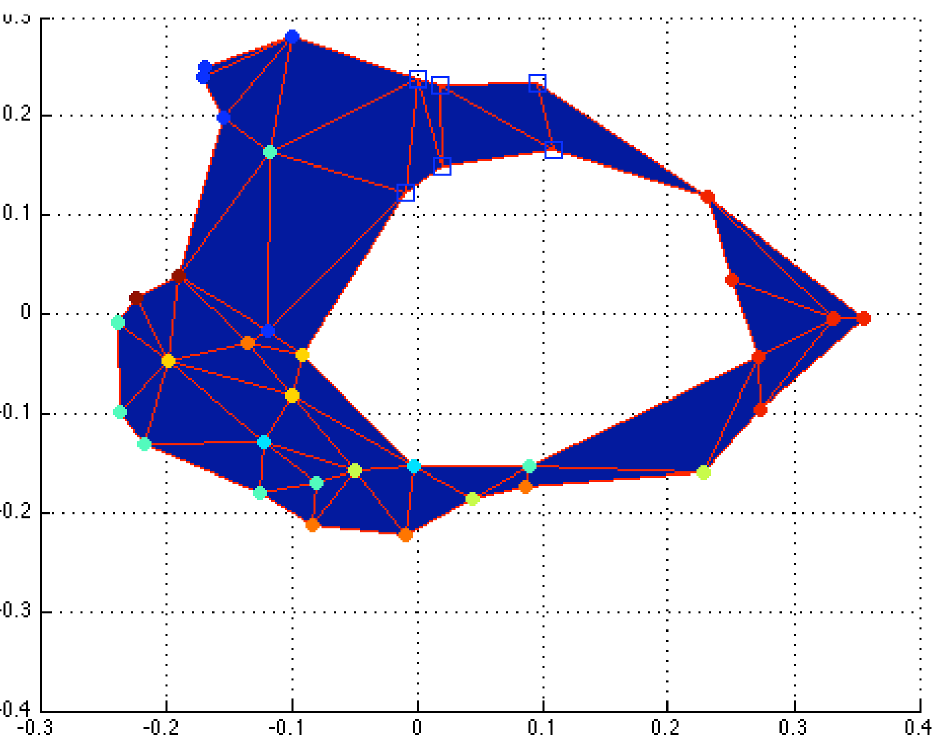
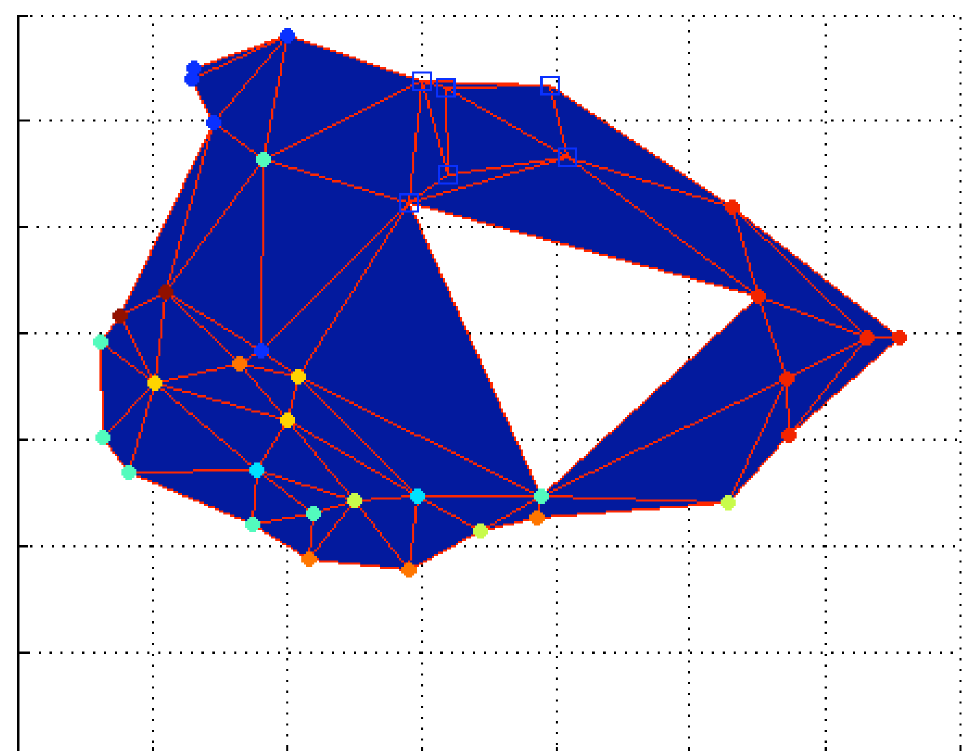
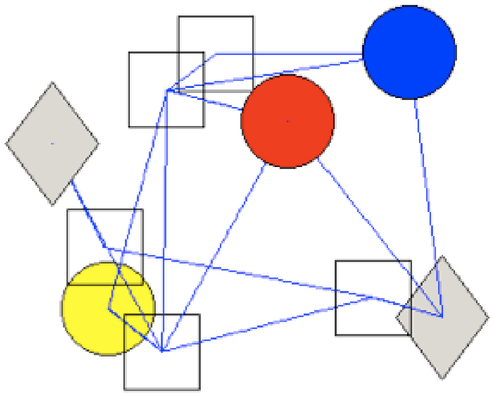
0
-0.1
-0.2
-0.3
-0.4
-0.3 -0.2 -0.1 0 0.1 0.2 0.3 0.4


0.2
0.1
0
-0.1
-0.2
-0.3
-0.3 -0.2 -0.1 0 0.1 0.2 0.3 0.4

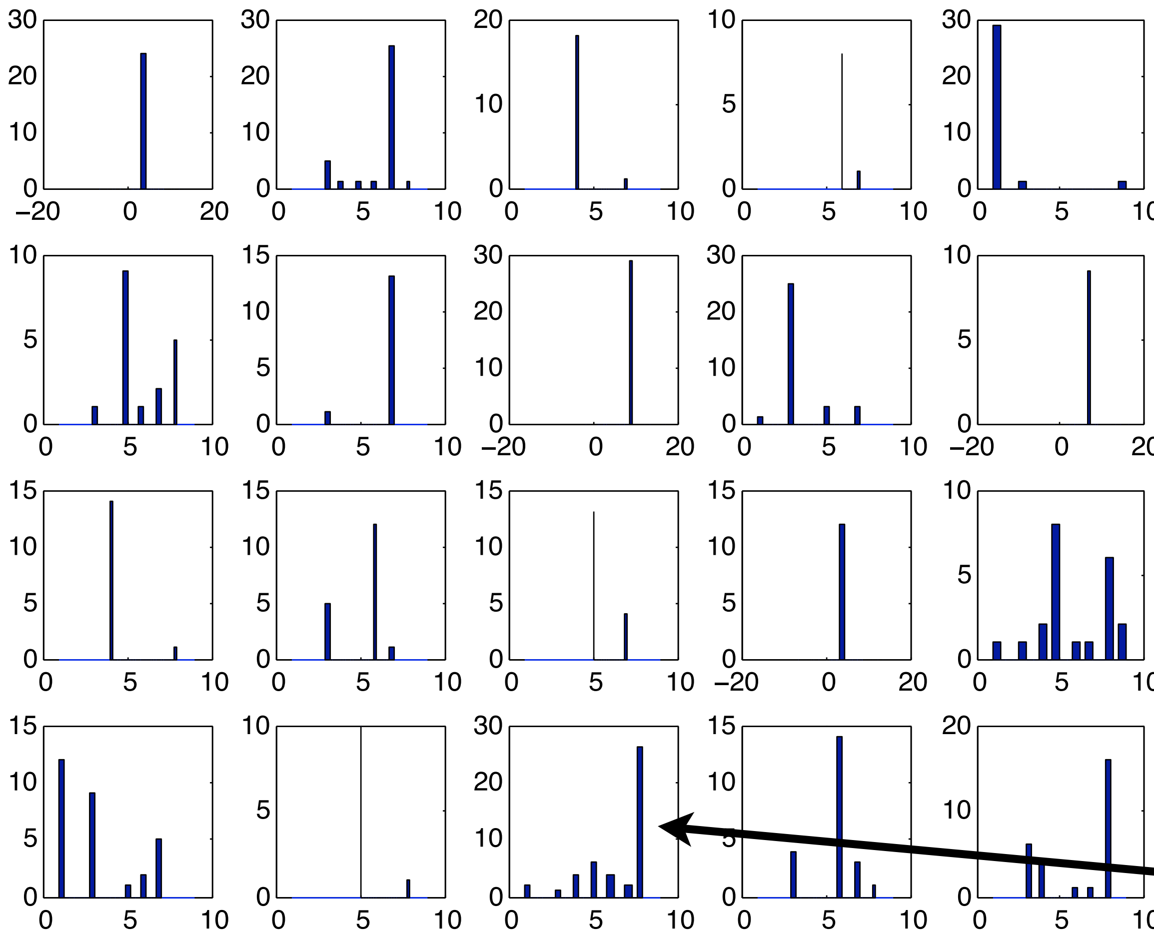


Scrub and examine

actions. For example, the diamond shaped clusters contain a mixture of multiple sectors. The first is predominantly Consumer Goods, Industrial Goods and Services, while the second is predominantly Financial, Healthcare, Services and Technology. However, both clusters contain significant commonalities. In the first, the equities in the Service sector are almost all related to the shipping industry, which obviously serves to distribute Consumer and Industrial Goods. In the second, the equities in the Financial, Services and Technology sectors are related to companies that either provide services or do business with healthcare companies (e.g. health insurance companies, drug companies, management services, healthcare based REITs, etc.). Equities in both of these clusters are drawn



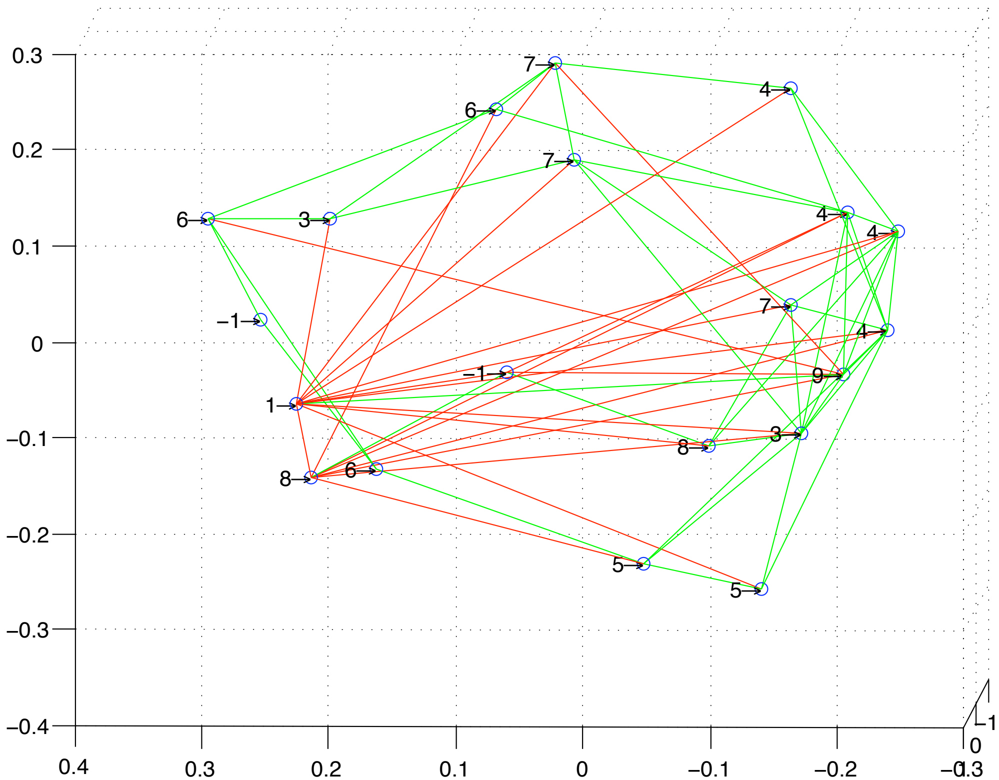
Other factors...sectors

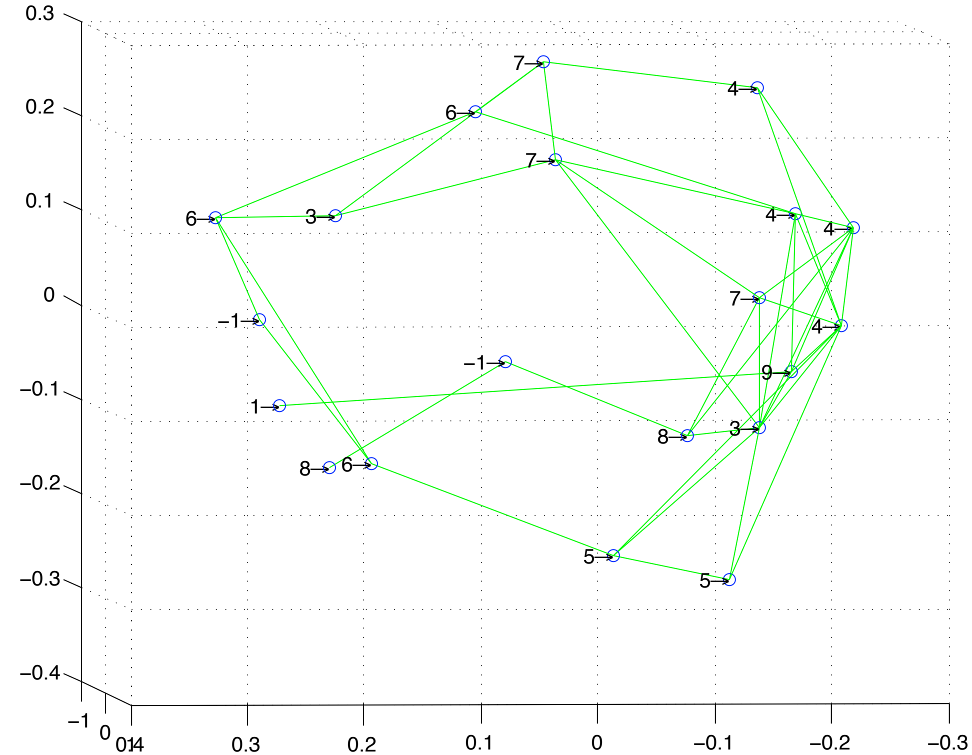


1 Basic Materials
2 Conglomerates
3 Consumer Goods
4 Financial
5 Healthcare
6 Industrial Goods
7 Services
8 Technology
9 Utilities

Guess?

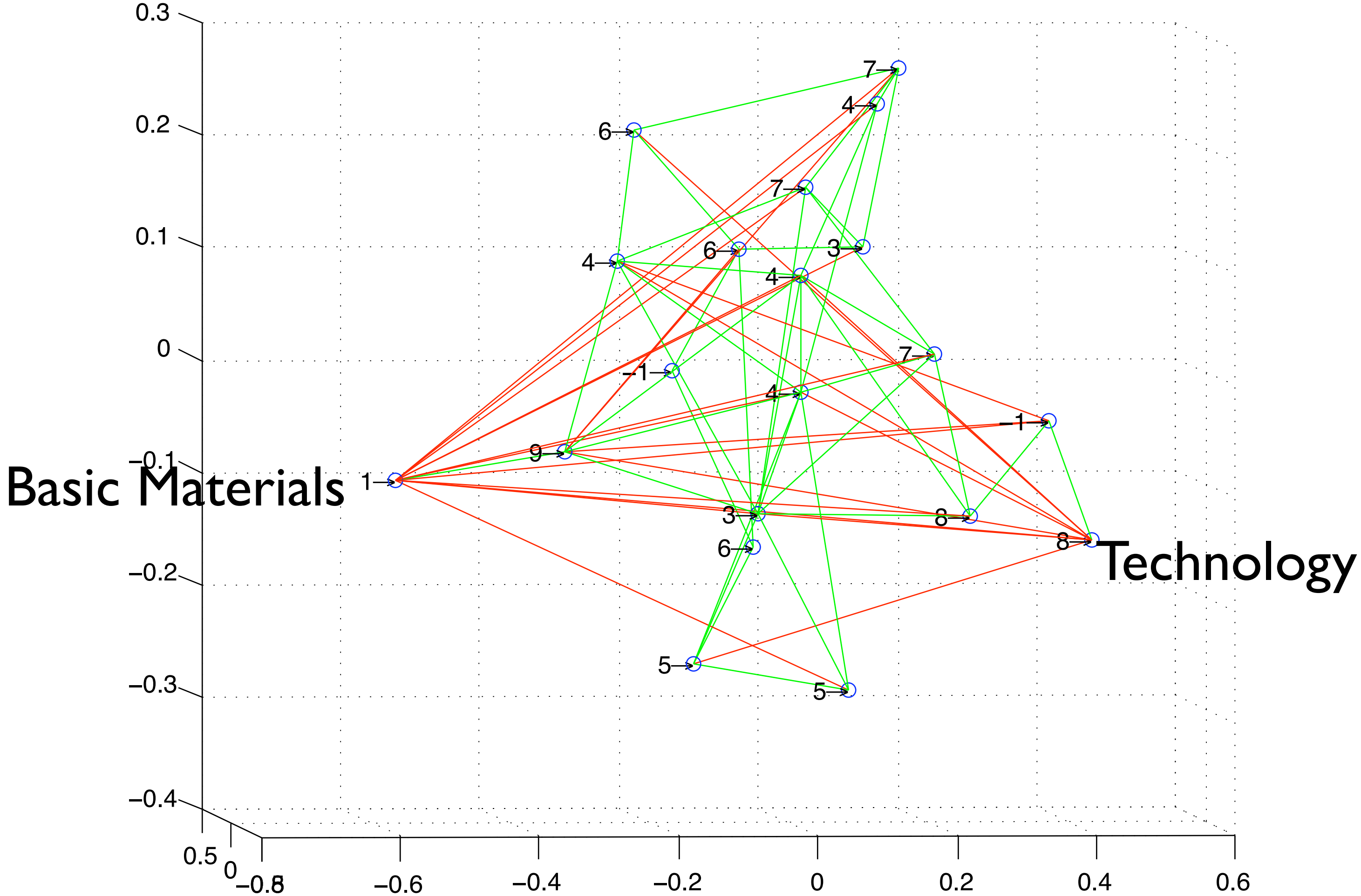
The dynamic picture is even more interesting....



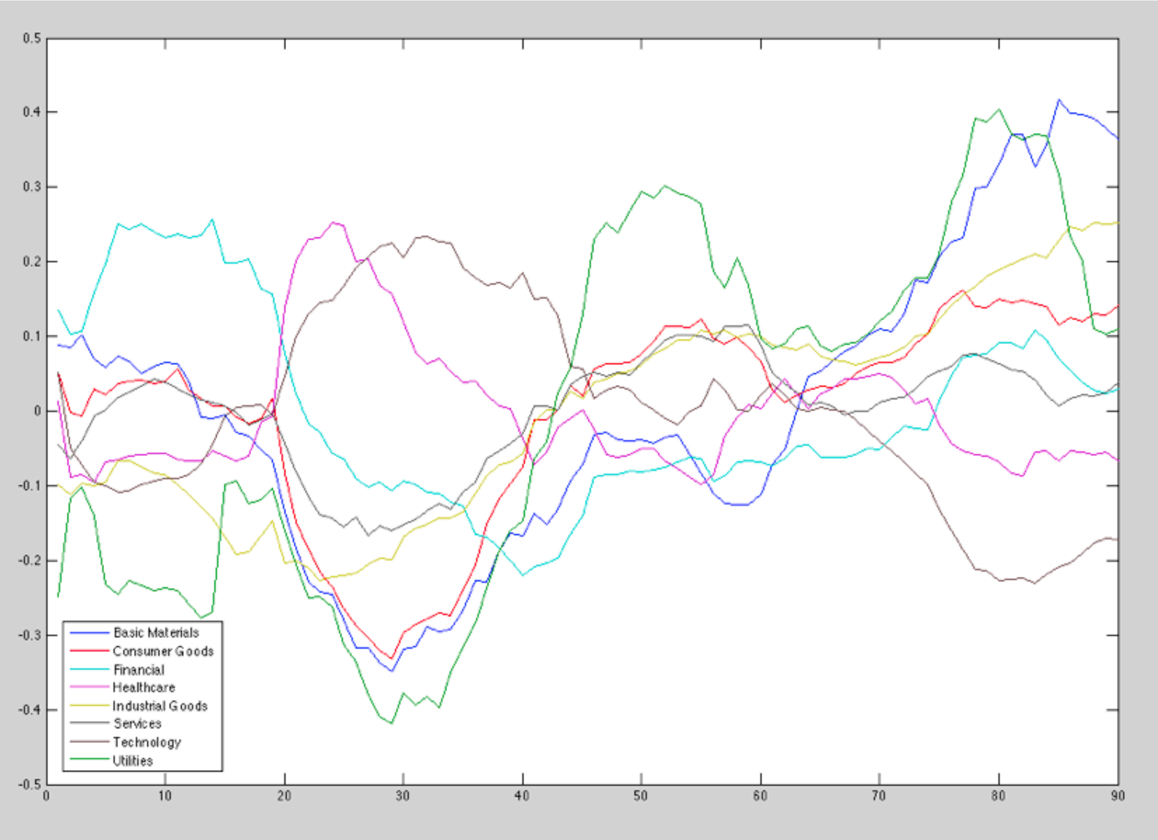


turn it on its side...

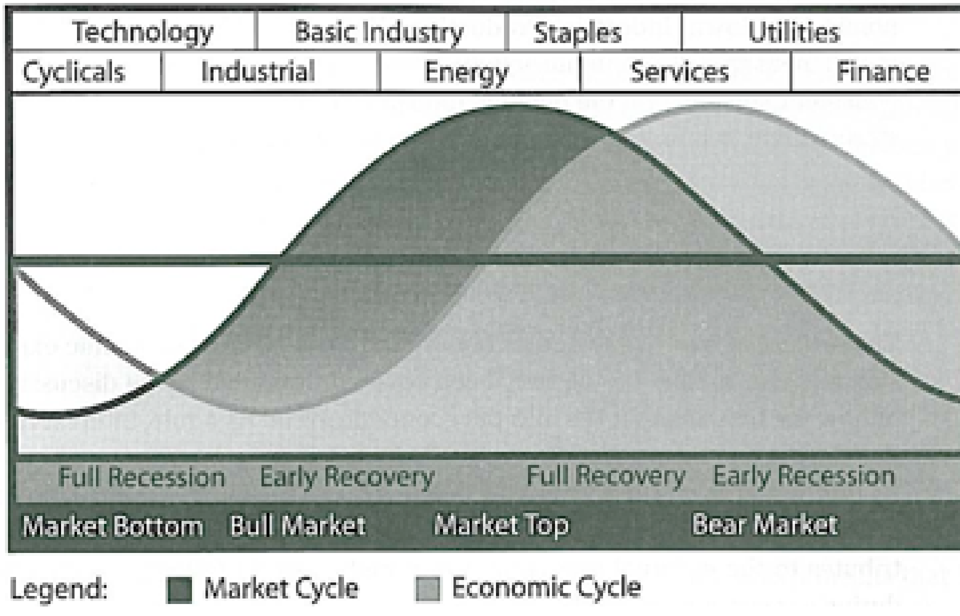
The Recent "Battle"



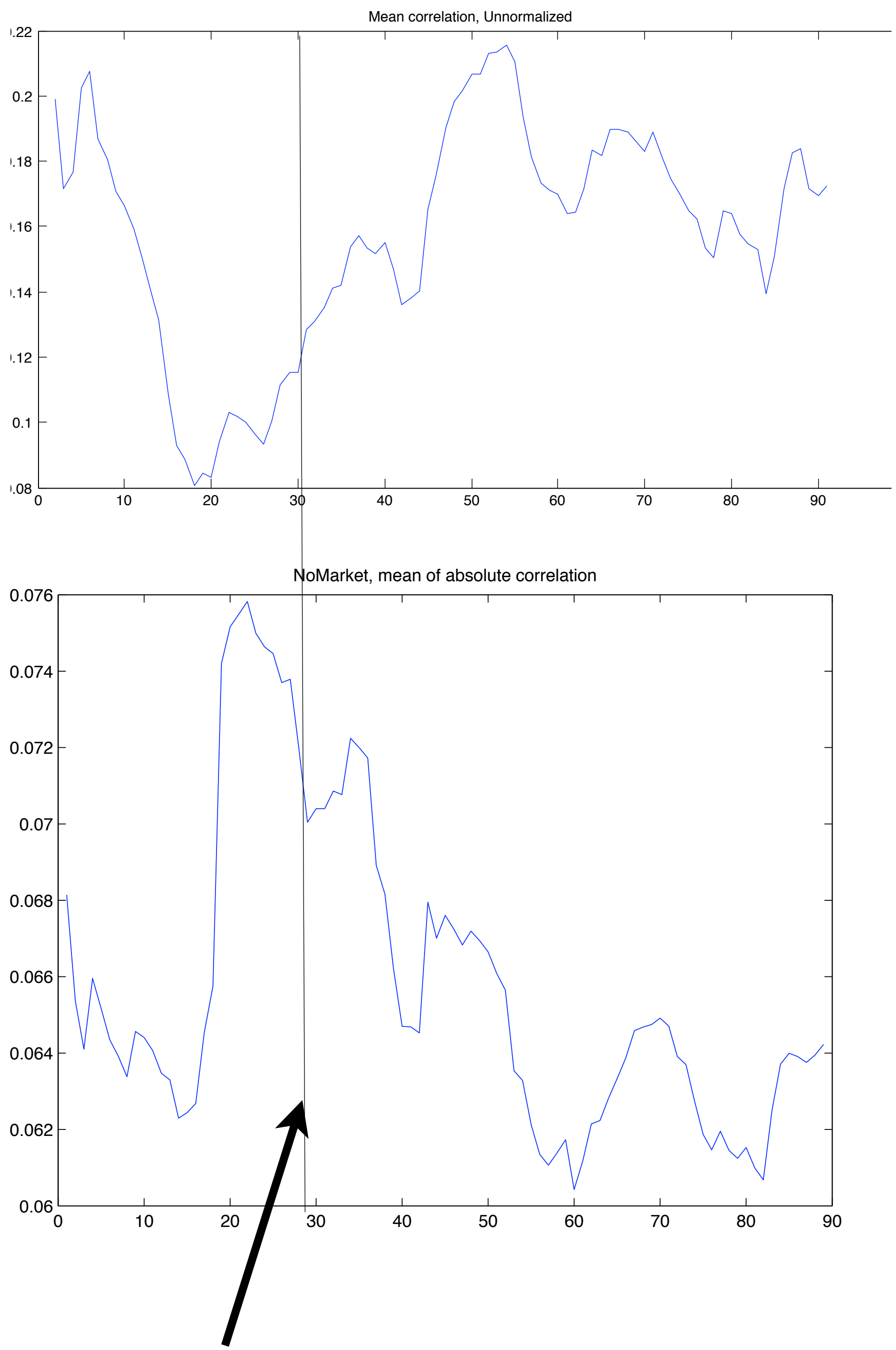
More importantly, generative models will allow us to deal with structured time series.



Correlation by Sector



Market Wisdom

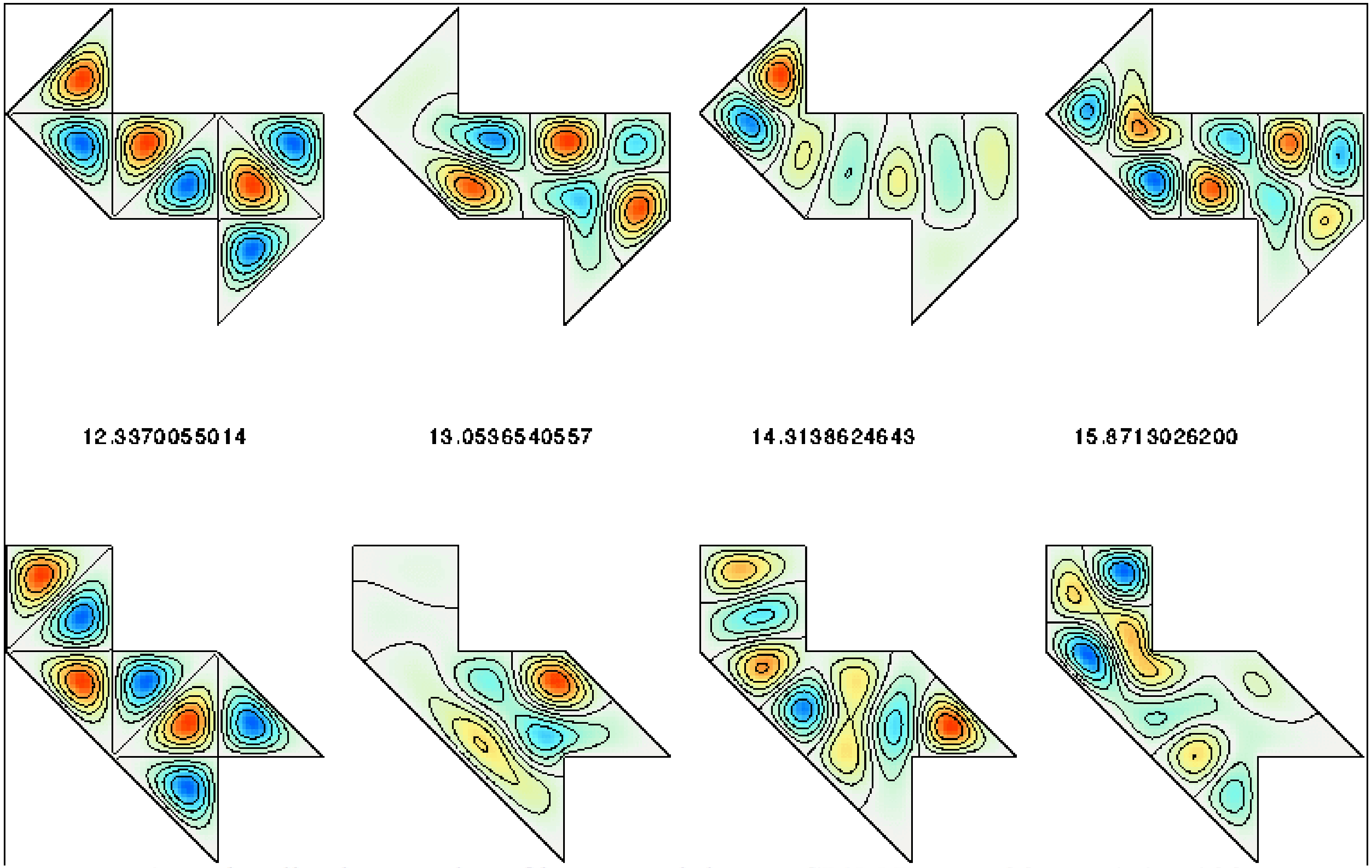


Full correlation pressure

With the equities market isolated

NASDAQ Tech Stock Crash

``Cover" credits:



Can you hear the shape of a drum?

T. A. Driscoll. Eigenmodes of isospectral drums. *SIAM Review* 39, pp. 1-17, 1997.

C. Gordon, D. Webb, and S. Wolpert, *Isospectral plane domains and surfaces via Riemannian orbifolds*, Invent. Math., 110 (1992), pp. 1–22.

M. Kac, *Can one hear the shape of a drum?*, Amer. Math. Monthly, 73 part II (1966), pp. 1–23.